THE SMARANDACHE SUM OF COMPOSITES BETWEEN FACTORS FUNCTION

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ABSTRACT

In this paper some basic properties of the Smarandache Sum of Composites Between Factors function are given, investigations are reported, conjectures are made, and open problems are given. As far as the author knows, this function is new and has never been investigated previously.

KEYWORDS:

Composites, Consecutive values, Even numbers, Factors, Factorials, Functions, Odd numbers, Palindromes, Primes, Reverse, Smarandache function, Squares, Square-free, Sum of composites, Triangular.

I INTRODUCTION

The Smarandache Sum of Composites Between Factors function SCBF(n) is defined as: The sum of composite numbers between the smallest prime factor of n and the largest prime factor of n (A074037) [1]. Example: SCBF(14) = 10 since 2*7=14 and the sum of the composites between 2 and 7 is: 4+6=10.

The first 50 values of SCBF(n) are:

n	SCBF(n)	n	SCBF(n)
1	0	26	49
2	0	27	0
3	0	28	10
4	0	29	0
5	0	30	4
6	0	31	0
7	0	32	0
8	0	33	37
9	0	34	94
10	4	35	6
11	0	36	0
12	0	37	0
13	0	38	112
14	10	39	49
15	4	40	4
16	0	41	0
17	0	42	10
18	0	43	0
19	0	44	37
20	4	45	4
21	10	46	175
22	37	47	0
23	0	48	0
24	0	49	0
25	0	50	4

II SOME PROPERTIES OF SCBF(n):

(A) SCBF(p) = 0, for any prime p, or p^k, and integers of the form 2^i*3^j, where i, j, and k are positive integers. This follows from the definition.

(B) If i and j are positive integers then SCBF(2^i*5^j)=4, SCBF(2^i*7^j)=10, etc. More generally, to allow other prime factors between the smallest and largest, if i and k are positive integers and j is a nonnegative integer, then SCBF(2^i*3^j*5^k)=4, etc. This follows from the definition.

(C) SCBF(n) is not a multiplicative function.
E.g. gcd(14,15) = 1. SCBF(14*15) = 10 and SCBF(14)*SCBF(15) = 40.

III INVESTIGATIONS AND OPEN PROBLEMS

Using PARI/GP [2], a software package for computer-aided number theory, the author has explored and compared SCBF(n) with some of the more common number theoretic functions as well as some of the more obscure functions in the hope of finding interesting results.

(A) There are solutions to SCBF(n) = n, although they are very rare. Dean Hickerson [3] found one such solution, not necessarily the smallest, which is: SCBF(245220126046) = 245220126046. His method was to search for a prime p for which the sum S of the composites from 2 to p is a multiple of 2p. His reasoning was that since S < p^2/2, S can't have any prime factors larger than p (or less than 2), so S satisfies SCBF(n) = n. According to [3] the probability that S is divisible by 2p is 1/(2p); since the sum of the reciprocals of the primes diverges slowly to infinity, there are probably infinitely many solutions of this type, although they will be very rare.

Is 245220126046 the smallest even solution to SCBF(n) = n? What is the smallest odd solution?

(B) When SCBF(n) is compared with some of the more common number theoretic functions, it is relatively easy to find solutions (although the following two sequences were not thoroughly analyzed). E.g. some solutions to bigomega(n) = SCBF(n), where bigomega(n) is the number of prime factors of n, (with repetition) are:

40,60,90,100,135,150,225,250,375,3584,5376,8064,

and solutions to SCBF(n) = d(n), where d(n) is the number of divisors of n are:

10,15,112,175,245,567,4802,7203,

(C) Solutions n such that SCBF(n) = S(n) where S(n) is the Smarandache function (A002034) [1], [4] are:

350,525,700,1050,1400,1575,1792,2100,2800 (A074055) [1]

Note that all of these numbers are of the forms: $2^i*3^j*5^k*7^l$, $2^i*5^j*7^k$, $3^i*5^j*7^k$, or 2^i*7^k, with S(x) and SCBF(x) being 10. What is the first term in the sequence not having the aforementioned forms? Are there an infinite number of solutions to the above functional equation?

(D) Some solutions n such that SCBF(n) is prime (A074054) [1] are:

a) 22,33,44,66,88,99,106,110,132,134,154,155,159,165,176,178

With the primes being:

b) 37,37,37,37,37,37,1049,37,37,1709,37,331,1049,37,37,3041,

Note that in sequence D.a. above, there are consecutive values listed. Are there an infinite number of consecutive values? Are there an infinite number of triple consecutive values such that SCBF(n) is a prime? For example:

SCBF(889) = 6397; SCBF(890) = 3041; SCBF(891) = 37.

Due to the abundance of solutions found from a computer search for sequence D.a, we are confident enough to conjecture that there are an infinite number of consecutive and triple consecutive solutions.

Concerning sequence D.b above, what is the first palindromic prime value? The author has found none for n <= 10000. Yet due to the erratic nature of the primes arising in the above list, we are confident enough to conjecture that there will be at least one palindromic prime solution in sequence D.b. What is the first prime in sequence D.b consisting of fifty digits?

(E) From a purely recreational viewpoint, it is often interesting to work with some of the more base 10 dependent functions to find surprising results. The rest of this paper deals with some of these base 10 dependent functions.

Let SFD(n) be the sum of factorials of the digits of n (A061602) [1]. Are there an infinite number of solutions to SCBF(n) = SFD(n)? The author has found only three solutions:

120,200,1000.

Example: 120 = 2^3*3*5 and 4 is the only composite between 2 and 5; 1!+2!+0! = 4.

(F) Let SDS(n) be the sum of squares of digits of n (A003132) [1]. Are there an infinite number of solutions n to SCBF(n) = SDS(n)? The author has found the following solutions:

20,200,2000,2754,5681,15028,19152,20000,25704,27945,31824,

Example: 2000 = 2^4*5^3 and 4 is the only composite between 2 and 5; 2^2+0^2+0^2+0^2 = 4.

(E) Let R(n) be the reversal of n. Are there an infinite number of solutions to SCBF(n) = R(n)?

IV CONCLUSION

The SCBF(n) function has been compared with various other number theoretic functions and fruitful avenues of research are still very much open. Different bases could be explored as well as making comparisons with other functions not mentioned here. The SCBF(n) function also suggests that other functions can be defined by summing different classes of numbers which lie between the smallest prime factor and the largest prime factor of any integer. For example, SSBF(n) could be the sum of the square free numbers between the smallest and largest prime factors of n. STBF(n) could be the sum of the triangular numbers between the smallest and largest prime factors of n. SPBF(n) could be the sum of the palindromes between the smallest and largest prime factors of n. The odd and even numbers could be summed between the smallest and largest prime factors of n as well. All of these functions should be investigated!

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