FLORENTIN SMARANDACHE

A Generalization of the Inequality of Minkowski

In Florentin Smarandache: “Collected Papers”, vol. I (second edition). Ann Arbor (USA): InfoLearnQuest, 2007.

Theorem : If p is a real number ≥ 1 and $a_i^{(k)} \in \mathbf{R}^+$ with $i \in \{1,2,...,n\}$ and $k \in \{1,2,...,m\}$, then:

$$\left(\sum_{i=1}^{n} \left(\sum_{k=1}^{m} a_i^{(k)} \right)^p \right)^{1/p} \leq \left(\sum_{k=1}^{m} \left(\sum_{i=1}^{n} a_i^{(k)} \right)^p \right)^{1/p}$$

Demonstration by recurrence on $m \in \mathbf{N}^*$.
First of all one shows that:

$\left(\sum_{i=1}^{n} \left(a_i^{(1)} \right)^p \right)^{1/p} \leq \left(\sum_{i=1}^{n} \left(a_i^{(1)} \right)^p \right)^{1/p}$, which is obvious, and proves that the inequality is true for $m = 1$.

(The case $m = 2$ precisely constitutes the inequality of Minkowski, which is naturally true!).

Let us suppose that the inequality is true for all the values less or equal to m

$$\left(\sum_{i=1}^{n} \left(\sum_{k=1}^{m+1} a_i^{(k)} \right)^p \right)^{1/p} \leq \left(\sum_{i=1}^{n} a_i^{(1)^p} \right)^{1/p} + \left(\sum_{i=1}^{n} \left(\sum_{k=2}^{m+1} a_i^{(k)} \right)^p \right)^{1/p} \leq$$

$$\leq \left(\sum_{i=1}^{n} \left(a_i^{(1)} \right)^p \right)^{1/p} + \left(\sum_{k=2}^{m+1} \left(\sum_{i=1}^{n} a_i^{(k)} \right)^p \right)^{1/p}$$

and this last sum is $\left(\sum_{k=1}^{m+1} \left(\sum_{i=1}^{n} a_i^{(k)} \right)^p \right)^{1/p}$ therefore the inequality is true for the level $m + 1$.