THE PRIMES IN THE SMARANDACHE POWER PRODUCT SEQUENCES OF THE FIRST KIND

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Abstract. In this paper we prove that if $r>1$ and r is not a power of 2, then the Smarandache r-power product sequence of the first kind contains only one prime 2.

Key words . Smarandache power product sequence, first kind, prime.

For any positive integers n , r with $r>1$, let $P(n,r)$ be the n-th power of degree r. Further, let

$$V(n,r)=\prod_{k=1}^{n} P(k,r)+1. \tag{1}$$

Then the sequence $V(r)=\{V(n,r)\}_{n=1}^{\infty}$ is called the Smarandache r-power product sequence of the first kind. In [2], Russo proposed the following question.

Question . How many terms in $V(2)$ and $V(3)$ are primes?

In fact, Le and Wu [1] showed that if r is odd, then $V(r)$ contains only one prime 2. It implies that $V(3)$ contains only one prime 2. In this paper we prove a general result as follows.

Theorem . If r is not a power of 2, then $V(r)$ contains only one prime 2.

Proof. Since $r>1$, if r is not a power of 2, then r has an odd prime divisor p . By (1), we get

$$V(n,r)=(n!)^r+1=((n!)^{r/p}+1)((n!)^{r(p-1)/p}-(n!)^{r(p-2)/p}+\ldots-(n!)^{r/p}+1),$$

Where r/p is a positive integer. Notice that if $n>1$, then $(n!)^{r/p}+1>1$ and $(n!)^{r(p-1)/p}-\ldots+1>1$. Therefore, we see from (2) that if $n>1$, then $V(n,r)$ is not a prime. Thus, the sequence $V(r)$ cotains only one prime $V(1,r)=2$. The theorem is proved.

References

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[2] F.Ruso, Some results about four Smarandache U-product sequences, Smarandache Notions J. 11(2000), 42-49.

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