BIPOLAR NEUTROSOPHIC MINIMUM SPANNING TREE NETWORK USING UPPER TRIANGULAR WEIGHT/COST ALGORITHM

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Abstract

A minimum spanning tree (MST) is a spanning tree in which the total weights of all of its edges are the lowest of all conceivable spanning trees for the network. MST is crucial in discipline of the operation research. Similarly, for the neutrosophic minimum spanning tree problem (NMST) the weights are unpredictable and inconsistent. The next form of NMST is considered the bipolar neutrosophic minimum spanning tree problem. Each edge of the standard NMST is assigned a bipolar weight based on the bipolar neutrosophic number (consisting of positive and negative membership degrees). This paper introduces the upper triangular weight algorithm for a given network and the optimal results are executed in MATLAB.

Keywords: : Minimum Spanning Tree, Bipolar NMST, Upper/Lower triangular weight algorithm

1. Introduction

The MST problem is an acyclic subgraph that connects all of its vertices/nodes to its edges with the least total associated weights. The most basic subgraph of a given network can be a spanning tree. In the majority of network design issues, explicitly or implicitly spanning trees are crucial. One such network issue, and the most well-known NP-hard issue in network optimization, is the traditional minimum spanning tree problem (MSTP). Roads, utilities, power, communication networks, etc. are only a few examples of authentic world issues where network design plays a crucial role in general MSTP.

The MSTP represents a spanning tree graph in weighted graphs that develops with the lowest cost/weight compared to other trees on a similar graph. The weight can indicate computer traffic, distance, cost, congestion, or any other subjective factors connected to the arcs in authentic world circumstances. The idea of the neutrosophic set generalizes the notion of the classical set, fuzzy set, conventional fuzzy, type-1 fuzzy, and intuitionistic fuzzy set (IFS). Inside the real and standard or non-standard unit interval]-0, 1+[, the neutrosophic sets are separated into three membership functions truth-membership function (T), indeterminate-membership function (I), and false-membership function (F). Additionally, BNSs (Bipolar Neutrosophic sets) can be used for real-world problems by Smarandache. Then they had a concept of bipolar neutrosophic sets [1] proposed the fuzzy notion to deal with uncertainty.

Smarandache [2] proposed the idea of a neutrosophic set (NS) to symbolize the uncertain, imprecise, lacking in detail, unpredictable, and ambiguous information that exists in our authentic world. Also, the author initially introduced the single-valued neutrosophic set (SVNS) to be used in industrial and scientific applications. Neutrosophic graphs are much more resilient, cost-effective, accurate, and applicable to model unpredictable problems compared to fuzzy graphs.

[17] described how the second-degree vertices' notation of the fuzzy graph works as well as a number of their attributes. However, compared to fuzzy graphs, neutrosophic graphs are significantly more robust, cost-effective, accurate, and suitable to simulate unpredictable situations as an enormous application in our world such as wireless communication networks, logistic networks, and social networks.
Many studies have focused on the T, I, and F components, leading to the definition of specific neutrosophic set cases such as simplified neutrosophic sets.[3] introduce the concept of single-valued neutrosophic sets and express the nature of the single-valued neutrosophic weight. A new efficient method to draw an MST to evaluate with several sample networks and the methodology of the effective algorithm is described by [4]. The neutrosophic sets' decisions provide the basis of the decision-making process. The weights, costs, distances, time.... decisions over the cities, states, traffics, relationships through the concepts. Neutosophic sets (NSs), which are employed in decision-making, are discussed by many authors, and neutrosophic numbers are utilized as weights. [5] discusses the similarity measures between interval neutrosophic sets and their use in multi-criteria decision-making then the author examined the concept in a multi-attribute trapezoidal neutrosophic set in 2015. A. [10] describe the neutrosophic soft sets with multi-criteria decision-making. Bipolar neutrosophic set [7], trapezoidal neutrosophic set [8], interval-valued neutrosophic sets [11], rough neutrosophic set with efficient theorems and conditions described [6].
The author [7] distinctions concerning the two bipolar neutrosophic numbers (BNNs) and explains the notions of each weight by the accuracy function and certainty function. [9] built a second minimum weight spanning tree of a network using a triangular intuitionistic fuzzy number as edge weight and tabulated the crips values to the provided graph. The bipolar neutrosophic weights for the MST were described as a matrix form and applying the score function in the matrix then final computation figures to get the optimal tree by [12] than for general neutrosophic minimum spanning tree be investigated with single-valued neutrosophic weights. The MST Algorithm with example and scoring functions w.r.t. bipolar neutrosophic graphs are provided [18]. The enormous author described the concepts of bipolar neutrosophic numbers in various problems [19], [15], [16], and [19]. The approximate balanced score function [20] will be introduced for neutrosophic graphs with weak edge weights, and then the score functions for neutrosophic graphs by Nancy et al. and Boumi et al. will be compared.

2. Methods

Upper/Lower weight algorithm of neutrosophic undirected graph for bipolar minimum spanning tree problem. The focus of this upper/lower weight algorithm is to help for finding the optimal tree with less computational time. Because while computing programming in MATLAB gives the results for the score matrix as well the optimal edges of the bipolar neutrosophic minimum spanning network here by following the steps below,
Input: consider bipolar minimum spanning network of G.
Output: Final optimal tree for G
Consider the given network G for solving the bipolar neutrosophic minimum spanning tree problem.
Step 1: Construct the given network G into $n \times n$ matrix form.[ie., M(A)].
Step 2: Follow the phase-1 pseudocode help to make the score matrix $S(\tilde{A})$ for M(A).
Step 3: In phase-2 the main algorithm starts.

- Consider the upper/lower score matrix $S(\tilde{A})$.
- For assigning a larger value for the diagonal element in U(A)/L(A).
- Construct all the edge values for the upper/lower triangular matrix.
- Sortrows all the edges.

Step 4: Draw the edges one by one according to sortrows also follow the rules of the neutrosophic minimum spanning tree.
Step 5: Finally, we get the final optimal tree for the given bipolar neutrosophic minimum spanning tree problem.
Step 6: Also find the accuracy and certainty for all the edges of the optimal network.
MATLAB Pseudocode:
Input: Taking $n \times n$ matrix for the given G.
Phase-1:
tic
Enter $T^p_A(X)$, $I^p_A(X)$, $F^p_A(X)$ and $T^n_A(X)$, $I^n_A(X)$, $F^n_A(X)$ all the entries of G edges or consider the upper/lower triangular edges.
Apply for all the p and n memberships in $S(\tilde{A})$.
toc
Phase-2:
tic
Using $S(\tilde{A})$ and creating U(A) matrix.
Taking triu $S(\tilde{A})$ followed by U(A).
for i=1:n
A(i,i)=999;
for j=1:n
if (i>j)
A(i,j)=999;
end
end
end
Choose the matrix upper/lower case named by U(A)/L(A).
Sortrows of U(A).
Collect all the sort rows.
toc
Output: Finally draw the optimal network.

3. Results

A numerical example using network G, which has a 6-node undirected complete and bipolar weighted graph, is taken into consideration to define the model as a Neutrosophic bipolar minimum spanning tree. Finding a least-cost route that connects every node with $|N|$=n=6 and prevents any other nodes from having more degrees than the allowed three is the challenge. Since the problem is defined on the undirected graph, the corresponding cost matrix becomes symmetric.
The cost and weight values typically presumptively presume non-negative quantity. But in bipolar case, Fig.1 shows the truth, indeterminacy, and falsity with positive and negative edges.

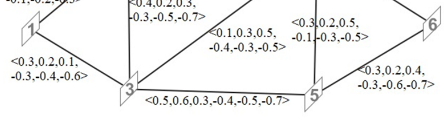

Figure 1: *Network G*

Using the Fig.1 the weight/cost matrices be framed as M(A), $< 0 >$ demonstrates the node's for dis-connectivity.
Calculating the values of the matrix M(A) using the phase-1 the score matrix is represented in matrix form (i.e.,).
$S((\tilde{A}))$=

$$\begin{pmatrix} 0 & 0.517 & 0.617 & 0 & 0 & 0 \\ 0.517 & 0 & 0.633 & 0.5 & 0 & 0 \\ 0.617 & 0.633 & 0 & 0.45 & 0.567 & 0 \\ 0 & 0.5 & 0.45 & 0 & 0.55 & 0.317 \\ 0 & 0 & 0.567 & 0.55 & 0 & 0.317 \\ 0 & 0 & 0 & 0.533 & 0.317 & 0 \end{pmatrix}$$

Thus, an upper triangular cost matrix U(A)/L(A) is considered for evaluating the new algorithm in phase-2.
$U(A)$=

$$\begin{pmatrix} 999 & 0.517 & 0.617 & 0 & 0 & 0 \\ - & 999 & 0.633 & 0.5 & 0 & 0 \\ - & - & 999 & 0.45 & 0.567 & 0 \\ - & - & - & 999 & 0.55 & 0.317 \\ - & - & - & - & 999 & 0.317 \\ - & - & - & - & - & 999 \end{pmatrix}$$

Fig.2 shows the optimal minimum spanning network for the given bipolar minimum spanning network G. The crisp minimum cost of the M(A) is 2.565 and the ordered pair for the bipolar neutrosophic tree is (2,4), (1,3), (1,2), (4,5), and (4,6).

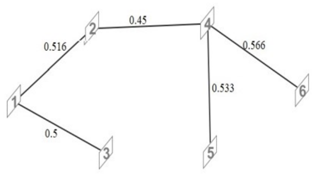

Figure 2: *Optimal solution for given G.*

The accuracy and certainty of every edge value be examined for bipolar NMST for the given network G as shown in Fig.3.

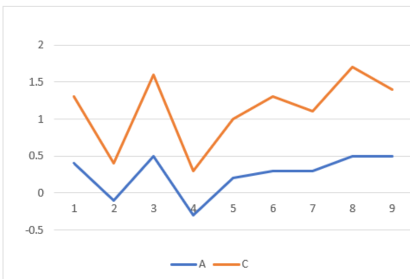

Figure 3: *shows the accuracy and certainty)*

4. Comparison Results

The BNMST computation details are provided in this section for solving the BNMST, and a precise new algorithm has been created. The algorithm is programmed in MATLAB-2019b and

Table 1: *Noted the Upper triangular algorithm with CPU run time*

Author's algorithm	n	Order Pairs of BNMST	Optimal values	Elapse time
Broumie et.al Algorithm	5	{2,1},{1,4},{4,3},{3,5}	1.8	-
Proposed Algorithm	5	{1,4},{3,5},{1,2},{3,4}	1.74	2.137817
Upender et.al Algorithm	6	{1,2},{2,4},{4,3},{4,6},{5,6}	2.327	-
Proposed Algorithm	6	{2,4},{1,3},{1,2},{4,5},{4,6}	2.565	0.026348

tested on a computer running the Microsoft Windows11 version 22H2, operating system with an Intel(R) Core (TM) 8250U processor running at 1.60 GHz or 1.80 GHz and 8 GB of Memory.
To determine the effectiveness of the Upper/Lower triangular edge weight algorithm extensive experimentation was done for investigation purposes searching some problems with varying n related to the BNMST problem by Broumie et.al and Upende et.al are compared with our algorithm and noted the running time of the algorithm in Table.1.

5. Conclusion and Future Work

Consider the bipolar minimum spanning tree network to design the optimal decision tree using the order pairs. For the symmetric version of BNMST, an upper/lower weight algorithm is created and tested in the MATLAB programming language also running time is calculated. The created method is validated using a relevant numerical example. Computational work with some examples has been done to assess the effectiveness of the algorithm.
The overall findings demonstrate the effectiveness of the developed algorithm and calculating the best optimal result for BNMST with help of upper/lower weight algorithm. Also, this model is more adaptable in addressing issues with real-time networks.

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