TWO SMARANDACHE SERIES

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Abstract . In this paper we consider the convergence for two Smarandache series .

Key words . Smarandache reciprocal series , convergence .

Let $A=\{a(n)\}_{n=1}^{\infty}$ and $B=\{b(n)\}_{n=1}^{\infty}$ be two Smarandache sequences . Then the series

$$S(A,B)=\sum_{n=1}^{\infty}\frac{a(n)}{b(n)}$$

is called the Smarandache series of A and B . Recently , Castillo [1] proposed the following two open problems .

Problem 1 . Is the series

$$S_1=\frac{1}{1}+\frac{1}{12}+\frac{1}{123}+\frac{1}{1234}+\cdots \tag{1}$$

convergent ?

Problem 2 . Is the series

$$S_2=\frac{1}{1}+\frac{12}{21}+\frac{123}{321}+\frac{1234}{4321}+\cdots \tag{2}$$

Convergent ?

In this paper we completely solve the mentioned problems as follows .

Theorem . The series S_1 is convergent and the series S_2 is divergent .

Proof . Let $r(n)=1/\overline{12\cdots n}$ for any positive integer n . Since

$$(3) \qquad \lim_{n\to\infty} \frac{r(n+1)}{r(n)} = \frac{12\cdots n}{12\cdots n(n+1)} < 1,$$

by D'Alembert's criterion, we see from (3) that S_1 is convergent.

Let $s(n)=12\cdots(n-1)n \,/\, n(n-1)\cdots 21$ for any positive integer n. If $n=10^t+1$, where t is a positive integer, then we hare

$$(4) \qquad S(n)=\frac{12\cdots(10\cdots01)}{(10\cdots01)\cdots21}>1.$$

Therefore, by (4), we get from (2) that

$$(5) \qquad S_2=\sum_{n=1}^{\infty} s(n) > \sum_{t=1}^{\infty} s(10^t+1) > \sum_{t=1}^{\infty} 1=\infty .$$

Thus, the series S_2 is divergent. The theorem is proved.

Reference

[1] J. Castillo, **Smarandache** series, Smarandache Notions J., http://www.gallup.unm.edu/~smarandache/SERIES.TXT.

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