The Smarandache Function, together with a sample of The Infinity of Unsolved Problems associated with it, presented by Mike Mudge.[1]

The Smarandache Function, S(n) (originated by Florentin Smarandache — *Smarandache Function Journal*, vol 1, no 1, December 1990. ISSN 1053-4792) is defined for all non-null integers, n, to be the smallest integer such that (S(n))! is divisible by n.

Note N! denotes the factorial function, N!=1x2x3x...xN: for all positive integer N. In addition 0! = 1 by definition.

S(n) is an even function. That is, S(n) = S(-n) since if (S(n))! is divisible by n it is also divisible by -n.

S(p) = p when p is a prime number, since no factorial less than p! has a factor p in this case where p is prime.

The values of S(n) in *Fig 1* are easily verified. For example, S(14) = 7 because 7 is the smallest number such that 7! is divisible by 14.

Problem (i) Design and implement an algorithm to generate and store/tabulate S(n) as a function of n.

Hint It may be advantageous to consider the STANDARD FORM of n, *viz* $n = e p_1^{a_1} p_2^{a_2} \ldots\ldots p_r^{a_r}$ where $e = \pm 1$, $p_1, p_2, \ldots p_r$ denote the distinct prime factors of n and $a_1, a_2, \ldots a_r$ are their respective multiplicities.

Problem (ii) Investigate those sets of consecutive integers (i,i+1,i+2,...i+x) for which S generates a monotonic increasing (or indeed monotonic decreasing) sequence.

Note For (1,2,3,4,5) S generates the monotonic increasing sequence 0,2,3,4,5; here i = 1 & x = 4.

If possible estimate the largest value of x.

Problem (iii) Investigate the existence of integers m,n,p,q & k with n≠m and p≠q for which:

either **(A)**: S(m) + S(m+1) +....S(m+p) = S(n) + S(m+1) +....S(n+q) or **(B)**:

$$\frac{S(m)^2 + S(m+1)^2 + \ldots S(m+p)^2}{S(n)^2 + S(n+1)^2 + \ldots S(n+p)^2} = k$$

Problem (iv) Find the smallest integer k for which it is true that <u>for all n less than some given n_0</u> at least one of:

S(n), S(n+1)....S(n+k-1) is:

A) a perfect square

B) a divisor of k^n

C) a factorial of a positive integer.

Conjecture what happens to k as n_0 tends to infinity: i.e. becomes larger and larger.

Problem (v) <u>Construct prime numbers of the form</u> $\overline{S(n)\,S(n+1)\ldots S(n+k)}$: where $\overline{abcdefg}$ denotes the integer formed by the concatenation <u>of a,b,c,d</u>,e,f & g. For example, trivially $\overline{S(2)\,S(3)}$ = 23 which <u>is prime, but no so trivially</u> $\overline{S(14)S(15)S(16)S(17)}$ = 75617, also prime!

Definition An A-SEQUENCE is an integer sequence $a_1, a_2, \ldots$ with $1 \le a_1 < a_2 < \ldots$ such that no a_i is the sum of distinct members of the sequence (other than a_i).

Problem (vi) Investigate the construction of A-SEQUENCES $a_1, a_2, \ldots$ such that the associated sequences $S(a_1), S(a_2), \ldots$ are also A-SEQUENCES.

Definition The k^{th} order forward finite differences of the Smarandache function are defined thus:

$D_S(x)$ = *modulus(S(x+1) - S(x)),

$D_S^{(k)}(x) = D(D(\ldots k\text{-times} D_S(x)\ldots))$

Problem (vii) Investigate the conjecture that $D_S^{(k)}(1)$ = 1 or 0 for all k greater than or equal to 2.

c.f. Gilbreath's conjecture on prime numbers, discussed in 'Numbers Count' *PCW* Dec 1983. * Here modulus is taken to mean the absolute value of (ABS.), modulus (y) = y if y is positive and modulus (y) = -y if y is negative.

The following selection of Diophantine Equations (i.e. solutions are sought in integer values of x) are taken from the Smarandache Journal and make up:

Problem (viii) If m & n are given integers, solve each of:

a) S(x) = S(x+1), conjectured to have no solution

b) S(mx+n) = x

c) S(mx+n) = m+nx

d) S(mx+n) = x!

e) $S(x^m) = x^n$

f) $S(x)^m = S(x^n)$

g) S(x) + y = x + S(y), x&y not prime

h) S(x) + S(y) = S (x+y)

i) S(x+y) = S(x)S(y)

j) S(xy) = S(x)S(y)

Review, July 1992
The Smarandache Function: a first visit? [2]

This topic is certain to be revisited in the near future, and the lack of space available here will certainly be remedied on that occasion. Suffice it to report that Jim Duncan computed up to S(1499999), the first million taking 50 hours in Lattice C on an Atari 1040ST. In Problem (ii), no evidence for a largest value of x was found, while in Problem (vii) the conjecture was verified for the first 32,000 values of S(n). The very worthy prizewinner is John McCarthy of 17 Mount Street, Mansfield, Notts NG19 7AT, who has extensively *investigated* the computation of S(n) up to 2^{32}; arriving at conclusions such as: 'several years of computing', 'at least 12Gb of disk space' and '3,303,302 pages of output'. John's concluding comment, 'Am I mad?', is clearly answered NO! by examining his specimen pages of output including those relating to 10-digit values of n. Listings supplied. Details from John directly upon request.

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1 *Republished from <Personal Computer World>, No.112, 420, July 1992 (with the author permission), because some of the following research papers are referring to these open problems.*

2 *Republished from <Personal Computer World>, No.117, 412, December 1992 (with the author permission).*