THE SYSTEM - GRAPHICAL ANALYSIS OF SOME NUMERICAL SMARANDACHE SEQUENCES

S.A. YASINSKIY, V.V. SHMAGIN, Y.V. CHEBRAKOV
Department of Mathematics, Technical University,
Nevsky 3-11, 191186, St-Petersburg, Russia
E-mail: chebra@phdeg.hop.stu.neva.ru

The system - graphical analysis results of some numerical Smarandache sequences are adduced. It is demonstrated that they possess of the big aesthetic, cognitive and applied significance.

1 Introduction

The analytical investigation[1] of some 6 numerical Smarandache sequences[2] permitted to state that the terms of these sequences are given by the following general recurrent expression

$$a_{\varphi(n)} = \sigma(a_n 10^{\psi(a_n)} + a_n + 1), \tag{1}$$

where $\varphi(n)$ and $\psi(a_n)$ — some functions; σ — operator. In this paper we will denote all numerical sequences, yielded by (1), as Smarandache sequences of 1st kind, and analyse ones by system - graphical methods. The main goal of the present research is to demonstrate that the system - graphical analysis results of numerical Smarandache sequences of 1st kind possess of the big aesthetic, cognitive and applied significance.

2 System - graphical analysis of some Smarandache sequences of 1st kind

1. Smarandache numbers

$$1, 12, 123, 1234, 12345, 123456, \ldots \tag{2}$$

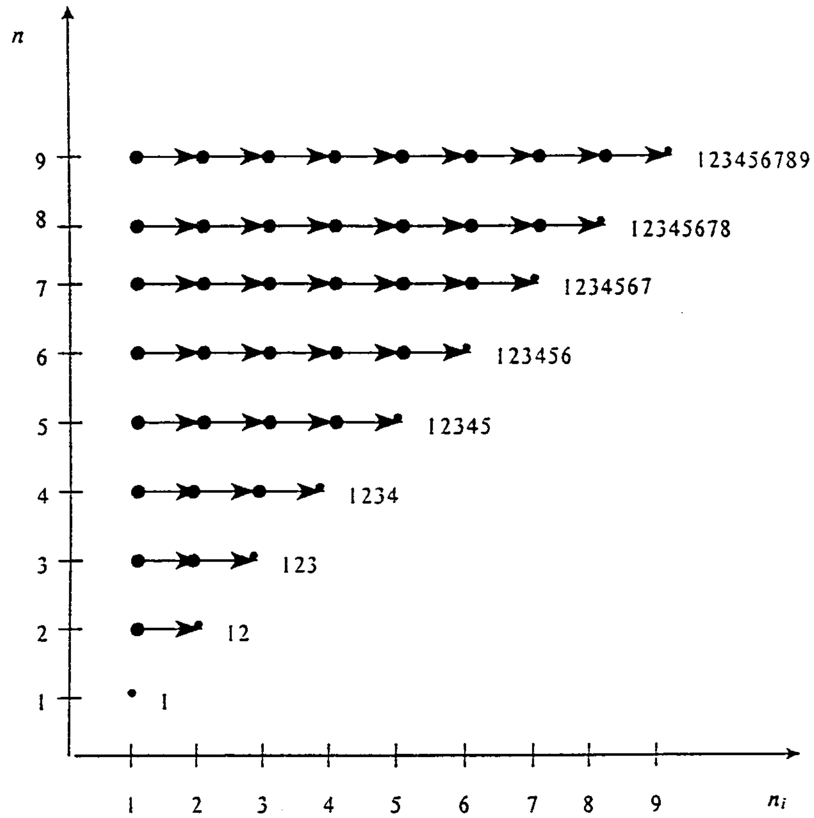


Fig. 1. Graphical image of the first nine terms of S_1-series.

we shall call *numbers of S_1-series*. Graphical image of the first nine terms of S_1-series is given in Fig. 1. For these numbers we introduce[1] an operator Λ^{-k}, making k-truncation the numbers (2) from the left and/or from the right: for instance, if $k = 1$ then $(\Lambda^{-k}\,123) = 23$ and $(123\Lambda^{-k}) = 12$.

It is evident from Fig. 1 that one may use numbers of S_1-series with Λ^{-k} operator as *a standardizative representation* of any quantity characteristics of investigated object in such cases when the values of these characteristics are limited from the left and/or the right and uniformly discrete. For instance, one may use mentioned standardization in visual control device of sound level in audio-techniques, in the information decoding and transmitting systems and so on.

2. Smarandache numbers

$$1, 11, 121, 1221, 12321, 123321, 1234321, \ldots \qquad (3)$$

we shall call *numbers of* S_2*-series*. The numbers (3) possess the mirror-symmetric properties evinced in graphical image of ones (see Fig. 2) by the presence of reverse motion arcs. It is easy to find that there are a great number of technical and physical objects, using the same principle of action as one showed in Fig. 2.

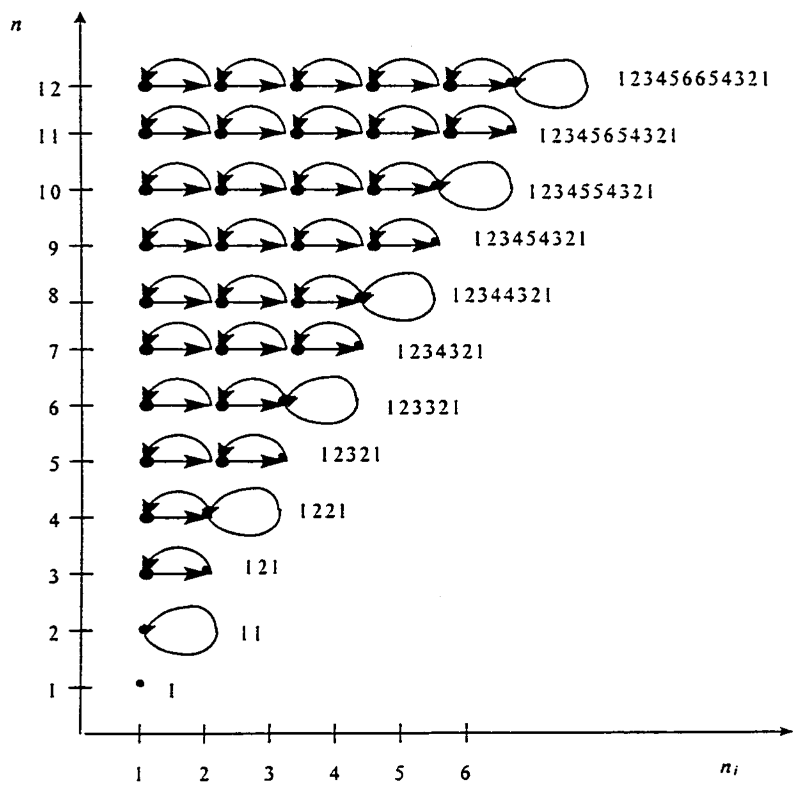


Fig. 2. Graphical image of the first twelve terms of S_2-series.

In particular, a standardizative representation in terms of Smarandache numbers of S_2-series can be made for reverse connection circuits in the different control and handling systems; for the suitable graphical representation of any systems in which for complete description of system state the knowledge of n last states is required; for coding information on effects of "staying waves" and so on.

3. Smarandache numbers

$$1, 212, 32123, 4321234, 543212345, 65432123456, \ldots \qquad (4)$$

we shall call *numbers of S_3-series*. By analysing graphical image of S_3-series terms given in Fig. 3 one can conclude this image is similar to that given in Fig. 2. Indeed, figures of S_3-series terms differ from the ones of S_2-series terms only by reverse orientation in space (a suitable interpretation for description reversible physical phenomenon) or by another initial state (— for the theory of automates).

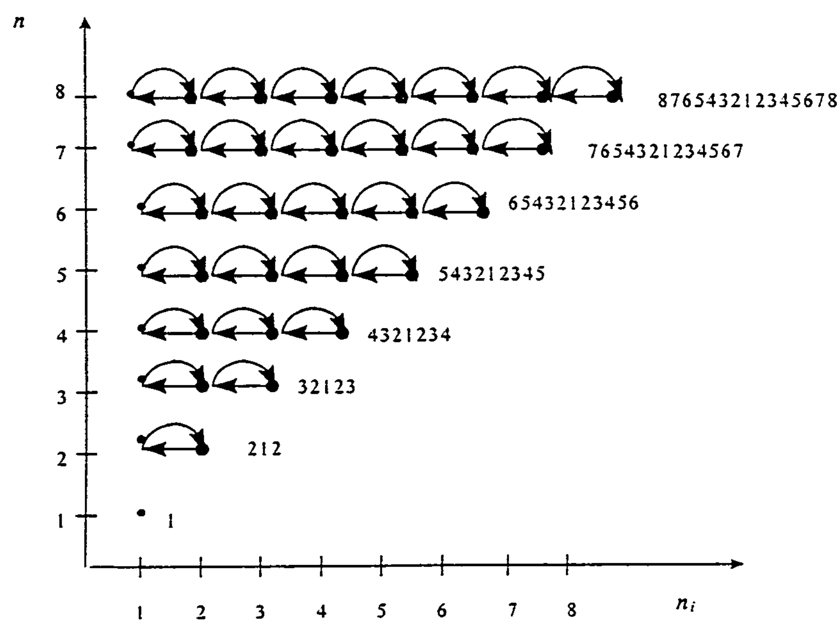


Fig. 3. Graphical image of the first eight terms of S_3-series.

Thus, the numbers of S_3-series can get the same applications in standardizative representations of quantity characteristics as the numbers of S_2-series, though they are less useful because its graphic image structure more poor than one that numbers of S_2-series have.

4. Smarandache numbers

$$1, 23, 456, 7891, 23456, 789123, 4567891, \ldots \qquad (5)$$

we shall call *numbers of S_4-series*. It's graphical image is given in Fig. 4. In distinction from the terms of considered Smarandache series the ones of S_4-series consist of only numbers from 1 to 9. Thus, after 17th term of S_4-series 23456789123456789 the successive ones do not enrich S_4-series since any sequence

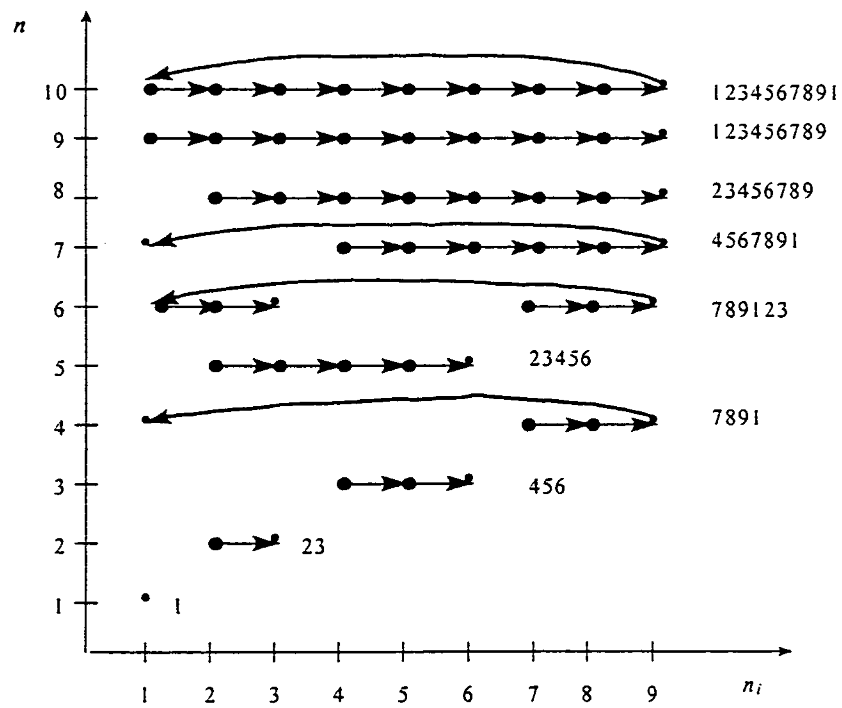


Fig. 4. Graphical image of the first ten terms of S_4-series.

from 9 or less different successive digits can be obtained from 17th term of S_4-series by truncation from the left and right Λ^{-k} operators. However, in spite of mentioned lack the standardizative representation of quantity characteristics of local computer nets by terms of S_4-series may be quite useful. In particular, such Smarandache numbers can reflect the principle of transmitting data packets from one local station to another. Besides one may use standardization by S_4-terms for description of recurrent relations between sequence elements or some processes, described usually by Markov's chains.

5. Smarandache numbers

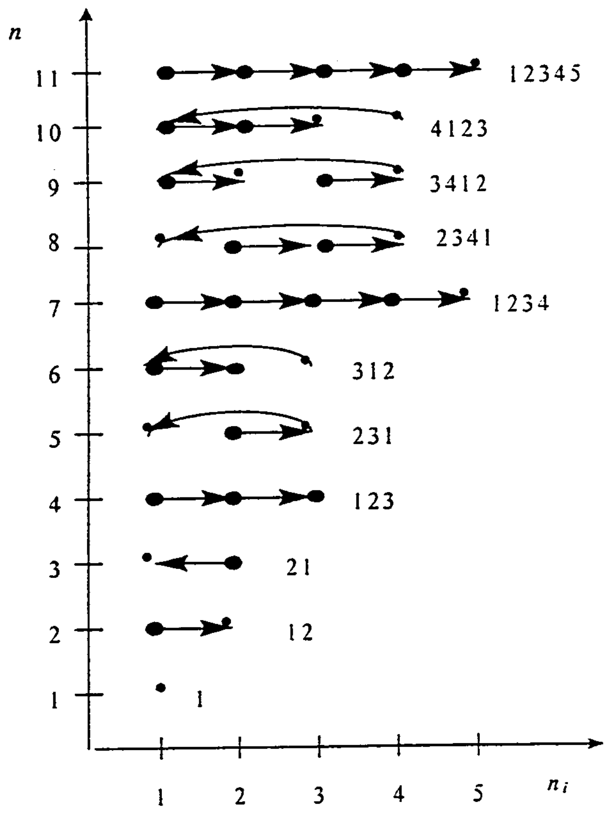


Fig. 5. Graphical image of the first eleven terms of S_5-series.

$$1, 12, 21, 123, 231, 312, 1234, 2341, 3412, 4123, 12345, \ldots \qquad (6)$$

we shall call *numbers of S_5-series*. It is evident that S_5-series contains all the numbers of S_1-series and some additional numbers. Successive-circular properties of this series are well-shaped in Fig. 5. The analysis of the S_5-series graphical image permits to find applications fields where S_5-series terms as standardizative representation of object characteristics can be used: these are fields where some look over several states of objects is required. For instance, technical diagnostics, systems of processes handling, theory of selecting and taking the decision may be named.

6. Smarandache numbers

$$12, 1342, 135642, 13578642, 13579108642, \ldots \qquad (7)$$

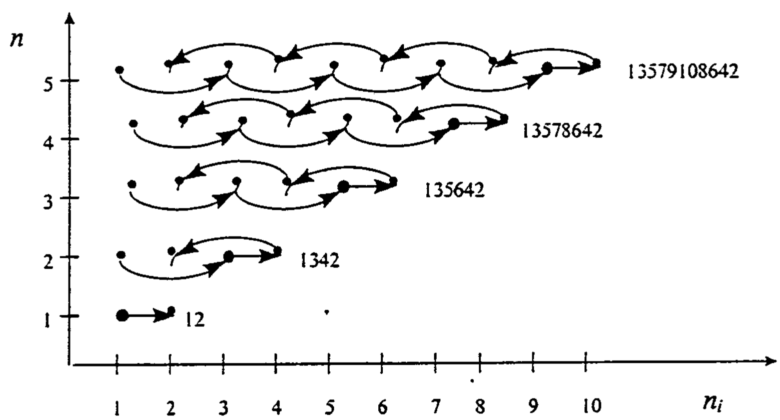


Fig. 6. Graphical image of the first five terms of S_6-series.

we shall call *numbers of S_6-series*. These numbers also as numbers (6) have circular properties with the uniform structure. Graphical image of (7) is given in Fig. 6. By analysing Fig. 6 one may note that presented images may describe the test procedure rounds the all elements of system with minimal steps and aim to finish the round in the element the nearest to the initial one. Besides it turns out that a term of (7) being divided into two subterms can serve as standardizative representation of two simultaneous processes. In particular, such standardization of S_6-series terms can be applied for parallel signal processing or parallel design processes.

3 System - graphical analysis of numbers of S_2- and S_3-series

Divide[1] a set of S_2-series numbers (3) into two different subsequences:

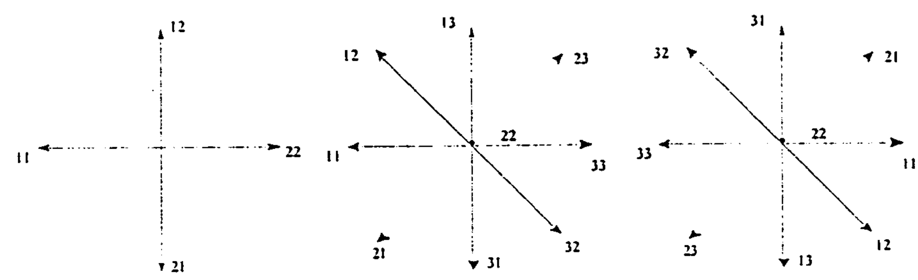
1) $a_1=1$, $a_2=121$, $a_3=12321$, $a_4=1234321, \ldots$

2) $b_1=11$, $b_2=1221$, $b_3=123321$, $b_4=12344321, \ldots$

The numbers of the first and the second subsequences we shall call A- and B-*numbers* correspondingly.

In Sect. 2 Smarandache numbers were presented in the proper constructions on the number axises. It is naturally to expect that employment not only number axises but the whole plane and different geometrical figures for representation Smarandache numbers will permit to reveal new interesting properties of ones, explain by ones a great number of technical and nature processes, study more deeply its peculiarities and preferences. In this section we consider the only mentioned above A- and B-numbers and also S_3-series numbers (4).

Firstly we explain how to construct numerical circumferences from Smarandache numbers:



a) b) c)

Fig. 7. The graphical images of the second (a) and the third terms of A- and B-subsequences and the third term (c) of S_3-series.

a) the first terms of A- and B-subsequences and of S_3-series convert into the point;

b) the graphical images of the second and the third terms of these sequences are given in Fig. 7 correspondingly.

By analysing Fig. 7 one can easy find the graphical forms of the representation for next terms of these sequences. Namely, to construct the graphical image of n-th term of mentioned above Smarandache sequences the following algorithm may be used:

1. To draw the circumference and two perpendicular lines crossed in the centre of circle.

2. To denote the tops of four rays going from the point of the cross as 11, $1n$, nn, $n1$ consequently in the forward of clock hand.

3. To divide every sector into $n-1$ equal parts by drawing additional rays with proper mark.

At such representation of Smarandache sequences terms these ones produce the subsequences. For example,

a) the second term of A- or B-subsequences produce the proper 2×2 series of subsequence

$$11, 22, 22, 11; \quad 12, 21, 21, 12; \tag{8}$$

b) the third term of A- or B-subsequences produce the proper 3×3 series of subsequence

$$\begin{array}{ll} 11, 22, 33, 33, 22, 11; & 12, 22, 32, 32, 22, 12; \\ 13, 22, 31, 31, 22, 13; & 21, 22, 23, 23, 22, 21; \end{array} \tag{9}$$

and so on.

Among splendid peculiarities of graphical images, depicted in Fig. 7, we point that the all Smarandache circumferences reveal Magic properties: they have a constant sum for the elements located in the diameters of the circumference. It is very interesting to confront the ancient Chinese hexagrams, located on the circumference (see Fig. 8a), with numbers of subsequence (9), by which the tops of diameters are marked (Fig. 8b).

By deleting commas in (8) and (9) and combining two-digit elements in single terms one obtains extended representation of Smarandache sequences terms.

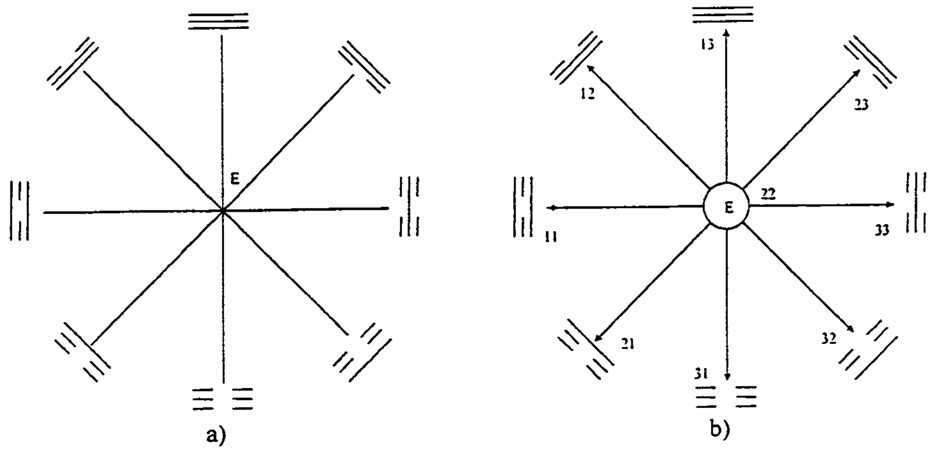


Fig. 8. The graphical confrontation of the ancient Chinese hexagrams with numbers of subsequence (9).

Thus, graphical images of Smarandache sequences terms, used as standardizative representations of the objects allow both decomposition of the object representation (analysis of the object) and combination of the object representation (synthesis of a new object). This graphical technique is similar to operators of truncation Λ^{-k} and extending Λ^{+k} of series terms[1], but more flexible.

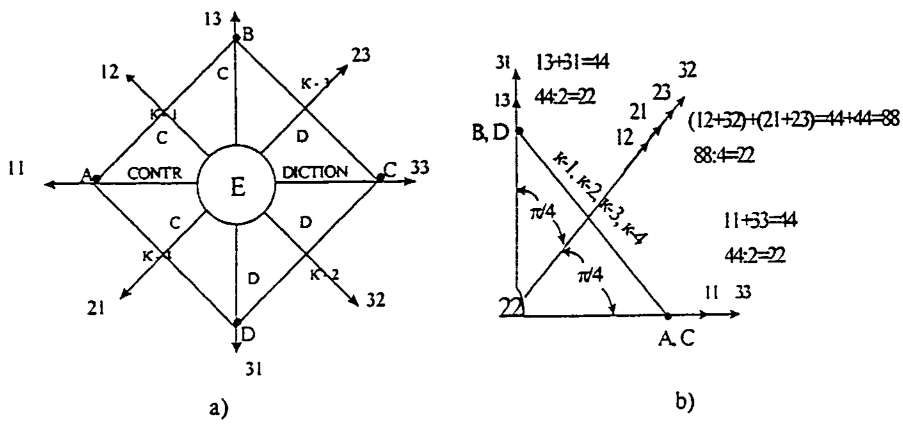


Fig. 9. The Aristotelian logical (a) and the reduced logical (b) squares.

Curtailment of these graphical images is seemed to be very interesting since it reflects the ways of simplification of real instruments and devices. For example, if one takes into account that Aristotelian logical square can be superposed into the graphical representation of *A*- or *B*-subsequences third term (see Fig. 9a) then the circumference may be reduced one fourth of the circle (Fig. 9b).

Very curious graphical images with Smarandache circumferences are revealed when one draws the track of point M and M' movement along the circumference, and this track will be shown not on the whole diagram, but on the reduced one to 1/8 of the circle (see Fig. 10).

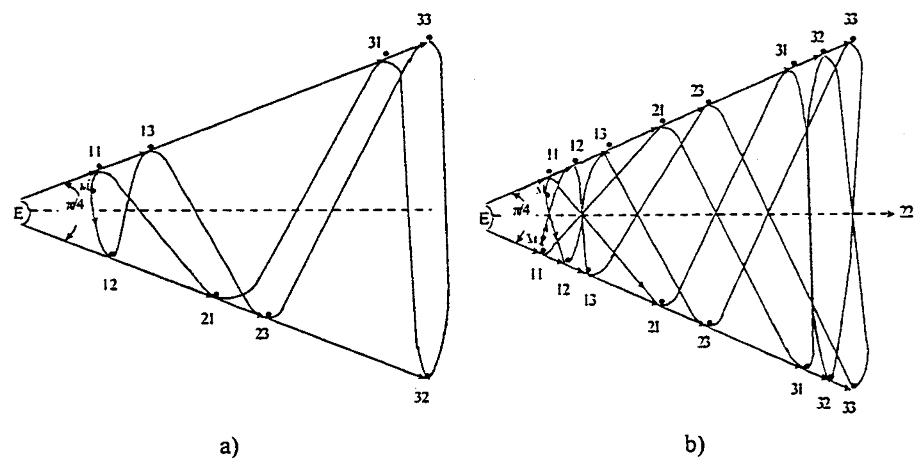


Fig. 10. The track of points M (a) and both M and M' (b) movement along the circumference.

Very important for understanding internal properties of Smarandache numbers are images given in Fig. 11(a, b), where the third term of *A*- or *B*-subsequences is depicted. Indeed, it shows the quantitative characteristics of Smarandache numbers even in such case when numerical information, adduced in Fig. 11(a, b), is absent. We pay attention, that in Fig. 11b circumference with unit diameter and the graphical quantitative characteristic of Smarandache number are depicted simultaneously.

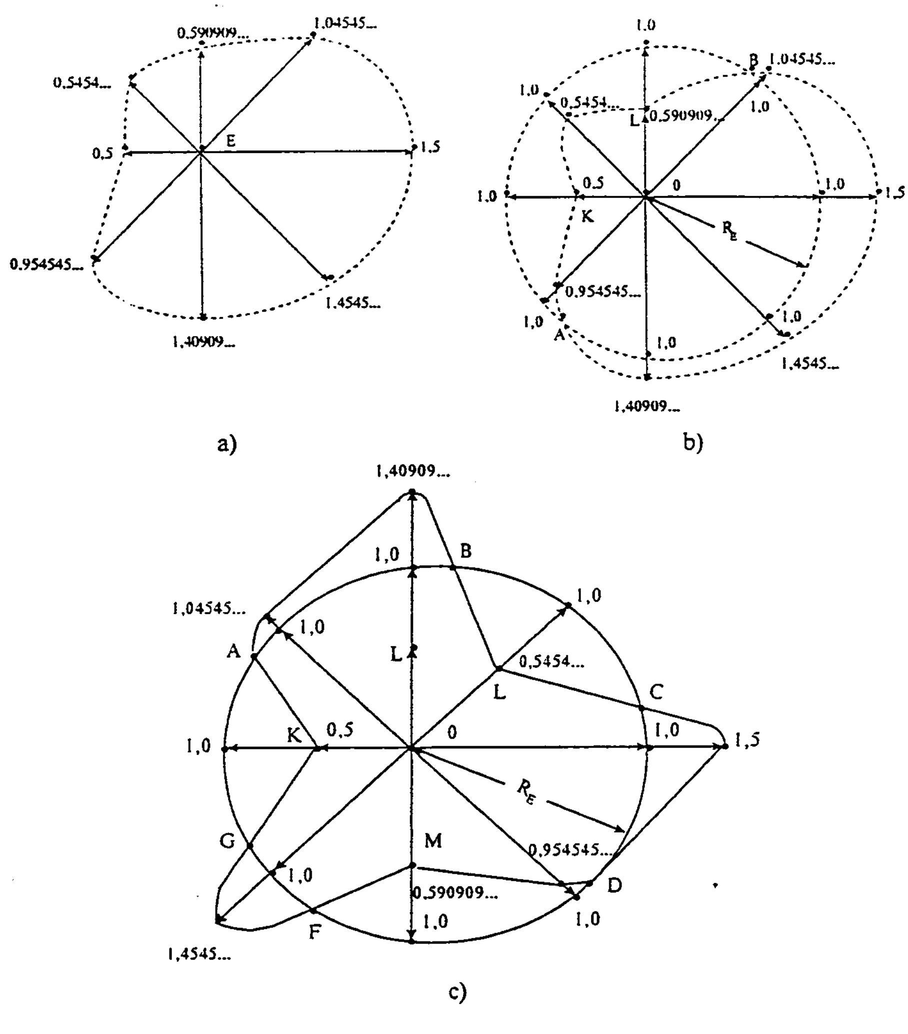


Fig. 11. The graphical image containing the quantitative characteristics of the third term of A- or B-subsequences.

Thus, the image of quantitative characteristic of Smarandache number in Fig. 11(a, b) one may interpret as any transformation of unit circumference. Taking such interpretation into account one can easy come from Fig. 11b to Fig. 11c. In the image depicted in Fig. 11c one can easy find a schematic picture of an aeroplane. Hence, it turns out that third term of the A- or B-

subsequences contains in the implicit form picture of an aeroplane. We assume that by using discussed system graphical analysis methods one may reveal some another unexpected graphical information, contained in some Smarandache numbers.

References

1. Y.V. Chebrakov, V.V. Shmagin (see this Proceedings).
2. C.Dumitrescu, V.Seleacu, *Some notions and questions in number theory* (Erhus University Press, Vail, 1995).