TWO FORMULAS FOR SMARANDACHE LCM RATIO SEQUENCES

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Abstract: In this paper we give two reduction formulas for Smarandache LCM ratio sequences $SLRS(3)$ and $SLRS(4)$.

Key words: Smarandache LCM ratio sequence; reduction formula

For any $t(t>1)$ positive integers $x_1,x_2,\cdots,x_t$, let $(x_1,x_2,\cdots,x_t)$ and $[x_1,x_2,\cdots,x_t]$ denote the greatest common divisor and the least common multiple of $x_1,x_2,\cdots,x_t$ respectively. Let r be a positive integer with $r>1$. For any positive integer n, let

$$T(r,n)=\frac{[n,n+1,\cdots,n+r-1]}{[1,2,\cdots,r]}. \qquad (1)$$

Then the sequence $SLRS(r)=\{T(r,n)\}_{n=1}^{\infty}$ is called the Smarandache LCM ratio sequence of degree r. It is easy to see that

$$T(2,n)=\frac{1}{2}n(n+1)$$

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for any positive integer n. In [2], Murthy asked that find reduction formulas for $T(r,n)$. In this paper we solve this open problem for r=3 or 4. We prove the following result.

Theorem. For any positive integer n, we have

$$T(3,n)=\begin{cases}\frac{1}{6}n(n+1)(n+2), \text{ if } n \text{ is odd,}\\ \frac{1}{12}n(n+1)(n+2), \text{ if } n \text{ is even}\end{cases} \tag{2}$$

and

$$T(4,n)=\begin{cases}\frac{1}{24}n(n+1)(n+2)(n+3), \text{ if } n\not\equiv 0 \pmod 3,\\ \frac{1}{72}n(n+1)(n+2)(n+3), \text{ if } n\equiv 0 \pmod 3.\end{cases} \tag{3}$$

The proof of our theorem depends on the following lemmas.

Lemma 1 ([1, Theorem 1.6.4]). For any positive integers a and b, we have $(a,b)[a,b]=ab$.

Lemma 2 ([1, Theorem 1.6.5]). For any positive integers s and $s<t$, we have

$$(x_1,x_2,\cdots,x_t)=((x_1,\cdots,x_x),(x_{s+1},\cdots,x_t))$$

and

$$[x_1,x_2,\cdots,x_t]=[[x_1,\cdots x_s],[x_{s+1},\cdots,x_t]].$$

Proof of theorem. By Lemmas 1 and 2, we get

$$[n,n+1,n+2]=[n,[n+1,n+2]]=\left[n,\frac{(n+1)(n+2)}{(n+1,n+2)}\right]. \tag{4}$$

Since $(n+1, n+2)=1$, we get from (4) that

$$[n,n+1,n+2]=[n,(n+1)(n+2)]. \tag{5}$$

Further, since $(n,n+1)=1$, we have

$$(n,(n+1)(n+2))=(n,n+2)=\begin{cases}1, \text{ if } n \text{ is odd,}\\ 2, \text{ if } n \text{ is even.}\end{cases} \tag{6}$$

Hence, by Lemma 1, we obtain from (5) and (6) that

$$[n,n+1,n+2]=\left[n,\frac{(n+1)(n+2)}{(n+1,n+2)}\right]$$

$$= \begin{cases} n(n+1)(n+2), & \text{if } n \text{ is odd}, \\ \frac{1}{2} n(n+1)(n+2), & \text{if } n \text{ is even}. \end{cases} \quad (7)$$

Since [1,2,3]=6, we get (2) by (7) immediately.

Similarly, we have

$$[n,n+1,n+2,n+3] = [[n,n+1],[n+2,n+3]]$$

$$= \left[\frac{n(n+1)}{(n,n+1)}, \frac{(n+2)(n+3)}{(n+2,n+3)}\right] = [n(n+1),(n+2)(n+3)]. \quad (8)$$

Since [1,2,3,4]=12 and

$$(n(n+1),(n+2)(n+3)) = \begin{cases} 2, \text{if } n \not\equiv 0 \pmod 3, \\ 6, \text{if } n \equiv 0 \pmod 3, \end{cases} \quad (9)$$

we obtain (3) by (8) immediately. The theorem is proved.

References

[1] G.H.Hardy and E.M.Wright, An introduction to the theory of numbers, Oxford University Press, Oxford, 1938.

[2] A.Murthy, Some notions on least common multiples, Smarandache Notions J. 12(2001), 307-308.