THE SMARANDACHE FUNCTION AND THE DIOPHANTINE EQUATION $X!+a = y^2$

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Abstract. For any positive integer n, let S(n) denote the Smarandache function of n. In this paper we prove that if a is a nonsquare positive integer, then all positive integer solutions (x,y) of the equation $x!+a=y^2$ satisfy $x< 2S(a)$.

Let N be the set of all positive integers. For any positive integer n, let S(n) denote the Smarandache function of n. Let a be a fixed positive integer. Recently, Dabrowschi[1] proved that if a is not a square, then the equation

(1) $$x!+a = y^2, \quad x,y \in N$$

has only finitely many solutions (x,y). In this paper we give an upper bound for the solutions of (1) as follows.

Theorem. If a not a square, then all solutions (x,y) of (1) satisfy $x<2S(a)$.

Proof. Since a is not a square, a has a prime factor p such that

(2) $$p^{2r+1} \| a,$$

were r is a nonnegative integer. We now suppose that (x,y) is a solution of (1) with $x\geq 2S(a)$. By the result of [2], we have $S(mn)\leq S(m)+S(n)$ for any positive integers m, n. It implies that $2S(a)\geq S(a^2)$. So have

(3) $$a^2 | x!.$$

Therefore, we see from (1) and (3) that

(4) $$a | y^2.$$

Further, by (2) and (4), we get

(5) $$ap | y^2$$

Since p is a prime factor of a, we see from (3) that

(6) $ap \mid x!$.

Thus, by (1), (5) and (6), we obtain $p \mid 1$, a contradiction. So we have $x<2S(a)$. The theorem is proved.

References

1.A. Dabrowski, On the diophantine equation $x!+A = y^2$, Nieuw Arch. Wisc. (4) 14 (1996), No.3, 321-324.

2.M.-H.Le, An inequality concerning the Smarandache function, Smarandache Notions J.