

condition holds: $(i, j) \in A$ and $i < j$. The activity times in the network are determined by T_{ij} .

Notations of CPM solution:

T_i^e =Earliest occurrence time of predecessor event i ,

T_i^l = Latest occurrence time of predecessor event i ,

T_j^e =Earliest occurrence time of successor event j ,

T_j^l = Latest occurrence time of successor event j ,

T_{ij}^e /Start= Earliest start time of an activity ij ,

T_{ij}^e /Finisht=Earliest finish time of an activity ij ,

T_{ij}^l /Start=Latest start time of an T_i^l activity ij ,

T_{ij}^l /Finisht= Latest finish time of an activity ij ,

T_{ij} = Duration time of activity ij ,

Earliest and Latest occurrence time of an event:

T_j^e =maximum $(T_j^e + T_{ij})$, calculate all T_j^e for j th event, select maximum value.

T_i^l =minimum $(T_j^l - T_{ij})$, calculate all T_i^l for i th event, select minimum value.

T_{ij}^e /Start= T_i^e ,

T_{ij}^e /Finisht= $T_i^e + T_{ij}$,

T_{ij}^l /Finisht= T_j^l ,

T_{ij}^l /Start= $T_j^l - T_{ij}$,

Critical path is the longest path in the network. At critical path, $T_i^e = T_i^l$, for all i .

Slack or Float is cushion available on event/ activity by which it can be delayed without affecting the project completion time.

Slack for i th event = $T_i^l - T_i^e$, for events on critical path, slack is zero.

From the previous steps we can conclude the proposed algorithm as follows:

1. To deal with uncertain, inconsistent and incomplete information about activity time, we considered activity time of CPM technique as triangular neutrosophic number.
2. Calculate membership functions of each triangular neutrosophic number, using equation 1, 2 and 3.
3. Obtain crisp model of neutrosophic CPM using equation (4) and (5) as we illustrated previously.
4. Draw CPM network diagram.
5. Determine floats and critical path, which is the longest path in network.
6. Determine expected project completion time.

IV. ILLUSTRATIVE EXAMPLES

To explain the proposed approach in a better way, we solved two numerical examples and steps of solution are determined clearly.

A. NUMERICAL EXAMPLE 1

An application deals with the realization of a road connection between two famous cities in Egypt namely Cairo and Zagazig. Linguistics terms such as "approximately between" and "around" can be properly represented by approximate reasoning of neutrosophic set theory. Here triangular neutrosophic

numbers are used to describe the duration of each task of project. As a real time application of this model, the following example is considered. The project manager wishes to construct a possible route from Cairo (s) to Zagazig (d). Given a road map of Egypt on which the times taken between each pair of successive intersection are marked, to determine the critical path from source vertex (s) to the destination vertex (d). Activities and their neutrosophic durations are presented in table 1.

TABLE 1. INPUT DATA FOR NEUTROSOPHIC CPM.

Activity	Neutrosophic Activity Time(days)	Immediate predecessors
A	About 2 days (1,2,3;0.8,0.5,0.3)	-
B	About 3 days (2,3,8;0.6,0.3,0.5)	-
C	About 3 days (1,3,10;0.9,0.7,0.6)	A
D	About 2 days (1,2,6;0.5,0.6,0.4)	B
E	About 5 days (2,5,11;0.8,0.6,0.7)	B
F	About 4 days (1,4,8;0.4,0.6,0.8)	C
G	About 5 days (3,5,20;0.8,0.3,0.2)	C
H	About 6 days (4,6,10;0.8,0.5,0.3)	D
I	About 7 days (5,7,15;0.3,0.5,0.4)	F,E
J	About 5 days (3,5,7;0.8,0.5,0.7)	H,G

Step 1: Neutrosophic model of project take the following form: $N = \langle E, A, \tilde{T} \rangle$, where E is asset of events (nodes) and $A \subset E \times E$ is a set of activities. \tilde{T} is a triangular neutrosophic number and stand for activity time.

Step 2: Obtaining crisp model of problem by using equations (4) and (5). Activities and their crisp durations are presented in table 2.

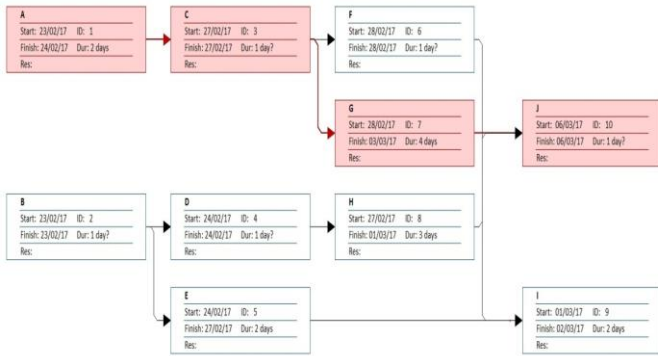
TABLE 2. INPUT DATA FOR CRISP CPM.

Activity	Activity Time(days)	Immediate predecessors
A	2	-
B	1	-
C	1	A
D	1	B
E	2	B
F	1	C
G	4	C
H	3	D
I	2	F,E
J	1	H,G

Step 3: Draw network diagram of CPM.

Network diagram of CPM using Microsoft Project 2010 presented in Fig.1.

Fig. 1. Network of activities with critical path



Step 4: Determine critical path, which is the longest path in the network.

From Fig.1, we find that the critical path is A-C-G-J and is denoted by red line.

Step 5: Calculate project completion time.

The expected project completion time = $t_A + t_C + t_G + t_J = 8$ days.

B. NUMERICAL EXAMPLE 2

Let us consider neutrosophic CPM and try to obtain crisp model from it. Since you are given the following data for a project.

TABLE 3. INPUT DATA FOR NEUTROSOPHIC CPM.

Activity	Neutrosophic Activity Time(days)	Immediate predecessors
A	$\tilde{2}$	-
B	$\tilde{4}$	A
C	$\tilde{5}$	A
D	$\tilde{8}$	B
E	$\tilde{6}$	C
F	$\tilde{10}$	D,E

Time in the previous table considered as a triangular neutrosophic numbers.

Let,
 $\tilde{2} = \langle(0,2,4); 0.8,0.6,0.4\rangle, \tilde{8} = \langle(4,8,15); 0.2,0.3,0.5\rangle,$
 $\tilde{4} = \langle(1,4,12); 0.2,0.5,0.6\rangle, \tilde{6} = \langle(2,6,18); 0.5,0.4,0.9\rangle,$
 $\tilde{5} = \langle(1,5,10); 0.8,0.2,0.4\rangle, \tilde{10} = \langle(2,10,22); 0.7,0.2,0.5\rangle.$

To obtain crisp values of each triangular neutrosophic number, we should calculate score function of each neutrosophic number using equation (4).

The expected time of each activity are presented in table 4.

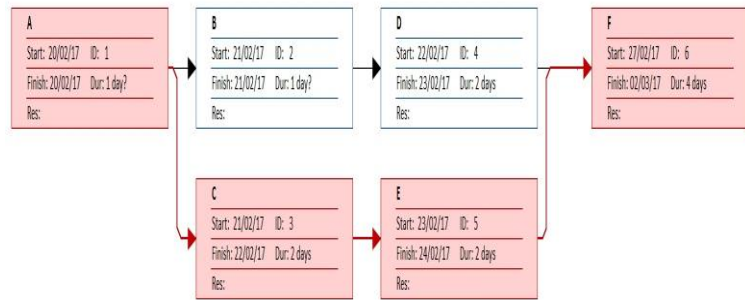
TABLE4. INPUT DATA FOR CRISP CPM.

Activity	Immediate Predecessors	Activity Time(days)
A	-	1
B	A	1
C	A	2
D	B	2
E	C	2
F	D,E	4

After obtaining crisp values of activity time we can solve the critical path method easily, and determine critical path efficiently.

To draw network of activities with critical path we used Microsoft project program.

Fig. 2. Network of activities with critical path



From Fig.2, we find that the critical path is A-C-E-F and is denoted by red line.

The expected project completion time = $t_A + t_C + t_E + t_F = 9$ days.

V. CONCLUSION

Neutrosophic set is a generalization of classical set, fuzzy set and intuitionistic fuzzy set because it not only considers the truth-membership and falsity- membership but also an indeterminacy function which is very obvious in real life situations. In this paper, we have considered activity time of CPM as triangular neutrosophic numbers and we used score function to obtain crisp values of activity time. In future, the research will be extended to deal with different project management techniques.

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