An indicator of inclusion with applications to computer vision

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ABSTRACT: In this paper we present an algorithmic process of necessary operations for the automatic movement of a predefined object from a video image in the target region of that image, intended to facilitate the implementation of specialized software applications in solving this kind of problems.

1. INTRODUCTION

The Algorithmic problem-solving procedures for automatic traveling objects within the video images were approached by us also in an earlier work, see [4]. The purpose of this paper is to point out a new method of solving these problems. As mentioned in the earlier work, the process which we shall indicate, will be based on the definition of an indicator of Extenics type specialized in signaling whether a particular set (of pixels, in our modeled case) is included in a target set on the monitor screen.

An aspect which has to be mentioned from the beginning is that both indicators, the one defined in [4], as well as the indicator that we shall define in this paper, fundamentally differ from the indicators currently used in the Extension theory because they make the leap from reporting the position of a single point in relation to one or two given sets to that of reporting the relationship between two sets - which is much more complex, thus establishing a factor that is meant to ensure the progress of this theory. The Extension theory, which we have referred to earlier, has been proposed by Professor Cai Wen in [5].

Because of the importance of this theory in both theoretical and practical field, it has been continuously extended, at the beginning by its founder himself, see [1, 6, 7], and then by other researchers from various fields of activity, see [2, 3, 4].

2. AN INDICATOR CAPABLE TO REPORT IF A SPECIFIC SET IS INCLUDED IN A GIVEN TARGET SET

This paragraph aims at presenting new results in order to complete and improve the existent Extension theory. The framework addressing these results is that of a metric space expressed through the doublet (X, d), where X is the set of points which make up the considered space, and d is the metric of this space.

For any two nonempty sets A and B from X we introduce the indicator

$$\Delta(A, B) = \sup\{\delta(a, B) \mid a \in A\}, \tag{1}$$

where we denote by $\delta(a, B)$ the usual distance from point $a \in A$ to set B, that is

$$\delta(a, B) = \inf\{d(a, b) \mid b \in B\}.$$

Observations: 1) Relation $\Delta(A, B) = \Delta(B, A)$ is not always true, in other words, the value of indicator $\Delta(A, B)$ depends, in general, on the order in which sets A and B are considered.

2) Indicator $\Delta(A, B)$ can also take infinite values.

3) In the case of two bounded sets A and B [1] indicator $\Delta(A, B)$ is finite.

Proposition: Indicator Δ defined by relation (1) has the following properties:

1) $\Delta(A, B) = 0 \Rightarrow A \subseteq \overline{B}$, where $\overline{B}$ is the closure of set B in topology induced by metric d on space X. Reciprocally, $A \subseteq \overline{B} \Rightarrow \Delta(A, B) = 0$.

2) $\Delta(A, B) = \Delta(B, A) = 0 \Rightarrow \overline{A} = \overline{B}$. Reciprocally, $\overline{A} = \overline{B} \Rightarrow \Delta(A, B) = \Delta(B, A) = 0$.

Demonstration: 1) $\Delta(A, B) = 0 \Leftrightarrow \delta(a, B) = 0\ \forall\, a \in A \Leftrightarrow A \subseteq \overline{B}$. Reciprocally, if $A \subseteq \overline{B}$ then $\delta(a, B) = 0\ \forall\, a \in A \Leftrightarrow \Delta(A, B) = 0$.

2) $\Delta(A, B) = 0 \Rightarrow A \subseteq \overline{B}$, and $\Delta(B, A) = 0 \Rightarrow B \subseteq \overline{A}$. From $A \subseteq \overline{B}$, and $B \subseteq \overline{A}$ we deduce that $\overline{A} \subseteq \overline{B}$, respectively $\overline{B} \subseteq \overline{A}$. Consequently $\overline{A} = \overline{B}$. Reciprocally, if $\overline{A} = \overline{B}$ then $\delta(a, B) = 0\ \forall\, a \in A$, and $\delta(b, A) = 0\ \forall\, b \in B$, hence, $\Delta(A, B) = \Delta(B, A) = 0$.

Observations: 1) Due to property 1) from the above proposition, indicator Δ is named indicator of inclusion.

2) On the set $\mathsf{C}(X)$ of all non-empty compact subsets of X,

$$H(A, B) = \max\{\Delta(A, B), \Delta(B, A)\},$$

represents the Hausdorff distance between sets A and B.

3. APPLICATIONS

Pointer Δ specified by us in this paper can be used to solve the problems posed by the development of software applications for an automated movement of a specific object “O” from a given video image “VIm” in a target region “R” of that image. In order to achieve this goal, we use an algorithm similar to the one that was defined in [4]. In very general terms, the new algorithm has the following content: through a set of isometries I_i, $i \in I$ of the plane, we move object O into different regions and positions of the image “VIm” by calculating, every time, the value of each indicator $\Delta(\mathsf{I}_i(\mathsf{O}), \mathsf{R})$. The determination of the indices $i_0 \in I$ for which $\Delta(\mathsf{I}_{i_0}(\mathsf{O}), \mathsf{R}) = 0$ indicates the solution to the problem. Indeed, in the situation presented within the application, object O and region R can be abstractized through the means of compact sets, and given these hypotheses, point 1) of the proposition enunciated earlier admits the following restatement:

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[1] A set Y from X is called bounded if its diameter $D(Y) = \sup\{d(y_1, y_2) \mid y_1, y_2 \in Y\}$ is bounded.

$$\Delta\left(I_{i_0}(O), R\right) = 0 \Leftrightarrow I_{i_0}(O) \subseteq R .$$

Observation: This algorithm can be easily adapted to solving some similar problems in the space of three dimensions, becoming even more useful in projecting artificial intelligence forms.

4 REFERENCES

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