THE SMARANDACHE-RIEMANN ZETA SEQUENCE

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Abstract. In this paper we prove that the Smarandach-Riemann sequence is not a sequence of integers. Moreover, no two integer terms of this sequence are relatively prime.

Key words: Riemann zeta function, Smarandache-Riemann zeta sequence

For any complex number s, let

(1) $$\zeta(s)=\sum_{k=1}^{\infty} k^{-s}$$

be the Riemann zeta function. For any positive integer n, let T_n be a number such that

(2) $$\zeta(2n)=\frac{\pi^{2n}}{T_n},$$

where π is ratio of the circumference of a circle to its diameter. Then the sequence $T=\{T_n\}_{n=1}^{\infty}$ is called the Smarandache-Riemann zeta sequence. In [2], Murthy believed that T is a sequence of integers. Simultaneous, he proposed the following conjecture:

Conjecture No two terms of T are relatively prime.

In this paper we prove the following results.

Theorem 1 If ord $(2, (2n)!) < 2n-2$, where ord $(2, (2n)!)$ is the order of prime 2 in $(2n)!$, then T_n is not an integer.

Theorem 2 No two integer terms of T are relatively prime.

Since ord (2, 14!)=11<12=2.7-2, by Theorem 1, we find that T is not a sequence of integers. However, by Theorem 2, the above-mentioned conjecture holds for all integer terms of T.

Proof of Theorem 1 It is a well known fact that

(3) $$\zeta(2n)=(-1)^{n-1}\frac{2^{2n-1}\pi^{2n}}{(2n)!}B_n, n\geq 1,$$

where B_n is a Bernoulli number (see [1]). Notice that

(4) $$B_n=(-1)^n\frac{a_n}{b_n}, n\geq 1,$$

where a_n and b_n are coprime positive integers satisfying

(5) $$2\,\|\,b_n, 3\mid b_n, n\geq 1.$$

By (2), (3) and (4), we get

(6) $$T_n=\frac{(2n)!b_n}{2^{2n-1}a_n}, n\geq 1.$$

Since gcd $(a_n, b_n)=1$ and b_n is even, we see that a_n is odd. Therefore, by (5) and (6), if ord $(2, (2n)!)+1<2n-1$, then T_n is not an integer. The theorem is proved.

Proof of Theorem 2 Let T_m and T_n be two integer terms of T with $m\neq n$. By (6), we get

(7) $$T_m=\frac{(2m)!b_m}{2^{2m-1}a_m}.$$

Since gcd (2, 3)=gcd (a_m, b_m)=gcd $(a_n, b_n)=1$, $3|b_m$ and $3|b_n$ by (5), we get from (6) and (7) that $3|T_n$ and $3|T_m$ respectively. It implies that gcd $(T_m, T_n)\geqslant 3>1$. The theorem is proved.

References

[1] G. H. Hardy and E. M. Wright, An introduction to the theory of numbers, Oxford Univ. Press, Oxford, 1937.

[2] A. Murthy, Some more conjectures on primes and divisors, Smarandache Notions J. 12(2001), 311-312.