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A POSITION INDICATOR WITH APPLICATIONS IN THE FIELD OF DESIGNING FORMS WITH ARTIFICIAL INTELLIGENCE(14)

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The indicators used so far within the Theory of Extension, see papers [1], [2] and [4], can be synthetically expressed by the notion of "position indicators". More exactly, these indicators can be grouped in two main sub-categories: point-set position indicators and point-two sets position indicators. The secondary goal of this paper is to define these classifications, while the primary goal is that to extend the two notions to the most general notion of set-set position indicator.

Key words: Hausdorff measure, Extension theory, position indicators, computer vision, artificial intelligence.

1. Introduction

The first part of this paper aims at defining the notion of point-set position indicator and that of point-two sets position indicator, at discussing the main examples of such indicators and their relevance for the applicative field.

Point-set Position Indicators

For any point $x \in \mathbb{R}^n$ and any set $A \subset \mathbb{R}^n$, formula $\delta(x,A) = \inf\{d(x,a) | a \in A\}$, where d is the Euclidean distance on $\mathbb{R}^n$, defines (in classical mathematics) the distance from point x to set A. Based on the properties characteristic for the notion of distance, every time we have $\delta(x,A) > 0$ we can conclude that point x lies outside set $\overline{A}$ (closure of set A in relation to the usual topology of space $\mathbb{R}^n$) at distance $\delta(x,A)$ from the nearest point of set A. On this

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account expression $\delta(x,A)$ somehow takes the role of position indicator of x towards A, but not entirely, because in the case $\delta(x,A)=0$ the sole information provided is that $x \in A$, with no further indication of how far or how close to frontier ∂A of set A this point lies. In many concrete situations the knowledge of these details can be more useful. For example, when A symbolizes the 2-dimensional representation of a risk zone for a human (a very deep lake, a region contaminated with toxic substances etc.) and x symbolizes the position vector of a human situated in the interior of that zone, it would be useful for that human to know how close or far the exit ways are. It thus, becomes necessary the extension of the classical notion of distance by taking into account some indicators that deal with more requirements. This aspect has been pointed out by other researchers in their papers, as well, see, for example [4]. The most general point-set position indicator aiming at fulfilling the requirements formulated above is given by [2] and takes the form

$$\mathfrak{s}(x,A)=\begin{cases} \delta(x,A), & x \in \complement A \\ -\delta(x,\complement A), & x \in A \end{cases}, \tag{1}$$

where $\complement A$ represents the absolute complement of A, i.e. $\complement A = \mathbb{R}^n \setminus A$. Given the way it has been defined, indicator $\mathfrak{s}(x,A)$ has the following properties: $x \in \complement \overline{A} \Leftrightarrow \mathfrak{s}(x, A) > 0$; $x \in \overset{o}{A}$[18] $\Leftrightarrow \mathfrak{s}(x, A) < 0$; $x \in \partial A$[19] $\Leftrightarrow \mathfrak{s}(x, A) = 0$, respectively.

Observations: 1) The indicator defined in expression (1) does not represent a distance; it expresses the distance from x to A only when point x is exterior to set A.

2) Other examples of point-set position indicators can be found in [1] and [4].

Point-two Sets Position Indicators

Paper [2], mentioned above, provides us with an example of a point-two sets indicator, as well. This takes the expression

[18] $\overset{o}{A}$ represents the interior of set A in the usual topology of space $\mathbb{R}^n$.

[19] ∂A represents the boundaries of set A, namely $\partial A = \overline{A} \setminus \overset{o}{A}$.

$$\mathfrak{S}(x,A,B)=\frac{\mathfrak{s}(x,A)}{\mathfrak{s}(x,B)-\mathfrak{s}(x,A)}, \tag{2}$$

where $x\in\mathbb{R}^n$, A and B are two sets from $\mathbb{R}^n$ with the property $\overline{A}\subset\overset{o}{B}$, and $\mathfrak{s}$ is indicator (1). This indicator has the following properties: $\mathfrak{S}(x,A,B)<-1 \Leftrightarrow x\in\complement\overline{B}$; $-1\le\mathfrak{S}(x,A,B)<0 \Leftrightarrow x\in\overline{B}\setminus\overline{A}$; $\mathfrak{S}(x,A,B)\ge 0 \Leftrightarrow x\in\overline{A}$.

Observations: 1) The properties presented earlier justify the designation of "point – two sets position indicator" given to indicator (2).
2) Other examples of point – two sets position indicators can be found in [1, 4].

2. Set-set Position Indicators

This paragraph focuses on the presentation of new results aiming at further developing and improving the existent theory of Extension. The frame we shall refer to in our discussion is that of any metric space $(X,\, d)$. The mathematical apparatus we would like to advance further on requires to take into consideration the notion of "Hausdorff measure". To ensure a better understanding of the concepts presented, we have synthetized the minimum of knowledge required in this regard in the appendix.

Let A and B be two non-empty sets from X. About set A we additionally assume that it admits a Hausdorff measure of dimension $r\ge 0$, $\mathcal{H}^r(A)$ is finite and nonzero. Under these conditions, by using indicator $\mathfrak{s}$ defined by the generalized relation (1) from the Euclidean metric space $\mathbb{R}^n$ for the actual metric space $(X,\, d)$, we are able to consider the expression

$$\mathsf{S}(A,\, B)=\frac{\mathcal{H}^r\left(\left\{a\in A\mid \mathfrak{s}(a,\, B)\le 0\right\}\right)}{\mathcal{H}^r(A)}, \tag{3}$$

which accurately defines the indicator we wanted to introduce.

This indicator fulfills several mathematical properties important to the applicative field:

Proposition 1: $\mathsf{S}(A,\, B)=0 \Leftrightarrow A\cap B=\varnothing$ $\mathcal{H}^r$- almost everywhere (or differently expressed, $\mathcal{H}^r(A\cap B)=0$).

Demonstration: $\mathsf{S}(A,\, B)=0 \Leftrightarrow \mathcal{H}^r\left(\left\{a\in A\mid \mathfrak{s}(a,\, B)\le 0\right\}\right)=0$. Since $\mathfrak{s}(a,\, B)\le 0$ is equivalent to relation $a\in\overline{B}$, we deduce that $\mathcal{H}^r(A\cap B)=0$.

Proposition 2: $\mathsf{S}(A, B) > 0 \Rightarrow A \cap \overline{B} \neq \varnothing$.

Demonstration: $\mathsf{S}(A, B) > 0 \Leftrightarrow \mathcal{H}^r(\{a \in A | \, s(a, B) \leq 0\}) > 0$. From relation $\mathcal{H}^r(\{a \in A | \, s(a, B) \leq 0\}) > 0$ we deduce that there is $a \in A$ so that $s(a, B) \leq 0$. But $s(a, B) \leq 0$ implies that $a \in \overline{B}$, so $A \cap \overline{B} \neq \varnothing$.

Proposition 3: If besides the initial hypothesis made over set A and B, we assume additionally that set $\overline{B}$ is measurable[20] with respect to Hausdorff measure $\mathcal{H}^r$ (regarded as an outer measure on $\mathcal{P}(X)$, the family of all subsets of X), then relation $\mathsf{S}(A, B) = 1$ is equivalently with $A \subseteq \overline{B}$ $\mathcal{H}^r$ - almost everywhere.

Demonstration: $\mathsf{S}(A, B) = 1 \Leftrightarrow \mathcal{H}^r(\{a \in A | \, s(a, B) \leq 0\}) = \mathcal{H}^r(A)$. The way how indicator s has been defined implies that $\{a \in A | \, s(a, B) \leq 0\} = A \cap \overline{B}$. Because set $\overline{B}$ is measurable we have $\mathcal{H}^r(A) = \mathcal{H}^r(A \cap \overline{B}) + \mathcal{H}^r(A \cap \complement\overline{B})$. But $\mathcal{H}^r(A \cap \overline{B}) = \mathcal{H}^r(\{a \in A | \, s(a, B) \leq 0\}) = \mathcal{H}^r(A) \Rightarrow \mathcal{H}^r(A \cap \complement\overline{B}) = 0$, namely $A \subseteq \overline{B}$ $\mathcal{H}^r$- almost everywhere.

Corollary: Let A and B be two closed nonempty sets from X for which there exists a Hausdorff measure of dimension $r \geq 0$ so that $\mathcal{H}^r(A)$ and $\mathcal{H}^r(B)$ are finite and nonzero. If sets A and B are $\mathcal{H}^r$- measurable and if $\mathsf{S}(A,B) = 1$ and $\mathsf{S}(B,A) = 1$, then $A = B$ $\mathcal{H}^r$- almost everywhere, and reciprocally.

Demonstration: From proposition 3 it results that $\mathsf{S}(A, B) = 1 \Leftrightarrow A \subseteq B$ $\mathcal{H}^r$- almost everywhere and $\mathsf{S}(B, A) = 1 \Leftrightarrow B \subseteq A$ $\mathcal{H}^r$- almost everywhere. Then $A = B$ $\mathcal{H}^r$- almost everywhere.

[20] By definition we say that set $\overline{B}$ is $\mathcal{H}^r$ measurable if for any $T \subseteq X$ relation $\mathcal{H}^r(T) = \mathcal{H}^r(T \cap \overline{B}) + \mathcal{H}^r(T \cap \complement\overline{B})$ takes place.

Observations: 1) From the definition of indicator S, (relation (3)) it can be easily deduced that $0 \le \mathsf{S}(A,B) \le 1$, for any pair of non-empty subsets A and B of space X for which set A admits a Hausdorff measure $\mathcal{H}^r$ of dimension $r \ge 0$, so that $\mathcal{H}^r(A) \ne 0$ and $\mathcal{H}^r(A) < \infty$.
2) The properties presented earlier within propositions 1 - 3 aim at justifying the designation of "set-set position indicator" that indicator (3) receives.
3) Another example of set-set position indicator can be found in [3].

3. Applications

Indicator S defined by us in this paper can be used as an example for computer vision while developing software applications regarding the automatic inclusion of a certain object $\mathcal{O}$ into a target region $\mathcal{R}$ of a given video image (*VIm*). To realize this, we propose an algorithm which in broad terms has the following content: by means of a set of isometries $\mathcal{J}_i$, $i \in I$ of the plan, we move object $\mathcal{O}$ to different regions and positions of image *VIm* by calculating the value of indicator $\mathsf{S}(\mathcal{J}_i(\mathcal{O}), \mathcal{R})$, each time. Finding that index $i_0 \in I$ for which $\mathsf{S}(\mathcal{J}_{i_0}(\mathcal{O}), \mathcal{R}) = 1$, is equivalent to finding the solution to the problem.

Observations: 1) In some cases solution $\mathcal{J}_{i_0}$ found by using the method presented above can not be fully satisfactory because relation $\mathcal{J}_{i_0}(\mathcal{O}) \subseteq \mathcal{R}$, concerned in $\mathsf{S}(\mathcal{J}_{i_0}(\mathcal{O}), \mathcal{R}) = 1$, (see proposition 3, applied in the case when the sets by which objects $\mathcal{O}$ and $\mathcal{R}$ are being abstractized, are supposed to be compact[21]) is only guaranteed by $\mathcal{H}^r$ - almost everywhere.
2) Just like the algorithm presented in [3], this algorithm can be easily adapted to solving any similar problem in a space with three dimensions, becoming, thus, more useful to the field of designing forms with artificial intelligence.

4. Appendix

Hausdorff measure

Let (X, d) be a metric space, Y a subset from X, and δ a strictly positive real number. A finite or countable collection of sets $\{U_1, U_2, ...\}$ from X with diameter $D(U_1) \le \delta$, $D(U_2) \le \delta$,.., for which $Y \subset U_1 \cup U_2 \cup \cdots$, is called δ-cover of set Y. By virtue of this notion, for any subset Y from X and for any two real numbers $r \ge 0$, and $\delta > 0$, we can define indicator

[21] Given the application we analyze, these hypotheses are as natural as possible

$$\mathcal{H}_\delta^r(Y) = \inf_{\{U_1, U_2, \ldots\} \in \mathcal{C}_\delta(Y)} \left\{ D^r(U_1) + D^r(U_2) + \cdots \right\},$$

where $\mathcal{C}_\delta(Y) = \left\{ \{U_1, U_2, \ldots\} \mid \{U_1, U_2, \ldots\} \text{ is a } \delta-\text{cover of } Y \right\}$. This indicator defines a decreasing function $\delta \to \mathcal{H}_\delta^r(Y)$. This property guaranties the existence of the limit

$$\mathcal{H}^r(Y) = \lim_{\delta \to 0} \mathcal{H}_\delta^r(Y),$$

which, by definition, is called the *r*-dimensional Hausdorff measure of Y.

Among the properties of the Hausdorff measure, $\mathcal{H}^r(\cdot)$, we mention:

1) $\mathcal{H}^r(\varnothing) = 0$;

2) $\mathcal{H}^r(Y_1) \leq \mathcal{H}^r(Y_2)$, if $Y_1 \subseteq Y_2,\ Y_1, Y_2 \in \mathcal{P}(X)$;

3) $\mathcal{H}^r\left(\bigcup_{n=1}^{\infty} Y_n\right) \leq \sum_{n=1}^{\infty} \mathcal{H}^r(Y_n)$, if $\left\{Y_n \mid n \in \mathbb{N}^*\right\} \subset \mathcal{P}(X)$, is any countable collection of sets.

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