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A Study on Hyperfuzzy Hyperrough Sets, Hyperneutrosophic Hyperrough Sets, and Hypersoft Hyperrough Sets with Applications in Cybersecurity

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Abstract

Rough Sets approximate subsets by defining lower and upper bounds, effectively capturing uncertainty through equivalence classes or indiscernibility relations. Additionally, concepts such as Fuzzy Sets, Neutrosophic Sets, and Soft Sets are well-known for addressing uncertainty, with numerous applications explored in various fields.

This paper extends these foundational concepts by introducing six advanced frameworks: the Hyperfuzzy Rough Set, Hyperfuzzy Hyperrough Set, HyperNeutrosophic Rough Set, HyperNeutrosophic Hyperrough Set, Hypersoft Hyperrough Set, and Multigranulation Hyperrough Set. These new models aim to enhance the theoretical understanding and practical handling of uncertainty.

Keywords: Hyperrough set, Rough set, Neutrosophic Set, Hyperfuzzy set, Hypersoft set

1 | Introduction in this Paper

1.1 | Fuzzy Sets, Neutrosophic Sets, Soft Sets, and Rough Sets

Set theory serves as a cornerstone of mathematics, offering a systematic approach for studying collections of objects, commonly referred to as "sets" [24, 146, 145, 69, 84, 73]. This paper examines key advancements in classical set theory, including Fuzzy Sets [152], Neutrosophic Sets [32, 127, 44, 43, 40, 41, 49, 33, 42, 31, 126, 28], and Plithogenic Sets [37, 47, 132, 45, 48, 34, 131], and explores their evolution into more advanced frameworks such as Hyperfuzzy Sets [63], HyperNeutrosophic Sets [35], and Hyperplithogenic Sets [35].

Fuzzy Sets enhance traditional set theory by allowing elements to have degrees of membership, enabling the representation of partial truths within the continuous interval $[0, 1]$ [152, 154, 157]. Neutrosophic Sets build upon this idea by introducing three independent parameters—truth, indeterminacy, and falsity—that each range

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independently within $[0, 1]$ [126, 127]. Extensions of these frameworks include the Hyperfuzzy Set [74, 139, 63] and the Hyperneutrosophic Set [35, 29], which offer additional flexibility and modeling power.

Soft Sets address uncertainty by associating attributes or parameters with subsets of a universal set[94, 97]. Enhanced versions of Soft Sets, such as the Hypersoft Set[130, 30, 46] and the SuperHypersoft Set[16, 56, 22, 134], enable more sophisticated multi-parameter modeling.

Rough Sets, in contrast, approximate subsets by defining lower and upper bounds, capturing uncertainty based on equivalence classes or indiscernibility relations[38, 169, 92, 148, 83, 25]. Advanced extensions, including the Multigranulation Rough Set [116, 163, 13, 89, 123, 86, 115] and the HyperRough Set [38, 36, 35], incorporate multi-relational and multi-attribute considerations.

This paper examines these foundational concepts and their extensions, aiming to promote further exploration and practical application of these advanced theoretical frameworks.

1.2 | Our Contribution in This Paper

In this work, we introduce and analyze the following concepts: Hyperfuzzy Rough Set, Hyperfuzzy Hyperrough Set, HyperNeutrosophic Rough Set, HyperNeutrosophic Hyperrough Set, Hypersoft Hyperrough Set, and Multigranulation Hyperrough Set. Furthermore, to explore their applications in cybersecurity, we provide several concrete examples demonstrating their relevance and effectiveness in this domain.

These concepts extend the scope of existing frameworks in their respective fields. We hope that the development of these new theoretical models will inspire further research and exploration, particularly in their application to solving complex problems.

2 | Preliminaries and Definitions

This section provides an introduction to the foundational concepts and definitions required for the discussions in this paper.

2.1 | Hyperrough set

A Rough Set approximates a subset using lower and upper bounds based on equivalence classes, capturing certainty and uncertainty in membership [105, 112, 109, 106, 106, 110, 107, 111, 108, 161, 121, 9, 17, 149]. The definitions are provided below.

Definition 1 (Rough Set Approximation)**.** [106] Let X be a non-empty universe of discourse, and let $R \subseteq X \times X$ be an equivalence relation (or indiscernibility relation) on X. The equivalence relation R partitions X into disjoint equivalence classes, denoted by $[x]_R$ for $x \in X$, where:

$$[x]_R = \{y \in X \mid (x, y) \in R\}.$$

For any subset $U \subseteq X$, the *lower approximation* $\underline{U}$ and the *upper approximation* $\overline{U}$ of U are defined as follows:

(1) *Lower Approximation* $\underline{U}$:

$$\underline{U} = \{x \in X \mid [x]_R \subseteq U\}.$$

The lower approximation $\underline{U}$ includes all elements of X whose equivalence classes are entirely contained within U. These are the elements that *definitely* belong to U.

(2) *Upper Approximation* $\overline{U}$:

$$\overline{U} = \{x \in X \mid [x]_R \cap U \neq \emptyset\}.$$

The upper approximation $\overline{U}$ contains all elements of X whose equivalence classes have a non-empty intersection with U. These are the elements that *possibly* belong to U.

The pair $(\underline{U}, \overline{U})$ forms the *rough set* representation of U, satisfying the relationship:

$$\underline{U} \subseteq U \subseteq \overline{U}.$$

The *HyperRough Set* is a concept that adapts the framework of the HyperSoft Set[130] to Rough Set theory. Its formal definition is provided below.

Definition 2 (HyperRough Set)**.** [35] Let X be a non-empty finite universe, and let $T_1, T_2, \dots, T_n$ be n distinct attributes with respective domains $J_1, J_2, \dots, J_n$. Define the Cartesian product of these domains as:

$$J = J_1 \times J_2 \times \cdots \times J_n.$$

Let $R \subseteq X \times X$ be an equivalence relation on X, where $[x]_R$ denotes the equivalence class of x under R.

A *HyperRough Set* over X is a pair (F, J), where:

- $F : J \to \mathcal{P}(X)$ is a mapping that assigns a subset $F(a) \subseteq X$ to each attribute value combination $a = (a_1, a_2, \dots, a_n) \in J$.
- For each $a \in J$, the rough set $(\underline{F(a)}, \overline{F(a)})$ is defined as:

$$\underline{F(a)} = \{x \in X \mid [x]_R \subseteq F(a)\}, \quad \overline{F(a)} = \{x \in X \mid [x]_R \cap F(a) \neq \emptyset\}.$$

The *lower approximation* $\underline{F(a)}$ represents the set of elements in X whose equivalence classes are entirely contained within $F(a)$, while the *upper approximation* $\overline{F(a)}$ includes elements whose equivalence classes have a non-empty intersection with $F(a)$.

Additionally, the following properties hold:

- $\underline{F(a)} \subseteq \overline{F(a)}$ for all $a \in J$.
- If $F(a) = \emptyset$, then $\underline{F(a)} = \overline{F(a)} = \emptyset$.
- If $F(a) = X$, then $\underline{F(a)} = \overline{F(a)} = X$.

2.2 | Hyperfuzzy Set

Intuitively, hyperfuzzy Set extends the idea of fuzzy sets [152, 153, 154, 155, 160, 156, 68, 157, 158, 159] into hierarchical structures, allowing for a more nuanced and flexible representation of uncertainty. The formal definition is provided below. A hyperfuzzy set generalizes the traditional fuzzy set framework [74, 103, 93, 120, 53, 118, 75, 67, 15, 139, 63].

Definition 3 (Fuzzy Set)**.** [152, 157] Let Y be a non-empty universe. A *fuzzy set* τ in Y is a function

$$\tau : Y \to [0, 1],$$

where $\tau(y)$ represents the degree of membership of y in the fuzzy set.

A *fuzzy relation* on Y is a fuzzy subset δ of $Y \times Y$. If τ is a fuzzy set in Y and δ is a fuzzy relation on Y, then δ is said to be a *fuzzy relation on* τ if

$$\delta(y, z) \leq \min\{\tau(y), \tau(z)\}, \quad \text{for all } y, z \in Y.$$

Definition 4 (Hyperfuzzy Set)**.** [74, 52, 103, 15, 139, 63, 35] Let X be a non-empty set. A *hyperfuzzy set* over X is a function

$$\tilde{\mu} : X \to \tilde{P}([0, 1]),$$

where $\tilde{P}([0, 1])$ denotes the collection of all non-empty subsets of the interval $[0, 1]$. This generalization allows each element $x \in X$ to be assigned a *set* of membership degrees rather than a single value, capturing a broader range of uncertainty.

Example 5 (Hyperfuzzy Set in Cybersecurity: Intrusion Suspicion Levels)**.** Cybersecurity is the practice of protecting systems, networks, and data from cyber threats, unauthorized access, attacks, or damage using policies, technologies, and monitoring [19, 7]. Network Intrusion is unauthorized access, exploitation, or compromise of a network's resources, data, or services by attackers using malicious techniques, requiring detection and prevention [99, 96, 91, 147].

Let

$$X = \{e_1, e_2, e_3\}$$

be a set of network events detected by a cybersecurity monitoring system. A hyperfuzzy set

$$\tilde{\mu} : X \to \tilde{P}([0,1])$$

assigns to each event a set of membership degrees that represent the level of suspicion regarding the event being an intrusion attempt. For example, we define:

$$\tilde{\mu}(e_1) = [0.6, 0.8],$$
$$\tilde{\mu}(e_2) = [0.3, 0.5],$$
$$\tilde{\mu}(e_3) = [0.7, 0.9].$$

Here, the interval for each event reflects the uncertainty in assessing its threat level; for instance, event e_1 is considered suspicious to a degree ranging from 0.6 to 0.8.

2.3 | Neutrosophic and HyperNeutrosophic Sets

Neutrosophic Sets extend Fuzzy Sets by introducing the concept of indeterminacy, which accounts for situations that are neither entirely true nor entirely false [127, 125, 79, 128, 129, 137, 138, 135, 136, 55, 133, 72, 49]. As an advanced generalization, the HyperNeutrosophic Set has been developed, offering a more comprehensive framework for handling complex uncertainty [35]. The relevant definitions are provided below.

Definition 6 (Neutrosophic Set)**.** [127] Let X be a given set. A Neutrosophic Set A on X is characterized by three membership functions:

$$T_A : X \to [0,1], \quad I_A : X \to [0,1], \quad F_A : X \to [0,1],$$

where for each $x \in X$, the values $T_A(x)$, $I_A(x)$, and $F_A(x)$ represent the degree of truth, indeterminacy, and falsity, respectively. These values satisfy the following condition:

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3.$$

Definition 7 (HyperNeutrosophic Set)**.** [51, 39, 57, 61, 59, 58, 50, 60, 54, 35] Let X be a non-empty set. A mapping $\tilde{\mu} : X \to \tilde{P}([0,1]^3)$ is called a *HyperNeutrosophic Set* over X, where $\tilde{P}([0,1]^3)$ denotes the family of all non-empty subsets of the unit cube $[0,1]^3$. For each $x \in X$, $\tilde{\mu}(x) \subseteq [0,1]^3$ represents a set of neutrosophic membership degrees, each consisting of truth (T), indeterminacy (I), and falsity (F) components, satisfying:

$$0 \leq T + I + F \leq 3.$$

Example 8 (HyperNeutrosophic Set in Cybersecurity: Comprehensive Threat Assessment)**.** Let

$$X = \{e_1, e_2, e_3\}$$

be a set of network events monitored for potential cyber attacks (cf.[113, 102, 20, 70]). A hyperneutrosophic set

$$\tilde{\nu} : X \to \tilde{P}([0,1]^3)$$

assigns to each event a set of neutrosophic membership triples (T, I, F), where:

- T represents the degree of truth (i.e., the confidence that the event is a genuine threat),
- I represents the degree of indeterminacy (i.e., the uncertainty or lack of clear evidence), and
- F represents the degree of falsity (i.e., the likelihood of the event being a false alarm).

For example, we may define:

$$\tilde{\nu}(e_1) = \{(T, I, F) : T \in [0.7, 0.8],\ I \in [0.1, 0.15],\ F \in [0.0, 0.05]\},$$
$$\tilde{\nu}(e_2) = \{(T, I, F) : T \in [0.4, 0.6],\ I \in [0.2, 0.3],\ F \in [0.1, 0.2]\},$$
$$\tilde{\nu}(e_3) = \{(T, I, F) : T \in [0.8, 0.9],\ I \in [0.05, 0.1],\ F \in [0.0, 0.05]\}.$$

For event e_1, the truth component between 0.7 and 0.8 indicates high confidence that it is a true threat, the indeterminacy between 0.1 and 0.15 captures moderate uncertainty, and the low falsity value (between 0.0 and 0.05) suggests a minimal likelihood of a false alarm.

2.4 | Hypersoft Set

A Soft Set offers a simplified framework for parameterized decision-making by mapping attributes or parameters to subsets of a universal set, effectively addressing uncertainty in a straightforward manner [94, 97, 143, 65, 6, 104, 8, 64]. Building on this concept, a Hypersoft Set enhances multi-attribute decision analysis by mapping combinations of multiple attributes to subsets of a universal set, allowing for a more nuanced and flexible approach [130, 1, 3, 71, 100, 5, 119, 46, 30].

A concise definition of the Hypersoft Set is provided below.

Definition 9 (Soft Set)**.** [94, 97] Let U be a universal set and A be a set of attributes. A soft set over U is a pair $(\mathcal{F}, S)$, where $S \subseteq A$ and $\mathcal{F} : S \to \mathcal{P}(U)$. Here, $\mathcal{P}(U)$ denotes the power set of U. Mathematically, a soft set is represented as:

$$(\mathcal{F}, S) = \{(\alpha, \mathcal{F}(\alpha)) \mid \alpha \in S, \mathcal{F}(\alpha) \in \mathcal{P}(U)\}.$$

Each $\alpha \in S$ is called a parameter, and $\mathcal{F}(\alpha)$ is the set of elements in U associated with α.

Definition 10 (Hypersoft Set)**.** [130] Let U be a universal set, and let $\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_m$ be attribute domains. Define $\mathcal{C} = \mathcal{A}_1 \times \mathcal{A}_2 \times \cdots \times \mathcal{A}_m$, the Cartesian product of these domains. A hypersoft set over U is a pair $(G, \mathcal{C})$, where $G : \mathcal{C} \to \mathcal{P}(U)$. The hypersoft set is expressed as:

$$(G, \mathcal{C}) = \{(\gamma, G(\gamma)) \mid \gamma \in \mathcal{C}, G(\gamma) \in \mathcal{P}(U)\}.$$

For an m-tuple $\gamma = (\gamma_1, \gamma_2, \ldots, \gamma_m) \in \mathcal{C}$, where $\gamma_i \in \mathcal{A}_i$ for $i = 1, 2, \ldots, m$, $G(\gamma)$ represents the subset of U corresponding to the combination of attribute values $\gamma_1, \gamma_2, \ldots, \gamma_m$.

Example 11 (Hypersoft Set in Cybersecurity)**.** Consider a cybersecurity scenario where the universal set

$$U = \{e_1, e_2, e_3, e_4\}$$

represents network events recorded by a monitoring system. Suppose we are interested in classifying these events based on two attributes:

- $\mathcal{A}_1$: **Attack Type** with domain $\{\text{DDoS[21]}, \text{Phishing[150]}\}$,
- $\mathcal{A}_2$: **Severity Level** with domain $\{\text{High}, \text{Low}\}$.

The combined parameter domain is then given by

$$\mathcal{C} = \mathcal{A}_1 \times \mathcal{A}_2 = \{(\text{DDoS}, \text{High}),\ (\text{DDoS}, \text{Low}),\ (\text{Phishing}, \text{High}),\ (\text{Phishing}, \text{Low})\}.$$

A *hypersoft set* is defined as a pair $(G, \mathcal{C})$ where

$$G : \mathcal{C} \to \mathcal{P}(U)$$

assigns to each parameter tuple a subset of events. For example, one might define:

$$G(\text{DDoS}, \text{High}) = \{e_1, e_2\},$$

$$G(\text{DDoS}, \text{Low}) = \{e_3\},$$

$$G(\text{Phishing}, \text{High}) = \{e_2, e_4\},$$

$$G(\text{Phishing}, \text{Low}) = \{e_3, e_4\}.$$

This hypersoft set categorizes network events according to both the type of attack and its severity, facilitating multi-attribute decision-making in cybersecurity.

2.5 | Hypersoft Rough Set

A hypersoft rough set uses lower and upper approximations over an approximation space to represent multi-attribute uncertainty in sets [141, 140, 76]. Note that an approximation space is a mathematical structure that models uncertainty, consisting of a universe of objects and an equivalence relation, enabling the definition of lower and upper approximations for subsets.

Definition 12 (Soft Rough Set)**.** (cf.[122, 87, 165, 2, 4, 23, 77, 95]) Let U be a universal set, A a set of parameters, and $P(U)$ the power set of U. Let R be an equivalence relation on U, inducing a partition $U/R = \{Y_1, Y_2, \dots, Y_m\}$ into equivalence classes. A soft set (F, A) on U is defined as $F : A \to P(U)$.

For $B \subseteq U$, the *Soft Rough Lower Approximation* $L(B)$ and *Soft Rough Upper Approximation* $U(B)$ are given by:

$$L(B) = \{u \in U \mid \exists e \in A \text{ such that } F(e) \subseteq B\},$$
$$U(B) = \{u \in U \mid \exists e \in A \text{ such that } F(e) \cap B \neq \emptyset\}.$$

The *Soft Rough Set* is represented as the pair:

$$(L(B), U(B)),$$

where $L(B)$ and $U(B)$ are the approximations of B with respect to the soft set.

Definition 13 (Hypersoft Rough Set)**.** [141, 140, 76] Let (X, R) be a Pawlak approximation space, where R is an equivalence relation on X. Given a Hypersoft Set (F, J) over X, the *Hypersoft Lower Approximation* $F_*(\mathbf{j})$ and *Hypersoft Upper Approximation* $F^*(\mathbf{j})$ of F with respect to R are defined for each $\mathbf{j} \in J$ as:

$$F_*(\mathbf{j}) = \{x \in X \mid [x]_R \subseteq F(\mathbf{j})\},$$
$$F^*(\mathbf{j}) = \{x \in X \mid [x]_R \cap F(\mathbf{j}) \neq \emptyset\},$$

where $[x]_R$ denotes the equivalence class of x under R.

The *Hypersoft Rough Set* is then the pair (F_*, F^*, J).

Example 14 (HyperSoft Rough Set in Cybersecurity)**.** Let the set of network events be

$$X = \{e_1, e_2, e_3, e_4\},$$

and assume an approximation space (X, R) where the equivalence relation R groups events with similar characteristics. For instance, suppose the equivalence classes are given by

$$[e_1]_R = \{e_1, e_2\} \quad \text{and} \quad [e_3]_R = \{e_3, e_4\}.$$

Consider the hypersoft set $(G, \mathcal{C})$ from the previous example, and focus on the parameter tuple

$$\gamma = (\text{DDoS}, \text{High}),$$

with

$$G(\gamma) = G(\text{DDoS}, \text{High}) = \{e_1, e_2\}.$$

The *Hypersoft Lower Approximation* $F_*(\gamma)$ and *Hypersoft Upper Approximation* $F^*(\gamma)$ of $G(\gamma)$ with respect to R are defined by:

$$F_*(\gamma) = \{x \in X \mid [x]_R \subseteq G(\gamma)\},$$
$$F^*(\gamma) = \{x \in X \mid [x]_R \cap G(\gamma) \neq \emptyset\}.$$

For the given data:

- For $x = e_1$: $[e_1]_R = \{e_1, e_2\} \subseteq \{e_1, e_2\}$ so $e_1 \in F_*(\gamma)$ and also $e_1 \in F^*(\gamma)$.
- For $x = e_2$: $[e_2]_R = \{e_1, e_2\} \subseteq \{e_1, e_2\}$ so $e_2 \in F_*(\gamma)$ and $e_2 \in F^*(\gamma)$.
- For $x = e_3$: $[e_3]_R = \{e_3, e_4\}$ has no intersection with $\{e_1, e_2\}$, hence $e_3 \notin F^*(\gamma)$.
- Similarly, $e_4 \notin F^*(\gamma)$.

Thus, the hypersoft rough set corresponding to $\gamma = (\text{DDoS}, \text{High})$ is given by:

$$F_*(\gamma) = \{e_1, e_2\}, \quad F^*(\gamma) = \{e_1, e_2\}.$$

This result illustrates that, for the chosen parameter combination, the approximation process precisely identifies the subset of network events associated with high-severity DDoS attacks, thereby aiding in the targeted analysis of cybersecurity threats.

2.6 | Fuzzy Rough Set and Neutrosophic Rough Set

The definitions of Fuzzy Rough Set[83, 161, 170, 162, 11, 142] and Neutrosophic Rough Set[168, 13, 167, 14, 12] are provided below. These concepts are known as extensions of the classical rough set theory, utilizing fuzzy sets and neutrosophic sets to handle uncertainty and indeterminacy.

Definition 15 (Fuzzy Rough Set)**.** [83, 161, 170] A *fuzzy rough set* is a mathematical model that combines the concepts of fuzzy set theory and rough set theory to handle uncertainty in data. It is particularly useful in cases where the boundary between classes or sets is not well-defined, leveraging the flexibility of fuzzy sets and the approximation capabilities of rough sets.

Let U be a finite, non-empty universe of discourse, and $A = (U, A)$ be an information system where A is a finite, non-empty set of attributes. Each attribute $a \in A$ is associated with a mapping $a : U \to V_a$, where V_a is the domain of a.

(1) *Fuzzy Indiscernibility Relation*
In fuzzy rough set theory, the indiscernibility relation R is replaced by a *fuzzy tolerance relation* $R : U \times U \to [0, 1]$, satisfying the following properties:

- *Reflexivity:* $R(x, x) = 1$ for all $x \in U$,
- *Symmetry:* $R(x, y) = R(y, x)$ for all $x, y \in U$,
- *T-Transitivity (Optional):* $T(R(x, y), R(y, z)) \leq R(x, z)$ for all $x, y, z \in U$, where T is a t-norm (e.g., minimum).

(2) *Lower and Upper Approximations*
For a fuzzy set $X : U \to [0, 1]$ and a fuzzy indiscernibility relation R, the *lower approximation* $R_{\#}(X)$ and *upper approximation* $R^{\#}(X)$ are defined as:

$$(R_{\#}(X))(y) = \inf_{x \in U} I(R(x, y), X(x)),$$
$$(R^{\#}(X))(y) = \sup_{x \in U} T(R(x, y), X(x)),$$

where I is a fuzzy implicator (e.g., $I(a, b) = \max(1 - a, b)$), and T is a t-norm.

(3) *Regions*
Based on the lower and upper approximations, the regions in a fuzzy rough set are defined as:

- *Positive Region:* The set of objects that can be certainly classified:
$$POS_B = \bigcup_{x \in U} R_{\#}([x]_d),$$
where $[x]_d$ is the equivalence class of x under a decision attribute d.
- *Boundary Region:* The set of objects that cannot be classified with certainty:
$$BND_B = R^{\#}([x]_d) \setminus R_{\#}([x]_d).$$

(4) *Dependency Degree*
The degree of dependency of the decision attribute d on a set of conditional attributes B is given by:

$$\gamma_B = \frac{|POS_B|}{|U|}.$$

This framework generalizes classical rough sets by incorporating the flexibility of fuzzy membership, making it suitable for modeling complex or vague relationships in data.

Definition 16. [168, 13] A *Neutrosophic Rough Set* (NRS) is a mathematical model that combines the concepts of neutrosophic sets and rough sets to handle uncertainty, indeterminacy, and incompleteness in data. It generalizes traditional rough set theory by incorporating truth-membership, indeterminacy-membership, and falsity-membership degrees from neutrosophic sets.

(1) *Single-Valued Neutrosophic Set (SVNS):*
A single-valued neutrosophic set A on a universe U is defined as:

$$A = (A_T, A_I, A_F),$$

where $A_T, A_I, A_F : U \to [0,1]$ represent the truth-membership function, indeterminacy-membership function, and falsity-membership function, respectively.

(2) *Single-Valued Neutrosophic Relation (SVNR):*
A single-valued neutrosophic relation R on $U \times U$ is defined as:

$$R = (R_T, R_I, R_F),$$

where $R_T, R_I, R_F : U \times U \to [0,1]$. The pair (U, R) is called a *single-valued neutrosophic approximation space (SVNAS).*

R satisfies the following properties:

- *Reflexivity:* $R(x,x) = (\top, 0, 0)$ for all $x \in U$, where $\top = (1,0,0)$,
- *Transitivity:* For all $x, y, z \in U$:

$$R_T(x,y) \wedge R_T(y,z) \leq R_T(x,z),$$
$$R_I(x,y) \vee R_I(y,z) \geq R_I(x,z),$$
$$R_F(x,y) \vee R_F(y,z) \geq R_F(x,z).$$

Let $A \in \text{SVNS}(U)$ and R be a SVNR. The *lower approximation* $Y_R^{\#}(A)$ and *upper approximation* $Y_R^{\#}(A)$ of A are defined as follows for all $x \in U$:

(1) *Lower Approximation:*

$$Y_R^{\#}(A)_T(x) = \inf_{y \in U} \min(R_T(x,y), A_T(y)),$$
$$Y_R^{\#}(A)_I(x) = \sup_{y \in U} \max(R_I(x,y), A_I(y)),$$
$$Y_R^{\#}(A)_F(x) = \sup_{y \in U} \max(R_F(x,y), A_F(y)).$$

(2) *Upper Approximation:*

$$Y_R^{\#}(A)_T(x) = \sup_{y \in U} \min(R_T(x,y), A_T(y)),$$
$$Y_R^{\#}(A)_I(x) = \inf_{y \in U} \max(R_I(x,y), A_I(y)),$$
$$Y_R^{\#}(A)_F(x) = \inf_{y \in U} \max(R_F(x,y), A_F(y)).$$

Neutrosophic Regions

- *Positive Region:*

$$POS_R(A) = \{x \in U \mid Y_R^{\#}(A)_T(x) = 1\}.$$

- *Boundary Region:*

$$BND_R(A) = \{x \in U \mid 0 < Y_R^{\#}(A)_T(x) < 1\}.$$

- *Negative Region:*

$$NEG_R(A) = \{x \in U \mid Y_R^{\#}(A)_T(x) = 0\}.$$

This model provides a powerful tool to analyze and process information in the presence of indeterminacy and incompleteness, extending the capabilities of classical rough sets.

3 | Results of This Paper

In this paper, we propose new definitions for various types of sets and briefly examine their relationships with existing concepts.

3.1 | Hyperfuzzy Rough Set

We now extend the classical/fuzzy rough set approach to the hyperfuzzy case.

Definition 17 (Hyperfuzzy Rough Set)**.** Let (X, R) be an approximation space, where X is a non-empty set and R is either:

- a crisp equivalence relation on X (i.e. $R \subseteq X \times X$), or
- a fuzzy tolerance (or T-transitive) relation $R\colon X \times X \to [0,1]$.

Let $\tilde{\mu}$ be a hyperfuzzy set over X, i.e. $\tilde{\mu} : X \to \mathcal{P}^*([0,1])$. We define the *hyperfuzzy rough lower approximation* $\tilde{\mu}_*(x)$ and the *hyperfuzzy rough upper approximation* $\tilde{\mu}^*(x)$ for each $x \in X$ as follows.

(1) *If R is a crisp equivalence relation* (Pawlak-type): For any $x \in X$, let $[x]_R = \{\, y \in X : (x, y) \in R\}$ be the equivalence class of x. We set

$$\tilde{\mu}_*(x) \;=\; \bigcap_{y\in[x]_R} \tilde{\mu}(y), \qquad \tilde{\mu}^*(x) \;=\; \bigcup_{y\in[x]_R} \tilde{\mu}(y).$$

In words, the lower approximation at x is the intersection of all membership-degree-sets $\tilde{\mu}(y)$ over y in $[x]_R$, while the upper approximation at x is the union of the same.

(2) *If R is a fuzzy relation $R\colon X \times X \to [0,1]$*: A more general approach (analogous to fuzzy rough sets) is given by:

$$\tilde{\mu}_*(x) \;=\; \bigcap_{y\in X} \Big(\tilde{\mu}(y) \;\cup\; \{\text{rules derived by } I(R(x,y),\cdot)\}\Big),$$

$$\tilde{\mu}^*(x) \;=\; \bigcup_{y\in X} \Big(\tilde{\mu}(y) \;\cap\; \{\text{rules derived by } T(R(x,y),\cdot)\}\Big),$$

where I is a suitable implicator, T is a t-norm, and we embed each scalar condition into a set-based condition on $\tilde{\mu}(y)$. For simplicity, many authors adopt the intersection/union definition of (1) for crisp equivalence classes, extended by weighting factors. Various formulations are possible.

The pair $(\tilde{\mu}_*,\, \tilde{\mu}^*)$ is called the *Hyperfuzzy Rough Set* induced by $\tilde{\mu}$ with respect to R.

Theorem 18. *A Hyperfuzzy Rough Set generalizes both:*

- *Fuzzy Rough Set: If each $\tilde{\mu}(x)$ is a singleton in $[0,1]$, then $\tilde{\mu}$ is just an ordinary fuzzy set, and the definitions in Definition 17 reduce exactly to those of a fuzzy rough set.*
- *Hyperfuzzy Set (with trivial approximation): If R is taken to be the universal (or identity) relation, then $\tilde{\mu}_*(x) = \tilde{\mu}^*(x) = \tilde{\mu}(x)$ for all x, so the hyperfuzzy rough set simply collapses to $\tilde{\mu}$ itself, i.e. we obtain the original hyperfuzzy set without further approximation.*

Proof: *(1)* When each $\tilde{\mu}(x)$ is a *single* real number in $[0,1]$, write $\tilde{\mu}(x) = \{\mu(x)\}$.

- For a crisp R,

$$\tilde{\mu}_*(x) = \bigcap_{y\in[x]_R} \{\mu(y)\} = \{\min\{\mu(y) : y \in [x]_R\}\} \quad \text{and} \quad \tilde{\mu}^*(x) = \bigcup_{y\in[x]_R} \{\mu(y)\} = \{\max\{\mu(y) : y \in [x]_R\}\},$$

  which recovers a known type of fuzzy rough approximation (one version of many).
- If R is a fuzzy relation, then depending on the chosen definitions (inf/sup using I, T), we again retrieve the classical fuzzy rough set formulas in Definition.

Hence the case of singleton-values recovers fuzzy rough sets.

(2) If R is universal (every pair is related) or identity (only each x is related to itself), both extremes yield that the lower and upper approximations of $\tilde{\mu}$ coincide with $\tilde{\mu}$ itself (up to minor details in the universal case). Thus we recover exactly the original hyperfuzzy set $\tilde{\mu}$ with no approximation enforced. □

Example 19 (Hyperfuzzy Rough Set in Cybersecurity)**.** Network Security is the practice of protecting network infrastructure from unauthorized access, misuse, malfunction, modification, destruction, or disruption using policies, technologies, and monitoring(cf.[26, 18, 124, 101]). Network Event is an occurrence in a network, such as traffic flow, connection attempts, security breaches, or anomalies, recorded for analysis, troubleshooting, and threat detection(cf.[166, 27, 82]).

Let

$$X = \{e_1, e_2, e_3, e_4\}$$

be a set of network events, where each event represents a potential intrusion attempt. An equivalence relation R is defined on X to group events with similar characteristics (for instance, events originating from the same source IP or exhibiting similar traffic patterns). For example, assume

$$[e_1]_R = \{e_1, e_2\} \quad \text{and} \quad [e_3]_R = \{e_3, e_4\}.$$

We define a hyperfuzzy set

$$\tilde{\mu} : X \to \mathcal{P}^*([0,1])$$

that assigns an interval of membership degrees to each event to quantify its level of suspiciousness. For example, let

$$\tilde{\mu}(e_1) = [0.6, 0.8], \quad \tilde{\mu}(e_2) = [0.5, 0.7], \quad \tilde{\mu}(e_3) = [0.2, 0.4], \quad \tilde{\mu}(e_4) = [0.3, 0.5].$$

The *lower approximation* and *upper approximation* of $\tilde{\mu}$ at an event $x \in X$ are computed as

$$\tilde{\mu}_*(x) = \bigcap_{y \in [x]_R} \tilde{\mu}(y), \quad \tilde{\mu}^*(x) = \bigcup_{y \in [x]_R} \tilde{\mu}(y).$$

For instance, for event e_1 (with $[e_1]_R = \{e_1, e_2\}$):

$$\tilde{\mu}_*(e_1) = [0.6, 0.8] \cap [0.5, 0.7] = \Big[\max(0.6, 0.5), \min(0.8, 0.7)\Big] = [0.6, 0.7],$$

$$\tilde{\mu}^*(e_1) = [0.6, 0.8] \cup [0.5, 0.7] = \Big[\min(0.6, 0.5), \max(0.8, 0.7)\Big] = [0.5, 0.8].$$

Here, the lower approximation $[0.6, 0.7]$ represents the degree of certainty that e_1 is suspicious, while the upper approximation $[0.5, 0.8]$ reflects the overall possibility of suspicion considering the uncertainty in similar events.

3.2 | Hyperfuzzy Hyperrough Set

The Hyperfuzzy Hyperrough Set is a set concept that combines the properties of a Hyperrough Set and a Hyperfuzzy Set. The formal definition is provided below.

Definition 20 (Hyperfuzzy Hyperrough Set)**.** Let (X, R) be an approximation space. Let

$$\tilde{F} : J \to (\mathcal{P}^*([0,1])),$$

where $J = J_1 \times \cdots \times J_n$, so that for each $a \in J$, $\tilde{F}(a)$ is a hyperfuzzy set on X:

$$\tilde{F}(a) \colon X \to \mathcal{P}^*([0,1]).$$

We define the *Hyperfuzzy Hyperrough Set* associated to $(\tilde{F}, J)$ by assigning to each $a \in J$ the *hyperfuzzy rough approximations* of $\tilde{F}(a)$:

$$\big(\tilde{F}(a)\big)_*, \quad \big(\tilde{F}(a)\big)^*,$$

exactly as in Definition 17, but done separately for each hyperfuzzy set $\tilde{F}(a)$.

Hence, for crisp R,

$$\big(\tilde{F}(a)\big)_*(x) = \bigcap_{y \in [x]_R} \tilde{F}(a)(y), \quad \big(\tilde{F}(a)\big)^*(x) = \bigcup_{y \in [x]_R} \tilde{F}(a)(y),$$

and similarly for a fuzzy R with an appropriate t-norm or implicator. The triple

$$\Big(\big(\tilde{F}(a)\big)_*,\ \big(\tilde{F}(a)\big)^*,\ J\Big)$$

is called the *Hyperfuzzy Hyperrough Set.*

Theorem 21. *A Hyperfuzzy Hyperrough Set generalizes both:*

- *HyperRough Set (when membership sets are crisp* $\{0, 1\}$ *or classical subsets, i.e. we track just* **0** *or* **1** *membership),*
- *Hyperfuzzy Rough Set (when* J *consists of a single parameter; i.e.* $|J| = 1$ *so we effectively have a single hyperfuzzy set* $\tilde{F}(a)$*).*

Proof: *(1)* If each $\tilde{F}(a)(x)$ is either $\{0\}$ or $\{1\}$ (i.e. crisp membership for each x), then $\tilde{F}(a)$ reduces to a standard indicator set $F(a) \subseteq X$, and the approximation definitions become those of the usual HyperRough Set.

(2) If J has exactly one element a, we only have $\tilde{F}(a)$ as a hyperfuzzy set on X, and the resulting structure is precisely that of a single Hyperfuzzy Rough Set. □

Example 22 (Hyperfuzzy Hyperrough Set in Cybersecurity)**.** Consider a multi-attribute scenario where the suspiciousness of a network event is evaluated based on two independent cybersecurity criteria: *IP Reputation* (cf.[85, 144]) and *Payload Anomaly*(cf.[80, 117]). Let the parameter space be

$$J = J_1 \times J_2,$$

where J_1 corresponds to IP Reputation and J_2 to Payload Anomaly. For simplicity, assume $J = \{(1, 1)\}$ (i.e., a single combined parameter). Define the mapping

$$\tilde{F} : J \to \Big(\mathcal{P}^*([0, 1])\Big)^X,$$

so that for each $a \in J$, $\tilde{F}(a)$ is a hyperfuzzy set on X. For example, assign

$$\tilde{F}(1, 1)(e_1) = [0.7, 0.9], \quad \tilde{F}(1, 1)(e_2) = [0.6, 0.8], \quad \tilde{F}(1, 1)(e_3) = [0.3, 0.5], \quad \tilde{F}(1, 1)(e_4) = [0.4, 0.6].$$

The hyperfuzzy hyperrough set is then obtained by applying the hyperfuzzy rough approximations separately for the hyperfuzzy set $\tilde{F}(1, 1)$. That is, for each $x \in X$,

$$\big(\tilde{F}(1, 1)\big)_*(x) = \bigcap_{y \in [x]_R} \tilde{F}(1, 1)(y), \quad \big(\tilde{F}(1, 1)\big)^*(x) = \bigcup_{y \in [x]_R} \tilde{F}(1, 1)(y).$$

For instance, for event e_1 (with $[e_1]_R = \{e_1, e_2\}$):

$$\big(\tilde{F}(1, 1)\big)_*(e_1) = [0.7, 0.9] \cap [0.6, 0.8] = \Big[\max(0.7, 0.6),\ \min(0.9, 0.8)\Big] = [0.7, 0.8],$$

$$\big(\tilde{F}(1, 1)\big)^*(e_1) = [0.7, 0.9] \cup [0.6, 0.8] = \Big[\min(0.7, 0.6),\ \max(0.7, 0.9)\Big] = [0.6, 0.9].$$

In this context, the lower approximation $[0.7, 0.8]$ indicates the degree to which the evidence (from both IP Reputation and Payload Anomaly) definitively supports the classification of e_1 as a threat, while the upper approximation $[0.6, 0.9]$ accommodates the inherent uncertainty in the evaluation. This multi-attribute approach enables more nuanced decision-making in intrusion detection.

3.3 | Hypersoft Hyperrough Set

The Hypersoft Hyperrough Set is a set concept that integrates the principles of Hypersoft Sets and Hyperrough Sets. The definitions and related details are presented below.

Definition 23 (Hypersoft Hyperrough Set)**.** Let (X, R) be an approximation space, where R is either crisp or fuzzy. Suppose we have a *hypersoft mapping*

$$G: \mathcal{C} \ \to \ \mathcal{P}(X),$$

where $\mathcal{C} = \mathcal{A}_1 \times \cdots \times \mathcal{A}_m$. We convert G into a *hyper* mapping by assigning to each $c \in \mathcal{C}$ not just a subset $G(c) \subseteq X$, but a *hyper-membership*

$$\widetilde{G}(c) : X \to \mathcal{P}^*([0, 1]),$$

i.e. for every $x \in X$, $\widetilde{G}(c)(x)$ is a non-empty subset of $[0, 1]$. Then, as in Definition 20, for each $c \in \mathcal{C}$, we define a hyperfuzzy rough approximation:

$$\big(\widetilde{G}(c)\big)_*(x) = \bigcap_{y \in [x]_R} \widetilde{G}(c)(y), \quad \big(\widetilde{G}(c)\big)^*(x) = \bigcup_{y \in [x]_R} \widetilde{G}(c)(y),$$

(crisp R version). The collection of all these approximations,

$$\left(\widetilde{G}_*,\, \widetilde{G}^*,\, \mathcal{C}\right),$$

is called the *Hypersoft Hyperrough Set.*

Theorem 24. *A Hypersoft Hyperrough Set generalizes both:*

- *Hypersoft Rough Set, by restricting* $\widetilde{G}(c)(x)$ *to* $\{0\}$ *or* $\{1\}$ *(or singletons in* $[0,1]$*) for all* x, c*;*
- *Hyperrough Set, by taking* $\mathcal{C} = J_1 \times \cdots \times J_n$ *in the standard hyperrough sense and not necessarily splitting attributes in a hypersoft manner.*

Proof: *(1)* If $\widetilde{G}(c)(x)$ is always a single membership degree in $\{0, 1\}$, we retrieve a crisp subset $G(c) \subseteq X$, hence the structure collapses to the standard Hypersoft Rough Set (where each c in $\mathcal{C}$ simply picks out $G(c)$ in X).

(2) If we force $\mathcal{C}$ to be precisely $J = J_1 \times \cdots \times J_n$, and interpret $\widetilde{G}$ exactly as in hyperrough definitions, we recover a Hyperrough approach. □

Example 25 (Cybersecurity Application: Intrusion Categorization using Hypersoft Hyperrough Set)**.** Let

$$X = \{e_1, e_2, e_3, e_4\}$$

be a set of network events detected by a security monitoring system. An equivalence relation R on X groups events with similar characteristics (e.g., originating IP address, time stamp[66], or traffic pattern[78]). For example, suppose

$$[e_1]_R = \{e_1, e_2\} \quad \text{and} \quad [e_3]_R = \{e_3, e_4\}.$$

Consider two cybersecurity attributes:

- $\mathcal{A}_1$ (Attack Type) with domain $\{\text{DDoS[21]}, \text{Phishing[150]}\}$,
- $\mathcal{A}_2$ (Severity Level) with domain $\{\text{High}, \text{Low}\}$.

Thus, the parameter space is

$$\mathcal{C} = \mathcal{A}_1 \times \mathcal{A}_2 = \{(\text{DDoS}, \text{High}),\ (\text{DDoS}, \text{Low}),\ (\text{Phishing}, \text{High}),\ (\text{Phishing}, \text{Low})\}.$$

We define a hypersoft mapping

$$G\colon \mathcal{C} \to \mathcal{P}(X)$$

and convert it into a hyper-membership mapping

$$\widetilde{G}\colon \mathcal{C} \to \{\, \widetilde{G}(c)\colon X \to \mathcal{P}^*([0,1])\,\}.$$

For each $c \in \mathcal{C}$ and $x \in X$, the value $\widetilde{G}(c)(x)$ (typically given as an interval in $[0, 1]$) represents the degree to which event x exhibits the characteristics described by c.

For instance, for $c = (\text{DDoS}, \text{High})$ we might assign:

$$\widetilde{G}(\text{DDoS}, \text{High})(e_1) = [0.8, 0.9], \quad \widetilde{G}(\text{DDoS}, \text{High})(e_2) = [0.75, 0.85],$$

$$\widetilde{G}(\text{DDoS}, \text{High})(e_3) = [0.3, 0.5], \quad \widetilde{G}(\text{DDoS}, \text{High})(e_4) = [0.2, 0.4].$$

Following Definition, for each $c \in \mathcal{C}$ the hyperfuzzy rough approximations are defined (for a crisp R) by

$$\big(\widetilde{G}(c)\big)_*(x) = \bigcap_{y \in [x]_R} \widetilde{G}(c)(y), \quad \big(\widetilde{G}(c)\big)^*(x) = \bigcup_{y \in [x]_R} \widetilde{G}(c)(y).$$

For example, for $c = (\text{DDoS}, \text{High})$ and event e_1 (with $[e_1]_R = \{e_1, e_2\}$), we have

$$\big(\widetilde{G}(\text{DDoS}, \text{High})\big)_*(e_1) = \widetilde{G}(\text{DDoS}, \text{High})(e_1) \cap \widetilde{G}(\text{DDoS}, \text{High})(e_2) = \Big[\max(0.8, 0.75),\ \min(0.9, 0.85)\Big] = [0.8, 0.85],$$

$$\big(\widetilde{G}(\text{DDoS}, \text{High})\big)^*(e_1) = \widetilde{G}(\text{DDoS}, \text{High})(e_1) \cup \widetilde{G}(\text{DDoS}, \text{High})(e_2) = \Big[\min(0.8, 0.75),\ \max(0.9, 0.85)\Big] = [0.75, 0.9].$$

The lower approximation $[0.8, 0.85]$ represents the degree to which the evidence from events in the same equivalence class certainly supports that e_1 is a high-severity DDoS attack, while the upper approximation $[0.75, 0.9]$ reflects the possibility considering the uncertainty among similar events.

Repeating similar computations for each $c \in \mathcal{C}$ and for every $x \in X$ yields the collection

$$\left(\widetilde{G}_*, \widetilde{G}^*, \mathcal{C}\right),$$

which is the *Hypersoft Hyperrough Set* that integrates multiple cybersecurity attributes with the inherent uncertainty in attack detection.

3.4 | HyperNeutrosophic Rough Set

We now define the *HyperNeutrosophic Rough Set*, extending the above concept to an approximation space. Informally, we take a HyperNeutrosophic Set and impose rough approximations in the neutrosophic sense, but now each x has a *set of possible* (T, I, F) membership values.

Definition 26 (HyperNeutrosophic Rough Set)**.** Let (X, R) be a *Neutrosophic Approximation Space*, meaning $R = (R_T, R_I, R_F)$ is a single-valued neutrosophic relation on $X \times X$. Suppose $\widetilde{N}\colon X \to \mathcal{P}^*([0,1]^3)$ is a *HyperNeutrosophic Set*. We define the *HyperNeutrosophic Lower Approximation* $\widetilde{N}_*(x)$ and the *HyperNeutrosophic Upper Approximation* $\widetilde{N}^*(x)$ for each $x \in X$ as follows (presented in a crisp-like version for clarity, then generalized to fuzzy-like below).

(1) *In a Crisp-Equivalence Scenario:* If R is an equivalence relation in a crisp sense (some authors encode neutrosophic equivalence as $\{0, 1\}$ membership in R_T, R_I, R_F), for each $x \in X$ let $[x]_R$ be its equivalence class. Then:

$$\widetilde{N}_*(x) = \bigcap_{y \in [x]_R} \widetilde{N}(y), \quad \widetilde{N}^*(x) = \bigcup_{y \in [x]_R} \widetilde{N}(y).$$

So the lower approximation is the intersection of all possible (T, I, F)-sets of y with $y \in [x]_R$, and the upper approximation is the union of them.

(2) *In a Fuzzy-Neutrosophic Relation Scenario:* If R_T, R_I, R_F take values in $[0, 1]$, one can generalize the intersection/union via suitable t-norm T_* and t-conorm S_*, or using fuzzy implicators $\mathbf{I}$, etc. in the style of neutrosophic rough sets. Concretely,

$$\widetilde{N}_*(x) = \bigcap_{y \in X} \Psi_{\text{lower}}(R(x, y), \widetilde{N}(y)), \quad \widetilde{N}^*(x) = \bigcup_{y \in X} \Psi_{\text{upper}}(R(x, y), \widetilde{N}(y)),$$

where $R(x, y) = (R_T(x, y), R_I(x, y), R_F(x, y))$ and the operators $\Psi_{\text{lower}}, \Psi_{\text{upper}}$ systematically combine the *relation strengths* with the sets of membership triples in $\widetilde{N}(y)$. Various forms exist, analogous to definitions for Neutrosophic Rough Sets or Fuzzy Rough Sets, but extended to set-of-triples membership.

In either approach, the pair

$$\left(\widetilde{N}_*, \widetilde{N}^*\right)$$

is called the *HyperNeutrosophic Rough Set* determined by $\widetilde{N}$ w.r.t. R.

Theorem 27 (HyperNeutrosophic Rough Set Generalizes Multiple Frameworks)**.** *A* HyperNeutrosophic Rough Set *strictly generalizes:*

(i) *HyperNeutrosophic Set (with trivial approximation): if R is taken to be the identity or universal relation, there is no real approximation restriction, and $\widetilde{N}_*(x) = \widetilde{N}^*(x) = \widetilde{N}(x)$ for all x.*

(ii) *Neutrosophic Rough Set: if each $\widetilde{N}(x)$ is a* singleton $\{(T_A(x), I_A(x), F_A(x))\}$*, then $\widetilde{N}$ recovers a standard single-valued neutrosophic set $A = (A_T, A_I, A_F)$, and the approximations reduce to those of Definition* **??***.*

(iii) *HyperFuzzy Rough Set: if we force $I_A(x) = 0$ and $F_A(x) = 0$ for all membership triples, effectively we only have $T_A(x) \in [0, 1]$; hence $\widetilde{N}(x)$ becomes a hyperfuzzy set in $[0, 1]$. The lower/upper approximations then coincide with hyperfuzzy rough sets previously introduced.*

Proof: *Case (i):* If R is identity ($R(x,x) = \top$ and $R(x,y) = \bot$ for $x \neq y$), each equivalence class is just $\{x\}$. Then

$$\widetilde{N}_*(x) = \widetilde{N}^*(x) = \widetilde{N}(x) \quad \text{for each } x.$$

Similarly, if R is universal, then each equivalence class is X itself, so

$$\widetilde{N}_*(x) = \bigcap_{y \in X} \widetilde{N}(y), \quad \widetilde{N}^*(x) = \bigcup_{y \in X} \widetilde{N}(y),$$

which is again a trivial collapse (all x have the same intersection/union). In both extremes, the rough approximations do not impose standard partition-based constraints, and we effectively retrieve $\widetilde{N}$ with no further approximation.

Case (ii): If each $\widetilde{N}(x) = \{(T_A(x), I_A(x), F_A(x))\}$ is a single triple, then $\widetilde{N}$ is precisely a single-valued neutrosophic set A. Applying the formula from Definition 26 yields standard Neutrosophic Rough set operations (intersection of singletons yields minimum or inf, union yields maximum or sup, etc.).

Case (iii): If we impose $I_A(x) = 0, F_A(x) = 0$ for all membership triples, effectively each triple (T, I, F) becomes $(T, 0, 0)$ with $T \in [0,1]$. Then $\widetilde{N}(x)$ reduces to a *hyperfuzzy set* $\tilde{\mu}(x) \subseteq [0,1]$. The neutrosophic $R = (R_T, R_I, R_F)$ can be replaced or restricted so that only R_T is relevant, yielding the hyperfuzzy rough set construction.

Thus, each of the three classical frameworks is embedded in the notion of a HyperNeutrosophic Rough Set. □

Example 28 (HyperNeutrosophic Rough Set in Cybersecurity)**.** Consider a set of network events

$$X = \{e_1, e_2, e_3, e_4\},$$

detected by a cybersecurity monitoring system. We assume that events with similar characteristics—such as source IP, time stamp, or traffic behavior—are grouped via a neutrosophic equivalence relation R. For simplicity, let

$$[e_1]_R = \{e_1, e_2\} \quad \text{and} \quad [e_3]_R = \{e_3, e_4\}.$$

Define a *HyperNeutrosophic Set*

$$\widetilde{N} : X \to \mathcal{P}^*([0,1]^3)$$

that assigns to each event a set of possible neutrosophic membership triples (T, I, F) representing, respectively, the degree of threat (truth), uncertainty (indeterminacy), and false alarm (falsity). For instance, let

$$\begin{aligned}
\widetilde{N}(e_1) &= \{(T,I,F) : T \in [0.7, 0.8],\ I \in [0.1, 0.2],\ F \in [0.0, 0.05]\}, \\
\widetilde{N}(e_2) &= \{(T,I,F) : T \in [0.65, 0.75],\ I \in [0.15, 0.25],\ F \in [0.05, 0.10]\}, \\
\widetilde{N}(e_3) &= \{(T,I,F) : T \in [0.3, 0.4],\ I \in [0.4, 0.5],\ F \in [0.1, 0.15]\}, \\
\widetilde{N}(e_4) &= \{(T,I,F) : T \in [0.35, 0.45],\ I \in [0.35, 0.45],\ F \in [0.05, 0.10]\}.
\end{aligned}$$

For a crisp neutrosophic relation R, the *HyperNeutrosophic Lower Approximation* and *Upper Approximation* at an event x are defined by

$$\widetilde{N}_*(x) = \bigcap_{y \in [x]_R} \widetilde{N}(y), \quad \widetilde{N}^*(x) = \bigcup_{y \in [x]_R} \widetilde{N}(y).$$

In particular, for e_1 (with $[e_1]_R = \{e_1, e_2\}$), we compute:

$$\begin{aligned}
\widetilde{N}_*(e_1) &= \Big([0.7, 0.8] \cap [0.65, 0.75],\ [0.1, 0.2] \cap [0.15, 0.25],\ [0.0, 0.05] \cap [0.05, 0.10]\Big) \\
&= \Big([0.7, 0.75],\ [0.15, 0.2],\ \{0.05\}\Big), \\
\widetilde{N}^*(e_1) &= \Big([0.7, 0.8] \cup [0.65, 0.75],\ [0.1, 0.2] \cup [0.15, 0.25],\ [0.0, 0.05] \cup [0.05, 0.10]\Big) \\
&= \Big([0.65, 0.8],\ [0.1, 0.25],\ [0.0, 0.10]\Big).
\end{aligned}$$

Here, the lower approximation $\widetilde{N}_*(e_1)$ represents the degree to which similar events certainly indicate an intrusion, while the upper approximation $\widetilde{N}^*(e_1)$ reflects the overall potential of e_1 being part of a threat, considering uncertainty.

3.5 | HyperNeutrosophic Hyperrough Set

We now move to the *HyperNeutrosophic Hyperrough Set*, extending multi-parameter or multi-attribute generalizations such as *HyperRough Sets* and *HyperFuzzy Hyperrough Sets* into the neutrosophic domain with a hyper-based membership notion.

Definition 29 (HyperNeutrosophic Hyperrough Set)**.** Let (X, R) be a neutrosophic approximation space as before. Let

$$\widetilde{G} : \mathcal{C} \to \Big\{ \text{HyperNeutrosophic Sets on } X \Big\},$$

where $\mathcal{C} = A_1 \times A_2 \times \cdots \times A_m$ is a multi-attribute parameter domain. In other words, for each $c \in \mathcal{C}$, $\widetilde{G}(c)$ is itself a *HyperNeutrosophic Set* on X, i.e.

$$\widetilde{G}(c) : X \to \mathcal{P}^*([0,1]^3).$$

We define the *HyperNeutrosophic Rough Approximations* of $\widetilde{G}(c)$ for each $c \in \mathcal{C}$ in the same manner as Definition 26, obtaining

$$(\widetilde{G}(c))_*(x), \quad (\widetilde{G}(c))^*(x), \quad \forall x \in X.$$

The triple

$$\Big(\widetilde{G}_*, \widetilde{G}^*, \mathcal{C} \Big)$$

is called a *HyperNeutrosophic Hyperrough Set*. Concretely, for each parameter tuple $c \in \mathcal{C}$, we have a *HyperNeutrosophic Rough Set* $(\widetilde{G}(c))_*, (\widetilde{G}(c))^*$, reflecting the multi-attribute notion of c.

Theorem 30 (HyperNeutrosophic Hyperrough Set Generalizes Several Frameworks)**.** *The* HyperNeutrosophic Hyperrough Set *strictly generalizes:*

(1) *HyperRough Set: If each $\widetilde{G}(c)(x)$ merely assigns $\{0\}$ or $\{1\}$ membership or crisp sets in X, we recover the usual HyperRough approach (where $F(c) \subseteq X$).*

(2) *HyperNeutrosophic Rough Set: If $\mathcal{C}$ has a single element c_0, then we only have a single* HyperNeutrosophic Set $\widetilde{G}(c_0)$ *to approximate. This collapses to the prior definition of a HyperNeutrosophic Rough Set in Definition 26.*

(3) *HyperFuzzy Hyperrough Set: By forcing all membership triples (T, I, F) to have $I = 0, F = 0$, we effectively embed the hyperfuzzy membership sets in $[0,1]$; the multi-parameter approach is that of a hyperrough set, so we recover the* HyperFuzzy Hyperrough Set*.*

Proof: *(1)* Suppose for each $c \in \mathcal{C}$, $\widetilde{G}(c)(x)$ is always $\{0\}$ or $\{1\}$ for each $x \in X$. Then it does not truly matter that we call it (T, I, F); each triple is effectively $(1, 0, 0)$ or $(0, 0, 0)$, or we can say it is a single crisp membership. Hence $\widetilde{G}(c)$ reduces to a function $G : \mathcal{C} \to \mathcal{P}(X)$. The approximation formulas become $\underline{G(c)}$, $\overline{G(c)}$ in the sense of a *HyperRough Set*.

(2) If $|\mathcal{C}| = 1$, then we have exactly one hyperneutrosophic set $\widetilde{N}$ on X, so we are back to the *HyperNeutrosophic Rough Set* as in Definition 26.

(3) By letting all (T, I, F) in $\widetilde{G}(c)(x)$ satisfy $I = 0$, $F = 0$, we effectively revert to membership degrees $T \in [0, 1]$ only. Then $\widetilde{G}(c)$ becomes a *hyperfuzzy set* for each c, and the multi-attribute approximation is exactly a *HyperFuzzy Hyperrough Set*.

Hence, each specialized framework is naturally embedded in the new *HyperNeutrosophic Hyperrough Set*. □

Example 31 (HyperNeutrosophic Hyperrough Set in Cybersecurity)**.** Now suppose that, in addition to the intrinsic uncertainty of each event, cybersecurity analysts consider multiple attributes to further classify threats. Let the attribute domains be:

- A_1: *Attack Type* (e.g., Malware[62] or Phishing),

- A_2: *Severity Level* (e.g., High or Low).

The parameter space is then

$$\mathcal{C} = A_1 \times A_2 = \{(\text{Malware}, \text{High}),\ (\text{Malware}, \text{Low}),\ (\text{Phishing}, \text{High}),\ (\text{Phishing}, \text{Low})\}.$$

For each $c \in \mathcal{C}$, define a *HyperNeutrosophic Set*

$$\widetilde{G}(c) : X \to \mathcal{P}^*([0,1]^3)$$

that evaluates the degree (expressed as sets of neutrosophic membership triples) to which an event x exhibits the characteristics associated with c. For example, for $c = (\text{Malware}, \text{High})$ we may assign

$$\begin{aligned}
\widetilde{G}(\text{Malware}, \text{High})(e_1) &= \{(T, I, F) : T \in [0.8, 0.9],\ I \in [0.05, 0.1],\ F \in [0.0, 0.05]\}, \\
\widetilde{G}(\text{Malware}, \text{High})(e_2) &= \{(T, I, F) : T \in [0.75, 0.85],\ I \in [0.1, 0.15],\ F \in [0.05, 0.1]\}.
\end{aligned}$$

Then, using the same approximation process as above, for an event e_1 (with $[e_1]_R = \{e_1, e_2\}$) we have:

$$\begin{aligned}
\left(\widetilde{G}(\text{Malware}, \text{High})\right)_*(e_1) &= \widetilde{G}(\text{Malware}, \text{High})(e_1) \cap \widetilde{G}(\text{Malware}, \text{High})(e_2) \\
&= \Big([0.8, 0.9] \cap [0.75, 0.85],\ [0.05, 0.1] \cap [0.1, 0.15],\ [0.0, 0.05] \cap [0.05, 0.1]\Big) \\
&= \Big([0.8, 0.85],\ \{0.1\},\ \{0.05\}\Big), \\
\left(\widetilde{G}(\text{Malware}, \text{High})\right)^*(e_1) &= \widetilde{G}(\text{Malware}, \text{High})(e_1) \cup \widetilde{G}(\text{Malware}, \text{High})(e_2) \\
&= \Big([0.8, 0.9] \cup [0.75, 0.85],\ [0.05, 0.1] \cup [0.1, 0.15],\ [0.0, 0.05] \cup [0.05, 0.1]\Big) \\
&= \Big([0.75, 0.9],\ [0.05, 0.15],\ [0.0, 0.10]\Big).
\end{aligned}$$

The collection

$$\left(\widetilde{G}_*,\ \widetilde{G}^*,\ \mathcal{C}\right)$$

forms the *HyperNeutrosophic Hyperrough Set*, integrating multi-attribute evaluations with neutrosophic uncertainty. This framework enables analysts to combine various cybersecurity indicators—such as attack type and severity—with uncertain, incomplete, or conflicting evidence, thereby supporting robust intrusion detection and classification.

3.6 | Multigranulation HyperRough Set

We explore the concept of the Multigranulation HyperRough Set. This set is defined as a generalization of both the Multigranulation Rough Set[116, 163, 13, 89, 123, 86, 115] and the Hyperrough Set.

Definition 32 (Optimistic Multigranulation Rough Set)**.** [81, 90] Let $I = \langle U, AT \rangle$ be an information system where $AT = \{a_1, a_2, \dots, a_m\}$ and $X \subseteq U$. The optimistic multigranulation lower and upper approximations of X are defined as:

$$AT^O(X) = \{x \in U : [x]_{a_1} \subseteq X \vee \cdots \vee [x]_{a_m} \subseteq X\},$$
$$AT^O(X) = \sim (AT^O(\sim X)),$$

where $[x]_{a_i}$ denotes the equivalence class of x with respect to a_i, and $\sim X$ is the complement of X.

Definition 33 (Pessimistic Multigranulation Rough Set)**.** [114, 151, 88, 10, 164, 98] Let $I = \langle U, AT \rangle$ be an information system where $AT = \{a_1, a_2, \dots, a_m\}$ and $X \subseteq U$. The pessimistic multigranulation lower and upper approximations of X are defined as:

$$AT^P(X) = \{x \in U : [x]_{a_1} \subseteq X \wedge \cdots \wedge [x]_{a_m} \subseteq X\},$$
$$AT^P(X) = \sim (AT^P(\sim X)),$$

where $[x]_{a_i}$ denotes the equivalence class of x with respect to a_i, and $\sim X$ is the complement of X.

We now unify these two notions by introducing a function from a parameter domain J to subsets of X, but *approximating each subset via multiple granules (equivalence relations)* in either an optimistic or pessimistic manner. Formally, we allow a family of relations $\{R_{a_1}, \dots, R_{a_m}\}$ on X (each R_{a_i} induced by attribute a_i), and apply multigranulation-style approximations to $F(j)$.

Definition 34 (Multigranulation HyperRough Set (MHRS))**.** Let X be a non-empty universe, $AT = \{a_1, \dots, a_m\}$ be a family of attributes, each inducing an equivalence relation R_{a_i} on X. Let J be a (possibly multi-attribute) parameter set, and let

$$F : J \to \mathcal{P}(X)$$

be a hyper-mapping. We define *optimistic* and *pessimistic Multigranulation HyperRough Sets* as follows.

(1) *Optimistic MHRS:* For each $j \in J$, define

$$\underline{F}^{O}(j) = \Big\{x \in X : ([x]_{a_1} \subseteq F(j)) \vee \cdots \vee ([x]_{a_m} \subseteq F(j))\Big\},$$

$$\overline{F}^{O}(j) = \sim\Big(\underline{F}^{O}(j^c)\Big) \quad \text{or equivalently} \quad \Big\{x \in X : ([x]_{a_1} \cap F(j) \neq \emptyset) \vee \cdots \vee ([x]_{a_m} \cap F(j) \neq \emptyset)\Big\},$$

where $[x]_{a_i}$ is the equivalence class of x with respect to a_i, and j^c indicates the parameter for the complement $F(j^c) = X \setminus F(j)$ if desired. The triple

$$\Big(\underline{F}^{O},\ \overline{F}^{O},\ J\Big)$$

is called the *Optimistic Multigranulation HyperRough Set* associated with F and $\{R_{a_1}, \dots, R_{a_m}\}$.

(2) *Pessimistic MHRS:* For each $j \in J$, define

$$\underline{F}^{P}(j) = \Big\{x \in X : ([x]_{a_1} \subseteq F(j)) \wedge \cdots \wedge ([x]_{a_m} \subseteq F(j))\Big\},$$

$$\overline{F}^{P}(j) = \sim\Big(\underline{F}^{P}(j^c)\Big) \quad \text{or equivalently} \quad \Big\{x \in X : ([x]_{a_1} \cap F(j) \neq \emptyset) \wedge \cdots \wedge ([x]_{a_m} \cap F(j) \neq \emptyset)\Big\}.$$

Then

$$\Big(\underline{F}^{P},\ \overline{F}^{P},\ J\Big)$$

is called the *Pessimistic Multigranulation HyperRough Set.*

In other words, for each parameter $j \in J$, we apply a multigranulation approximation (optimistic or pessimistic) to the subset $F(j) \subseteq X$. The result is a pair of approximations $(\underline{F}^{O/P}(j),\ \overline{F}^{O/P}(j))$ that collectively form a hyperrough structure, but with multiple underlying granules $\{R_{a_1}, \dots, R_{a_m}\}$ instead of a single R.

We next show that the Multigranulation HyperRough Set (MHRS) generalizes both the usual *Multigranulation Rough Set* (when J is a singleton) and the *HyperRough Set* (when $m = 1$).

Theorem 35 (MHRS Generalizes Multigranulation Rough Set)**.** *If $J = \{j_0\}$ is a singleton parameter domain, then the optimistic/pessimistic Multigranulation HyperRough Set of Definition 34 reduces to the classical optimistic/pessimistic multigranulation rough set.*

Proof: With $J = \{j_0\}$, the hyper-mapping F is effectively $F(j_0) \subseteq X$. Then:

$$\underline{F}^{O}(j_0) = \{\, x \in X : [x]_{a_1} \subseteq F(j_0) \ \vee \cdots \vee\ [x]_{a_m} \subseteq F(j_0)\},$$

which is precisely $\underline{AT^{O}}(F(j_0))$, up to notation. Similarly for $\overline{F}^{O}(j_0)$ and for the pessimistic case $\underline{F}^{P}(j_0), \overline{F}^{P}(j_0)$. Thus we recover the standard multigranulation rough set approach for a single subset $F(j_0)$. □

Theorem 36 (MHRS Generalizes HyperRough Set)**.** *If $m = 1$, i.e. $AT = \{a_1\}$ with a single equivalence relation R_{a_1}, then the Multigranulation HyperRough Set becomes the standard HyperRough Set with respect to R_{a_1}.*

Proof: If $m = 1$, then the condition $[x]_{a_1} \subseteq F(j)$ is the only relevant one in either the optimistic or pessimistic definition. The "optimistic" version collapses to:

$$\underline{F}^{O}(j) = \{\, x \in X : [x]_{a_1} \subseteq F(j)\}, \quad \overline{F}^{O}(j) = \{\, x \in X : [x]_{a_1} \cap F(j) \neq \emptyset\},$$

which are precisely the classical Pawlak rough approximations (or Crisp HyperRough approximations) for $F(j)$ under R_{a_1}. The same occurs with the pessimistic definition, but since $m = 1$, the logical $\wedge$ or $\vee$ are trivial. Hence we retrieve the HyperRough Set notion. □

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Data Availability

This paper does not involve any data analysis.

Ethical Approval

This article does not involve any research with human participants or animals.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Disclaimer

This study primarily focuses on theoretical aspects, and its application to practical scenarios has not yet been validated. Future research may involve empirical testing and refinement of the proposed methods. The authors have made every effort to ensure that all references cited in this paper are accurate and appropriately attributed. However, unintentional errors or omissions may occur. The authors bear no legal responsibility for inaccuracies in external sources, and readers are encouraged to verify the information provided in the references independently. Furthermore, the interpretations and opinions expressed in this paper are solely those of the authors and do not necessarily reflect the views of any affiliated institutions.

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