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A CASE STUDY OF THE IMPACT OF CARBON EMISSIONS AND INFLATION ON NONLINEAR DENSE NEUTROSOPHIC FUZZY INVENTORY SYSTEM OF VARYING DEMAND WITH DELAYED DETERIORATION

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Abstract. In a comprehensive consensus, humans should reduce carbon emissions on the way to minimise the adverse effects of global warming. Manufacturing firms contribute a significant amount of carbon emissions to the environment. This article examines the impact of carbon emissions on non-instantaneously degrading commodities with price-dependent demand and advertisements since many nations are implementing emission reduction policies. The paper also focuses on the effects of inflation, where shortages are partially backlogged and partially lost in sales. This paper develops a new concept of non-linear triangular dense neutrosophic numbers with its basic properties. Further, the classifications of symmetry and asymmetry are introduced, and thereafter De-neutrosophication technique is applied for crispification. Since the effect of some parameters like carbon emissions, advertising, and inflation are uncertain, we have considered it in this new form described above to grab the uncertain characters of the underlying parameters. The classification of the uncertain parameter based on the symmetric and asymmetric nature and linear-nonlinear nature of triangular dense neutrosophic numbers have also been investigated here. Additionally, the effect of the model is examined under different situations for both linear and non-linear triangular dense fuzzy, dense intuitionistic, and dense neutrosophic. Finally, a numerical example is considered to illustrate the model, and it is observed that the model is optimum when the parameters are considered in Asymmetric Convex Non-Linear numbers for a shorter replenishment cycle. Also, a comparative study has been done by performing sensitivity analysis through the case study and provides managerial insight into this outcome.

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1. Introduction

Carbon dioxide emissions are the prime cause of the change in the global environment. It poses a peril to the world's ecological system and mankind. It's highly predictable that the world needs an urgent reduction in carbon emissions to avoid the adverse effects of climate change. Carbon emission or carbon footprinting is the total amount of greenhouse gas emission that leads to global warming. Intergovernmental Panel on Climate Change, a United Nations body, assess and reports on climate change, especially global warming. Global warming

Keywords. Carbon emission, advertisement and price dependent demand, non-instantaneous deterioration, triangular dense neutrosophic number, de-neutrosophication.

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is now driven mainly by rising carbon emissions, which have grown due to deforestation or the widespread use of non-renewable resources. So, global warming and carbon footprinting are interdependent. Many academics have raised the issue of carbon footprinting over the years, believing it to be a constant figure. But in reality, the quantity fluctuates from time to time. So, we cannot predict the exact value properly using the real data. Thus, it is essential to incorporate the concept of uncertainty to capture the data.

The conventional paradigm anticipated that demand would always remain constant; nonetheless, further research revealed that this may not be the case everywhere. It may be depending on price and stock, ramp-type, linear, or quadratic functions. It is noted that ads have several effects on society in this digital economy. Several firms produce similar products; in this case, buyers are more likely to purchase the products that have the most advertising. Once more, consumers like buying high-quality products at lower prices. As a result, it is shown that two crucial factors influencing an item's demand are its pricing and its marketing. Therefore, decreases in the price of each item and increases in advertising result in more demand, and *vice versa*.

Deterioration is the indispensable property of the items. Nowadays, there is global warming, and the temperature of the environment increases due to the above phenomenon. So, the deterioration rate of the perishable items also increases. Since deterioration depends on preserving facilities and environmental conditions available in a warehouse, different warehouses may have different deterioration rates. As the deterioration phenomenon is considered, a unit of inventory stored incurs holding and deterioration costs. When the phenomena of degradation is taken into account, storing an inventory unit results in holding and deterioration expenses. Additionally, it is noted that the item's degeneration does not begin at the beginning of the supply. It is observed that the item's degradation is not instantaneous; rather, it starts at a certain point in time when the stock is taken.

In different innovative inventory models, it is necessary to have adequate real-life data for predicting the various costs and their management. Still, due to technical difficulties, finding the real-life data available for analysis is very challenging. Recently, researchers have become more interested in neutrosophic data as it can grab all three uncertainty components: truth, false, and hesitation, in a compact way of representing uncertain parameters. In the case of inventory management problems, decision-makers have commonly utilised their proper membership grade values for different attributes based on examined results. But in reality, the data that was prophesied a day ago may not be valuable for tomorrow. Thus, it isn't easy to detect the original data. In most cases, the actual data is veiled due to national /international regulations. We have theoretically developed the non-linear triangular neutrosophic dense set idea to resolve the inventory management problem in these circumstances. The presence of non-linear components can easily grab any unknown gradation cases, and it is more reliable, efficient and prominent than any other dense structure.

1.1. Literature review

It is observed that packed food items like beverages, juice, and sliced vegetables have carbon emissions while they are in the inventory. Some studies focus on the carbon emissions measuring technique in the EOQ and supply chain models in a deterministic setting. Tiwari *et al.* [66] worked on sustainable inventory management with deteriorating and imperfect quality items with the effect of carbon emission. Li and Hai [35] observed the effect of carbon emission cost in one warehouse and multi-retailer systems. Huang *et al.* [28] considered different carbon emission policies in a supply chain model. Nowadays, many researchers, namely Sarkar *et al.* [58], Wu *et al.* [68], Gautam and Khanna [23], Tiwari *et al.* [66], Lu *et al.* [37], Taleizadeh *et al.* [63], Das *et al.* [8,12,13], Durga Bhavani *et al.* [20], etc. are working more on carbon emission and its footprinting along with cap and trade of carbon emission in crips and fuzzy environment. The literature on carbon footprint management in nonlinear dense neutrosophic numbers is scarce. Thus, in this paper, we have considered the effect of carbon emission as the dense neutrosophic number in the inventory modelling.

It is observed that the demand for the items and price are interrelated. If the demand for the commodity increases for specific reasons, then the price of the item also increases. Also, if the price of the item is increased, the demand decreases. Thus, many researchers are considering price as one of the demand parameters. Khan *et al.* [31] developed a model where demand is deemed to be advertisement and selling price dependent with advanced payment for perishable items. Chowdhury *et al.* [6] considered the sensitive demand for perishable commodities in stock and advertisement. Pan *et al.* [50] worked on a multi-stage sustainable supply chain model with optimal pricing and advertisement-dependent demand. Researchers have recently focused on uncertain markets, whether stochastic or fuzzy demand. Hovelaque and Bironneau [27], Nouira *et al.* [44], etc., worked on carbon emission-sensitive demand. Many researchers, such as Mashud *et al.* [41], Pan *et al.* [49], and Priyan *et al.* [53], have worked on deterministic demand. Thus, it is observed that demand for the items increases if the advertisement of the item is increased. Therefore, in this paper, we have considered both advertisement and price as a determining factor for demand.

In the past decades, many researchers like Saha and Chakrabarti [56], and Yadav *et al.* [70], etc. worked on instantaneous deterioration. But generally, deterioration of items takes place a few times after the date of commencement of orders. So, now a days, researchers Yadav and Swami [69], Jaggi *et al.* [29], Adak and Mahapatra [2], etc., are mainly focusing on the effect of non-instantaneous deterioration. Also, in present day, the time value of money and inflation are an important factors which severely impact the current monetary environment of the market. In the last decades, researchers considered this effect in the inventory as inflation. Dey *et al.* [18] considered a dynamic demand inventory model with lead time and inflation. Kundu and Chakrabarti [34] developed a fuzzy multi-stage supply chain inventory model with carbon emissions and inflation. Many researchers, such as Gupta and Singh [24], Singh and Rana [60], and many others, have considered the effect of inflation and the time value of money in their research papers.

Benlap [3] attempted to construct four-valued uncertain parameters, namely truth, false, Unknown and Contradiction, based on bi-lattice concepts where each component was interrelated. Further, Smarandache [61] established the idea of a neutrosophic set and its extension in the research field, which can efficiently tackle any real-life problem. As researches goes on, Wang *et al.* [67] developed a single-valued neutrosophic set. Chakraborty *et al.* [21, 22] developed the concepts of triangular and trapezoidal neutrosophic sets with their classification. Mandal and Pramanik [39], Biswas *et al.* [4, 5], etc., have established different ranking methods of neutrosophic sets to convert them into crisp sets. Haque *et al.* [25, 26] focused on exponential and logarithmic operation law-based decision-making problems. Peng *et al.* [51] incorporated the idea of a multivalued power operator in NS theory. Pramanik and Mallick [52] designed a VIKOR method for multi-attribute group decision-making (MAGDM) in a trapezoidal neutrosophic environment. Sahin *et al.* [57] focused on weighted arithmetic and geometric operators in the SVTN arena, an application in decision-making problems. Abdel-Basset *et al.* [1] developed a type II neutrosophic number and used it in decision-making problems. Ye [71] manifested the concept of trapezoidal neutrosophic number (TNN) along with SVNS. Deli and Subas [17] developed a new ranking skill of TNN in decision-making techniques. Liang *et al.* [36] introduced the score and accuracy function of TNN with the help of the centre of gravity skill. Pal *et al.* [45, 46] used the neutrosophic number to explain the inventory management problem. Dey and Beg [15] have recently proposed a new idea of a dense fuzzy set in the research domain. The above discussion clearly indicates this topic is an essential field of research nowadays. After that, Maity *et al.* [38] established a heptagonal fuzzy dense set and applied it to the inventory management problem. Further, Dey and Beg [14] introduced a triangular neutrosophic dense set and its application in the research domain, which plays an essential role in the modern age of science. Many researchers like Das *et al.* [11], Tavana *et al.* [65], Shaw *et al.* [59], and Durga Bhavani and Mahapatra [20] have applied the fuzzy or neutrosophic numbers in an inventory system. Many researchers are working in this field, such as Das *et al.* [9–11], etc.

More literature surveys are cited in Table 1.

1.2. Motivation

From the literature review, we have observed that the researchers mostly focus on EOQ model in the crisp domain or sometimes in the fuzzy/neutrosophic domain by considering the linear cases. However, they have not studied the effect of non-linear neutrosophic characters in case of uncertain parameters. A dense fuzzy number is an essential, useful and much more reliable parameter to handle the uncertain portion in real life. We need non-linear neutrosophic dense cases in several situations especially if the model parameters contain any geometrical convexity or concavity to form the EOQ model in real-life scenarios. Non-linearity arises whenever

Table 1. Review of related literature.

Author	Model type	Demand type	Deterioration	Inflation	Carbon emission	Linear uncertainty	Non-linear uncertainty
Kumar *et al.* [33]	Supply chain	Constant	No	No	Yes	Fuzzy	No
Rani *et al.* [55]	Green Supply chain	Carbon	Yes	No	Yes	Fuzzy	No
Márquez *et al.* [40]	EOQ	Price sensitive	No	No	Yes	No	No
Karthick and Uthayakumar [30]	Supply chain	Uncertain	Yes	No	Yes	Pentagonal fuzzy	No
Mishra *et al.* [42]	Supply chain	Linear and non-linear price-dependent	Yes, Preservation technology	No	Yes	No	No
De *et al.* [16]	EPL	Constant	Yes	No	Yes	Fuzzy	No
Das and Roy [8]	Multi-objective	Constant	No	Yes	Yes, Variable	Neutrosophic	No
Mishra and Mishra [43]	EOQ	Constant	Yes, Non-instantaneous	No	Yes	No	No
Sundararajan *et al.* [62]	EOQ	Price determination	Yes, Non-instantaneous	Yes	Yes	No	No
Taleizadeh *et al.* [64]	EOQ	Price sensitive	No	No	Yes	No	No
Pal *et al.* [47, 48]	EOQ	Ramp type	Yes	Yes	No	Triangular fuzzy	No
Dai *et al.* [7]	Supply chain	Yes	Yes	No	No	Fuzzy constrain	No
Our model	EOQ	Price and advertisement dependent	Yes, Non-instantaneous	Yes	Yes	Triangular neutrosophic dense fuzzy	Yes

the model parameters have some gradation values in the EOQ model, and a dense number indicates the condense of the uncertain parameter. Since neutrosophic character captures all three uncertain components (truth, false, hesitation) thus it is very useful for the EOQ model parameters in real-life situations. Also, since there are numerous cases where we need to tackle the uncertainties of the parameters in the mathematical model by considering non-linear triangular dense neutrosophic. So, we have incorporated a generalised NLTDNN to make it more flexible on the decision maker's (retailer's) choice in this EOQ model formulation. In this paper, we have developed a non-linear triangular dense neutrosophic number; its classifications based on symmetry asymmetry and also incorporated the De-neutrosophication technique for Crispification of it. Also, in the existing literature all the researcher has considered the model in crips and then compared it with fuzzy. None of the literature considered the model as uncertain. But in this paper, we have considered the uncertain parameters like carbon emission, advertisement, and inflation in triangular dense neutrosophic environment to formulate the model. It is also noticed that in case of real life the effect of price, advertisement dependent demands are uncertain in several situations. Additionally, the deterioration of the items is non-instantaneous where part of the shortages is backordered and part of the sales are lost. In this scenario, we have also incorporated the effect of inflation and carbon emission in this EOQ inventory model which plays an essential role in this modern age.

1.3. Novelties

Recently, researchers have shown their attentiveness to evolving theories connecting to a dense neutrosophic domain and incessantly promote its numerous applications in distinct branches of the neutrosophic arena, like the EOQ inventory model. In this research paper, our supreme motto is to focus on some blurred topics in non-linear triangular dense neutrosophic environments, which are listed as follows:

(1) Demonstration of non-linear triangular dense neutrosophic number and its classification based on symmetry and asymmetry.
(2) Establishment of technique for De-neutrosophication of a non-linear triangular dense neutrosophic number using the Centroid method.
(3) Formulation of the EOQ model considering essential parameters in the NLTDNN arena.
(4) Considering carbon emission in the inventory modelling for non-instantaneous deteriorating items.
(5) Incorporated managerial facts through a case study and coordinating analytical and numerical analysis.

1.4. Construction of the paper

The research paper is structured as follows:

Section 2 has developed the new non-linear triangular dense neutrosophic number and its properties and classifications based on symmetry and asymmetry. In Section 3, the assumption and notation used in the model are explained and discussed along with the case study. In Section 4, the EOQ model is formulated in an imprecise environment, and its costs are calculated. Section 5 represents the model in a neutrosophic environment, and the de-neutrosophication is done by the centroid method. The data received by the case study is used for a numerical explanation of the model. A comparative analysis is also done in Section 6. Finally, in Section 7, the paper is concluded, and future extension of the paper is discussed.

2. Mathematical definitions

2.1. Neutrosophic set

A set $\tilde{S}_{Neu}$ is the universe of discourse. A neutrosophic set $\tilde{S}_{Neu}$ is defined as $\widetilde{S}_{Neu} = \{\langle\mu; [\alpha_{\widetilde{S_{Neu}}}(\mu), \beta_{\widetilde{S_{Neu}}}(\mu), \gamma_{\widetilde{S_{Neu}}}(\mu)]\rangle \vdots \mu \in S\}$, where $\alpha_{\widetilde{S_{Neu}}}(\mu) : S \to -]0,1[+$ stand for truthiness, $\beta_{\widetilde{S_{Neu}}}(\mu) : S \to\to -]0,1[+$ stands for hesitation and $\gamma_{\widetilde{S_{Neu}}}(\mu) : S \to\to -]0,1[+$ represents the falseness in the decision making course of act. Where, $[\alpha_{\widetilde{S_{Neu}}}(\mu), \beta_{\widetilde{S_{Neu}}}(\mu), \gamma_{\widetilde{S_{Neu}}}(\mu)]$ satisfies the inequality,

$$-0 \leq \alpha_{\widetilde{S_{Neu}}}(\mu) + \beta_{\widetilde{S_{Neu}}}(\mu) + \gamma_{\tilde{S_{Neu}}}(\mu) \leq 3+.$$

2.2. Triangular dense fuzzy number (TDFN) with asymmetry

A TDFN with an asymmetry $\check{A}\langle g_1(1-\frac{\varphi_1}{1+n}), g_1, g_1(1+\frac{\sigma_1}{1+n})\rangle$, where φ_1, $\sigma_1 \in [0,1]$ is defined as follows:

$$\mu_{\check{A}}(x) = \frac{x - g_1\left(1-\frac{\varphi_1}{1+n}\right)}{\frac{\rho_1 p_1}{1+n}} \quad \text{when} \quad g_1\left(1-\frac{\varphi_1}{1+n}\right) \leq x \leq g_1$$
$$= \frac{g_1\left(1+\frac{\sigma_1}{1+n}\right) - x}{\frac{\sigma_1 g_1}{1+n}} \quad \text{when } g_1 \leq x \leq g_1\left(1+\frac{\sigma_1}{1+n}\right).$$

2.3. Triangular dense neutrosophic number (TDNN) with symmetry

A TDNN with symmetry $\breve{A} = \langle g_1(1-\frac{\varphi_1}{1+n}), g_1, g_1(1+\frac{\varphi_1}{1+n}); h_1(1-\frac{\varphi_2}{1+n}), h_1, h_1(1+\frac{\varphi_2}{1+n}); i_1(1-\frac{\varphi_3}{1+n}), i_1, i_1(1+\frac{\varphi_3}{1+n})\rangle$, where φ_1, $\varphi_2\varphi_3 \in [0,1]$ is defined as follows:

$$
\begin{aligned}
T_{\breve{A}}(x) &= \frac{x - g_1\left(1 - \frac{\varphi_1}{1+n}\right)}{\frac{\varphi_1 g_1}{1+n}} \text{ when } \ g_1\left(1 - \frac{\varphi_1}{1+n}\right) \le x \le g_1 \\
&= \frac{g_1\left(1 + \frac{\varphi_1}{1+n}\right) - x}{\frac{\varphi_1 g_1}{1+n}} \text{ when } g_1 \le x \le g_1\left(1 + \frac{\varphi_1}{1+n}\right) \\
I_{\breve{A}}(x) &= \frac{h_1 - x}{\frac{\varphi_2 h_1}{1+n}} \text{ when } \ h_1\left(1 - \frac{\varphi_2}{1+n}\right) \le x \le h_1 \\
&= \frac{x - h_1}{\frac{\varphi_2 h_1}{1+n}} \text{ when } \ h_1 \le x \le h_1\left(1 + \frac{\varphi_2}{1+n}\right) \\
F_{\breve{A}}(x) &= \frac{i_1 - x}{\frac{\varphi_3 i_1}{1+n}} \text{ when } \ i_1\left(1 - \frac{\varphi_3}{1+n}\right) \le x \le i_1 \\
&= \frac{x - i_1}{\frac{\varphi_3 i_1}{1+n}} \text{ when } \ i_1 \le x \le i_1\left(1 + \frac{\varphi_3}{1+n}\right).
\end{aligned}
$$

2.4. Triangular dense neutrosophic number (TDNN) with asymmetry

A TDNN with an asymmetry $\breve{A} = \langle g_1(1-\frac{\varphi_1}{1+n}), g_1, g_1(1+\frac{\sigma_1}{1+n}); h_1(1-\frac{\varphi_2}{1+n}), h_1, h_1(1+\frac{\sigma_2}{1+n}); i_1(1-\frac{\varphi_3}{1+n}), i_1, i_1(1+\frac{\sigma_3}{1+n})\rangle$, where φ_1, $\sigma_1, \varphi_2, \sigma_2, \varphi_3, \sigma_3 \in [0,1]$ is defined as follows (Fig. 1):

$$
\begin{aligned}
T_{\breve{A}}(x) &= \frac{x - g_1\left(1 - \frac{\varphi_1}{1+n}\right)}{\frac{\varphi_1 g_1}{1+n}} \text{ when } g_1\left(1 - \frac{\varphi_1}{1+n}\right) \le x \le g_1 \\
&= \frac{g_1\left(1 + \frac{\sigma_1}{1+n}\right) - x}{\frac{\sigma_1 g_1}{1+n}} \text{ when } g_1 \le x \le g_1\left(1 + \frac{\sigma_1}{1+n}\right) \\
I_{\breve{A}}(x) &= \frac{h_1 - x}{\frac{\varphi_2 h_1}{1+n}} \text{ when } h_1\left(1 - \frac{\varphi_2}{1+n}\right) \le x \le h_1 \\
&= \frac{x - h_1}{\frac{\sigma_2 h_1}{1+n}} \text{ when } h_1 \le x \le h_1\left(1 + \frac{\sigma_2}{1+n}\right) \\
F_{\breve{A}}(x) &= \frac{i_1 - x}{\frac{\varphi_3 i_1}{1+n}} \text{ when } i_1\left(1 - \frac{\varphi_3}{1+n}\right) \le x \le i_1 \\
&= \frac{x - i_1}{\frac{\sigma_3 i_1}{1+n}} \text{ when } i_1 \le x \le i_1\left(1 + \frac{\sigma_3}{1+n}\right).
\end{aligned}
$$

2.5. Non-linear triangular dense neutrosophic number (NLTDNN) with symmetry

A NLTDNN with symmetry $\breve{A} = [\langle g_1(1-\frac{\varphi_1}{1+n}), g_1, g_1(1+\frac{\varphi_1}{1+n}); h_1(1-\frac{\varphi_2}{1+n}), h_1, h_1(1+\frac{\varphi_2}{1+n}); i_1(1-\frac{\varphi_3}{1+n}), i_1, i_1(1+\frac{\varphi_3}{1+n})\rangle]_{\langle\langle m_1, m_2; n_1, n_2; s_1, s_2\rangle\rangle}$, where $\varphi_1, \varphi_2, \varphi_3 \in [0,1]$ is defined as follows (Fig. 2):

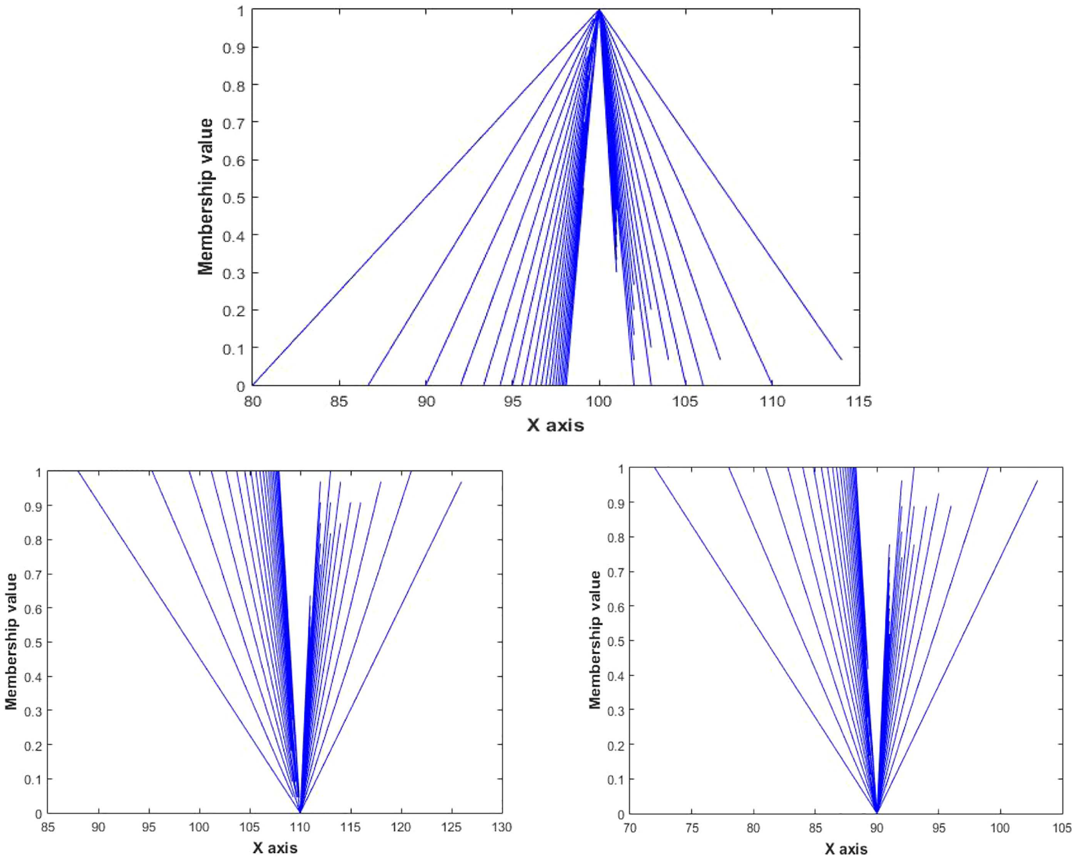


FIGURE 1. Graphical representation of triangular dense neutrosophic number (TDNN).

$$T_{\breve{A}}(x) = \left\{ \frac{x - g_1\left(1 - \frac{\varphi_1}{1+n}\right)}{\frac{\varphi_1 g_1}{1+n}} \right\}^{m_1} \text{ when } g_1\left(1 - \frac{\varphi_1}{1+n}\right) \le x \le g_1$$

$$= \left\{ \frac{g_1\left(1 + \frac{\varphi_1}{1+n}\right) - x}{\frac{\varphi_1 g_1}{1+n}} \right\}^{m_2} \text{ when } g_1 \le x \le g_1\left(1 + \frac{\varphi_1}{1+n}\right)$$

$$I_{\breve{A}}(x) = \left\{ \frac{h_1 - x}{\frac{\varphi_2 h_1}{1+n}} \right\}^{n_1} \text{ when } h_1\left(1 - \frac{\varphi_2}{1+n}\right) \le x \le h_1$$

$$= \left\{ \frac{x - h_1}{\frac{\varphi_2 h_1}{1+n}} \right\}^{n_2} \text{ when } h_1 \le x \le h_1\left(1 + \frac{\varphi_2}{1+n}\right)$$

$$F_{\breve{A}}(x) = \left\{ \frac{i_1 - x}{\frac{\varphi_3 i_1}{1+n}} \right\}^{s_1} \text{ when } i_1\left(1 - \frac{\varphi_3}{1+n}\right) \le x \le i_1$$

$$= \left\{ \frac{x - i_1}{\frac{\varphi_3 i_1}{1+n}} \right\}^{s_2} \text{ when } i_1 \le x \le i_1\left(1 + \frac{\varphi_3}{1+n}\right).$$

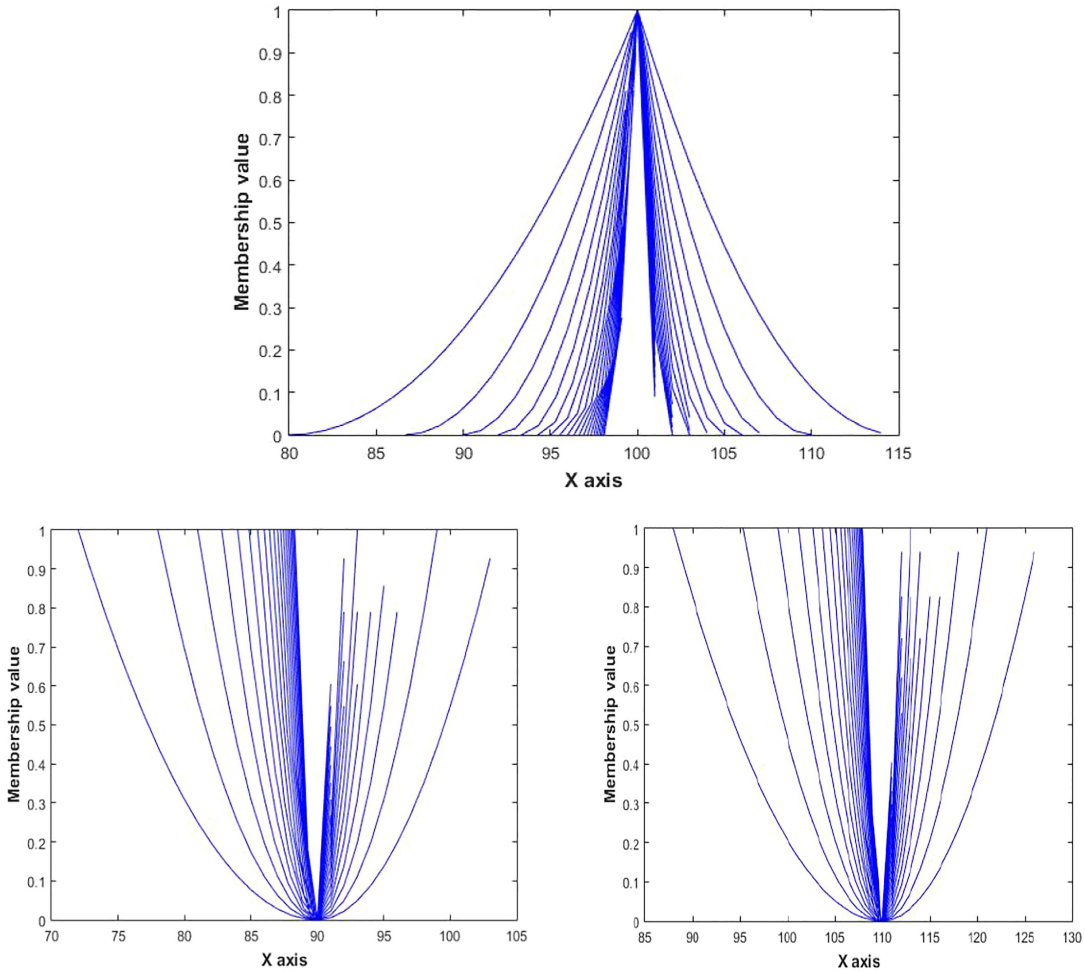


FIGURE 2. This depicts the geometrical representation of a non-linear triangular dense neutrosophic number (NLTDNN).

2.6. Non-linear triangular dense neutrosophic number (NLTDNN) with asymmetry

A NLTDNN with an asymmetry $\breve{A} = [\langle g_1(1-\frac{\varphi_1}{1+n}), g_1, g_1(1+\frac{\sigma_1}{1+n}); h_1(1-\frac{\varphi_2}{1+n}), h_1, h_1(1+\frac{\sigma_2}{1+n}); i_1(1-\frac{\varphi_3}{1+n}), i_1, i_1(1+\frac{\sigma_3}{1+n})\rangle]_{\langle(m_1,m_2;n_1,n_2;s_1,s_2)\rangle}$, where φ_1, $\sigma_1\varphi_2\sigma_2\varphi_3\sigma_3 \in [0,1]$ is defined as follows:

$$T_{\breve{A}}(x) = \left\{\frac{x - g_1\left(1-\frac{\varphi_1}{1+n}\right)}{\frac{\varphi_1 g_1}{1+n}}\right\}^{m_1} \quad \text{when } g_1\left(1-\frac{\varphi_1}{1+n}\right) \le x \le g_1$$
$$= \left\{\frac{g_1\left(1+\frac{\sigma_1}{1+n}\right) - x}{\frac{\sigma_1 g_1}{1+n}}\right\}^{m_2} \quad \text{when } g_1 \le x \le g_1\left(1+\frac{\sigma_1}{1+n}\right)$$
$$I_{\breve{A}}(x) = \left\{\frac{h_1 - x}{\frac{\varphi_2 h_1}{1+n}}\right\}^{n_1} \quad \text{when } h_1\left(1-\frac{\varphi_2}{1+n}\right) \le x \le h_1$$

$$= \left\{\frac{x-h_1}{\frac{\sigma_2 h_1}{1+n}}\right\}^{n_2} \text{ when } h_1 \le x \le h_1\left(1+\frac{\sigma_2}{1+n}\right)$$

$$F_{\breve{A}}(x) = \left\{\frac{i_1-x}{\frac{\varphi_3 i_1}{1+n}}\right\}^{s_1} \text{ when } i_1\left(1-\frac{\varphi_3}{1+n}\right) \le x \le i_1$$

$$= \left\{\frac{x-i_1}{\frac{\varphi_3 i_1}{1+n}}\right\}^{s_2} \text{ when } i_1 \le x \le i_1\left(1+\frac{\sigma_3}{1+n}\right).$$

2.7. Defuzzification/de-neutrosophication skill

The idea of defuzzification is that we can convert any fuzzy numbers into crisp one. We cannot compare any two fuzzy numbers in general, but if we can convert it into crisp one then we can compare it easily. Now, using the idea of defuzzification we can also mapped any fuzzy number into real one. This is a very useful and crucial technique in the field of research. Several defuzzification techniques are also established in the field of fuzzy research area. Neutrosophic number is an extension of a fuzzy number and we need to map it into real one to compare it or to declare some final results of it in several real-life scenarios. Few researchers already worked on it and developed some suitable de-neutrosophication skills here. Here we need know how we can convert any non-linear dense neutrosophic number into crisp one. Few deneutrosophication are established on linear dense field but no such skills are applied on the non-linear dense zone till now.

2.8. De-neutrosophication centroid method for non-linear triangular dense neutrosophic number

Here, we consider a non-linear triangular dense neutrosophic number with asymmetry $\breve{A} = [\langle p_1(1-\frac{\rho_1}{1+n}), p_1, p_1(1+\frac{\sigma_1}{1+n}); q_1(1-\frac{\rho_2}{1+n}), q_1, q_1(1+\frac{\sigma_2}{1+n}); r_1(1-\frac{\rho_3}{1+n}), r_1, r_1(1+\frac{\sigma_3}{1+n})\rangle]_{\langle(m_1,m_2;n_1,n_2;s_1,s_2)\rangle}$. Then, according to the centroid method the De-neutrosophication value becomes $D_{Neu} = \frac{T_{\breve{A}}(x)+I_{\breve{A}}(x)+F_{\breve{A}}(x)}{3}$, where $T_{\breve{A}}(x), I_{\breve{A}}(x), F_{\breve{A}}(x)$ is computed as follows:

$$T_{\breve{A}}(x) = \frac{\sum_{n=0}^{N}\left\{\int_a^b x\mu(x)\,\mathrm{d}x + \int_b^c x\mu(x)\,\mathrm{d}x\right\}}{\sum_{n=0}^{N}\left\{\int_a^b \mu(x)\,\mathrm{d}x + \int_b^c \mu(x)\,\mathrm{d}x\right\}}$$

$$= \frac{\sum_{n=0}^{N}\left\{\int_{p_1}^{p_1}\left(1-\frac{\rho_1}{1+n}\right) x\left\{\frac{x-p_1\left(1-\frac{\rho_1}{1+n}\right)}{\frac{\rho_1 p_1}{1+n}}\right\}^{m_1}\mathrm{d}x + \int_{p_1}^{p_1\left(1+\frac{\sigma_1}{1+n}\right)} x\left\{\frac{p_1\left(1+\frac{\sigma_1}{1+n}\right)-x}{\frac{\sigma_1 p_1}{1+n}}\right\}^{m_2}\mathrm{d}x\right\}}{\sum_{n=0}^{N}\left\{\int_{p_1}^{p_1}\left(1-\frac{\rho_1}{1+n}\right)\left\{\frac{x-p_1\left(1-\frac{\rho_1}{1+n}\right)}{\frac{\rho_1 p_1}{1+n}}\right\}^{m_1}\mathrm{d}x + \int_{p_1}^{p_1\left(1+\frac{\sigma_1}{1+n}\right)}\left\{\frac{p_1\left(1+\frac{\sigma_1}{1+n}\right)-x}{\frac{\sigma_1 p_1}{1+n}}\right\}^{m_2}\mathrm{d}x\right\}}$$

$$= \frac{\sum_{n=0}^{N}\left\{\frac{p_1\left(1-\frac{\rho_1}{1+n}\right)\left(\frac{p_1\cdot\rho_1}{1+n}\right)}{m_1+1} + \frac{\left(\frac{p_1\cdot\rho_1}{1+n}\right)^2}{m_2+2} + \frac{p_1\left(1+\frac{\sigma_1}{1+n}\right)\left(\frac{p_1\cdot\sigma_1}{1+n}\right)}{m_2+1} - \frac{\left(\frac{p_1\cdot\sigma_1}{1+n}\right)^2}{m_2+2}\right\}}{\sum_{n=0}^{N}\left\{\frac{\left(\frac{p_1\cdot\rho_1}{1+n}\right)^2}{m_1+2} + \frac{\left(\frac{p_1\cdot\sigma_1}{1+n}\right)^2}{m_2+2}\right\}}$$

$$I_{\breve{A}}(x) = \frac{\sum_{n=0}^{N}\left\{\int_d^e x\mu(x)\,\mathrm{d}x + \int_e^f x\mu(x)\,\mathrm{d}x\right\}}{\sum_{n=0}^{N}\left\{\int_d^e \mu(x)\,\mathrm{d}x + \int_e^f \mu(x)\,\mathrm{d}x\right\}}$$

$$= \frac{\sum_{n=0}^{N}\left\{\int_{q_1\left(1-\frac{\rho_2}{1+n}\right)}^{q_1} x\left\{\frac{q_1-x}{\frac{\rho_2 q_1}{1+n}}\right\}^{n_1}\mathrm{d}x + \int_{q_1}^{q_1\left(1+\frac{\sigma_2}{1+n}\right)} x\left\{\frac{x-q_1}{\frac{\sigma_2 q_1}{1+n}}\right\}^{n_2}\mathrm{d}x\right\}}{\sum_{n=0}^{N}\left\{\int_{q_1\left(1-\frac{\rho_2}{1+n}\right)}^{q_1}\left\{\frac{q_1-x}{\frac{\rho_2 q_1}{1+n}}\right\}^{n_1}\mathrm{d}x + \int_{q_1}^{q_1\left(1+\frac{\sigma_2}{1+n}\right)}\left\{\frac{x-q_1}{\frac{\sigma_2 q_1}{1+n}}\right\}^{n_2}\mathrm{d}x\right\}}$$

$$= \frac{\sum_{n=0}^{N}\left\{\frac{q_1\left(\frac{q_1\cdot\rho_2}{1+n}\right)}{n_1+1}-\frac{\left(\frac{q_1\cdot\rho_2}{1+n}\right)^2}{n_1+2}+\frac{q_1\left(\frac{q_1\cdot\sigma_2}{1+n}\right)}{n_2+1}+\frac{\left(\frac{q_1\cdot\sigma_2}{1+n}\right)^2}{n_2+2}\right\}}{\sum_{n=0}^{N}\left\{\frac{\left(\frac{q_1\cdot\rho_2}{1+n}\right)^2}{n_1+2}+\frac{\left(\frac{q_1\cdot\sigma_2}{1+n}\right)^2}{n_2+2}\right\}}$$

$$F_{\breve{A}}(x) = \frac{\sum_{n=0}^{N}\left\{\int_g^h x\mu(x)\,\mathrm{d}x+\int_h^k x\mu(x)\,\mathrm{d}x\right\}}{\sum_{n=0}^{N}\left\{\int_g^h \mu(x)\,\mathrm{d}x+\int_h^k \mu(x)\,\mathrm{d}x\right\}}$$

$$= \frac{\sum_{n=0}^{N}\left\{\int_{r_1\left(1-\frac{\rho_3}{1+n}\right)}^{r_1} x\left\{\frac{r_1-x}{\frac{\rho_3 r_1}{1+n}}\right\}^{s_1}\mathrm{d}x+\int_{r_1}^{r_1\left(1+\frac{\sigma_3}{1+n}\right)} x\left\{\frac{x-r_1}{\frac{\sigma_3 r_1}{1+n}}\right\}^{s_2}\mathrm{d}x\right\}}{\sum_{n=0}^{N}\left\{\int_{r_1\left(1-\frac{\rho_3}{1+n}\right)}^{r_1}\left\{\frac{r_1-x}{\frac{\rho_3 r_1}{1+n}}\right\}^{s_1}\mathrm{d}x+\int_{r_1}^{r_1\left(1+\frac{\sigma_3}{1+n}\right)}\left\{\frac{x-r_1}{\frac{\sigma_3 r_1}{1+n}}\right\}^{s_2}\mathrm{d}x\right\}}$$

$$= \frac{\sum_{n=0}^{N}\left\{\frac{r_1\left(\frac{r_1\cdot\rho_3}{1+n}\right)}{s_1+1}-\frac{\left(\frac{r_1\cdot\rho_3}{1+n}\right)^2}{s_1+2}+\frac{r_1\left(\frac{r_1\cdot\sigma_3}{1+n}\right)}{s_2+1}+\frac{\left(\frac{r_1\cdot\sigma_3}{1+n}\right)^2}{s_2+2}\right\}}{\sum_{n=0}^{N}\left\{\frac{\left(\frac{r_1\cdot\rho_3}{1+n}\right)^2}{s_1+2}+\frac{\left(\frac{r_1\cdot\sigma_3}{1+n}\right)^2}{s_2+2}\right\}}.$$

Thus, the De-neutrosophic value becomes,

$$D_{Neu} = \frac{T_{\breve{A}}(x)+I_{\breve{A}}(x)+F_{\breve{A}}(x)}{3}$$

$$= \frac{\frac{\sum_{n=0}^{N}\left\{\frac{p_1\left(1-\frac{\rho_1}{1+n}\right)\left(\frac{p_1\cdot\rho_1}{1+n}\right)}{m_1+1}+\frac{\left(\frac{p_1\cdot\rho_1}{1+n}\right)^2}{m_2+2}+\frac{p_1\left(1+\frac{\sigma_1}{1+n}\right)\left(\frac{p_1\cdot\sigma_1}{1+n}\right)}{m_2+1}-\frac{\left(\frac{p_1\cdot\sigma_1}{1+n}\right)^2}{m_2+2}\right\}}{\sum_{n=0}^{N}\left\{\frac{\left(\frac{p_1\cdot\rho_1}{1+n}\right)^2}{m_1+2}+\frac{\left(\frac{p_1\cdot\sigma_1}{1+n}\right)^2}{m_2+2}\right\}}+\frac{\sum_{n=0}^{N}\left\{\frac{q_1\left(\frac{q_1\cdot\rho_2}{1+n}\right)}{n_1+1}-\frac{\left(\frac{q_1\cdot\rho_2}{1+n}\right)^2}{n_1+2}+\frac{q_1\left(\frac{q_1\cdot\sigma_2}{1+n}\right)}{n_2+1}+\frac{\left(\frac{q_1\cdot\sigma_2}{1+n}\right)^2}{n_2+2}\right\}}{\sum_{n=0}^{N}\left\{\frac{\left(\frac{q_1\cdot\rho_2}{1+n}\right)^2}{n_1+2}+\frac{\left(\frac{q_1\cdot\sigma_2}{1+n}\right)^2}{n_2+2}\right\}}+\frac{\sum_{n=0}^{N}\left\{\frac{r_1\left(\frac{r_1\cdot\rho_3}{1+n}\right)}{s_1+1}-\frac{\left(\frac{r_1\cdot\rho_3}{1+n}\right)^2}{s_1+2}+\frac{r_1\left(\frac{r_1\cdot\sigma_3}{1+n}\right)}{s_2+1}+\frac{\left(\frac{r_1\cdot\sigma_3}{1+n}\right)^2}{s_2+2}\right\}}{\sum_{n=0}^{N}\left\{\frac{\left(\frac{r_1\cdot\rho_3}{1+n}\right)^2}{s_1+2}+\frac{\left(\frac{r_1\cdot\sigma_3}{1+n}\right)^2}{s_2+2}\right\}}}{3}. \tag{1}$$

3. Explanation of the formulation of the model and case study

3.1. Assumption

(1) The demand for packed food and beverages depends on advertisement and selling price. Generally, it is perceived that the sale of a product increases if the commercialism of the items is high and if the selling price of the product is inversely proportional to the item's demand. So, demand $D(\tilde{A},p)=\tilde{A}^{bt}(\alpha_1-\alpha_2 p)$, where $b,\alpha_1,\alpha_2>0$, b is the frequency of advertisement.
(2) The part of the shortages is backordered, and the sales details are lost.
(3) The non-instantaneous deteriorating rate $\theta(t)$ of the items is

$$\theta(t)=\begin{cases}0 & 0\le t\le t_0\\ \gamma t & t_0\le t\le t_1\end{cases}$$

where $0<\gamma\ll 1$ and t_0 is the maximum lifetime of the items.
(4) The effect of inflation is imprecise, so it is considered a triangular dense neutrosophic number $(\tilde{k})$.

(5) As the items have carbon emission, which occurs from the beginning of the inventory, and since the amount is uncertain, it is considered in neutrosophic environment ($\tilde{c}$).
(6) The model is considered under a finite time horizon (H) where $H = T/m$, where m is the number of replenishment cycles.
(7) Each cycle length is T_i, where $i = 1, 2, 3, \dots m$ such that $T_m = mT = H$.
(8) As inflation, carbon emission, and advertisement effects are uncertain, it is taken as a dense neutrosophic fuzzy number.

3.2. Notations

a_1	Base parameter of the selling price
a_2	Scale parameter for selling price
b	Frequency of advertisement
p	Selling price
$\tilde{A}$	Nonlinear neutrosophic Advertisement factor (uncertain)
$\tilde{k}$	Inflation rate (uncertain)
c_1	Purchase cost per unit item per unit time
c_2	Deterioration cost per unit item per unit time
c_3	Shortage cost per unit item per unit time
c_4	Lost in sale cost per unit item per unit time
$\tilde{c}$	Rate of carbon footprint per unit inventory per unit time (uncertain)
h	Holding cost per unit item per unit time
$I(t)$	Inventory level at time "t"
I_0	Maximum Inventory ordered at each cycle
I_s	Maximum shortages in each cycle
t_1	The time when the inventory finishes (decision variable)
T	Replenishment time
t_0	The maximum lifetime of the items

3.3. Case study

In an ABC retail store in Kolkata, West Bengal, the manager of the shop orders items from the distributor, considering that the customer demand for the items is as per the advertisement and price of the item. As a result, the demand for such items may be hypothetical in such a situation. It is noted that the deterioration of the items is not instantaneous, *i.e.*, after a certain time period, the items deteriorate, and the usage of the product generates carbon emission. This realistic scenario arises in the retail store of grocery items and packed food items (like juice and beverages, chips, chocolates, etc.) where the deterioration is non-instantaneous, and the carbon emission occurs in the inventory. Thus, the actual demand depends not only on the price and advertisement of the items but also on the indirect effect of carbon emission. As per the situation, there is high demand, hence the shortages incurred. Thus, once there is no items in the stock, few customers wait for the items to arrive, reflecting those demands are back-ordered, and some of the customers leave the store, resulting in a loss in sale for the retailer. Hence, the market condition deduces that the total cost might vary on the uncertainty of the parameters such as carbon emission, the effect of advertisement and inflation in the market. After extensive discussion with the store executive, our research squad got the following data set in the Table 2.

As per the situation, we come across some basic questions in our research:

(1) What will be the optimum number of cycles and the time to hold the inventory so that the cost is minimal?
(2) Which methodology is more practical, crips, fuzzy, intuitionistic, or neutrosophic under the uncertainty in carbon emission, Advertisement and inflation?
(3) Under a Neutrosophic environment, which methodology is useful, whether linear symmetric, linear asymmetric, non-linear symmetric or non-linear asymmetric?

TABLE 2. Data set from a retailer for case study.

Set up cost $O_c = 500$	Holding cost per unit item per unit time $(h) = \$2$	Purchasing cost per unit item $(c_1) = \$4$	Deterioration cost per unit item $(c_2) = \$0.5$	Shortage cost per unit item $(c_3) = \$1$
Goodwill cost for lost in sale $(c_4) = \$4$	Amount of carbon emission per unit inventory per unit time $\tilde{c} = \langle(0.01, 0.1, 0.16), (0.01, 0.1, 0.15), (0.02, 0.1, 0.19)\rangle$ The crips value $c = 0.1(10\%)$	Selling price per unit item $(p) = \$30$	Total replenishment time $(T) = 10$ years	Each replenishment cycle $m = 2, 3, 4, 5$ in years
Fraction of the items back ordered $(\alpha = 0.12)$	Deterioration fraction $(\gamma) = 0.01$	Annual market demand function $D = D(\tilde{A}, p) = \tilde{A}^{bt}(\alpha_1 - \alpha_2 p)$, $D(\tilde{A}, p) = \tilde{A}^{0.4t}(22 - 0.5p)$, where advertisement factor $\tilde{A} = \langle(14.6, 15, 15.3), (14.6, 15, 15.2), (14.5, 15, 15.4)\rangle$	Time upto no deterioration $t_0 = 0.6$ years	Inflation rate $(\tilde{k}) = \langle(0, 0.05, 0.1), (0.01, 0.05, 0.15), (0.02, 0.05, 0.2)\rangle$ The crisps value is $k = 0.05$

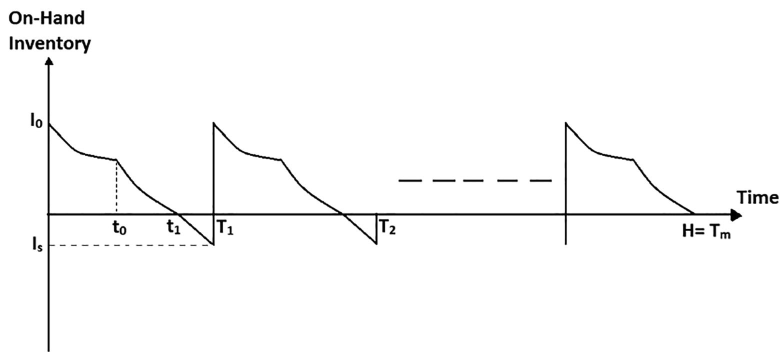


FIGURE 3. Inventory in hand.

4. Model formulation

Till today, researchers have considered carbon emission or carbon footprinting to be a constant value [54,63], etc. But here, we have considered carbon footprinting a rate per unit item in an uncertain environment. This is considered because the retailer has to maintain various facilities to provide a suitable environment for the preservation of items in the inventory. These facilities have their own carbon footprint which is uncertain. So, we have considered the effect of carbon footprint in formulating the model in an uncertain environment. Also, since the items deteriorate non-instantaneously, there is a carbon emission from the scrap items, directly affecting the inventory's carbon footprint.

4.1. Mathematical formulation of inventory model

From Figure 3, it is evident that the items are ordered at $t = 0$ and up to $t = t_0$, the items don't undergo deterioration. Thus, the amount of inventory initially ordered is I_0 units. In order to maintain various facilities, some uncertain quantity $\tilde{c}$ unit of carbon emission occurs throughout the tenure $[0, T_1]$. Again, from $[t_0, t_1]$, the items undergo deterioration $\theta(t)$ and at $t = t_1$ the inventory level reaches zero. Further, the demand in the market prevails, and thus, the items undergo shortages till $t = T_1$, with maximum I_s amount of shortages. The following differential equations can be used to produce the above descriptions of the proposed inventory system:

$$\begin{aligned}
&\frac{\mathrm{d}I(t)}{\mathrm{d}t} + \tilde{c}I(t) = -D\Big(\tilde{A}, p\Big), && 0 \le t \le t_0 \text{ where } I(0) = I_0, I_0 \text{ is the maximum inventory}\\
&\frac{\mathrm{d}I(t)}{\mathrm{d}t} + (\theta(t) + \tilde{c})I(t) = -D\Big(\tilde{A}, p\Big), && t_0 \le t \le t_1 \text{ where } I(t_1) = 0\\
&\frac{\mathrm{d}I(t)}{\mathrm{d}t} + \tilde{c}I(t) = -D\Big(\tilde{A}, p\Big), && t_1 \le t \le T_1 \text{ where } I(T_1) = I_s,
\end{aligned}$$

where I_s is the maximum shortages.

The given differential equation contains the fuzzy parameter $\tilde{c}$ and $\tilde{A}$. To solve it, we have defuzzified it to c and A, respectively, using equation (1)

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t} + cI(t) = -D(A, p), \quad 0 \le t \le t_0 \tag{2}$$

where $I(0) = I_0$, I_0 is the maximum inventory

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t} + (\theta(t) + c)I(t) = -D(A, p), \qquad t_0 \le t \le t_1 \text{ where } I(t_1) = 0 \tag{3}$$

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t} + cI(t) = -D(A, p), \qquad t_1 \le t \le T_1 \text{ where } I(T_1) = I_s. \tag{4}$$

After solving equations (2)–(4) we get equations (5)–(7)

$$I(t) = \frac{\big(A^{bt} - 1\big)(a_2p - a_1)}{b \log A} + I_0, \qquad 0 \le t \le t_0 \tag{5}$$

$$I(t) = (a_2p - a_1)e^{-\gamma t}\left[\int_{t_0}^{t} A^{bt} e^{\frac{\gamma t^2}{2} + ct}\,\mathrm{d}t - \int_{t_0}^{t_1} A^{bt} e^{\frac{\gamma t^2}{2} + ct}\,\mathrm{d}t\right], \qquad t_0 \le t \le t_1 \tag{6}$$

$$I(t) = \frac{\gamma(a_2p - a_1)}{b \log A}\big[A^{bt} - A^{bT_1}\big] + I_s, \qquad t_1 \le t \le T_1. \tag{7}$$

Now equating equations (5) and (6) at $t = t_1$, the maximum inventory (I_0) required to order is

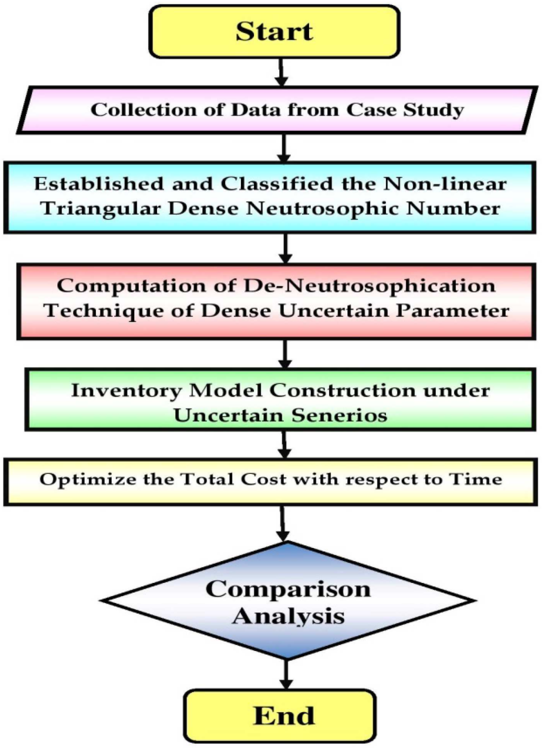
$$\frac{\big(A^{bt} - 1\big)(a_2p - a_1)}{b \log A} + I_0 = (a_2p - a_1)e^{-\gamma t}\left[-\int_{t_0}^{t_1} A^{bt} e^{\frac{\gamma t^2}{2} + ct}\,\mathrm{d}t\right]$$

$$\text{Thus, } I_0 = \frac{\big(A^{bt} - 1\big)(\alpha_1 - \alpha_2 p)}{b \log A} - (a_2p - a_1)e^{-\gamma t}\left[\int_{t_0}^{t_1} A^{bt} e^{\frac{\gamma t^2}{2} + ct}\,\mathrm{d}t\right]. \tag{8}$$

Again, from equation (7) at $t = t_1$, the maximum shortages (I_s) that occurred is

$$I_s = \frac{\gamma(\alpha_1 - \alpha_2 p)}{b \log A}\big[A^{bt_1} - A^{bT_1}\big]. \tag{9}$$

Flowchart of the work



4.2. Present worth of holding cost (HC)

To keep the items in stock during $[0, t_1]$, a cost is incorporated; hence, we calculate the cost of holding during that period. At $t = t_1$ no items are in the stock, so from $[t_1, T_1]$, the inventory undergoes shortages, and there are no items in store. At $t = T_1$, the retailer re-orders the items and the stock is again filled up to overcome the shortages and fulfil the next cycle's future demand, indicating no items are stored during backorder. Hence, the present worth of holding cost HC is given by

$$\begin{aligned}
\mathrm{HC} &= h\left\{\int_0^{t_0} I(t)e^{-kt}\,\mathrm{d}t + \int_{t_0}^{t_1} I(t)e^{-kt}\,\mathrm{d}t\right\} \\
&= h\Bigg(\frac{A^{bt_1}}{(b\mathrm{Log}(A))^2}\left(1+\frac{\gamma}{2(b\mathrm{Log}[A])^2}-\frac{\gamma}{2b\mathrm{Log}[A]}\left(t_1-\frac{1}{b\mathrm{Log}[A]}\right)\right) \\
&\quad + \frac{3\gamma A^{bt_1}}{(b\log[A])^2}\left(t_1^2-2\left(t_1-\frac{1}{b\mathrm{Log}[A]}\right)\right)+\gamma(c-k)\left(\frac{t_1^4}{8}-\frac{t_1}{2b\mathrm{Log}[A]}\right) \\
&\quad + \frac{A^{bt_1}}{b\mathrm{Log}(A)}\left(t_1-\frac{1}{b\mathrm{Log}[A]}\right)\left(\frac{1+\frac{\gamma}{2(b\mathrm{Log}[A])^2}-\frac{c-k}{b\mathrm{Log}[A]}}{b\mathrm{Log}[A]}\right) \\
&\quad + \frac{\left(c-k-\frac{\gamma}{2}\right)A^{bt_1}}{(b\mathrm{Log}(A))^2}\left(t_1^2-2\left(t_1-\frac{1}{b\mathrm{Log}[A]}\right)\right) \\
&\quad + \frac{\gamma A^{bt_1}}{4b\mathrm{Log}(A)}\left(t_1^3-3t_1^2+6\left(t_1-\frac{1}{b\mathrm{Log}[A]}\right)\right)
\end{aligned}$$

$$
\begin{aligned}
&-\left(\frac{A^{bt_0}}{(b\mathrm{Log}(A))^2}\left(1+\frac{\gamma}{2(b\mathrm{Log}[A])^2}-\frac{\gamma}{2b\mathrm{Log}[A]}\left(t_0-\frac{1}{b\mathrm{Log}[A]}\right)\right)\right.\\
&+\frac{3\gamma A^{bt_0}}{(b\log[A])^2}\left(t_0^2-2\left(t_0-\frac{1}{b\mathrm{Log}[A]}\right)\right)+\gamma(c-k)\left(\frac{t_0^4}{8}-\frac{t_0}{2b\mathrm{Log}[A]}\right)\\
&+\frac{A^{bt_0}}{b\mathrm{Log}(A)}\left(t_0-\frac{1}{b\mathrm{Log}[A]}\right)\left(\frac{1+\frac{\gamma}{2(b\mathrm{Log}[A])^2}-\frac{c-k}{b\mathrm{Log}[A]}}{b\mathrm{Log}[A]}\right)\\
&+\frac{\left(c-k-\frac{\gamma}{2}\right)A^{bt_0}}{(b\mathrm{Log}(A))^2}\left(t_0^2-2\left(t_0-\frac{1}{b\mathrm{Log}[A]}\right)\right)\\
&\left.+\frac{\gamma A^{bt_0}}{4b\mathrm{Log}(A)}\left(t_0^3-3t_0^2+6\left(t_0-\frac{1}{b\mathrm{Log}[A]}\right)\right)\right)\Bigg).
\end{aligned}
$$

4.3. Present worth of purchase cost (PC)

To fulfil the inventory demand, the retailer has to order the items and purchase them from the manufacturer. The retailer not only has to purchase a fixed amount (I_0) but also has to purchase the items which is required to fulfil the backorder. Thus, the present worth of the purchase cost is

$$
\begin{aligned}
\mathrm{PC}&=c_1I_0+c_1\int_0^{T_1-t_1}e^{-kt}A^{bt}(a_1-a_2p)\,\mathrm{d}t\\
&=c_1\left(I_0+\frac{(a_1-a_2p)}{b\log(A)}(A^{b(T_1-t_1)}-1)\left(1-k(T_1-t_1)+\frac{k}{(b\log(A))}\right)\right).
\end{aligned}
$$

4.4. Present worth of deterioration cost (DC)

As we have considered non-instantaneous deterioration, the items do not deteriorate as soon as they are ordered. So, there is no deterioration during $[0,t_0]$. Also, as the items undergo shortages, there is no deterioration during $[t_1,T_1]$. So the total number of deteriorations is

$$
d=I_0-\int_{t_0}^{t_1}A^{bt}(a_1-a_2p)\,\mathrm{d}t.
$$

Thus, the present worth of deterioration cost is

$$
\mathrm{DC}=c_2d=c_2\left(I_0-\frac{\left((a_1-a_2p)\left(A^{bt_1}-A^{bt_0}\right)\right)}{b\log(A)}\right).
$$

4.5. Present worth of shortage cost (SC)

As the inventory finishes at $t=t_1$, there is no stock during $[t_1,T_1]$, which leads to adverse effects in the inventory. Thus, the present worth of cost imposed due to shortages is

$$
\begin{aligned}
\mathrm{SC}&=c_3\int_{t_1}^{T_1}-I(t)e^{-kt}\,\mathrm{d}t\\
&=\alpha c_3\left(\frac{a_1-a_2p}{b\log(A)}\right)\left[\frac{A^{bT_1}-A^{bt_1}}{b\log(A)}\left(1-k(T_1-t_1)+\frac{k}{(b\log(A))}\right)\right.\\
&\quad\left.-\left(A^{bT_1}-I_s\right)(T_1-t_1)\left(1-\frac{k}{2}(T_1+t_1)\right)\right].
\end{aligned}
$$

4.6. Present worth of lost cost (LC)

Due to shortages, a part of the inventory is back-ordered, and the rest of the parts get lost in the sale. The fraction which is back-ordered is fulfilled after reordering. But the amount which is lost in sales incurred a cost to the inventory. Therefore, the present worth of lost in sale cost is

$$\begin{aligned}\mathrm{LC} &= c_4(1-\alpha)(a_1-a_2p)\big(A^{bT_1}-A^{bt_1}\big)e^{-kt}\\ &= c_4(1-\alpha)(a_1-a_2p)(A^{bT_1}-A^{bt_1})(1-kT_1).\end{aligned}$$

Neglecting the higher power of k as $k \ll 1$.

4.7. Present worth of setup cost (STC)

In order to set up the store, the cost incorporated is given by $\mathrm{STC} = O_C$.

4.8. Total cost

Thus, the total cost of the given inventory is

$$\begin{aligned}
\mathrm{TC}(t_1,m) &= \mathrm{STC} + (\mathrm{HC}+\mathrm{PC}+\mathrm{DC}+\mathrm{SC}+\mathrm{LC})\sum_{i=0}^{m} e^{-(i-1)kt}\\
&= \mathrm{STC} + (\mathrm{HC}+\mathrm{PC}+\mathrm{DC}+\mathrm{SC}+\mathrm{LC})\left(\frac{1-e^{-kmt}}{1-e^{-kt}}\right)\\
&= O_c + \Bigg(h\Bigg(\frac{A^{bt_1}}{(b\mathrm{Log}(A))^2}\left(1+\frac{\gamma}{2(b\mathrm{Log}[A])^2}-\frac{\gamma}{2b\mathrm{Log}[A]}\left(t_1-\frac{1}{b\mathrm{Log}[A]}\right)\right)\\
&\quad + \frac{3\gamma A^{bt_1}}{(b\log[A])^2}\left(t_1^2-2\left(t_1-\frac{1}{b\mathrm{Log}[A]}\right)\right) + \gamma(c-k)\left(\frac{t_1^4}{8}-\frac{t_1}{2b\log(A)}\right)\\
&\quad + \frac{A^{bt_1}}{b\log(A)}\left(t_1-\frac{1}{b\log(A)}\right)\left(\frac{1+\frac{\gamma}{2(b\log(A))^2}-\frac{c-k}{b\log(A)}}{b\log[A]}\right) + \frac{(c-k-\frac{\gamma}{2})A^{bt_1}}{(b\log(A))^2}\left(t_1^2-2\left(t_1-\frac{1}{b\log(A)}\right)\right)\\
&\quad + \frac{\gamma A^{bt_1}}{4b\log(A)}\left(t_1^3-3t_1^2+6\left(t_1-\frac{1}{b\log(A)}\right)\right) - \left(\frac{A^{bt_0}}{(b\log(A))^2}\left(1+\frac{\gamma}{2(b\log(A))^2}-\frac{\gamma}{2b\log(A)}\left(t_0-\frac{1}{b\log(A)}\right)\right)\right.\\
&\quad + \frac{3\gamma A^{bt_0}}{(b\log(A))^2}\left(t_0^2-2\left(t_0-\frac{1}{b\log(A)}\right)\right) + \gamma(c-k)\left(\frac{t_0^4}{8}-\frac{t_0}{2b\log(A)}\right) + \frac{A^{bt_0}}{b\log(A)}\left(t_0-\frac{1}{b\log(A)}\right)\\
&\quad \times\left(\frac{1+\frac{\gamma}{2(b\log(A))^2}-\frac{c-k}{b\log(A)}}{b\log(A)}\right) + \frac{(c-k-\frac{\gamma}{2})A^{bt_0}}{(b\log(A))^2}\left(t_0^2-2\left(t_0-\frac{1}{b\log(A)}\right)\right)\\
&\quad + \left.\frac{\gamma A^{bt_0}}{4b\log(A)}\left(t_0^3-3t_0^2+6\left(t_0-\frac{1}{b\log(A)}\right)\right)\right)\Bigg) + c_1\Bigg(I_0+\frac{(a_1-a_2p)}{b\log(A)}\left(A^{b(T_1-t_1)}-1\right)(1-k(T_1-t_1)\\
&\quad + \frac{k}{(b\log(A))}\Bigg)\Bigg) + c_2\left(I_0-\frac{((a_1-a_2p)(A^{bt_1}-A^{bt_0}))}{b\log(A)}\right) + \alpha c_3\left(\frac{a_1-a_2p}{b\log(A)}\right)\\
&\quad \times\left[\frac{A^{bT_1}-A^{bt_1}}{b\log(A)}\left(1-k(T_1-t_1)+\frac{k}{(b\log(A))}\right) - \left(A^{bT_1}-I_s\right)(T_1-t_1)\left(1-\frac{k}{2}(T_1+t_1)\right)\right]\\
&\quad + c_4(1-\alpha)(a_1-a_2p)\left(A^{bT_1}-A^{bt_1}\right)(1-kT_1)\Big)\left(\frac{1-e^{-kmt}}{1-e^{-kt}}\right).
\end{aligned} \tag{10}$$

Thus, the total cost TC is the function of t_1 and m. Now to minimise the total cost TC (t_1, m) by using the necessary condition $\frac{\mathrm{dTC}(t_1,m^*)}{\mathrm{d}t_1} = 0$ for a positive integer m. To obtain the optimum value m^*, first optimize the total cost with respect to t_1 for different values of m. Now, the optimum value of $m = m^*$ is the value where the total cost is minimum. To guarantee that the objective function is convex, a sufficient condition $\frac{\mathrm{d}^2\mathrm{TC}}{\mathrm{d}t_1^2} \geq 0$ should satisfy.

5. Effect of the dense neutrosophic fuzzy number on the proposed model

A dense neutrosophic fuzzy number is a new mathematical concept to deal with imprecise and uncertain concepts of any inventory model. In our model, we have considered advertisement ($\tilde{A}$), inflation ($\tilde{k}$) and carbon emission ($\tilde{c}$) as dense neutrosophic fuzzy numbers. As in realism, the above parameters are indeterminate and can vary with time due to various external factors. However, dealing with triangular dense number is not possible analytically. So, we have used equation (1) to defuzzify our model. Thus, it is difficult to optimise if we consider the advertisement, inflation and carbon emission as the neutrosophic number and consider the total cost (10). Thus, we have used the de-neutrosophication form (1) and obtained the de-neutrosophic value of the above parameters as $A_{Neu}, k_{Neu}, c_{Neu}$.

$$
\begin{aligned}
&= O_c + \Bigg(h\Bigg(\frac{A_{Neu}^{bt_1}}{(b\log(A_{Neu}))^2}\Bigg(1+\frac{\gamma}{2(b\log[A_{Neu}])^2}-\frac{\gamma}{2b\log[A_{Neu}]}\Bigg(t_1-\frac{1}{b\log[A_{Neu}]}\Bigg)\Bigg)\\
&\quad + \frac{3\gamma A_{Neu}^{bt_1}}{(b\log[A_{Neu}])^2}\Bigg(t_1^2-2\Bigg(t_1-\frac{1}{b\log[A_{Neu}]}\Bigg)\Bigg)+\gamma(c_{Neu}-k_{Neu})\Bigg(\frac{t_1^4}{8}-\frac{t_1}{2b\log[A_{Neu}]}\Bigg)\\
&\quad + \frac{A_{Neu}^{bt_1}}{b\log(A_{Neu})}\Bigg(t_1-\frac{1}{b\log[A_{Neu}]}\Bigg)\Bigg(\frac{1+\frac{\gamma}{2(b\log A_{Neu})^2}-\frac{c_{Neu}-k_{Neu}}{b\log[A_{Neu}]}}{b\mathrm{Log}[A_{Neu}]}\Bigg)\\
&\quad + \frac{(c_{Neu}-k_{Neu}-\frac{\gamma}{2})A_{Neu}^{bt_1}}{(b\log(A_{Neu}))^2}\Bigg(t_1^2-2\Bigg(t_1-\frac{1}{b\log[A_{Neu}]}\Bigg)\Bigg)\\
&\quad + \frac{\gamma A_{Neu}^{bt_1}}{4b\log(A_{Neu})}\Bigg(t_1^3-3t_1^2+6\Bigg(t_1-\frac{1}{b\log[A_{Neu}]}\Bigg)\Bigg)\\
&\quad - \Bigg(\frac{A_{Neu}^{bt_0}}{(b\log(A_{Neu}))^2}\Bigg(1+\frac{\gamma}{2(b\log[A_{Neu}])^2}-\frac{\gamma}{2b\log[A_{Neu}]}\Bigg(t_0-\frac{1}{b\log[A_{Neu}]}\Bigg)\Bigg)\\
&\quad + \frac{3\gamma A^{bt_0}}{(b\log[A_{Neu}])^2}\Bigg(t_0^2-2\Bigg(t_0-\frac{1}{b\mathrm{Log}[A_{Neu}]}\Bigg)\Bigg)+\gamma(c_{Neu}-k_{Neu})\Bigg(\frac{t_0^4}{8}-\frac{t_0}{2b\log[A_{Neu}]}\Bigg)\\
&\quad + \frac{A_{Neu}^{bt_0}}{b\log(A_{Neu})}\Bigg(t_0-\frac{1}{b\log[A_{Neu}]}\Bigg)\Bigg(\frac{1+\frac{\gamma}{2(b\log[A_{Neu}])^2}-\frac{c_{Neu}-k_{Neu}}{b\log[A_{Neu}]}}{b\log[A_{Neu}]}\Bigg)+\frac{(c_{Neu}-k_{Neu}-\frac{\gamma}{2})A_{Neu}^{bt_0}}{(b\log(A_{Neu}))^2}\\
&\quad \times\Bigg(t_0^2-2\Bigg(t_0-\frac{1}{b\log[A_{Neu}]}\Bigg)\Bigg)+\frac{\gamma A_{Neu}^{bt_0}}{4b\log(A_{Neu})}\Bigg(t_0^3-3t_0^2+6\Bigg(t_0-\frac{1}{b\log[A_{Neu}]}\Bigg)\Bigg)\Bigg)\Bigg)\\
&\quad + c_1\Bigg(I_0+\frac{(a_1-a_2p)}{b\log(A_{Neu})}\Big(A_{Neu}^{b(T_1-t_1)}-1\Big)\Bigg(1-k_{Neu}(T_1-t_1)+\frac{k_{Neu}}{(b\log(A_{Neu}))}\Bigg)\Bigg)\\
&\quad + c_2\Bigg(I_0-\frac{\Big((a_1-a_2p)\Big(A_{Neu}^{bt_1}-A_{Neu}^{bt_0}\Big)\Big)}{b\log(A_{Neu})}\Bigg)+\alpha c_3\Bigg(\frac{a_1-a_2p}{b\log(A_{Neu})}\Bigg)\Bigg[\frac{A_{Neu}^{bT_1}-A_{Neu}^{bt_1}}{b\log(A_{Neu})}\\
&\quad \times\Bigg(1-k_{Neu}(T_1-t_1)+\frac{k_{Neu}}{(b\log(A_{Neu}))}\Bigg)-\Big(A_{Neu}^{bT_1}-I_s\Big)(T_1-t_1)\Bigg(1-\frac{k_{Neu}}{2}(T_1+t_1)\Bigg)\Bigg]\\
&\quad + c_4(1-\alpha)(a_1-a_2p)\Big(A_{Neu}^{bT_1}-A_{Neu}^{bt_1}\Big)(1-k_{Neu}T_1)\Bigg)\Bigg(\frac{1-e^{-k_{Neu}mt}}{1-e^{-k_{Neu}t}}\Bigg). \qquad (11)
\end{aligned}
$$

6. Analysis of numerical outcome

The above data set we received from the retail store is used to calculate the TC *vs.* t_1 for different values of m.

TABLE 3. TC *vs.* t_1 for different values of m.

	TC	t_1
$m = 2$	8896.44	2.693
$m = 3$	2933.6	1.859
$m = 4$	1901.54	1.447
$m = 5$	1528.56	1.205

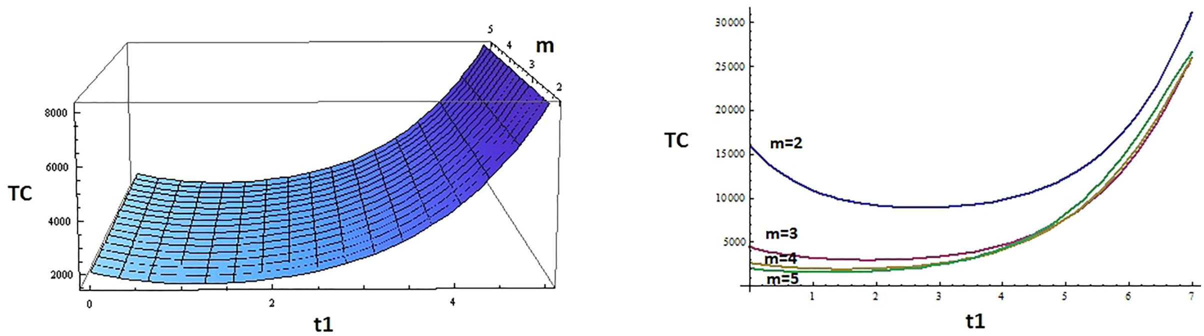


FIGURE 4. 3D and 2D view of TC *vs.* t_1 for various values of m.

From Table 3 and Figure 4, it is detected that the total cost is minimum if $m = 5$, *i.e.*, for the short replenishment cycle. This is because, for the short cycle, the items are held for less time, and the effect of inflation is not more. Also, for $m = 2$, it is shown that in a long replenishment cycle, the value of money becomes dominant and hence the total cost increases.

In Tables 4–6, let us now observe that if we consider the variables A, k and c as dense neutrosophic numbers, how do the parameters vary and affect the total cost?

From the above Table 4 and Figure 5, it is observed that the total cost is minimum when we consider advertisement (A) as Symmetric Concave Non-Linear. As observed, the advertisement parameter decreases for all values of m (no. of cycle), leading to a decrease in total cost. This is because if the frequency of advertisement decreases, demand decreases, implying fewer items are ordered in every cycle; hence, various (holding, deterioration, shortages) costs decrease, leading to a reduction in total cost. In comparison with Table 3, the advertisement factor decreases as we take a dense neutrosophic number. Thus, the total price also decreases due to a decrease in demand.

Table 5 and Figure 6, shows that carbon emission decreases if we take the dense neutrosophic number, which is required in the realistic scenario. Also, the retailer's total cost is minimal when considering carbon emission (c) as Asymmetric Concave Non-Linear. In this case, the carbon emission parameter decreases for all values of m (no. of the cycle), which leads to a decrease in total cost. This is because when carbon emissions decline, products decay less, resulting in more stock in the inventory to sell, leading to more profits and lower overall costs. Also, if we compare with Table 3, we can see that the total cost is less if we contemplate dense asymmetric neutrosophic numbers compared to crisps.

From the above Table 6 and Figure 7, the total cost is minimum when we consider inflation (k) as Asymmetric Convex Non-Linear. In this case, the k increases for all values of m (no. of cycle), leading to a decrease in total cost. This is because if k increases, the value of money decreases, thus decreasing the total cost. Also, if we compare with Table 3, we can see that the total cost is less if we consider dense neutrosophic numbers compared to crisps.

TABLE 4. Advertisement (A) as different dense neutrosophic number.

	$m=2$		$m=3$		$m=4$		$m=5$	
	TC*	t_1^*	TC*	t_1^*	TC*	t_1^*	TC*	t_1^*
Symmetric linear $A=7.5$	3067.62	2.579	1708.64	1.732	1369.38	1.328	1226.58	1.096
Symmetric convex non-linear $m_i = n_i = s_i = 0.5, i = 1, 2$ $A=8.33$	3534.66	2.6	1831.47	1.753	1427.49	1.347	1260.45	1.112
Symmetric concave non-linear $m_i = n_i = s_i = 2, i = 1, 2$ $A=6.67$	2644.59	2.552	1589.85	1.706	1311.96	1.307	1193.15	1.077
Asymmetric linear $A=10.1$	4678.08	4.635	2105.28	1.79	1552.62	1.38	1332.85	1.142
Asymmetric convex non-linear $(m_i = n_i = s_i = 0.5, i = 1, 2)$ $A=11.26$	5536.68	2.653	2292.61	1.81	1635.06	1.399	1379.96	1.159
Asymmetric concave non-linear $m_i = n_i = s_i = 2, i = 1, 2$ $A=8.94$	3906.02	2.614	1924.09	1.767	1470.48	1.359	1285.42	1.123

TABLE 5. Carbon emission (c) as different dense neutrosophic number.

	$m=2$		$m=3$		$m=4$		$m=5$	
	TC*	t_1^*	TC*	t_1^*	TC*	t_1^*	TC*	t_1^*
Symmetric linear $c=0.15$	8904.53	2.705	2945.7	1.844	1911.44	1.419	1536.39	1.172
Symmetric convex non-linear $m_i = n_i = s_i = 0.5, i = 1, 2$ $c=0.167$	8907.24	2.71	2949.78	1.839	1914.71	1.41	1538.93	1.161
Symmetric concave non-linear $m_i = n_i = s_i = 2, i = 1, 2,$ $c=0.133$	8901.79	2.701	2941.6	1.849	1908.12	1.429	1533.79	1.183
Asymmetric linear $c=0.0679$	8891.17	2.686	2925.76	1.869	1894.97	1.464	1523.23	1.226
Asymmetric convex non-linear $(m_i = n_i = s_i = 0.5, i = 1, 2)$ $c=0.086$	8894.15	2.69	2930.18	1.863	1898.69	1.454	1526.27	1.214
Asymmetric concave non-linear $m_i = n_i = s_i = 2, i = 1, 2$ $c=0.0497$	8888.16	2.682	2921.29	1.874	1891.17	1.474	1520.11	1.238

Let us look at the influence of the model if the above parameters are considered under three situations, namely nonlinear triangular dense fuzzy, the nonlinear triangular dense intuitionistic, nonlinear triangular dense neutrosophic system, and all of the above in a linear triangular dense neutrosophic number for the shortest replenishment cycle ($m = 5$). We considered this cycle because Table 3 shows the minimum cost occurs if we take a small replenishment cycle. It is also detected numerically and mathematically that in the case of a dense symmetric situation, the defuzzied, de-intuitionistic and de-neutrosophication values are same as that of the truth value. Thus, we do not consider the symmetric case in Tables 7 and 8.

TABLE 6. Inflation (k) as different dense neutrosophic number.

	$m=2$		$m=3$		$m=4$		$m=5$	
	TC*	t_1^*	TC*	t_1^*	TC*	t_1^*	TC*	t_1^*
Symmetric linear $k=0.128$	4556.48	2.3	2056.81	1.607	1467.21	1.265	1231.88	1.067
Symmetric convex non-linear $m_i=n_i=s_i=0.5, i=1,2$ ($k=0.142$)	3894.74	2.232	1930.33	1.564	1405.99	1.235	1190.55	1.043
Symmetric concave non-linear $m_i=n_i=s_i=2, i=1,2$ ($k=0.114$)	5248.68	2.368	2191.37	1.65	1532.82	1.297	1276.33	1.091
Asymmetric linear $k=0.1$	5974.54	2.435	2334.82	1.693	1603.24	1.328	1324.22	1.115
Asymmetric convex non-linear ($m_i=n_i=s_i=0.5, i=1,2$) $k=0.144$	3802.51	2.222	1912.88	1.558	1397.58	1.23	1184.88	1.04
Asymmetric concave non-linear $m_i=n_i=s_i=2, i=1,2$ $k=0.057$	8452.44	2.654	2840.59	1.834	1854.8	1.429	1496.4	1.192

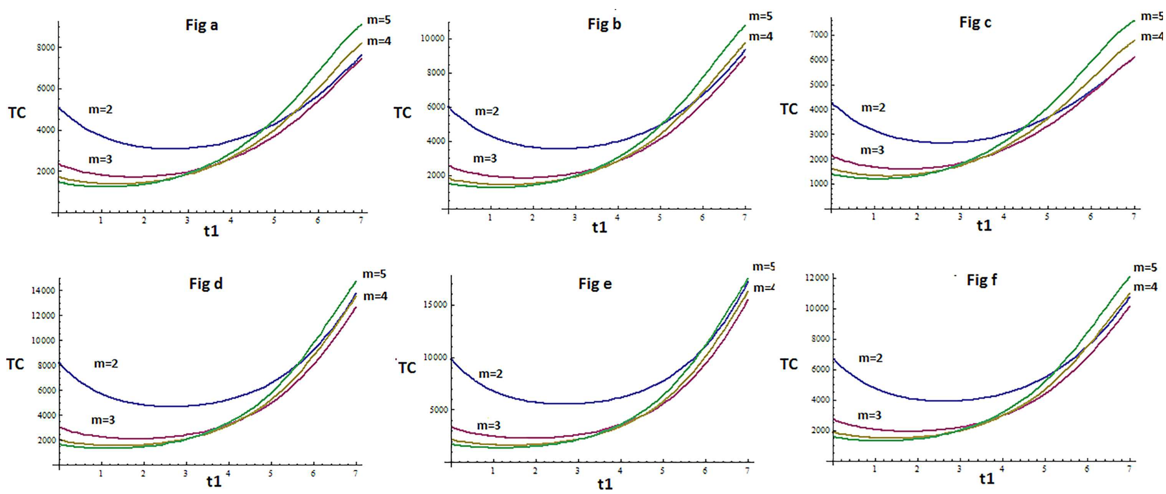


FIGURE 5. Different values of A: (a) Symmetric linear. (b) SCvNL ($m_i=n_i=s_i=0.5$). (c) SCcNL ($m_i=n_i=s_i=2$). (d) Asymmetric linear. (e) AsCvNL ($m_i=n_i=s_i=0.5$). (f) AsCcNL ($m_i=n_i=s_i=2$).

From Tables 4, 7 and 8, we can see that advertisement (A) decreases in dense neutrosophic compared to dense fuzzy and dense intuitionistic. Thus, the total cost is lower in neutrosophic environments than in other environments. Our aim is to decrease the expenditure on advertisement and optimise the total cost. Thus, if the advertisement factor increases, the total cost increases due to the expenditure for launching the advertisement in the market. Also, the demand increases; thus, the total cost also increases.

From Tables 6 to 8, it is also observed that the value of inflation (k) increases if we consider fuzzy, which is not required. Our aim is to reduce inflation because an upsurge in inflation means the value of money decreases, which is undesirable. Thus, considering neutrosophic is desirable as compared to other environments.

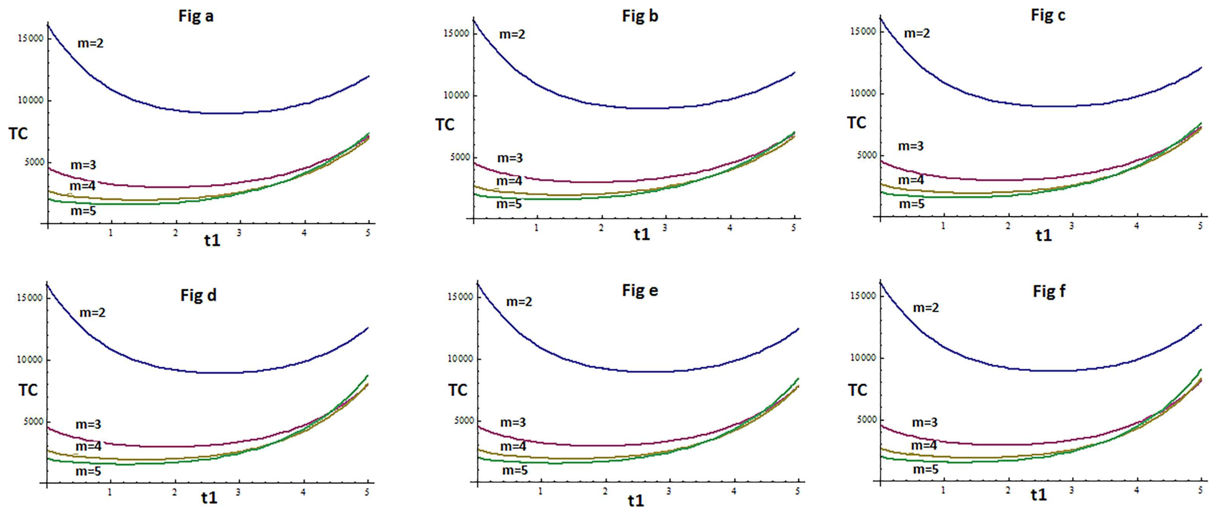


FIGURE 6. Different values of c: (a) Symmetric linear. (b) SCvNL ($m_i = n_i = s_i = 0.5$). (c) SCcNL ($m_i = n_i = s_i = 2$). (d) Asymmetric linear. (e) AsCvNL ($m_i = n_i = s_i = 0.5$). (f) AsCcNL ($m_i = n_i = s_i = 2$).

TABLE 7. Parameters in triangular dense fuzzy environment.

	A			k			c		
	Value of A	TC	t_1	Value of k	TC	t_1	Value of c	TC	t_1
Asymmetric linear	12.32	1422.61	1.174	0.367	786.9	0.623	0.192	1542.55	1.146
Asymmetric convex non-linear ($m_i = n_i = s_i = 0.5, i = 1, 2$)	13.63	1474.75	1.189	0.384	767.67	0.577	0.128	1533.02	1.186
Asymmetric concave non-linear $m_i = n_i = s_i = 2, i = 1, 2$	11.01	1369.84	1.156	0.35	807	0.664	0.256	1551.25	1.108

TABLE 8. Parameters in triangular dense intuitionistic environment.

	A			k			c		
	Value of A	TC	t_1	Value of k	TC	t_1	Value of c	TC	t_1
Asymmetric linear	10.88	1364.57	1.154	0.254	943.92	0.853	0.154	1537	1.17
Asymmetric convex non-linear ($m_i = n_i = s_i = 0.5, i = 1, 2$)	12.08	1412.99	1.17	0.273	912.88	0.819	0.13	1533.33	1.185
Asymmetric concave non-linear $m_i = n_i = s_i = 2, i = 1, 2$	9.68	1315.7	1.136	0.237	973.81	0.883	0.178	1540.54	1.155

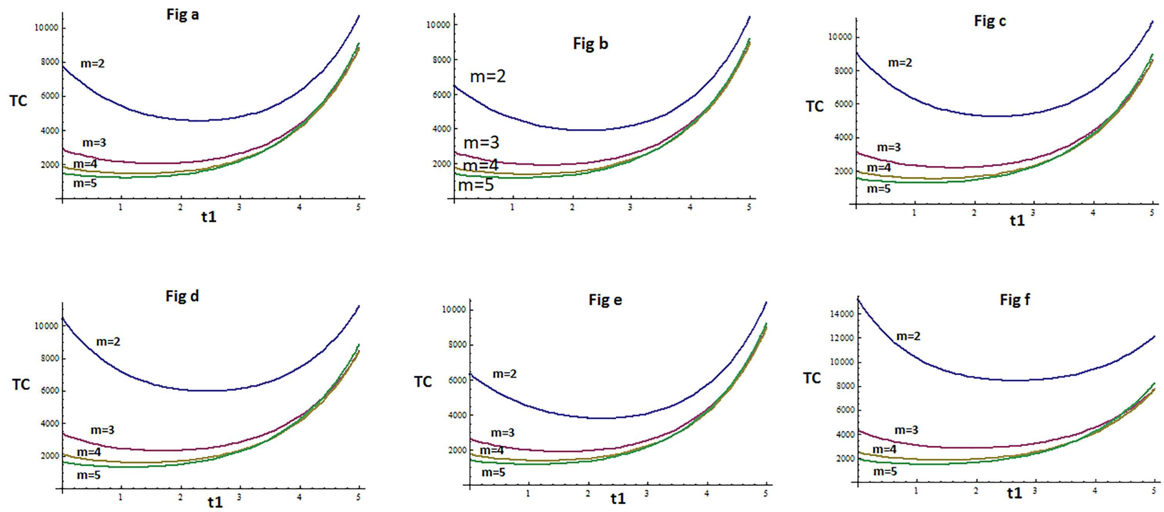


FIGURE 7. Different values of k: (a) Symmetric linear. (b) SCvNL ($m_i = n_i = s_i = 0.5$). (c) SCcNL ($m_i = n_i = s_i = 2$). (d) Asymmetric linear. (e) AsCvNL ($m_i = n_i = s_i = 0.5$). (f) AsCcNL ($m_i = n_i = s_i = 2$).

If we consider Tables 5, 7 and 8 and compare the data, we can see carbon emission (c) decreases in the neutrosophic environment compared to other environments, which is desirable in reality also, as the carbon emission increases, the total cost increases as more money is spent to maintain the inventory.

7. Conclusions and discussions

In this paper, we have considered the EOQ model under a dense neutrosophic environment. The article considers important parameters like advertisement, inflation and carbon emission as dense neutrosophic numbers because these factors are unpredictable. The change in these parameters depends on the customer's choice and stock market fluctuations. The paper considers price and advertisement-dependent demand under the effect of carbon emission and inflation. The article also considers the realistic scenario of non-instantaneous deterioration, as the groceries or the packed food items deteriorate after a specific time. Also, the model is discussed under shortages and finite time horizons.

A case study is considered, and after numerical analysis, it is discussed that the present EOQ model works much better in uncertain environments than in crisp environments, comparing Table 3 with Tables 4–8. It is seen that the total cost is less for different values of m (*i.e.*, for all replenishment cycles) in the uncertain environment as compared with the crisps environment. The model works better if we consider dense neutrosophic numbers than any other environment, which is observed in Tables 3–8 above and Figures 2–6. This is desirable as, in reality, advertisement, inflation, and carbon emission can't be fixed numbers, and they vary from time to time, *i.e.*, they are uncertain with time. Thus, considering nonlinear dense neutrosophic numbers makes our model profitable.

7.1. Managerial implications

The suggested inventory model offers effective decision-support to inventory management for determining the cost-optimal quantities of replenishment orders while considering crucial aspects affecting demand rates through consumer purchase behaviour.

The presented analysis illustrated through a case study aids tactical management in making decisions about handling changes in the sales environment brought on by the reasons discussed. The findings of our research can be effectively applied to scenarios comparable to inventory management.

For instance, the decisions about inventory replenishment for scenarios when a production company produces a perishable good for sale in its retail stores can also be handled. An excellent example of this is the sale of sweets made with milk. This model is helpful for packed food items, beverages, newly launched mobile phones, electronic gadgets, etc. From this study, the retailer can predict how to handle the uncertain environment and hence can make relevant decisions to maintain business strategies.

Online retailers can also use the approach because e-marketing companies make a substantial effort to promote some common effect.

7.2. Limitation and future proposal

In this article, the effect of lead time and trade credit has not received so much importance due to other significant parameters in this model. This is one of the limitations of this study.

The model can be further extended by incorporating it in the supply chain model or by collaborating with interdisciplinary ideas of control theory. The uncertainty of the model can also be represented using a stochastic process. We could further extend the model by considering the disruption management in the model which will make our model more robust and profitable.

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