THE SMARANDACHE PERIODICAL SEQUENCES

by

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1. Let N be a positive integer with not all digits the same, and N' its digital reverse.

Then, let N1 = abs (N - N'), and N1' its digital reverse. Again, let N2 = abs (N1 - N1'), N2' its digital reverse, and so on, where abs x is the absoluth value of x.

After a finite number of steps one finds an Nj which is equal to a previous Ni, therefore the sequence is periodical (because if N has, say, n digits, all other integers following it will have n digits or less, hence their number is limited, and one applies the Dirichlet's box principal).

For examples:

a. If one starts with N = 27, then N' = 72;
abs (27 - 72) = 45; its reverse is 54;
abs (45 - 54) = 09, ...
thus one gets: 27, 45, 09, 81, 63, 27, 45, ... ;
the Lenth of the Period LP = 5 numbers (27, 45, 09, 81, 63),
and the Lenth of the Sequence 'till the first repetition
occurs LS = 5 numbers either.

b. If one starts with 52, then one gets:
52, 27, 45, 09, 81, 63, 27, 45, ... ;
then LP = 5 numbers, while LS = 6.

c. If one starts with 42, then one gets:
42, 18, 63, 27, 45, 09, 81, 63, 27, ... ;
then LP = 5 numbers, while LS = 7.

For the sequences of integers of two digits, it seems like:
LP = 5 numbers (27, 45, 09, 81, 63; or a circular permutation of them), and 5 <= LS <= 7.

Question 1:

Find the Lenth of the Period (with its corresponding numbers), and the Lenth of the Sequence 'till the first repetition occurs for:
the integers of three digits, and the integers of four digits.
(It's easier to write a computer program in these cases to check the LP and LS.)

An example for three digits:
321, 198, 693, 297, 495, 099, 891, 693, ... ;
(similar to the previous period, just inserting 9 in the middle of each number).
Generalization for sequences of numbers of n digits.

2. Let N be a positive integer, and N' its digital reverse.

For a given positive integer c, let N1 = abs (N' - c), and N1' its digital reverse. Again, let N2 = abs (N1' - c), N2' its digital reverse, and so on.

After a finite number of steps one finds an Nj which is equal to a previous Ni, therefore the sequence is periodical (same proof).

For example:
If N = 52, and c = 1, then one gets:
52, 24, 41, 13, 30, 02, 19, 90, 08, 79, 68, 85, 57, 74, 46, 63, 35, 52, ... ;
thus LP = 18, LS = 18.

Question 2:

Find the Lenth of the Period (with its corresponding numbers), and the Lenth of the Sequence 'till the first repetition occurs (with a given non-null c) for:
the integers of two digits,
and the integers of three digits.
(It's easier to write a computer program in these cases to check the LP and LS.)

Generalization for sequences of numbers of a n digits.

3. Let N be a positive integer with n digits a1a2 ... an, and c a given integer > 1.

Multiply each digit ai of N by c, and replace ai with the last digit of the product ai x c, say it is bi. Note N1 = b1b2 ... bn, do the same procedure for N1, and so on.

After a finite number of steps one finds an Nj which is equal to a previous Ni, therefore the sequence is a periodical (same proof).

For example:
If N = 68 and c = 7:
68, 26, 42, 84, 68, ... ;
thus LP = 4, LS = 4.

Question 3:

Find the Lenth of the Period (with its corresponding numbers), and the Lenth of the Sequence 'till the first repetition occurs (with a given c) for:
the integers of two digits,
and the integers of three digits.
(It's easier to write a computer program in these cases to check the LP and LS.

Generalization for sequence of numbers of n digits.

4.1. Smarandache generalized periodical sequence:

Let N be a positive integer with n digits a1a2 ...an. If f is a function defined on the set of integers with n digits or less, and the values of f are also in the same set, then:

there exist two natural numbers i < j such that

$$f(f(\ldots f(s) \ldots)) = f(f(f(\ldots f(s) \ldots))),$$

where f occurs i times in the left side, and j times in the right side of the previous equality.

Particularizing f, one obtains many periodical sequences.

Say:

If N has two digits a1a2, then: add 'em (if the sum is greater than 10, add the resulted digits again), and substract 'em (take the absolute value) -- they will be the first, and second digit respectively of N1. And same procedure for N1.

Example:

75, 32, 51, 64, 12, 31, 42, 62, 84, 34, 71, 86, 52, 73, 14, 53, 82, 16, 75, ..

4.2. More General:

Let S be a finite set, and f: S ----> S a function. Then:

for any element s belonging to S, there exist two natural numbers i < j such that

$$f(f(\ldots f(s) \ldots)) = f(f(f(\ldots f(s) \ldots))),$$

where f occurs i times in the left side, and j times in the right side of the previous equality.

Reference:

F. Smarandache, "Sequences of Numbers", University of Craiova Symposium of Students, December 1975.