Florentin Smarandache, Another Way to Divide Two Complex Numbers: Identification Method, Octogon Mathematical Journal, Vol. 31. No. 2, 951-953, October 2023.

Another Way to Divide Two Complex Numbers: the Identification Method

Florentin Smarandache
University of New Mexico
Mathematics, Physics, and Natural Science Division
705 Gurley Ave., Gallup, NM 87301, USA

Abstract

In this short note to design another method to divide two complex numbers and present a numerical example.

1. Identification Method

Let $z_1 = a + bi$ and $z_2 - c + di$ be two complex numbers, where *a, b, c, d* are real numbers, and $i = \sqrt{-1}$.

$\frac{z_1}{z_2} = \frac{a+bi}{c+di} = x + yi$, where x and y are real numbers that we need to find out by identification method.

Whence:

$$a + bi \equiv (c+di)(x+yi) = cx + cyi + dxi + dyi^2 = cx + cyi + dxi + dy(-1) =$$
$$= (cx - dy) + (dx + cy)i$$

Therefore, by identification, one gets the following 2×2 linear system:

$$\begin{cases} cx - dy = a \\ dx + cy = b \end{cases}$$

Then

$$x = \frac{\begin{vmatrix} a & -d \\ b & c \end{vmatrix}}{\begin{vmatrix} c & -d \\ d & c \end{vmatrix}} = \frac{ac+bd}{c^2+d^2}$$

$$y = \frac{\begin{vmatrix} c & a \\ d & b \end{vmatrix}}{\begin{vmatrix} c & -d \\ d & c \end{vmatrix}} = \frac{bc - ad}{c^2 + d^2}$$

where the above notation

$$\begin{vmatrix} \bullet & \bullet \\ \bullet & \bullet \end{vmatrix}$$

means determinant of a 2×2 matrix.

2. Example using the Identification Method

For example, let $z_1 = 4 - 3i, z_2 = -1 + 2i$,

whence *a = 4, b = -3, c = -1,* and *d = 2;*

then:

$$\frac{z_1}{z_2} = \frac{ac + bd}{c^2 + d^2} + \frac{bc - ad}{c^2 + d^2} i = \frac{4 \cdot (-1) + (-3) \cdot 2}{(-1)^2 + 2^2} + \frac{(-3) \cdot (-1) - 4 \cdot 2}{(-1)^2 + 2^2} i = \frac{-10}{5} + \frac{-5}{5} i =$$
$$= -2 - i.$$

3. The Classical Division of Complex Numbers

It is based on multiplying both the numerator and denominator with the conjugate of the denominator [1].
The conjugate of *c + di* is *c - di*.

$$\frac{z_1}{z_2} = \frac{a + bi}{c + di} = \frac{(a + bi)(c - di)}{(c + di)(c - di)} = \frac{ac - adi + bci - bdi^2}{c^2 + d^2} = \frac{ac - adi + bci + bd}{c^2 + d^2} =$$
$$= \frac{(ac + bd) + (bc - ad)i}{c^2 + d^2} = \frac{(ac + bd)}{c^2 + d^2} + \frac{(bc - ad)i}{c^2 + d^2} = \frac{ac + bd}{c^2 + d^2} + \frac{bc - ad}{c^2 + d^2} i$$

Therefore, we get the same solution by both methods.

Reference

[1] James Stewart, Lothar Redlin, Saleem Watson, Complex Numbers, Section 1.5, pp. 126-131, in Algebra and Trigonometry, Fourth Edition, Cengage Learning, Boston, USA.