THE POWERS IN THE SMARANDACHE SQUARE PRODUCT SEQUENCES

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Abstract. In this paper we prove that the Smarandache square product sequences of the first kind and the second kind do not contain powers.

Key words. Smarandache square product sequence, power.

For any positive integer n, let $A(n)$ be the n-th square. Further, let

$$P(n)=\prod_{k=1}^{n} A(k)+1 \tag{1}$$

and

$$Q(n)=\prod_{k=1}^{n} A(k)-1. \tag{2}$$

Then the sequences $P=\{P(n)\}_{n=1}^{\infty}$ and $Q=\{Q(n)\}_{n=1}^{\infty}$ are called the Smarandache square product sequences of t he first kind and the second kind respectively (see [3]). In this paper we consider the powers in P and Q. We prove the following result.

Theorem. The sequences P and Q do not contain powers.

Proof. If $P(n)$ is a power, then from (1) we get

$$(n!)^2+1=a^r, \tag{3}$$

where a and r are positive integers satisfying $a>1$ and $r>1$. It implies that the equation.

(4) $$X^2+1=Y^m, m>1,$$

has a positive integer solution $(X,Y,m)=(n!,a,r)$. However, by the result of [2], the equation (4) has no positive integre solution (X,Y,m). Thus, the sequence P does not contain powers.

Similarly, by(2), if $Q(n)$ is a power, the we have

(5) $$(n!)^2-1=a^r,$$

where a and r are positive integres satisfying $a>1$ and $r>1$, It implies that the equation

(6) $$X^2-1=Y^m, X>1, m>1,$$

has a positive integer solution $(X,Y,m)=(n!,a,r)$. By the result of [1], (5) has only the solution $(X,Y,m)=(3,2,3)$. Notice that $1!=1, 2!=2$ and $n!\geqslant 6$ for $n\geqslant 3$. Therefore, (4) is impossible. The theoerm is proved.

References

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