THE SMARANDACHE φ-SEQUENCE

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Abstract: In this paper we completely determine the Smarandache φ-sequence.

Key words: Smarandache φ-sequence; Euler totient function; diophantine equation

For any positive integer n, let $\varphi(n)$ be the Euler totient function of n. Further, let the set

$$A=\{n|n=k\varphi(n), \text{ where } k \text{ is a positive integer}\}. \quad (1)$$

Then, all elements n of A form the Smarandache φ-sequence (see [2]). In this paper we completely determine this sequence as follows.

Theorem. Let $\{a(x)\}_{x=1}^{\infty}$ be the Smarandache φ-sequence. Then we have

$$a(x)=\begin{cases} 1, & \text{if } x=1, \\ 2, & \text{if } x=2, \\ 2^{(x+1)/2}, & \text{if } x>1 \text{ and } x \text{ is odd}, \\ 2^{x/2-1}.3, & \text{if } x>1 \text{ and } x \text{ is even}. \end{cases} \quad (2)$$

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Proof. We first consider the elements of A. We see from (1) that these elements are solutions of the equation

$$n=k\varphi(n). \tag{3}$$

Clearly, $(n,k)=(1,1)$ is a positive integer of (3). If $n>1$, let

$$n=p_1^{\alpha_1}p_2^{\alpha_2}\cdots p_s^{\alpha_s} \tag{4}$$

be the factorization of n. By [1, Theorem 62], we have

$$\varphi(n)=p_1^{\alpha_1-1}p_2^{\alpha_2-1}\cdots p_s^{\alpha_s-1}(p_1-1)(p_2-1)\cdots(p_s-1). \tag{5}$$

Substitute (4) and (5) into (3), we get

$$p_1p_2\cdots p_s=k(p_1-1)(p_2-1)\cdots(p_2-1). \tag{6}$$

If n is even, then $p_1=2$ and $p_2,\cdots,p_s$ are odd primes. Since p_i-1 $(i=2,\cdots,s)$ are even integer, we find fron (6) that either $s=1$ and $k=2$ or $s=2$, $p_2=3$ and $k=3$. It follows that (3) has positive integer solutions $(n,k)=(2^r,2)$ and $(2^r.3,3)$, where r is a positive integer.

If n is odd, then (6) is impossible, since $p_j(j=1,2,\cdots,s)$ are odd primes and p_j-1$(j=1,2,\cdots,s)$ are even integers.

Thus, by the above analysis, we obtain (2) immediately.

References

[1] G.H.Hardy and E.M.Wright, An introduction to the theory of numbers, Oxford University Press, Oxford, 1938.

[2] A.Murthy, Some new Smarandache sequences, functions and partitions, Smarandache Notions J. 11(2000), 179-183.