THE SQUARES IN THE SMARANDACHE HIGHER POWER PRODUCT SEQUENCES

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Abstract . In this paper we prove that the Smarandache higeher power product sequences of the first kind and the second kind do not contain squares.

Key words . Smarandache product sequence , higher power , square.

Let r be a positive integer with $r>3$,and let $A(n)$ be the n-th powew of degree r . Further, let

(1) $$p(n) = \prod_{k=1}^{n} A(k)+1$$

and

(2) $$Q(n) = \prod_{k=1}^{n} A(k)-1 .$$

Then the sequences $P=\{P(n)\}_{n=1}^{\infty}$ and $Q=\{Q(n)\}_{n=1}^{\infty}$ are called the Smarandache higher power product sequences of the first kind and the second kind respectively . In this paper we consider the squares in P and Q . We prove the following result .

Theoerm . For any positive integer r with $r>3$,the sepuences P and Q do not contain squares.

Proof . By (1), if $P(n)$ is a square , then we have

(3) $$(n!)^r+1=a^2,$$

where a is a positive integer . It implies that the equation

(4) $$x^m+1=y^2,\quad m>3$$

has a poitive integer solution $(x,y,m)=(n!,a,r)$. However, by the result of [1], the equation (4) has no positive integer solution (x,y,m) . Thus, the sequence P does not contain squares .

Similarly,by(2),if $Q(n)$ is a square , then we have

(5) $$(n!)^r-1=a^2,$$

where a is a positive integer . It implies that the equation

(6) $$x^m-1=y^2, m>3.$$

has a positive integer solution $(x,y,m)=(n!,a,r)$. However, by the result of [2] , it is impossible. Thus, the sequence Q does not contain squares . The theorem is proved.

References

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