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RESEARCH ARTICLE

A Critical Evaluation of the Criticisms against Neutrosophic Statistical Methods

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ABSTRACT

Neutrosophic statistical analysis has gained attention for incorporating the degree of indeterminacy when analyzing imprecise and interval data under uncertainty—an aspect often overlooked by classical statistics, fuzzy statistical analysis, and interval statistics. Recently, critical discussions have emerged regarding the use and applications of neutrosophic statistics, with some questioning its usefulness and validity. In this paper, we present a critical assessment of the existing literature, focusing on areas where misunderstandings and misinterpretations of neutrosophic statistical methods have occurred. We also examine flawed comparisons made between the results of neutrosophic statistics and interval statistics. Furthermore, substantial issues have been identified in the current simulation, both in its application and in the comparison of results. The objective of this paper is to highlight these key concerns and provide clarity for researchers working in this area, helping them to better understand the appropriate applications of neutrosophic statistical methods.

Keywords: Classical statistics, interval-statistics, simulation, neutrosophic data, interval-data

1. Introduction

Classical statistics are typically applied when data exhibit randomness, while fuzzy logic-based statistical analysis is suited for imprecise or interval data. As [1] explained, classical set theory involves precise classifications where an element either belongs to a set or it does not, whereas fuzzy set theory allows an element to simultaneously belong and not belong to a set to varying degrees. Interval statistical analysis addresses data represented by intervals through arithmetic operations. However, these approaches—classical, fuzzy, and interval statistics—neglect the degree of indeterminacy, an important aspect when

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analyzing uncertain data. To address this gap, neutrosophic statistical analysis was introduced by [2], extending traditional methods by incorporating degrees of truth, falsity, and indeterminacy. This framework generalizes classical statistics, fuzzy analysis, and interval statistics, offering more comprehensive tools for handling complex, imprecise, and interval data. [3] and [4] demonstrated the efficiency of neutrosophic methods over classical statistics through basic neutrosophic operations, while [5] highlighted their advantages over interval statistics with several practical examples. Further contributions include [6] work on data generation from the Erlang distribution, comparing neutrosophic, fuzzy, and intuitionistic fuzzy approaches. The neutrosophic statistical methodology has since garnered significant attention across various fields, resulting in numerous applications and developments: Aslam et al. [7] proposed a two-stage acceptance sampling plan; [8] introduced the neutrosophic negative binomial distribution and a corresponding data generation algorithm; [9] worked on the neutrosophic gamma distribution with applications; [10] developed the neutrosophic log-logistic distribution; and [11] proposed a neutrosophic t-test for regression analysis. Other notable works include proposals of neutrosophic Burr XII and Lindly distributions by [12] and [13], respectively; repetitive sampling control charts for gamma distributions by [14]; and contributions to experimental design by [15]. Reviews and applications span fields such as civil engineering [16], medicine [17], and various statistical designs and estimators, including the neutrosophic completely randomized design [18], ratio-type estimators under cost functions [19], Greco-Latin designs for interval data [20], and augmented randomized complete designs for missing data [21]. Additional works include neutrosophic rank-set sampling [22], calibrated cumulative distribution functions [23], median-based robust estimators [24], and the Horvitz-Thomson estimator [25], all illustrating the growing impact and versatility of neutrosophic statistical analysis.

By examining the extensive literature on neutrosophic statistics, it is evident that this emerging field offers valuable tools for analyzing data characterized by imprecision and intervals. Recently, Woodall et al. [26] published a critical analysis of neutrosophic statistical methods, including a simulation procedure based on uniform distributions over interval data and parameters. In this paper, we critically review their work, addressing several misunderstandings and misinterpretations regarding neutrosophic statistical methods. We also highlight flawed comparisons between neutrosophic statistics and interval statistics found in their study. Additionally, we identify significant issues in their proposed simulation procedures, particularly concerning result interpretation. The goal of this paper is to clarify these concerns and guide researchers toward the proper application of neutrosophic statistical methods. The remainder of the paper is structured as follows: Section 2 discusses operations involving neutrosophic numbers. Section 3 addresses cases where the degree of indeterminacy is unknown. Challenges in neutrosophic arithmetic are explored in Section 4. A critical assessment of neutrosophic probability distributions is presented in Section 5. Section 6 provides an evaluation of neutrosophic design of experiments. Issues related to the dependence of sums of squares are examined in Section 7. Section 8 contrasts neutrosophic intervals with those used in interval statistics. In Section 9, we offer critical evaluations of probability theory, fuzzy theory, grey theory, and neutrosophic theory. The advantages of neutrosophic statistics over traditional approaches are discussed in Section 10. Section 11 clarifies misinterpretations of the findings of Woodall et al. [26]. Fundamental concerns in simulating neutrosophic regression analysis are addressed in Section 12. Section 13 discusses the neutrosophic uniform distribution. Finally, concluding remarks are provided in Section 14.

2. Operations with neutrosophic numbers

By following [3], [5] and Aslam [6], we presented the operations for the neutrosophic numbers in this section. Let us define two neutrosophic numbers (NNs) $N_1 = a_1 + b_1 I$ and $N_2 = a_2 + b_2 I$ where $I \epsilon [I_L, I_U]$. It is worth to state that these two NNs have two parts namely determinate parts a_1 and a_2, and indeterminate parts, $b_1 I$ and $b_2 I$, with indeterminacy represented by I. By following component-wise addition rule, the addition of the two NNs $N_1 = a_1 + b_1 I$ and $N_2 = a_2 + b_2 I$ is expressed as follows:

$$N_1 + N_2 = (a_1 + b_1 I) + (a_2 + b_2 I) = (a_1 + a_2) + (b_1 + b_2) I \tag{1}$$

By following the rule of component-wise subtraction, the subtraction is given by:

$$N_1 - N_2 = (a_1 + a_1 I) - (b_2 + b_2 I) \tag{2}$$

The simplified form is given by

$$N_1 - N_2 = (a_1 - b_2) + (a_1 - b_2) I \tag{3}$$

Using the distributive property of multiplication, it is given by

$$N_1 . N_2 = (a_1 + b_1 I) \times (a_2 + b_2 I) = a_1 . a_2 + a_1 . b_2 . I + a_2 . b_1 . I + b_1 . b_2 . I^2 \tag{4}$$

After simplification, we have

$$N_1 . N_2 = a_1 . a_2 + (a_1 b_2 + a_2 b_1) I + b_1 . b_2 . I^2 \tag{5}$$

By divided two NNs, we have

$$\frac{N_1}{N_2} = \frac{(a_1 + b_1 I)}{(a_2 + b_2 I)} \tag{6}$$

It was important to note that in neutrosophic statistical analysis, a neutrosophic number represented indeterminacy and uncertainty, and these numbers did not always follow the logic of classical numbers. For example, a situation where a lower number was greater than an upper number might indicate a contradiction in classical numerical analysis. However, in neutrosophic analysis, such situations reflected indeterminacy, uncertainty, or incomplete information.

3. Indeterminacy is unknown

Usually, in simulations aimed at examining the effect of indeterminacy on the required statistical analysis, various fixed values of the degree of indeterminacy I were used. However, in practice, the degree of indeterminacy was often unknown and needed to be estimated from the available real data under study. The purpose of determining the degree of indeterminacy was to establish the upper limit of the neutrosophic numbers. In this approach, by setting $I = 0$, the lower value of the neutrosophic number was obtained. By substituting the estimated value of I, the approximate upper values of the neutrosophic numbers could then be determined as follows:

$$I = \frac{(b_1 - a_1)}{b_1} \tag{7}$$

The simulation conducted by Woodall et al. [26] did not adequately account for the indeterminacy present in real-world data, leading to a deviation between the simulation results and those observed in actual datasets.

4. Resolving challenges in neutrosophic arithmetic

In this section, we responded to the questions raised by [27], as well as [26], regarding operations with neutrosophic numbers. Following [28], we supposed that the form of the neutrosophic number was defined by:

$$N = a + bI \tag{8}$$

Where a, b are real numbers and $I =$ indeterminacy (is a real set, not necessarily interval). The confusion made by Woodall et al. [26] herein is that: from the fact that the single true real value v is in the set I, or $v \in I$, it does not result that v is in $a + bI = N$ as well, i.e. $v \notin a + bI$, or $v \notin N$, but:

$$a + bv \in a + bI, \text{ or } a + bv \in N. \tag{9}$$

According to [26] "As an example, consider the two neutrosophic numbers [4, 6] and [2, 4] represented as 4 + 2I and 4−2I, respectively. Then using the approach of Smarandache [42], the average of these two neutrosophic numbers would be $4 + 0I$, or simply the precise value 4. This result does not seem reasonable". The answer to this question is explained as follows.

Let $I = [0, 1]$, $N1 = 4 + 2I = [4, 6]$ and $N2 = 4 - 2I = [2, 4]$. Their average is $(N1 + N2)/2 = [4 + 2, 6 + 4]/2 = [3, 5]$. Because the true value $v \in I = [0, 1]$, one gets that: $4 + 2v \in N1$ (or $4 + 2v$ belongs to $N1$) and $4 - 2v \in N2$ (or $4 - 2v$ belongs to $N2$). If we average them, we get: $[(4 + 2v) + (4 - 2v)]/2 \in (N1 + N2)/2$ (or $[(4 + 2v) + (4 - 2v)]/2 \in (N1 + N2)/2$ or $8/2 \in [N1 + N2]/2$ or $4 \in ([4, 6] + [2, 4])/2 = [6, 10]/2 = [3, 5]$ which is perfectly reasonable.

5. A critical assessment of neutrosophic probability distributions

Woodall et al. [26, 27] raised questions regarding the method for generating data from neutrosophic statistical distributions. According to [26] "No probability distribution is given for the unknown parameter value within the neutrosophic interval, so presumably one would use a uniform density. Randomly generating a data value from a neutrosophic density would then seem to be based on the generation of parameter values followed by the generation of a value from the resulting probability distribution. This framework would have a straightforward Bayesian interpretation with a uniform prior distribution". [29] presented several neutrosophic statistical distributions. It is important to note that Woodall et al. [26] incorrectly attempted to conflate the theory of neutrosophic statistical distributions with the Bayesian approach. In the theory of neutrosophic distributions, similar to classical statistics, parameters are fixed values and cannot be treated as random variables under the assumption of a known statistical distribution. Instead, in neutrosophic statistical distributions, parameters are represented as intervals, and sometimes they take more complex forms that include degrees of truth, falsity, and indeterminacy. Therefore, any value within the interval represents a possible value for the parameter. The lack of a defined probability distribution over the interval reflects the uncertainty about the

Table 1. The difference between Bayesian and neutrosophic methods.

Bayesian Methods	Neutrosophic Methods
Parameters are considered random variables having some prior distribution. In Bayesian statistics, the parameters are treated as random variables.	Parameters are considered as the value within the given interval of the parameters. The parameters are not considered random variables with the known probability distribution.
The prior probability distribution is used to present beliefs about the parameters.	In neutrosophic statistical methods, the focus is on the degree of indeterminacy or incomplete information.
For the uniform distribution, it is assumed equal belief in all values within the parameters a and b	We cannot assign any probability distribution even if we cannot uniform distribution over the intervals.
In these analyses, all values are considered are equally likely based on the prior belief.	It does not depend on any weight but deals with the degree of indeterminacy.
It uses the Bayes' theorem to form the posterior distribution	Bayes' theorem is not required to update the posterior distribution but the degree of indeterminacy is handled directly.

true value of the parameter. Neutrosophic probability distributions are thus particularly useful when there is uncertainty in the data or the parameters of statistical distributions. Additionally, unlike Bayesian methods, we cannot incorporate further information about the probability distribution. Let's consider the example of the exponential distribution discussed by [29], where they defined the neutrosophic average μ_N To range between 0.67 and 2. For this exponential distribution, the neutrosophic rate parameters are given by:

$$\lambda_N = \frac{1}{\mu_N} = \frac{1}{[0.67, 2]} = [0.5, 1.50] \tag{10}$$

It is important to note that specifying the probability distribution for the rate parameter, which ranges from 0.5 to 1.50, is unnecessary. This indicates that all values within this interval are potentially valid for the rate of change. Unlike Bayesian methods, in neutrosophic theory, we do not treat the parameter as a random variable over the given range of rate parameters. In neutrosophic statistics, the focus is on evaluating the degree of indeterminacy rather than variability and probability. While decision-makers using Bayesian methods may consider a uniform distribution, this is not required for neutrosophic statistical distributions. The differences between Bayesian methods and the neutrosophic framework are outlined in Table 1.

6. Critical evaluation of neutrosophic design of experiments

Woodall et al. [26] conducted a simulation study and compared the results with the work of [30] and [31]. Woodall et al. [26] discussed the simulation from [30]. Using real data from [30], presented a simulation study to obtain the results of the analysis of variance (ANOVA) along with the corresponding p-values. They illustrated, through figures, that the neutrosophic bounds were unnecessarily wide. In this section, we critically assess this simulation procedure, highlighting the main shortcomings and drawbacks in both the simulation process and the results obtained. Woodall et al. [26] showed that the variations in empirical p-values were lower compared to the p-values obtained from neutrosophic statistics, where the distribution over the interval was assumed to follow a uniform distribution. This comparison raised significant concerns about the validity of the simulation process and its application in comparing the results. It is important to note that experimental designs are typically based on the assumption of normality. If the variables with lower and upper values follow a normal distribution, the data within each interval will also follow a normal distribution. Using a uniform distribution to generate data for p-value analysis is misleading and statistically incorrect, as the two distributions have different statistical properties. Additionally, a primary drawback is the comparison of p-values derived from the uniform distribution with those obtained from the F-distribution, which is clearly incorrect and misleading. Furthermore, the width of the neutrosophic bounds depends on the indeterminacy in the real data. If the real data exhibits significant indeterminacy, we would expect the neutrosophic statistical results to produce wider intervals for the p-values. It is worth considering that if the p-values from real data are already wide, there is no need to artificially reduce them. Forcing a reduction in the width of p-values would significantly impact the power of the test. In summary, the width of p-values depends on the degree of indeterminacy in the real data, which was not accounted for in the simulation. An increase in indeterminacy reflects greater uncertainty in the data, leading to wider neutrosophic statistical results. Therefore, we conclude that the approach presented by [26] is flawed, with unnecessary drawbacks, and cannot be recommended for neutrosophic statistical analysis. Woodall et al. [26] revisited the advantages of their simulation by comparing it with the results obtained from the real data [31]. It is important to note that the simulation discussed in [26] was again based on data generated from a uniform distribution, and no detailed simulation process was provided, making it difficult for readers to obtain precise and accurate information about the simulation. Additionally, no code related to this simulation was included in Woodall et al. [26]. Woodall et al. [26] argued that the bounds provided by [31] are too narrow and should encompass all possible values of the statistics. It is important to note that the lower bounds represent the classical statistics, while the upper bounds capture the indeterminate part. For example, from [31], we observe that the neutrosophic p-values are 0.194 and 0.138. These results indicate that the p-value for the classical statistics is 0.194, while the p-value for the indeterminate part is 0.138. This can be interpreted as suggesting that, under uncertainty for this real data, the p-value may range from 0.194 to 0.138. In neutrosophic statistical analysis, the lower and upper bounds are not necessarily ordered from smallest to largest, as they represent different states of information. Thus, the p-value interval captures all possibilities, accounting for both certain and indeterminate factors. In neutrosophic statistics, the bounds are defined by the lower bound (representing the determinate value) and the upper bound (representing the indeterminate value), together covering the entire range of real data under study. For example, if a p-value of 0.14 is observed, it falls within the interval $0.138 < 0.14 < 0.194$. This result can be interpreted as indicating that the uncertainty in the data or model may shift the p-value anywhere between 0.138 and 0.194. The variation in p-values within the given interval depends on the degree of indeterminacy. If the degree of indeterminacy is low, the p-value will be closer to 0.138, suggesting stronger evidence against the null hypothesis. On the other hand, if the degree of indeterminacy is high, the p-value will be closer to the upper bound, indicating weaker evidence against the null hypothesis. In conclusion, the p-values ranging from 0.138 to 0.194 are influenced by the degree of indeterminacy. A lower degree of indeterminacy results in p-values closer to the lower bound, while a higher degree of indeterminacy leads to p-values closer to the upper bound. Therefore, neutrosophic statistical analysis encompasses a comprehensive range of p-values, effectively capturing both certain and uncertain situations. It is important to note that any value within the interval yields statistical results that remain within the neutrosophic interval. In cases where indeterminacy is absent, the analysis begins from the lower bound (the determinate part), which is included in the neutrosophic interval—an aspect misinterpreted by Woodall et al. [26]. [31] presented the neutrosophic statistical results based on real data, demonstrating that a narrow interval is expected when the degree of indeterminacy is low, whereas wider intervals are appropriate when indeterminacy is high. However, Woodall et al. [26] made a surprising claim that the analysis using the real data should result in wider intervals, based on their simulation outcomes. This interpretation is misleading and reflects an artificial understanding of neutrosophic analysis. We clarify this point to ensure decision-makers are not misinformed by such inaccurate conclusions.

7. Issues related to dependence of sum of squares

Woodall et al. [26] argued that "interval-valued ANOVA sums of squares are not independent since the treatment sums of squares and the error sum of squares must sum to the total sum of squares. If the total sum of squares takes a high value in its interval of possible values, for example, then the other sums of squares will tend to be high." This claim as it refers to the sum of squares for the lower and upper values of each interval, and then it might be valid. However, for neutrosophic statistical analysis, this issue can be fixed as follows. Suppose that we have neutrosophic sum of square for total (NSST), neutrosophic sum of square for error (NSSE) and the neutrosophic sum of square of treatment ($NSST_r$) and these are defined by $NSST \in [S_L, S_U]$, $NSST_r \in [T_L, T_U]$ and $NSSE \in [E_L, E_U]$, then using the lower values of these sum of square, compute $\gamma_L = T_L/S_L$ and $\gamma_U = T_U/S_U$ and re-calculate $NSST_r \in [\gamma_L.S_L, \gamma_U.S_U]$ and $NSSE \in [(1-\gamma_L).S_L, (1-\gamma_U).S_U]$. This will be always produce the results $NSST = NSST_r + NSSE$. This procedure will be explained with the help of one example, suppose that $NSST \in [120, 150]$, $NSST_r \in [70, 90]$ and $NSSE \in [50, 60]$. This leads to $\gamma_L = 70/120 = 0.583$ and $\gamma_U = 90/150 = 0.60$. Suppose that we will select $NSST = 135$ within 120 and 150 and $\gamma = 0.59$ within 0.583 and 0.60. Then, re-calculations of $NSST_r$ and $NSSE$ are as follows: $NSST_r = [0.59 \times 135] = 79.65$ and $NSSE = [(1-0.59) \times 135] = 55.35$ and $79.65 + 55.35 = 135$ and it will be held exactly. This methodology can be used to resolve the issue when the sum of NSSE and NSSTr does not equal NSST.

8. Understanding the difference between neutrosophic and interval-statistics intervals

In this section, we explore the differences between neutrosophic intervals and traditional interval-statistics intervals to their application, interpretation, and justification in practice. To illustrate these distinctions, we use an example involving neutrosophic p-values, though the method is generalizable to any set of p-values. Consider a neutrosophic p-value interval, such as (0.03, 0.10), and a significance level of $\alpha = 0.05$ for testing a null hypothesis. In classical statistical inference, the null hypothesis is typically rejected if the p-value is less than 0.05. Comparing the lower bound of the neutrosophic interval (0.03) with $\alpha = 0.05$, we find strong evidence against the null hypothesis, suggesting it should be rejected. However, the upper bound of the interval (0.10) exceeds α, implying support for the null hypothesis. This variability—or spread—within the p-value introduces indeterminacy in decision-making. According to neutrosophic theory, this uncertainty is handled by assessing three degrees: the truth-membership (T), representing the extent to which the null hypothesis is rejected; the indeterminacy-membership (I), indicating the level of uncertainty in the decision; and the falsity-membership (F), reflecting the extent to which the null hypothesis is not rejected. We now proceed to present the application, interpretation, and justification for using neutrosophic p-value intervals by applying the approach proposed by [2], which states that if

$$min\left\{neutrosophic\ p\text{-}vale\right\} < \alpha < max\left\{neutrosophic\ p\text{-}vale\right\},$$

then there is an indeterminacy in decision-making. Thus, the probability of rejecting the null hypothesis at α is given by

$$T = \frac{\alpha\text{-}min\left\{neutrosophic\ p\text{-}vale\right\}}{max\left\{neutrosophic\ p\text{-}vale\right\} - min\left\{neutrosophic\ p\text{-}vale\right\}} \tag{11}$$

Thus, the probability of not rejecting the null hypothesis at α is given by

$$F = \frac{max\left\{neutrosophic\ p\text{-}vale\right\} - \alpha}{max\left\{neutrosophic\ p\text{-}vale\right\} - min\left\{neutrosophic\ p\text{-}vale\right\}} \tag{12}$$

For the given p-values, the probability of rejecting the null hypothesis at $\alpha = 0.05$ is given by

$$T = \frac{0.05 - 0.03}{0.10 - 0.03} = \frac{0.02}{0.07} = 0.2857$$

The probability of not rejecting the null hypothesis at $\alpha = 0.05$ is given by

$$F = \frac{0.10 - 0.05}{0.10 - 0.03} = \frac{0.05}{0.07} = 0.7142$$

Due to uncertainty, the neutrosophic form of the p-values.

$$p\text{-}value = 0.03 + 0.10I;\ I \in\ [0, 0.70] \tag{13}$$

It is important to note that the neutrosophic form of p-values reduces to the classical p-value when the indeterminacy component $I = 0$. However, when $I\ > 0$, the truth-membership (T) and falsity-membership (F) satisfy the condition $T + F = 1$, and the presence of indeterminacy necessitates a modification in how results are interpreted. To align with the probabilistic framework of classical statistics, we proceed by normalizing the truth and falsity components. Let T', F' and I' denote the normalized values of T, F and I, respectively, which are computed as follows:

$$\text{Total} = 0.2857 + 0.7142 + 0.70 = 1.6999$$

$$T' = \frac{0.2857}{1.6999} = 0.17, \quad F' = \frac{0.7142}{1.6999} = 0.42 \quad \text{and} \quad I' = \frac{0.70}{1.6999} = 0.41$$

Based on the three neutrosophic components, the interpretation of the null hypothesis under uncertainty is as follows: there is an 17% degree of belief that the null hypothesis under investigation should be rejected, a 42.0% degree of belief that it should not be rejected, and a 41% degree of indeterminacy. This high level of indeterminacy reflects significant ambiguity in the data and overlapping evidence, indicating that a clear decision—either to reject or not reject the null hypothesis—cannot be confidently made. Now, consider the interpretation of the same neutrosophic p-value interval (0.03, 0.10) using interval statistics. In classical interval analysis, such an interval suggests that the exact p-value is unknown but lies somewhere between 0.03 and 0.10. Under this approach, the null hypothesis is rejected if the entire interval lies below the significance level $\alpha = 0.05$. However, if the interval straddles α, as in this case, the decision becomes inconclusive, and additional caution or further data collection may be warranted. In summary, neutrosophic statistical analysis provides more nuanced information for decision-making compared to interval statistics. While interval statistics merely indicate the possible range of the p-value, neutrosophic analysis quantifies the degrees of belief, doubt, and uncertainty associated with the decision. These interpretive aspects have not been addressed in the work of Woodall et al. [26], which appears to focus primarily on simulations based on interval statistics rather than incorporating the richer framework of neutrosophic statistics.

9. Critical evaluations of probability theory, fuzzy theory, grey theory, and neutrosophic theory

In this section, we critically analyze the differences between probability theory, fuzzy theory, grey theory, and neutrosophic theory, focusing on their assumptions, degrees of indeterminacy, and applications. Although all these methods address uncertainty in data, they differ in significant ways. As noted by Woodall et al. [26], “Thus, one would expect considerable overlap in the methodologies of these four areas of inquiry. Each of the four disciplines has a vast literature and a widening scope, so comparing them in detail would be difficult.” To begin, probability theory assumes that data are random in nature and that uncertainty follows a stochastic process. Events are assigned probabilities between 0 and 1, and their implementation typically requires a well-defined probability distribution. Probability theory is widely applied wherever randomness governs data generation. Fuzzy theory, by contrast, deals with imprecise observations, using membership functions ranging from 0 to 1 to model the degree of truth. It is commonly applied in situations requiring human-like reasoning or where information is vague or expressed in linguistic terms, resulting in imprecise or interval-valued data. Grey system theory, on the other hand, addresses systems that contain both known (white) and unknown (black) information, collectively referred to as grey information. This approach is suited to problems where information is incomplete or partially known, with applications in fields such as engineering, decision-making, and forecasting. Further details on these theories can be found in the relevant literature in [32]. On the other hand, neutrosophic statistics is founded on neutrosophic logic, which addresses uncertainty by considering the degrees of truth, falsity, and indeterminacy [2]. [33] and [34] emphasized the limitations of both classical and fuzzy logic, demonstrating the effectiveness of neutrosophic methods in handling complex and uncertain data. Furthermore, [6] explored the distinctions between neutrosophic statistical analysis and fuzzy statistical methods, specifically in the context of the Erlang distribution.

10. Advantages of neutrosophic statistics over traditional methods

Woodall et al. [26] argued that it remains unclear when and where probability theory, fuzzy theory, and grey system theory should be applied. In response, we discuss the advantages of neutrosophic statistics over these three methods, particularly in terms of information handling, flexibility, and applicability. It is important to note that the use of these four approaches depends on the specific situation. However, by examining their theoretical foundations, it becomes evident that neutrosophic statistics, grounded in neutrosophic logic, extends beyond the capabilities of probability theory, fuzzy logic, and grey system theory by providing a broader framework for managing uncertainty, specifically through the incorporation of the degree of indeterminacy. Neutrosophic statistics is preferable when dealing with indeterminate data, as it explicitly accounts for indeterminacy in decision-making processes. Furthermore, it is well-suited for situations where the state of uncertainty in the data may shift depending on changing circumstances. Unlike fuzzy and grey systems, which are limited when faced with direct contradictions in data, neutrosophic statistics accommodates such complexities by allowing simultaneous consideration of truth, falsity, and indeterminacy. Additionally, while probability theory and fuzzy logic handle uncertainty related to randomness and partial truth, they do not adequately address uncertainty arising from ignorance or hesitation. In summary, probability theory, fuzzy theory, and grey system theory lack a comprehensive perspective on indeterminacy, an essential factor when conducting statistical analysis under uncertainty. Neutrosophic statistics overcomes these limitations, offering a more flexible and inclusive approach.

11. Misinterpretations of the results

In this section, we identify key flaws in the simulation approach adopted by Woodall et al. [26] to compare their results with those obtained using neutrosophic statistics. Specifically, they applied a uniform distribution across the neutrosophic interval data derived from real datasets. However, in neutrosophic statistics, the use of simulated data is often unnecessary when real-world datasets are available. Unlike classical simulations, neutrosophic analysis does not assume a uniform distribution over each interval. Instead, it directly accounts for the inherent indeterminacy present in real-world data. A fundamental aspect of the neutrosophic analysis is that the width of each interval reflects the degree of indeterminacy unique to each case and is not randomly generated. The lower bound of an interval represents the determinate part of the data, while the upper bound captures the indeterminate component. A critical flaw in Woodall et al.'s simulation lies in their failure to distinguish between the analysis of these components when using real-world data. The simulated data, often centered around the midpoint of each interval, can artificially narrow the range and distort the analysis. This leads to a significant deviation from the results derived from actual data, where the full spectrum of uncertainty is preserved. Thus, the issue is not simply about the width of the intervals but about accurately representing the data's inherent indeterminacy—such as in the case of interval-based p-values. In essence, comparing results from uniformly simulated interval data to those obtained through neutrosophic analysis of real datasets is misleading. The simulation fails to capture the full uncertainty present in the data and reduces the analysis to a crisp framework, thereby neglecting the core principles of neutrosophic statistics, particularly the role of indeterminacy.

12. Fundamental issues in the simulation of neutrosophic regression analysis

In this section, we once again highlight the fundamental shortcomings in the simulation conducted by [26] in the context of neutrosophic regression analysis. [35] contributed to the development of neutrosophic multiple regression, while [36] extended this framework by introducing neutrosophic regression with dummy variables. As previously noted, the core objective of neutrosophic regression analysis is to capture the degree of indeterminacy, particularly when working with real-world data. The width of the neutrosophic interval is directly influenced by the level of indeterminacy. [36] conducted a simulation study incorporating various values of indeterminacy, utilizing the properties of neutrosophic random variables. Their results demonstrated that the degree of indeterminacy significantly impacts the behavior of dummy variable regression. In contrast, Woodall et al. [26] failed to account for indeterminacy, neglecting its effect on the interval width—a key aspect of neutrosophic analysis. The simulation presented by [36] can be viewed as a generalization of the simulation proposed by Woodall et al. [26]. Moreover, it is methodologically unsound to directly compare results derived from simulation studies with those obtained from real data sets, as done by Woodall et al. [26].

13. Neutrosophic uniform distribution

[27] pointed out that "*pointed out that the method for generating random data from neutrosophic distributions has not been carefully explained. No probability distribution is given for the unknown parameter value within the neutrosophic interval, so presumably one would use a uniform density*". It is important to note that, in neutrosophic statistical analysis, assigning a probability distribution to the unknown parameter within the neutrosophic interval is not required. This fundamental distinction sets neutrosophic statistics apart from classical statistical methods, as discussed in detail in earlier sections. Using the framework of neutrosophic random variables proposed by [36], we define the neutrosophic uniform distribution as follows: If X_L Follows the uniform distribution with parameters a and b that is $X_L \sim U(a, b)$, then neutrosophic random variable $X_N \sim U(a(1+I_N), b(1+I_N))$, then the probability density function (pdf) of the neutrosophic uniform distribution is given by

$$f(X_N) = \frac{1}{(b-a)(1+I_N)} \tag{14}$$

It is important to note that the probability density function (pdf) of the neutrosophic uniform distribution reduces to the classical uniform distribution when $I_N = 0$. The statement by Woodall et al. [26], *"A more straightforward and informative approach would be to assume a probability density on each of the data intervals. One can use any distribution over these intervals, but without more information, we assumed uniform densities,"* which appears to be incorrect and potentially misleading, as the application of the uniform distribution under classical statistical assumptions lacks proper justification in this context.

14. Concluding remarks

In this paper, we presented a critical assessment of the existing literature, focusing on areas where misunderstandings and misinterpretations of neutrosophic statistical methods had occurred. We also examined flawed comparisons made between the results of neutrosophic statistics and interval statistics. Furthermore, significant issues were identified in the simulation procedures proposed by Woodall et al. [26], particularly in how they were used to generate and compare results. Based on a comprehensive review of the simulation presented by [26], we conclude that the simulation contains significant flaws and is grounded in an incorrect analysis that does not align with the principles of neutrosophic statistical analysis. The evidence presented in this paper supports our assertion that Woodall et al. [26] focused exclusively on simulations based on interval statistics, without any connection to the framework or principles of neutrosophic statistics. In light of these observations, the simulation by Woodall et al. [26] cannot be recommended for the neutrosophic statistical analysis. The objective of this paper was to highlight these key concerns and provide clarity for researchers working in this area, helping them to better understand the appropriate applications of neutrosophic statistical methods.

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Declarations

Ethics approval and consent to participate

The results/data/figures in this manuscript have not been published elsewhere, nor are they under consideration by another publisher. All the material is owned by the authors, and/or no permissions are required.

Consent for publication

This article does not contain any studies with human participants or animals performed by any of the authors.

Availability of data and materials

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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