A NEW APPROACH TO ALGEBRAIC CODING THEORY THROUGH THE APPLICATIONS OF SOFT SETS

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Abstract. Algebraic codes play a significant role in the minimisation of data corruption which caused by deffects such as inference, noise channel, crosstalk, and packet loss.In this paper, we introduce soft codes (soft linear codes) through the application of soft sets which is an approximated collection of codes. We also discuss several types of soft codes such as type-1 soft codes, complete soft codes etc. Further, we constrcut the soft generator matrix and soft parity check matrix for the soft linear codes. Moreover, we develop two techinques for the decoding of soft codes.

1. Intorduction

The transmission and storage of larage amounts of data reliably and without error is a significant part of the modern communication systems. Algebriac codes are used for data compression, cryptography, error correction and for network coding. The theory of codes was first focused by Shanon in 1948 and then gradually developed by time to time by different reserchers. There are many types of codes which is important to its algebriac structures such as Linear block codes, Hamming codes, BCH codes [40] and so on. The most common type of code is a linear code over the field F_q. Recently a variety of codes over finite rings have been studied. The linear codes over finite rings are initiated by Blake in a series of papers [10, 11], Spiegel [43, 44] . Huber defined codes over Gaussian integers [21, 22, 23]. Shankar [40] studied BCH codes over rings of residue integers. Satyanarayana [38] consider analyses of codes over $\mathbb{Z}_n$ by viewing their properties under the Lee metric. Some more literature can be studied in [8, 14, 15, 16, 17, 19, 25, 26, 30, 32] .

Zadeh in his seminal paper [47] introduced the innovative concept of fuzzy sets in 1965. A fuzzy set is characterized by a membership function whose values are defined in the unit interval $[0, 1]$ and thus fuzzy set perhaps is the most suitable framewrok to model uncertain data. Fuzzy sets have a several interesting applications in the areas such as signal processing, decision making, control theory, reasoning, pattern recognition, computer version and so on. The theory of fuzzy set is a significantly used in medical diagnosis, social science, engineering etc. The algebraic strucutres in the context of fuzzy sets have been studied such as fuzzy groups, fuzzy rings, fuzzy semigroups fuzzy codes etc. Some more study on fuzzy set can be found in [46, 48].

The theory of rough sets [33, 34] was first introduced by Pawlak in 1982 which is another significant mathematical tool to hanlde vague data and information. The theory of rough sets mainly based upon equivalence classes to approximate crisp

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sets. Rough sets has several applications in data mining, machine learning, medicine, data analysis, expert systems and cognitive analysis etc. Some more literature can be found on rough sets in [18, 24, 35, 36, 37, 41, 45]. Algebraic structures can also be studied in the context of rough set such rogh groups [9], rough semigroups and so on.

The complexities of modelling uncertain data is the main problem in engineering, environmental science, economics, social sciences, health and medical sciences etc. Classical theories are not always successful as the uncertainties are of several types which appearing in these domains. The fuzzy set theory [31], probability theory, rough set theory [33, 34] etc are well known and useful mathematical tools which describe uncertainty but each of them has its own limitation pointed out by Molodstov. Therefore, Molodstov intorduced the theory of soft sets [31] to model vague and uncertain information. A soft set a parameterized collection of subsets of a universe of discourse. This mathematical tool is free from parameterization inadequacy, syndrome of fuzzy set theory, rough set theory, probability theory and so on. Soft set theory has been applied successfully in several areas such as, smoothness of functions, game theory, operation reaserch, Riemann integration, Perron integration, and probability. Maji et. al [27, 28, 29] gave the application of soft sets in decision making problem. Recently soft set theory attained much attention of the researchers since its appearance and start studying soft algebraic strucutres. Aktas and Cagman [1] introduced soft groups which laid down the foundations to study algebraic structures in the contex of soft sets. Some properties and algebra may be found in [2] . Feng et al. studied soft semigroups in [20]. A huge amount of literature can be seen in [3, 4, 5, 6, 7, 39, 42] .

The main purpose of this paper is to introduce alegebraic soft coding theory which extend the notion of a code to soft sets. A soft code is a parameterized collection of codes. Different types of error correcting codes have been extend to constructuct soft error correcting codes. A variety of soft codes can be found by applying soft sets to codes. Soft linear codes have been discussed mainly in this paper. The novel concept of soft dimension have been introduced which infact a generalization of the dimension of a code and the concept of soft minimum distance is introduced here. Soft codes of *type* 1 has been established in this paper. The important notions of soft generator matrix as well as soft parity check matrix have been constructed to study more features of soft linear codes. Further, the notions of soft complete codes are introduced in this paper and in the end two soft decoding algorithm has been costructed in this paper.

The organization of this paper is as follows: In section 2, basic concpets of soft sets and cods are presented. In section 3, the important notions of soft codes are given with the study of some of their basic properties and features. In section 4, soft generator matrix and soft parity check matrix are presented. Section 5 is about the soft decoding of soft codes. In this, two examples are presented for the verification of soft decoding process. Conclusion is given in section 6.

2. Basic concepts

2.1. Codes.

Definition 1. [25]. *Let A be a finite set of q symbols where $(q > 1)$ and let $V = A^n$ be the set of n-tuples of elements of A where n is some positive integer greater than*

1.*In fact* V *is a vecotr space over* A*. Now let* C *be a non empty subset of* V*. Then* C *is called a* q*-ary code of lenght* n *over* A*.*

Definition 2. [25]*. Let* F^n *be a vector space over the field* F*, and* $x, y \in F^n$ *where* $x = x_1x_2...x_n, y = y_1y_2...y_n$*. The Hamming distance between the vectors* x *and* y *is denoted by* $d(x, y)$*, and is defined as* $d(x, y) = |i : x_i \neq yi|$*.*

Definition 3. [25]*. The minimum distance of a code* C *is the smallest distance between any two distinct codewords in* C *which is denoted by* $d(C)$*; that is* $d(C) = \min\{d(x, y) : x, y \in C,\ x \neq y\}$*.*

Definition 4. [25]*. Let* F *be a finite field and* n *be a positive integer. Let* C *be a subspace of the vector space* $V = F^n$*. Then* C *is called a linear code over* F*.*

Definition 5. [25]*. The linear code* C *is called linear* $[n, k]$*-code if* $dim(C) = k$*.*

Definition 6. [25]*. Let* C *be a linear* $[n, k]$*-code. Let* G *be a* $k \times n$ *matrix whose rows form basis of* C*. Then* G *is called generator matrix of the code* C*.*

Definition 7. [25]*. Let* C *be an* $[n, k]$*-code over* F*. Then the dual code of* C *is defined to be*

$$C^{\perp} = \{y \in F^n : x \cdot y = 0 \text{ for all } x \in C\}$$

Definition 8. [25]*. Let* C *be an* $[n, k]$*-code and let* H *be the generator matrix of the dual code* $C^{\perp}$*. Then* H *is called a parity-check matrix of the code* C*.*

Definition 9. [25]*. A code* C *is called self-orthogonal code if* $C \subset C^{\perp}$*.*

Definition 10. [25]*. Let* C *be a code over the field* F *and for every* $x \in F^n$*, the coset of* C *is defined to be*

$$C_c = \{x + c : c \in C\}$$

Definition 11. [25]*. Let* C *be a linear code over F. The coset leader of a given coset* C *is defined to be the vector with least weight in that coset.*

Definition 12. [25]*. If a codeword* x *is transmitted and the vector* y *is received, then* $e = y - x$ *is called error vector. Therefore a coset leader is the error vector for each vector* y *lying in that coset.*

2.2. **Soft set.** Throughout this subsection U refers to an initial universe, E is a set of parameters, $P(U)$ is the power set of U, and $A \subset E$. Molodtsov [31]. defined the soft set in the following manner:

Definition 13. [31]*. A pair* (F, A) *is called a soft set over* U *where* F *is a mapping given by* $F : A \longrightarrow P(U)$*.*

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $a \in A$, $F(a)$ may be considered as the set of a-elements of the soft set (F, A), or as the set of a-approximate elements of the soft set.

Definition 14. [28]*. For two soft sets* (F, A) *and* (H, B) *over* U*,* (F, A) *is called a soft subset of* (H, B) *if*

(1) $A \subseteq B$ and
(2) $F(e) \subseteq G(e)$, for all $e \in A$.

This relationship is denoted by $(F,A) \tilde{\subset} (H,B)$. Similarly (F,A) is called a soft superset of (H,B) if (H,B) is a soft subset of (F,A) which is denoted by $(F,A) \tilde{\supset} (H,B)$.

Definition 15. [28]. *Two soft sets (F,A) and (H,B) over U are called soft equal if (F,A) is a soft subset of (H,B) and (H,B) is a soft subset of (F,A).*

3. Soft code

Definition 16. *Let K be a finite field and $V = K^n$ be a vector space over K where n is a positive integer. Let $P(V)$ be the power set of V and (F,A) be a soft set over V. Then (F,A) is called soft linear code over V if and only if $F(a)$ is subspace of V which is a linear code.*

Example 1. *Let $K = K_2$ and $V = K_2^3$ is a vector space over K_2 and let (F,A) be a soft set over $V = K_2^3$. Then clearly (F,A) is a soft linear code over $V = K_2^3$, where*

$$\begin{aligned} F(a_1) &= \{000, 111\}, \\ F(a_2) &= \{000, 110, 101, 011, 111\}. \end{aligned}$$

Definition 17. *Let (F,A) be a soft code over the field K. Then the codeword of (F,A) is called soft codeword, which is denoted by Y_s.*

Definition 18. *Let (F,A) be a soft code over $V = K^n$. Then D_s is called soft dimension of (F,A) if*

$$D_s = (\dim(F(a)) \text{ forall } a \in A)$$

The soft dimension D_s of the soft code is simply an m-tuple, where m is the number of parameters in the parameter set A.

Example 2. *Let (F,A) be a soft code defined in above example. Then the soft dimension is as follows,*

$$\begin{aligned} D_s &= (\dim(F(a_1)) = 1, \dim(F(a_2)) = 2) \\ &= (1,2) \end{aligned}$$

Definition 19. *A soft linear code (F,A) over V of soft dimension D_s is called soft linear $[n, D_s]$-code.*

Definition 20. *Let (F,A) be a soft code over V. Then the soft minimum distance of (F,A) is denoted by $S_d(F,A)$ and is defined to be*

$$S_d(F,A) = \begin{pmatrix} d(F(a)) : d(F(a)) \text{ is the minimum distance of the code } F(a), \\ \text{for all } a \in A. \end{pmatrix}$$

Example 3. *Let $K = K_2$ be a field and $V = K_2^3$ is a vector space over K_2 and let (F,A) be a soft set over $V = K_2^3$. Then clearly (F,A) is a soft code over $V = K_2^3$, where*

$$\begin{aligned} F(a_1) &= \{000, 111\}, \\ F(a_2) &= \{000, 110, 101, 011\}. \end{aligned}$$

the minimum distance of the code $F(a_1) = 3$ and $F(a_2) = 2$. Thus the soft minimm distance of the soft code (F, A) is given as

$$\begin{aligned} S_d(F, A) &= (d(F(a_1)) = 3, d(F(a_2)) = 2) \\ &= (3, 2) \end{aligned}$$

Definition 21. *A soft code (F, A) in V over the field K is called soft code of type 1, if the dimension of $F(a)$ is same, for all $a \in A$.*

Example 4. *Let (F, A) be a soft code in V over the field K_2^2, where*

$$F(a_1) = \{00, 01\}, \ F(a_2) = \{00, 10\}$$

The sofd dimension D_s of (F, A) is as follows,

$$D_s = (1, 1)$$

Theorem 1. *Every soft code of type 1 is trivially a soft code but the converse is not true.*

For converse, we take the following example.

Example 5. *Let $K = K_2$ and $V = K_2^3$ is a vector space over K_2 and let (F, A) be a soft set over $V = K_2^3$. Then (F, A) is a soft linear code over $V = K_2^3$, where*

$$\begin{aligned} F(a_1) &= \{000, 111\}, \\ F(a_2) &= \{000, 110, 101, 011, 111\}. \end{aligned}$$

Then clearly (F, A) is not soft code of *type*1.

4. Soft Generator Matrix and Soft Parity Check Matrix

Definition 22. *Let (F, A) be a soft linear $[n, D_s]$-code. Let G_s be the super matrix whose elements are the generator matrices of the soft code (F, A), corresponding to each $a \in A$.*

Then G_s is termed as the soft generator matrix of the soft linear code (F, A).

Example 6. *Let $K = K_2$ be a field and $V = K_2^3$ is a vector space over K_2 and let (F, A) be a soft set over $V = K_2^3$. Then clearly (F, A) is a soft code over $V = K_2^3$, where*

$$\begin{aligned} F(a_1) &= \{000, 111\}, \\ F(a_2) &= \{000, 110, 101, 011\}. \end{aligned}$$

$F(a_1)$ has generator matrix $G_{F(a_1)} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$ and $F(a_2)$ has generator matrix $G_{F(a_2)} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$.

Then the soft generator matix of the soft code (F, A) is as follows.

$$G_s = \left[\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \middle\| \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \right]$$

Remark 1. *The soft generator matrix G_s of a soft linear code (F, A) is not unique.*

Definition 23. *Let (F, A) be a soft $[n, D_s]$-code over the field K and the vector space V. Then the soft dual code of (F, A) is defined to be*

$$(F, A)^{\perp} = \left\{ F(a)^{\perp} : F(a)^{\perp} \text{ is the dual code of } F(a), \text{ for all } a \in A \right\}$$

Example 7. *Let $K = K_2$ be a field and $V = K_2^3$ is a vector space over K_2 and let (F, A) be a soft set over $V = K_2^3$. Then clearly (F, A) is a soft code over $V = K_2^3$, where*

$$\begin{aligned} F(a_1) &= \{000, 111\}, \\ F(a_2) &= \{000, 110, 101, 011\}. \end{aligned}$$

Then the soft dual code of (F, A) is $(F, A)^{\perp}$, where

$$\begin{aligned} F(a_1)^{\perp} &= \{000, 110, 101, 011\}, \\ F(a_2)^{\perp} &= \{000, 111\}. \end{aligned}$$

Definition 24. *A soft linear code (F, A) in $V = K^n$ over the field K is called complete-soft code or simply if for all $a \in A$, the dual of $F(a)$ also exist in (F, A).*

Example 8. *Let $K = K_2$ be a field and $V = K_2^3$ is a vector space over K_2 and let (F, A) be a soft set over $V = K_2^3$. Then clearly (F, A) is a soft code over $V = K_2^3$, where*

$$\begin{aligned} F(a_1) &= \{000, 111\}, \\ F(a_2) &= \{000, 110, 101, 011\}. \end{aligned}$$

Then clearly (F, A) is a complete-soft code because the dual of $F(a_1)$ is $F(a_2)$ which is present in (F, A) and also the dual of $F(a_2)$ is $F(a_1)$.

Theorem 2. *All complete-soft codes are trivially soft codes but the converse is not true in general.*

Theorem 3. *A complete-soft code (F, A) over the field K and the vector space V is the parametrized collection of the codes C with its dual code $C^{\perp}$.*

Definition 25. *Let (F, A) be a soft code over the field K and the vector space V and $(F, A)^{\perp}$ be the soft dual code of (F, A). Then the soft dimension of the soft dual code $(F, A)^{\perp}$ is denoted by $(D_s)^{\perp}$ and is defined as*

$$(D_s)^{\perp} = \left(\begin{array}{c} \dim\left(F(a)^{\perp}\right) : \dim\left(F(a)^{\perp}\right) \text{ is the dimnesion of the dual code } F(a), \\ \text{for all } a \in A. \end{array} \right)$$

Example 9. *In previuos example the soft dimension of the dual code $(F, A)^{\perp}$ is following*

$$\begin{aligned} (D_s)^{\perp} &= \left(\dim\left(F(a_1)^{\perp}\right) = 2, \dim\left(F(a_1)^{\perp}\right) = 1\right) \\ &= (2, 1) \end{aligned}$$

Definition 26. *Let (F, A) be a soft over the field K and the vector space V and let H_s be the soft generator matix of the soft dual code $(F, A)^{\perp}$. Then H_s is called the soft parity check matrix of the soft code (F, A).*

Example 10. *In above example the soft dual code of (F, A) is $(F, A)^{\perp}$, where*

$$\begin{aligned} F(a_1)^{\perp} &= \{000, 110, 101, 011\}, \\ F(a_2)^{\perp} &= \{000, 111\}. \end{aligned}$$

The soft generator matrix of $(F,A)^{\perp}$ is $S(H)$, where

$$H_s = \left[\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} | \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \right]$$

Where $H_{F(a_1)} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ is a parity check matrix of $G_{F(a_1)}$ and $H_{F(a_2)} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$ is a parity check matrix of $G_{F(a_2)}$.

Remark 2. *A soft parity check matrix of soft linear code (F,A) is not unique.*

Theorem 4. *Let (F,A) be a soft code over the field K and the vector space V. Let G_s and H_s be the soft generator matrix and soft parity check matix of the soft code (F,A). Then*

$$G_s H_s^T = 0$$

Definition 27. *A soft linear code (F,A) in V over the field K is called soft self dual code if $(F,A)^{\perp} = (F,A)$.*

Definition 28. *Let (F,A) be a soft code over the field K and the vector space V. Let G_s be the soft generator matrix of (F,A). Then the soft canonical generator matrix of (F,A) is denoted by G_s^* and is defined as*

$$G_S^* = \begin{bmatrix} G_{F(a)}^* \end{bmatrix}$$

where $G_{F(a)}^*$ is the canonical generator matrix of $F(a)$, for all $a \in A$.

Example 11. *Let (F,A) be a soft code over $V = K_2^5$, where*

$$\begin{aligned} F(a_1) &= \left\{ \begin{array}{c} 00000, 10010, 01001, 00110, 11011, \\ 10100, 01111, 11101 \end{array} \right\}, \\ F(a_2) &= \{00000, 11111, 10110, 01001\}. \end{aligned}$$

The soft canonical generator matrix of (F,A) is as under,

$$G_s^* = \left[\begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \middle\| \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix} \right]$$

where $G_{F(a_1)}^* = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$, $G_{F(a_2)}^* = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$ are respectively the canonical generator matrices of $F(a_1)$ and $F(a_2)$.

Theorem 5. *Let (F,A) be a soft code. If (F,A) has a soft canonical generator matrix*

$$G_s^* = \left[G_{F(a)}^* = \begin{bmatrix} I_k \vdots A \end{bmatrix}, \text{ for all } a \in A \right]$$

Then

$$H_s^* = \left[H_{F(a)}^* = \begin{bmatrix} -A^T \vdots I_{n-k} \end{bmatrix}, \text{ for all } a \in A \right]$$

is the soft canonical parity check matrix of (F,A). Conversely, if

$$H_s^* = \left[H_{F(a)}^* = \begin{bmatrix} B \vdots I_{n-k} \end{bmatrix}, \text{ for all } a \in A \right]$$

is the soft canonical parity check matrix of (F,A), then

$$G_s^* = \left[G_{F(a)}^* = \left[\begin{array}{c} I_k \vdots -B^T \end{array} \right], \text{for all } a \in A \right].$$

5. Soft Decoding Algorithems

5.1. Soft Standard Array Decoding.

Definition 29. *Let (F,A) be a soft linear code or soft subspace of $V = K^n$ over the field K and for every $x \in K^n$,*

$$(F,A)_C = \{x + F(a)\,, \text{ for all } a \in A\}$$

is called the soft coset of (F,A) and is denoted by $(F,A)_C$.

Example 12. *Let Let $K = K_2$ be a field and $V = K_2^3$ is a vector space over K_2 and let (F,A) be a soft set over $V = K_2^3$. Then clearly (F,A) is a soft code over $V = K_2^3$, where*

$$\begin{aligned} F(a_1) &= \{000, 111\}, \\ F(a_2) &= \{000, 110, 101, 011\}. \end{aligned}$$

Then the soft cosets of the soft code (F,A) are as follows.
$(F,A)_{C_1} = \{000 + F(a_1), 000 + F(a_2)\} = (F,A)$,
$(F,A)_{C_2} = \{100 + F(a_1), 100 + F(a_2)\} = \{\{100, 011\}, \{100, 010, 001, 111\}\}$,
$(F,A)_{C_3} = \{010 + F(a_1), 010 + F(a_2)\} = \{\{010, 101\}, \{100, 010, 001, 111\}\}$,
$(F,A)_{C_4} = \{001 + F(a_1), 001 + F(a_2)\} = \{\{001, 110\}, \{100, 010, 001, 111\}\}$.

Definition 30. *Let (F,A) be a soft linear code in $V = K^n$ over the field K. The soft coset leader of a given soft coset $(F,A)_C$ is denoted by E_i and is defined to be*

$$E_i = \{u : u \text{ is the coset leader of } F(a)\,, \text{ for all } a \in A\}$$

Example 13. *In above example,the following are the soft coset leaders of the soft coset (F,A),*

$$E_1 = \{000, 000\},$$

$$E_2 = \{100, 100\},$$

$$E_3 = \{010, 010\},$$

$$E_4 = \{001, 001\}.$$

and so on.

Definition 31. *A set of standard arrays of the corresponding parametrized codes is called soft standard array for the soft linear code (F,A).*

Example 14. *Let $K = K_2$ be a field and $V = K_2^3$ is a vector space over K_2 and let (F,A) be a soft set over $V = K_2^3$. Then clearly (F,A) is a soft code over $V = K_2^3$, where*

$$\begin{aligned} F(a_1) &= \{000, 111\}, \\ F(a_2) &= \{000, 110, 101, 011\}. \end{aligned}$$

Then the soft cosets of (F, A) are as follows.
$(F,A)_{C_1} = \{000 + F(a_1), 000 + F(a_2)\} = (F, A)$,
$(F,A)_{C_2} = \{100 + F(a_1), 100 + F(a_2)\} = \{\{100, 011\}, \{100, 010, 001, 111\}\}$,
$(F,A)_{C_3} = \{010 + F(a_1), 010 + F(a_2)\} = \{\{010, 101\}, \{100, 010, 001, 111\}\}$,
$(F,A)_{C_4} = \{001 + F(a_1), 001 + F(a_2)\} = \{\{001, 110\}, \{100, 010, 001, 111\}\}$.
The following are the coset leaders of the soft linear code (F, A),

$$E_1 = \{000, 000\},$$

$$E_2 = \{100, 100\},$$

$$E_3 = \{010, 010\},$$

$$E_4 = \{001, 001\}.$$

The soft standard array for the soft linear code (F, A) is following,

$$\left\{ \begin{matrix} 000 & 111 \\ 100 & 011 \\ 010 & 101 \\ 001 & 110 \end{matrix}, \begin{matrix} 000 & 110 & 101 & 011 \\ 100 & 010 & 001 & 111 \\ 010 & 100 & 111 & 001 \\ 001 & 111 & 100 & 010 \end{matrix} \right\}$$

By soft standard array,we can decode a set of codewords at a time, because we decode codeword corresponding to each parameter.

Let we want to decode a soft codeword $Y_s = \{110, 111\}$ in the soft standard array. We find the positions of the codewords in the parameterized standard arrays. Since 110 occures in the second column of the first parametrized array and the top entry in that column is 111, so 110 is decoded as 111. Now look at the position of 111 in the second parameterized array. Since 111 is occure in all the three columns,so 111 can be decoded as all the non-trivial codewords. By taking any one codewrods,we get the dcoded codewords. Let we take 111 occures in the first column and the top entry in that column is 110. Hence the set of codewords $\{110, 111\}$ is decoded as $\{111, 110\}$.

5.2. **Soft Syndrome Decoding.**

Definition 32. *Let (F, A) be a soft code over K with soft parity check matrix H_S. For any vector $Y_s \subseteq K^n$. The soft syndrome of Y_s is defined as $S(Y_s) = Y_s (H_s)^T$.*

Theorem 6. *Let (F, A) be a soft code over the field K. For $Y_s \subseteq K^n$, the soft codeword nearest to Y_s is given by $X_s = Y_s - E_s$, where E_s is the soft coset leader.*

Let (F, A) be a soft code over the field K with soft parity check matrix $S(H)$. For soft syndrome, we first find all the soft coset of the soft code (F, A) and then find the soft coset leaders E_S which are infact the collection of coset leaders corresponding to each parameterized code. Now we compute the soft syndrome for all the soft coset leaders and then make a table of soft coset leaders with their soft syndroms. To decode a soft codeword say Y_S, we simply find the soft syndrome of that soft codeword and then compare their soft syndrome with soft coset leader syndrome. After comparing their soft syndromes, we then subtract the soft coset leader from the soft decoded word. Hence Y_S is soft decoded as $X_S = Y_S - E_s$.

Example 15. *Let* (F, A) *be a soft code in* $V = K_2^3$ *over the field* $K = Z_2$*, where*

$$\begin{aligned} F(a_1) &= \{000, 111\}, \\ F(a_2) &= \{000, 110, 101, 011\}. \end{aligned}$$

The soft parity check matrix of (F, A) is H_s, where

$$H_s = \left[\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} | \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \right]$$

The transpose of H_s is following,

$$(H_s)^T = \left[\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \Bigg| \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right]$$

Then the soft cosets of (F, A) are as follows.
$(F, A)_{C_1} = \{000 + F(a_1), 000 + F(a_2)\} = (F, A)$,
$(F, A)_{C_2} = \{100 + F(a_1), 100 + F(a_2)\} = \{\{100, 011\}, \{100, 010, 001, 111\}\}$,
$(F, A)_{C_3} = \{010 + F(a_1), 010 + F(a_2)\} = \{\{010, 101\}, \{100, 010, 001, 111\}\}$,
$(F, A)_{C_4} = \{001 + F(a_1), 001 + F(a_2)\} = \{\{001, 110\}, \{100, 010, 001, 111\}\}$.
The following are the soft coset leaders E_i of the soft code (F, A),

$$E_1 = \{000, 000\},$$

$$E_2 = \{100, 100\},$$

$$E_3 = \{010, 010\},$$

$$E_4 = \{001, 001\}.$$

Soft syndrome table of soft coset leader is given as follows.

soft coset leader	soft syndrome
$\{000, 000\}$	$\{00, 0\}$
$\{100, 100\}$	$\{10, 1\}$
$\{010, 010\}$	$\{11, 1\}$
$\{001, 001\}$	$\{01, 1\}$

We want to decod a soft codeword $Y_s = \{110, 101\}$. First we find the soft syndrom of the soft codeword.

$$S(Y_s) = y(H_s)^T = [110, 100] \left[\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \Bigg| \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right] = [01, 1]$$

Since $S(\{110, 100\}) = \{01, 1\} = S(\{001, 001\})$.
The decoded soft codeword is

$$\begin{aligned} S(Y_s) - S(E_4) &= X_s \\ S(\{110, 100\}) - S(\{001, 001\}) &= \{110 - 001, 100 - 001\} \\ &= \{111, 101\}. \end{aligned}$$

Hence the soft codesword $Y_s = \{110, 100\}$ is decoded as $X_s = \{111, 101\}$. By similar passion we can find all the soft decoded codewords.

6. Conclusion

In this paper, we introduced the iportant notions of soft codes (soft linear codes) in the context of soft set. Soft codes are basically an approximated family of codes over a vector space. Soft codes posses a varaity of several new types of codes. We also presented soft generator matix and soft parity check matrix of soft code. In fact super matrix is the soft generator matrix and soft parity check matrix of the soft codes. Furthermore, the techniques of soft decoding of soft codes are given in this paper. A lot of research can be conducted in this area of soft code. In the future, we"ll construct soft polynomial codes.

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