

**The 10th International Combinatorics and Graph Theory
Conference (CGT 2025)}, May 23-25, 2025, Xian, P.R.China**

Combinatorics

**-- A Mathematical Approach for Holding on
the Realty of Thing in the Universe**

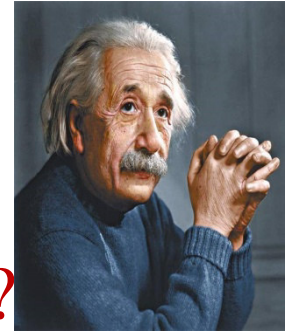
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>>Questions:

Q1: What is the original intention of mathematics?

A1: Mathematics is a formal system which can model the evolution of thing by its element so that the behavior of thing is consistent with that of the element.

Q2: Can we really understand the behavior of thing by mathematics now?

Einstein's complaint for mathematics on the reality:

As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality.

Why is so happening? Because we are lack of deep thinking on the combination!



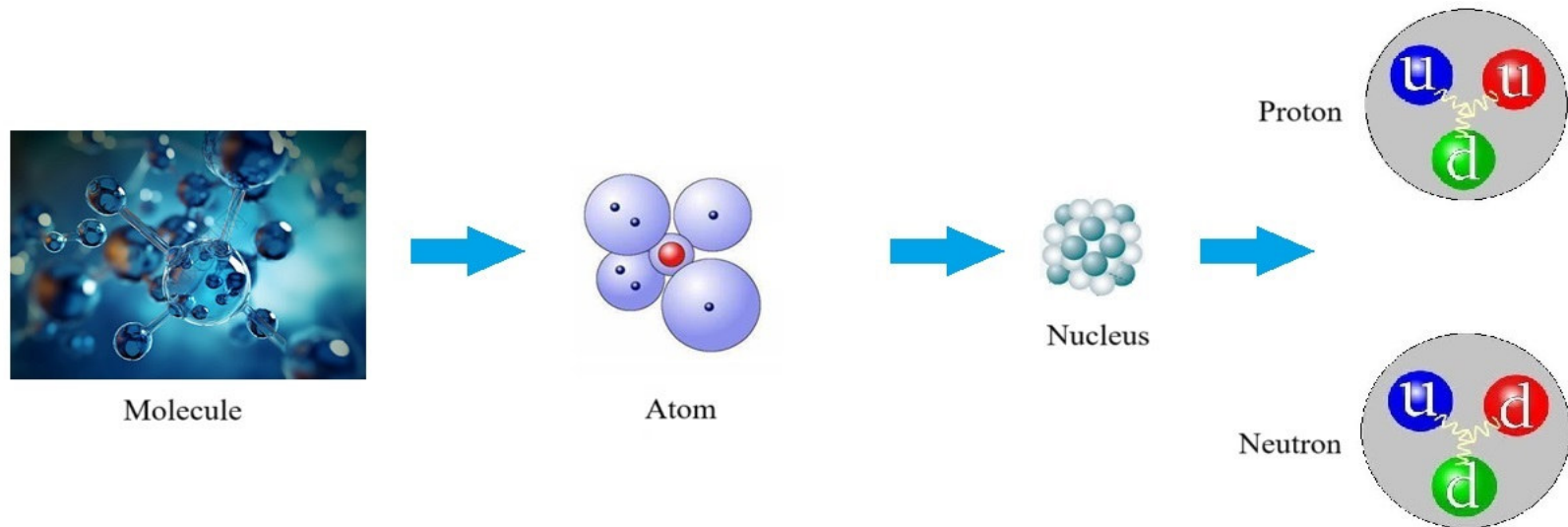
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1. Introduction

1.1. Combination Prior to Reductionism



Reductionism: subdividing matter $T \rightarrow$ molecule \rightarrow atom \rightarrow nucleus \rightarrow proton and neutron \rightarrow elementary particles

living $L \rightarrow$ biological macromolecule \rightarrow cells or genes

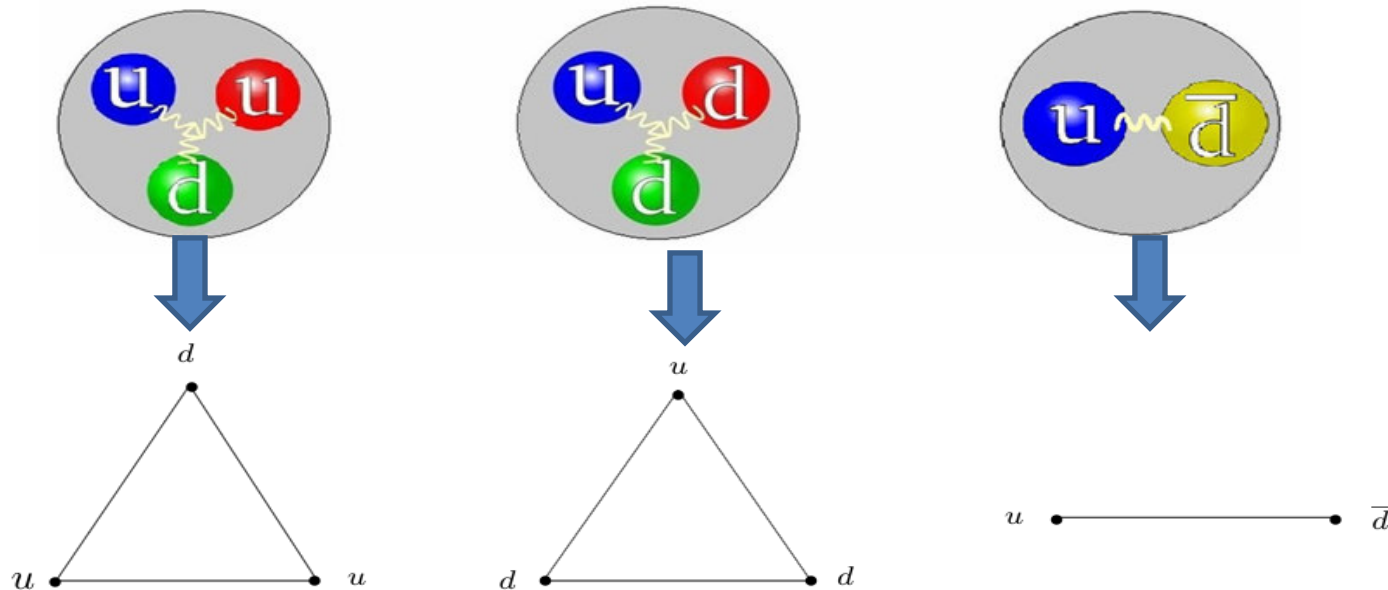
What is the result by reductionism? **$T =$ complex network.**

For example, the complex networks with 3.6×10^{13} cells of male and 2.8×10^{13} cells of female.

Assumption in reductionism without proof:

The reality of matter T can be held if the behavior of elementary particles can be recognized locally by humans.

For example, models on proton, neutron, meson by quarks.



Is this assumption right and so, one can hold on the reality of matter T by reductionism?

The answer is not certain unless all elementary particles are in stationary or synchronization.

>> Fable of blind men with an elephant

For holding the whole with the local, we all learned a famous fable of the **blind men with an elephant** in elementary school.

the 1st one touched the elephant's tooth and claimed it is like a big smooth radish.

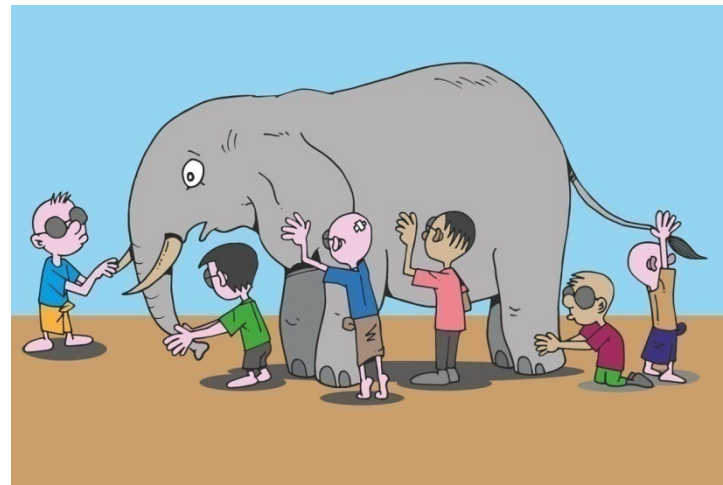
the 2nd one touched the elephant's trunk and claimed it is like a tube.

the 3rd one touched the elephant's ear and claimed it is like a big fan,

the 4th one touched the elephant's belly and claimed it is like a wall.

the 5th one touched the elephant's leg and claimed it is like a big pillar.

the 6th one touched the elephant's tail and claimed it is like a piece of rope.



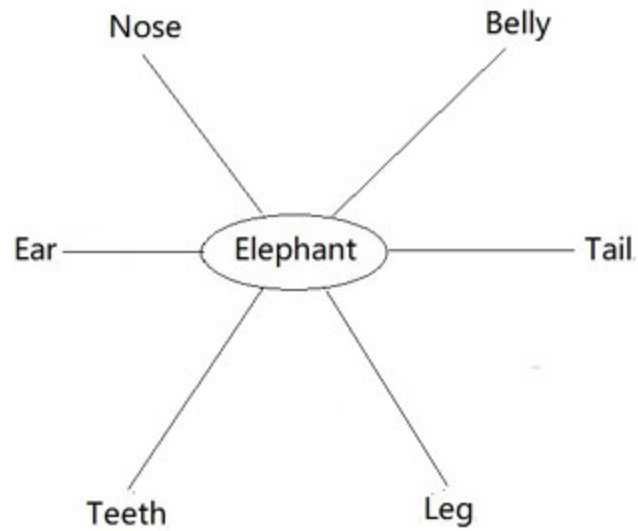
They fell into an endless argument! Why?



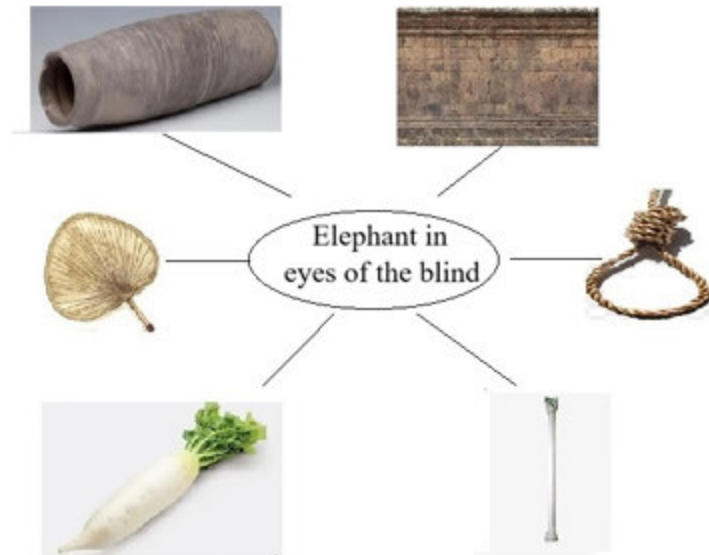
A sophist told them: “Why you are thinking about the shape of elephant different is because each of you touches the different part of the elephant's body. Essentially, an elephant has all characteristics that you are talking about!”

>> **What is the elephant shape in eyes of the sophist by that of the blind men?** The answer is a union of characteristics recognized locally by the 6 blind men, i.e., the combination of all the local to form a whole:

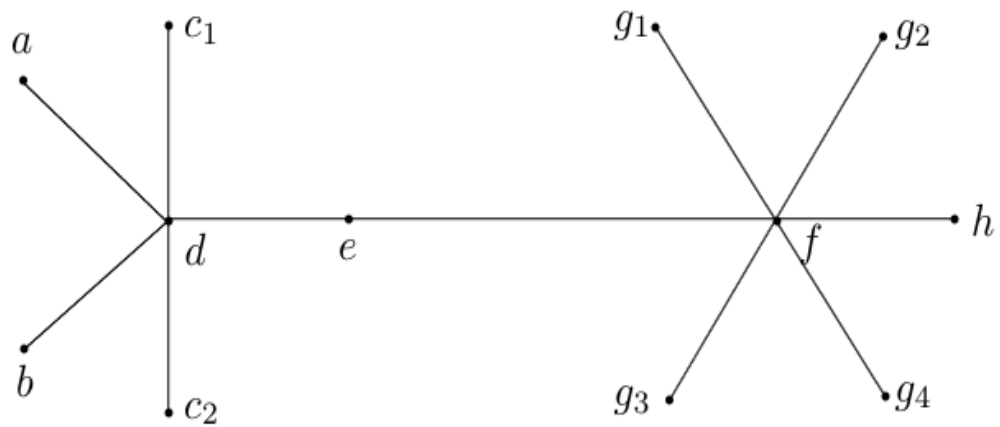
$$\text{An elephant} = \{4 \text{ big pillars}\} \cup \{1 \text{ gross rope}\} \cup \{1 \text{ tubes}\} \\ \cup \{2 \text{ big fans}\} \cup \{1 \text{ big wall}\} \cup \{2 \text{ big radishes}\}$$



Recognizing



with a combinatorial structure



>>Smarandache Multispace

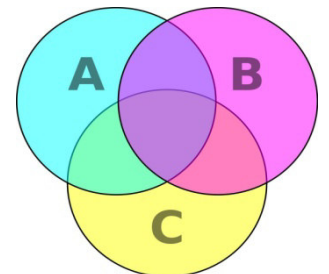
Generally, let the observable characteristics be $\chi_1, \chi_2, \dots, \chi_n$ in reductionism of thing T and denote the mathematical reality of this $T_{\mathcal{M}}$ by $T_{\mathcal{M}}$. One recognizes the mathematical reality of thing T by a union

$$T_{\mathcal{M}} = \bigcup_{i=1}^n \mathcal{R}(\chi_i)$$

of local recognitions $\mathcal{R}(\chi_i)$ of human for integers $1 \leq i \leq n$, called a Smarandache multispace.

F.Smarandache, **A Unifying Field in Logics--Neutrosopy: Neturosophic Probability, Set, and Logic**, American research Press, Rehoboth, 1999.

Linfan Mao, **Smarandache Multi-Space Theory**, The Education Publisher Inc., USA, 2011.



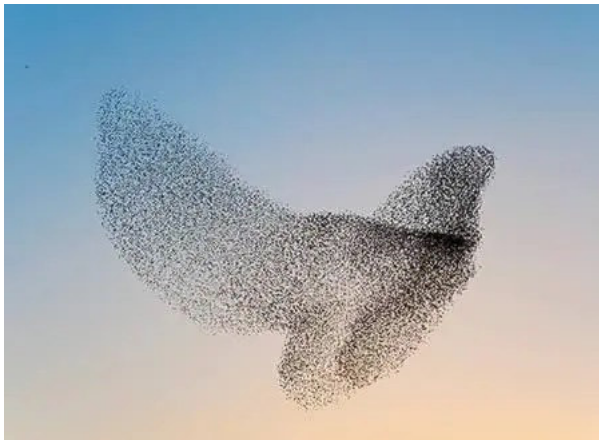
1.2. Combinatorics Implied in Contradictory System

Usually, human quarrelling is because of the differences in recognition on one thing, which leads to contradiction, even by mathematics.

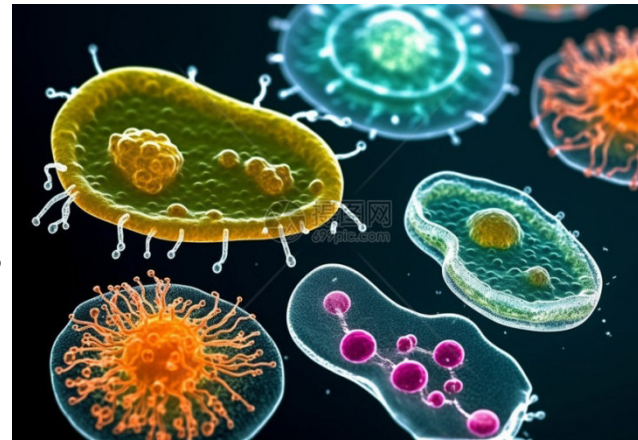
Is a contradictory system in mathematics worthless in recognition?

The answer is certainly Not because we can not asserted so, and the contradiction is essentially caused by the modeling of human, not the nature of thing T, particularly the self-organized system such as biological population, cell system, gene, etc.

Birds

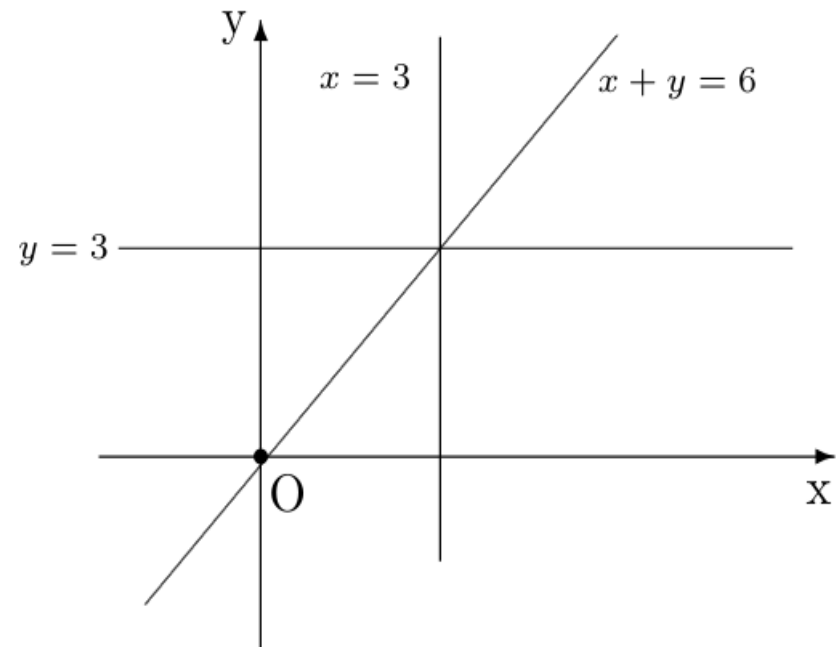
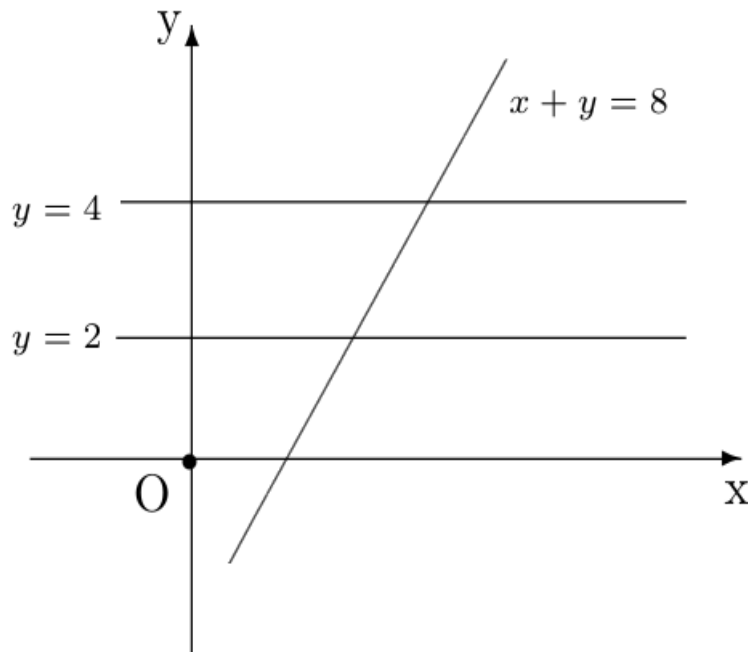


Cells





For example, let $A = \{C_1, C_2, C_3\}$ and $B = \{C'_1, C'_2, C'_3\}$ be two groups consisting of three Tom cats chasing three Jerry mice along three straight lines on Euclidean plane \mathbb{R}^2



How to modeling the running behavior of all cats in groups A or B on plane \mathbb{R}^2 ? A natural idea is to describe the running behavior of two groups by moving orbits, i.e., linear equations in A or B.

$$(LES_3^N) \begin{cases} y & = & 4 \\ y & = & 2 \\ x + y & = & 8 \end{cases} \quad (LES_3^S) \begin{cases} x & = & 3 \\ y & = & 3 \\ x + y & = & 6 \end{cases}$$

However, (LES_3^N) is non-solvable and (LES_3^S) has a solution $(3,3)$. Can we conclude that the running behavior of cats in A are nothing unless an empty set and cats in B are all still at the point $(3,3)$ without moving? Of course Not!

In this case, the running behavior of cats should be orbits $\text{Orb}(A)$ or $\text{Orb}(B)$, which is a union of points of cats in A or B passing on plane \mathbb{R}^2 .

$$\text{Orb}(A) = \{(x, y) : y = 4\} \cup \{(x, y) : y = 2\} \cup \{(x, y) : x + y = 8\},$$

$$\text{Orb}(B) = \{(x, y) : x = 3\} \cup \{(x, y) : y = 3\} \cup \{(x, y) : x + y = 6\},$$

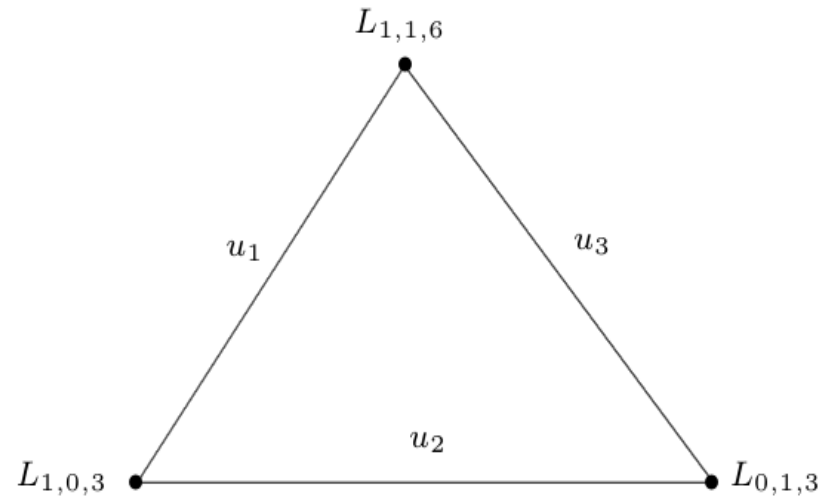
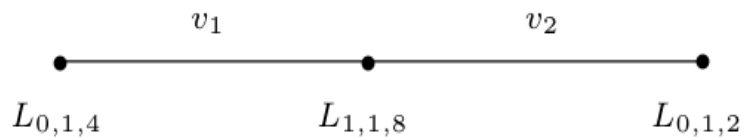
where each of the orbits $\text{Orb}(A)$ and $\text{Orb}(B)$ is a Smarandache multispace, i.e., combinatorial one.

Denote the points of a cat running on straight line $ax+by=c$ by the set $L_{a,b,c} = \{(x, y) | ax + by = c, a \neq 0 \text{ or } b \neq 0\}$ and

$$v_1 = L_{0,1,4} \cap L_{1,1,8}, \quad v_2 = L_{0,1,2} \cap L_{1,1,8}, \quad L_{0,1,2} \cap L_{0,1,4} = \emptyset,$$

$$u_1 = L_{1,0,3} \cap L_{1,1,6}, \quad u_2 = L_{0,1,3} \cap L_{1,0,3}, \quad u_3 = L_{0,1,3} \cap L_{1,1,6}$$

Then, the running of cats in groups A, B can be characterized by combinatorics, i.e., labeled graphs P_3^L, C_3^L respectively



Generally, let \tilde{S} be a Smarandache multispace on n distinct spaces S_1, S_2, \dots, S_n for an integer $n \geq 1$. Define a labeled graph $G^L[\tilde{S}]$ associated with \tilde{S} by

$$V(G^L[\tilde{S}]) = \{S_1, S_2, \dots, S_n\}$$

$$E(G^L[\tilde{S}]) = \{(S_i, S_j) \mid S_i \cap S_j \neq \emptyset, 1 \leq i \neq j \leq n\}$$

and labels on the vertex S_i , edge (S_i, S_j) for integers $1 \leq i \neq j \leq n$ respectively by

$$L : S_i \rightarrow L(S_i) = S_i \text{ and } L : (S_i, S_j) \rightarrow L(S_i, S_j) = S_i \cap S_j$$

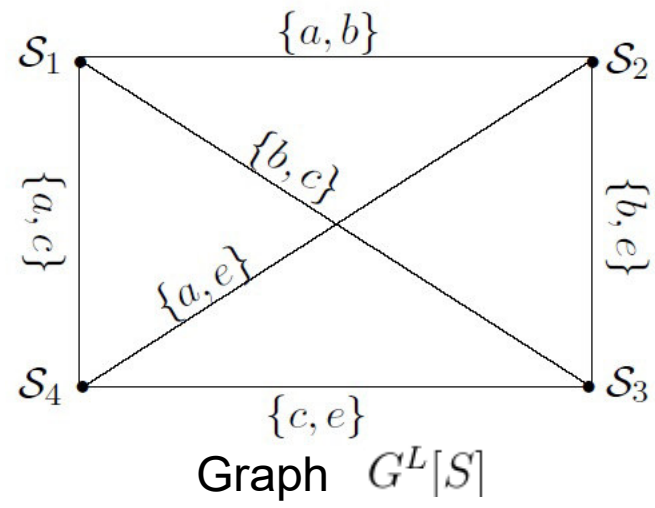
where S_1, S_2, \dots, S_n are mathematical systems. For example, let

$$S_1 = \{a, b, c\}, S_2 = \{a, b, e\}, S_3 = \{b, c, e\}, S_4 = \{a, c, e\},$$

and

$$\tilde{S} = \bigcup_{i=1}^4 S_i = \{a, b, c, d, e\}$$

Then, its graph $G^L[S]$ is shown.



2. Mathematical Reality

Usually, a physical state $\psi(t, x)$ of thing is characterized by differential equation

$$\mathcal{F}(t, x_1, x_2, x_3, \psi_t, \psi_{x_1}, \psi_{x_2}, \dots, \psi_{x_1 x_2}, \dots) = 0$$

In physics, but **is it a solvable equation?** The answer is Not certain by **Godel's Incompleteness Theorem:**

There exist always statements in a formal system S that can neither be proved nor disproved so long as S contains the Peano's axioms of arithmetic, namely it can be only characterized by non-solvable system of solvable equations, i.e., non-harmonious group. Then,

What is a non-harmonious group?





>>Non-Harmonious Group

Definition 2.1([29]) *A non-harmonious group \mathcal{S} is such a system \mathcal{S} consisting of elements P_i , $1 \leq i \leq m, m \geq 2$ with interactions that P_i is constrained on a system of equations*

$$(ES_m) \begin{cases} \mathcal{F}_{P_1}(\mathbf{x}, \mathbf{y}) = 0 \\ \mathcal{F}_{P_2}(\mathbf{x}, \mathbf{y}) = 0 \\ \dots\dots\dots \\ \mathcal{F}_{P_m}(\mathbf{x}, \mathbf{y}) = 0 \end{cases} \quad (2.1)$$

at time t , where $\mathcal{F}_i(\mathbf{x}^0, \mathbf{y}^0) = 0$ and \mathcal{F}_i satisfies the existence condition of implicit function theorem in a neighborhood U of point $(\mathbf{x}^0, \mathbf{y}^0)$ in Euclidean space \mathbb{R}^n for integers $1 \leq i \leq m$.

[29] Linfan Mao, Dynamic network with e-index applications, *International J.Math.Combin.*, Vol.4 (2020), 1-35.

>>Non-harmonious group is non-solvable system of equations.

What is condition of the non-solvable or solvable of system?

Let $S_{\mathcal{F}_i} \subset \mathbb{R}^n$ with $\mathcal{F}_i : S_{\mathcal{F}_i} \rightarrow 0$ be solution manifolds of the i th equation in system (2.1). Then,

$$\bigcap_{i=1}^m S_{\mathcal{F}_i} = \emptyset \quad \text{or} \quad \bigcap_{i=1}^m S_{\mathcal{F}_i} \neq \emptyset.$$

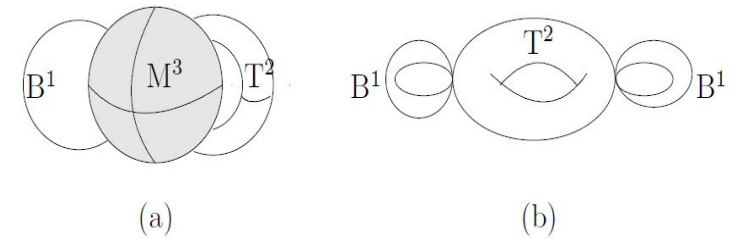
Is the condition of non-solvable of solvable in geometry.

What is meaning of system (2.1) has or has no solution?

Answer: The non-solvable case of (2.1) indicates only that there is no overlap state in elements P_1, P_2, \dots, P_m at time t , not implies the state of P_i existing or not.

How to characterize the group behavior of P_1, P_2, \dots, P_m ?

Answer: Smarandache multispace $\bigcup_{i=1}^m S_{\mathcal{F}_i}$



>> Combinatorial manifold solution of system (2.1)

Definition 2.2([29]) *For any integer $m \geq 1$, the G -solution of system (2.1) on non-harmonious group S is a labeled graph G^L with vertex and edge sets defined by*

$$V(G^L) = \{S_{\mathcal{F}_i}, 1 \leq i \leq m\},$$

$$E(G^L) = \{ (S_{\mathcal{F}_i}, S_{\mathcal{F}_j}) \mid \text{if } S_{\mathcal{F}_i} \cap S_{\mathcal{F}_j} \neq \emptyset \text{ for integers } 1 \leq i, j \leq m \}$$

and labels on vertices and edges of G by

$$L : S_{\mathcal{F}_i} \rightarrow S_{\mathcal{F}_i}, \quad (S_{\mathcal{F}_i}, S_{\mathcal{F}_j}) \rightarrow S_{\mathcal{F}_i} \cap S_{\mathcal{F}_j}, \quad 1 \leq i \neq j \leq m.$$

Theorem 2.3([29]) *For any integer $m \geq 1$, a G -solution G^L of system (2.1) on a non-harmonious group S is always existing.*

Linfan Mao, Geometrical theory on combinatorial manifolds, *JP J. Geometry and Topology*, Vol.7, No.1(2007),65-114.

Linfan Mao, Combinatorial fields -- An introduction, *International J. Math. Combin.*, Vol.3 (2009), 1-22.

Linfan Mao, Dynamic network with e-index applications, *International J. Math. Combin.*, Vol.4 (2020), 1-35.

For example, let

$$(LEq_m^1) \quad AX = B, \quad (LDES_m^1) \quad \begin{cases} \dot{X} = A_1 X \\ \dot{X} = A_2 X \\ \dots\dots\dots \\ \dot{X} = A_m X \end{cases}$$

be respectively a non-solvable system of linear equations and ordinary differential equations. Two linear equations

$$a_{i1}x_1 + a_{i2}x_2 + \dots + a_{in}x_n = b_i,$$

$$a_{j1}x_1 + a_{j2}x_2 + \dots + a_{jn}x_n = b_j$$

are called parallel if there exists a constant c such that

$$c = \frac{a_{j1}}{a_{i1}} = \frac{a_{j2}}{a_{i2}} = \dots = \frac{a_{jn}}{a_{in}} \neq \frac{b_j}{b_i},$$

Classify these equation $L_i, 1 \leq i \leq m$ to parallel families

$$\mathcal{C}_1, \mathcal{C}_2, \dots, \mathcal{C}_s$$

Theorem 2.4([16]) *Let (LEq_m^1) be a linear equation system for integers $m, n \geq 1$. Then*

$$G[LEq_m^1] \simeq K_{n_1, n_2, \dots, n_s} \quad (2.5)$$

with $n_1 + n + 2 + \dots + n_s = m$, where \mathcal{C}_i is the parallel family with $n_i = |\mathcal{C}_i|$ for integers $1 \leq i \leq s$ in (LEq_m^1) and the system (LEq_m^1) is non-solvable if $s \geq 2$.

[16]Linfan Mao, Non-solvable spaces of linear equation systems, *International J. Math. Combin.*, Vol.2, 2012, 9-23.

We can label each vertex of G-solution to get a basis graph of $(LDES_m^1)$ by its base.

Theorem 2.6([17]) *Let H be a basis graph. Then, there is a unique linear homogeneous differential equation system $(LDES_m^1)$ with G-solution H .*

For example, let $(LDES_6^1)$ be a system of homogeneous differential equations (1) – (6). It is easily to know the bases of (1) – (6) are respectively

$$\{e^t, e^{2t}\}, \{e^{2t}, e^{3t}\}, \{e^{3t}, e^{4t}\}, \{e^{4t}, e^{5t}\}, \{e^{5t}, e^{6t}\}, \{e^{6t}, e^t\}$$

with a basis graph $G^L[LDES_6^1]$ shown in Figure 9.

$$(LDES_6^1) \begin{cases} \ddot{x} - 3\dot{x} + 2x = 0 & (1) \\ \ddot{x} - 5\dot{x} + 6x = 0 & (2) \\ \ddot{x} - 7\dot{x} + 12x = 0 & (3) \\ \ddot{x} - 9\dot{x} + 20x = 0 & (4) \\ \ddot{x} - 11\dot{x} + 30x = 0 & (5) \\ \ddot{x} - 7\dot{x} + 6x = 0 & (6) \end{cases}$$

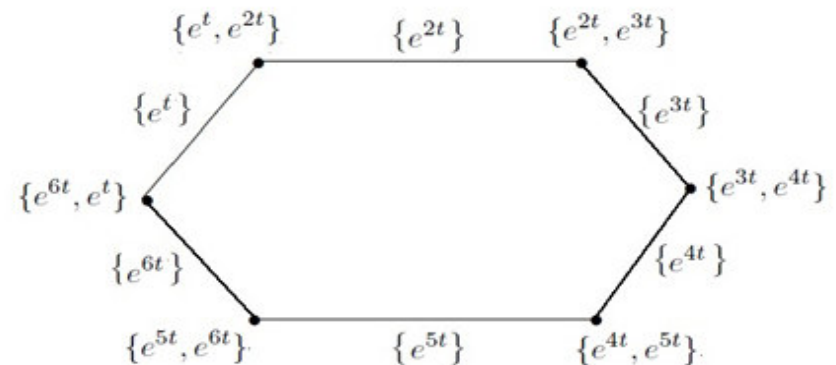


Figure 9

Furthermore, two linear spaces are isomorphic if and only if they have the same dimension. We can replace each basis by its dimension, obtain a basis graph $G^L[LDES_m^1]$ of systems $(LDES_m^1)$ labeled with integers, called an *integral graph* and then, classify systems $(LDES_m^1)$ of linear differential equations by integral graphs.

Theorem 2.7([17]) *Let $(LDES_m^1)$, $(LDES_m^1)'$ be two systems of linear homogeneous differential equation with integral graphs H , H' , respectively. Then $(LDES_m^1) \stackrel{\mathcal{L}}{\simeq} (LDES_m^1)'$ if and only if the integral graph $H = H'$.*

[17] Linfan Mao, Global stability of non-solvable ordinary differential equations with applications, International J. Math. Combin., Vol.1, 2013, 1-37.





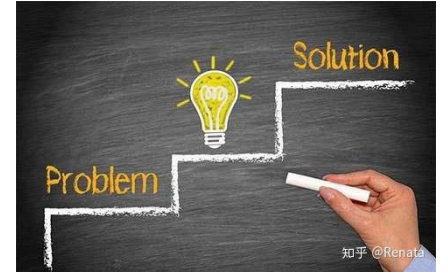
Similarly, let $(PDES_m^1)$ be a system of partial differential equations with

$$\begin{cases} F_1(x_1, x_2, \dots, x_n, u, u_{x_1}, \dots, u_{x_n}, u_{x_1x_2}, \dots, u_{x_1x_n}, \dots) = 0 \\ F_2(x_1, x_2, \dots, x_n, u, u_{x_1}, \dots, u_{x_n}, u_{x_1x_2}, \dots, u_{x_1x_n}, \dots) = 0 \\ \dots \\ F_m(x_1, x_2, \dots, x_n, u, u_{x_1}, \dots, u_{x_n}, u_{x_1x_2}, \dots, u_{x_1x_n}, \dots) = 0 \end{cases}$$

on a function $u(x_1, \dots, x_n, t)$. Then, its *symbol* is determined by

$$\begin{cases} F_1(x_1, x_2, \dots, x_n, u, p_1, \dots, p_n, p_1p_2, \dots, p_1p_n, \dots) = 0 \\ F_2(x_1, x_2, \dots, x_n, u, p_1, \dots, p_n, p_1p_2, \dots, p_1p_n, \dots) = 0 \\ \dots \\ F_m(x_1, x_2, \dots, x_n, u, p_1, \dots, p_n, p_1p_2, \dots, p_1p_n, \dots) = 0, \end{cases}$$

i.e., substitute $p_1^{\alpha_1}, p_2^{\alpha_2}, \dots, p_n^{\alpha_n}$ into $(PDES_m^1)$ for the term $u_{x_1^{\alpha_1}x_2^{\alpha_2}\dots x_n^{\alpha_n}}$, where $\alpha_i \geq 0$ for integers $1 \leq i \leq n$ and a non-solvable system $(PDES_m^1)$ is *algebraically contradictory* if its symbol is non-solvable. Otherwise, *differentially contradictory*. Then, we can characterize the non-solvable systems of partial differential equations of first order.



Theorem 2.9([20]) *A Cauchy problem on system $(PDES_m^1)$ of partial differential equations of first order with initial values $x_i^{[k^0]}, u_0^{[k]}, p_i^{[k^0]}, 1 \leq i \leq n$ for the k th equation in $(PDES_m^1)$, $1 \leq k \leq m$ such that*

$$\frac{\partial u_0^{[k]}}{\partial s_j} - \sum_{i=0}^n p_i^{[k^0]} \frac{\partial x_i^{[k^0]}}{\partial s_j} = 0 \quad (2.9)$$

is uniquely G -solvable, i.e., $G^L[PDES]$ is uniquely determined.

[20]Linfan Mao, Cauchy problem on non-solvable system of first order partial differential equations with applications, *Methods and Applications of Analysis*, Vol. 22, 2 (2015) , 171–200.

What is the role of G-solution on non-harmonious groups?

Answer: the global stability of non-harmonious group!

Let $G^L(t)$ be a G -solution of (ES_m) with initial values $G^L(t_0)$ and let $\omega : V(G^L[ES_m]) \rightarrow \mathbb{R}$ be a functional. A system (ES_m) is said to be ω -stable if there exists a number $\delta(\varepsilon)$ for any number $\varepsilon > 0$ such that

$$\left\| \omega \left(G^{L_1(t)} - L_2(t) \right) \right\| < \varepsilon \quad (2.10)$$

or furthermore, *asymptotically ω -stable* if

$$\lim_{t \rightarrow \infty} \left\| \omega \left(G^{L_1(t)} - L_2(t) \right) \right\| = 0 \quad (2.11)$$

if the initial values hold with $\|L_1(t_0)(v) - L_2(t_0)(v)\| < \delta(\varepsilon)$ for $\forall v \in V(\overset{\cdot}{G})$.

Theorem 2.10([21]) *If there is a Liapunov ω -function $L(\omega(t)) : \mathcal{O} \rightarrow \mathbf{R}$ on $G^L[ES_m]$ of system (ES_m) , then it is ω -stable, and furthermore, if $\dot{L}(\omega(t)) < 0$ for $G^L[ES_m] \neq \mathbf{O}$, then it is asymptotically ω -stable.*

[21]Linfan Mao, Mathematics with natural reality -- Action flows, *Bull.Cal.Math. Soc.*, Vol.107, 6(2015), 443-474.

For a linear system ($LDES_m^1$) of differential equations, we can further introduce the sum-stable and prod-stable on ($LDES_m^1$), i.e., a system $G^L[LDES_m^1]$ is *sum-stable* or *asymptotically sum-stable* if for all solutions $Y_v(t)$, $v \in V(G^L)$ of linear differential equations in ($LDES_m^1$) with $|Y_v(0) - X_v(0)| < \delta_v$ exists

$$\left| \sum_{v \in V(H^L)} Y_v(t) - \sum_{v \in V(H^L)} X_v(t) \right| < \varepsilon \quad (2.12)$$

for all $t \geq 0$ or furthermore,

$$\lim_{t \rightarrow 0} \left| \sum_{v \in V(H^L)} Y_v(t) - \sum_{v \in V(H^L)} X_v(t) \right| = 0 \quad (2.13)$$

and *prod-stable* or *asymptotically prod-stable* if for all solutions $Y_v(t)$, $v \in V(G)$ of ($LDES_m^1$) with $|Y_v(0) - X_v(0)| < \delta_v$ exists

$$\left| \prod_{v \in V(G)} Y_v(t) - \prod_{v \in V(G)} X_v(t) \right| < \varepsilon \quad (2.14)$$

for all $t \geq 0$, or furthermore,

$$\lim_{t \rightarrow 0} \left| \prod_{v \in V(G)} Y_v(t) - \prod_{v \in V(G)} X_v(t) \right| = 0. \quad (2.15)$$

Theorem 2.11([17]) *A zero \mathbf{O} -solution of system $(LDES_m^1)$ of linear homogenous differential equation is asymptotically sum-stable if and only if $\text{Re}\alpha_v < 0$ for each $\bar{\beta}_v(t)e^{\alpha_v t} \in \mathcal{B}_v$ with vertex $v \in G^L[LDES_m^1]$.*

Theorem 2.12([17]) *A zero \mathbf{O} -solution of systems $(LDES_m^1)$ of linear homogenous differential equation is asymptotically prod-stable if and only if*

$$\sum_{v \in V(G)} \text{Re}\alpha_v < 0 \tag{2.16}$$

for each $\bar{\beta}_v(t)e^{\alpha_v t} \in \mathcal{B}_v$ with vertex $v \in G^L[LDES_m^1]$.

[17]Linfan Mao, Global stability of non-solvable ordinary differential equations with applications, *International J. Math. Combin.*, Vol.1, 2013, 1-37.

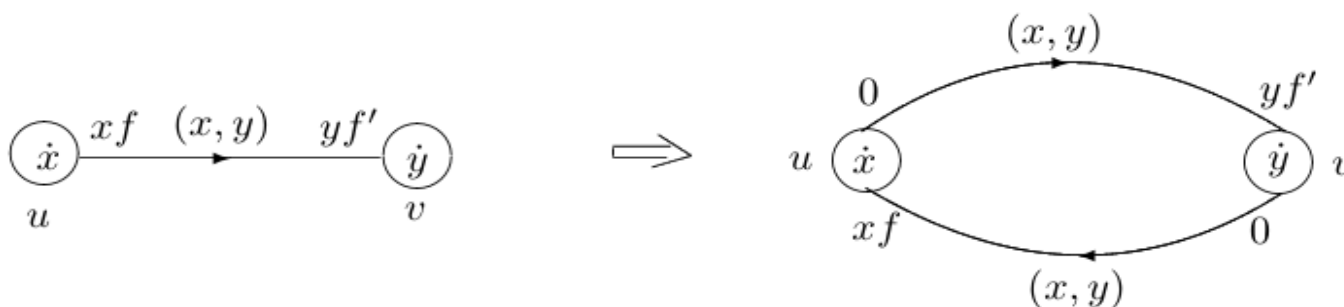


A food web is a complex network of food chains, i.e., "eating or being eaten" among various, more suitable characterized by labeled directed graphs.

Theorem 2.13([22]) *A food web \vec{G}^L with initial value \vec{G}^{L_0} is globally stable or asymptotically stable if and only if there is an Eulerian multi-decomposition*

$$(\vec{G} \cup \overleftarrow{G})^{\hat{L}} = \bigoplus_{i=1}^s \vec{H}_i^L \quad (2.17)$$

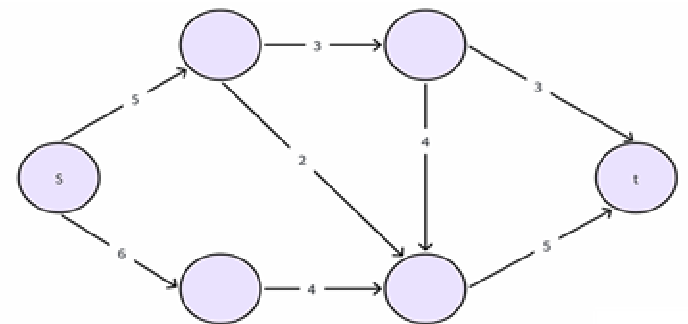
with solvable stable or asymptotically stable conservative equations on labeling Eulerian subgraphs \vec{H}_i^L for integers $1 \leq i \leq s$, where \overleftarrow{G} is the digraph reversing orientation on every edge in \vec{G} , $(\vec{G} \cup \overleftarrow{G})^{\hat{L}}$ is a labeled graph with labeling $\hat{L} : V(\vec{G} \cup \overleftarrow{G}) = L(V(\vec{G}))$ and $\hat{L} : E(\vec{G} \cup \overleftarrow{G}) \rightarrow L(E(\vec{G} \cup \overleftarrow{G}))$ by $\hat{L} : (u, v) \rightarrow \{0, (x, y), yf'\}$, $(v, u) \rightarrow \{xf, (x, y), 0\}$ if $L : (u, v) \rightarrow \{xf, (x, y), yf'\}$ for $\forall (u, v) \in E(\vec{G})$ such as those shown in Figure 10.



[22] Linfan Mao, Biological n-system with global stability, *Bull. Cal. Math. Soc.*, Vol.108, 6(2016), 403-430.

3. Mathematical Combinatorics

A G-solution $G^L[ES_m]$ of system (ES_m) characterize the state of non-harmonious group without direction on edges, i.e., not the action within itself. However, all things are in constantly moving and evolved by their internal action under the external with elements moving in thing, which is only in one-way. Whence, we generalize G-solution $G^L[ES_m]$ to a directed situation for modeling substance flow on the evolution of thing T, holding with conservation at each vertex, i.e., a generalization of network N to continuity flow, which is vectors in Banach space over a topological graph with end-operator actions, i.e., *mathematical combinatorics*.



Definition 3.1([23]) A continuity flow $(\vec{G}; L, \mathcal{A})$ is an oriented topological graph \vec{G}^L in space \mathcal{S} associated with a mapping $L : v \rightarrow L(v)$, $(v, u) \rightarrow L(v, u)$, 2 end-operators $A_{vu}^+ \in \mathcal{A} : L(v, u) \rightarrow L^{A_{vu}^+}(v, u)$ and $A_{uv}^+ \in \mathcal{A} : L(u, v) \rightarrow L^{A_{uv}^+}(u, v)$ on a Banach space \mathcal{B} over a field \mathcal{F} such as those shown in Figure 11 with $L(v, u) = -L(u, v)$, $A_{vu}^+(-L(v, u)) = -L^{A_{vu}^+}(v, u)$ for $\forall(v, u) \in E(\vec{G}^L)$ and meanwhile, holding with the continuity equation

$$\sum_{u \in N_G^-(v)} L^{A_{uv}^+}(u, v) - \sum_{u \in N_G^+(v)} L^{A_{uv}^+}(u, v) = L(v) \quad (3.1)$$

at any vertex $v \in V(\vec{G}^L)$ of topological graph \vec{G}^L , where $N_G^-(v), N_G^+(v)$ are respectively the in-neighborhood and out-neighborhood of vertex $v \in V(\vec{G}^L)$, namely all vertices in $N_G^-(v) \subset N_G(v)$ or $N_G^+(v) \subset N_G(v)$ flow into or out of the vertex v and $N_G^-(v) \cup N_G^+(v) = N_G(v)$.

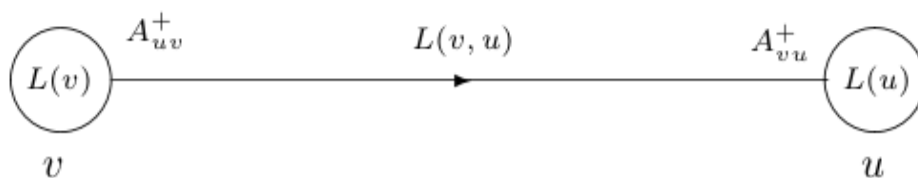


Figure 11

[23]Linfan Mao, Complex system with flows and synchronization, *Bull. Cal. Math. Soc.*, Vol.109, 6(2017), 461–484.

Notice that the continuity equations on vertices of \vec{G}^L form a non-solvable system (ES_m) of equations. For example, let the $L : (v, u) \rightarrow L(v, u) \in \mathbb{R}^n \times \mathbb{R}^+$ with end-operators $A_{vu}^+ = a_{vu} \frac{\partial}{\partial t}$ and $a_{vu} : \mathbb{R}^n \rightarrow \mathbb{R}$ for any edge $(v, u) \in E(\vec{G})$ in Figure 12 following.

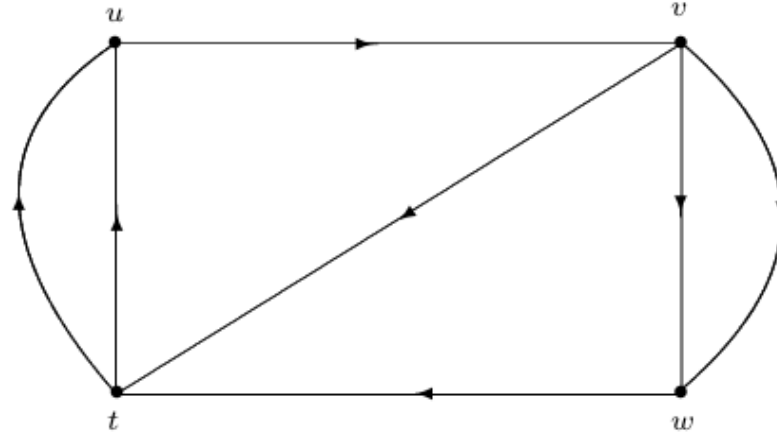


Figure 12

Then, the continuity equations on vertices of \vec{G}^L are partial differential equations

$$\left\{ \begin{array}{l} a_{tu^1} \frac{\partial L(t, u)^1}{\partial t} + a_{tu^2} \frac{\partial L(t, u)^2}{\partial t} = a_{uv} \frac{\partial L(u, v)}{\partial t} \\ a_{uv} \frac{\partial L(u, v)}{\partial t} = a_{vw^1} \frac{\partial L(v, w)^1}{\partial t} + a_{vw^2} \frac{\partial L(v, w)^2}{\partial t} + a_{vt} \frac{\partial L(v, t)}{\partial t} \\ a_{vw^1} \frac{\partial L(v, w)^1}{\partial t} + a_{vw^2} \frac{\partial L(v, w)^2}{\partial t} = a_{wt} \frac{\partial L(w, t)}{\partial t} \\ a_{wt} \frac{\partial L(w, t)}{\partial t} + a_{vt} \frac{\partial L(v, t)}{\partial t} = a_{tu^1} \frac{\partial L(t, u)^1}{\partial t} + a_{tu^2} \frac{\partial L(t, u)^2}{\partial t} \end{array} \right.$$

Now, could we really establish mathematics on continuity flows G^L by view it as a mathematical element? The answer is definite by considering \vec{G}^L to be a family of vectors underlying a topological graph \vec{G} with addition, multiplication and scalar multiplication for continuity flows $\vec{G}^L, \vec{G}'^{L'}$ and $\lambda \in \mathcal{F}$ defined by

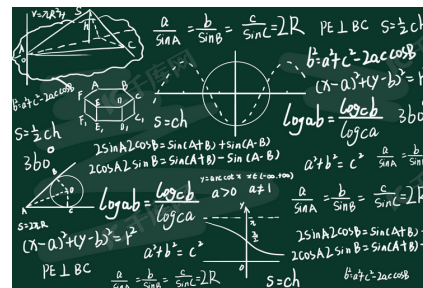
$$G^L + G'^{L'} = (G \setminus G')^L \cup (G \cap G')^{L+L'} \cup (G' \setminus G)^{L'}, \quad (3.2)$$

$$G^L \cdot G'^{L'} = (G \setminus G')^L \cup (G \cap G')^{L \cdot L'} \cup (G' \setminus G)^{L'}, \quad (3.3)$$

$$\lambda \cdot G^L = G^{\lambda \cdot L}. \quad (3.4)$$

where, for any vertex $v \in V(G)$ and edge $(v, u) \in E(G)$, $L(v), L'(v), L(v, u), L'(v, u) \in \mathcal{B}$, $L + L' : v \rightarrow L(v) + L'(v)$, $(v, u) \rightarrow L(v, u) + L'(v, u)$, $L \cdot L' : v \rightarrow L(v) \cdot L'(v)$, $(v, u) \rightarrow L(v, u) \cdot L'(v, u)$, $\lambda \cdot L : v \rightarrow \lambda \cdot L(v)$, $(v, u) \rightarrow \lambda \cdot L(v, u)$, $L(v) \cdot L'(v)$ and $L(v, u) \cdot L'(v, u)$ denotes the *Hadamard product* of vectors in Banach space \mathcal{B} , namely

$$(x_1, x_2, \dots, x_n) \cdot (y_1, y_2, \dots, y_n) = (x_1 y_1, x_2 y_2, \dots, x_n y_n). \quad (3.5)$$



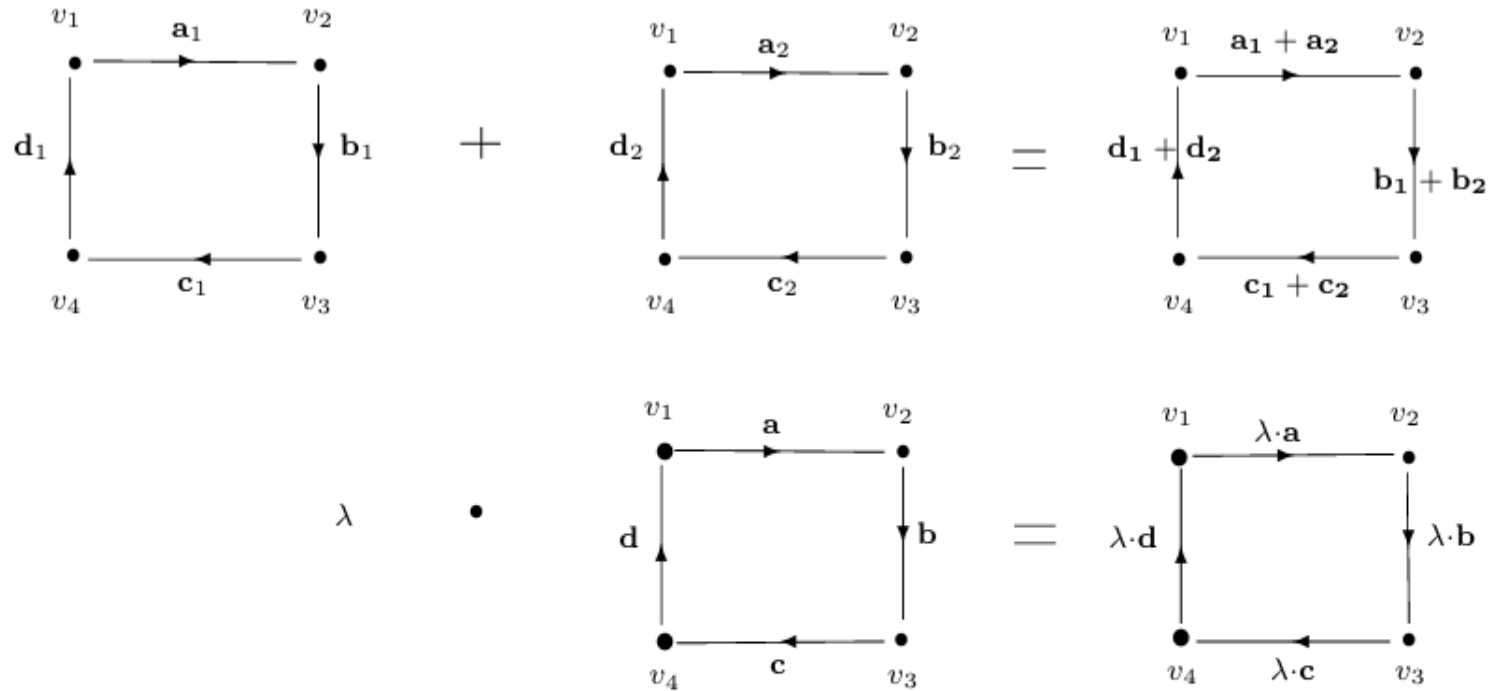


Figure 13

Notice that the addition “+”, scalar multiplication “ \cdot ” on vectors \mathbf{a} , \mathbf{b} can be viewed as the operations on a particular continuity flow \vec{G}^L , i.e., path \vec{P}_{n+1}^L , which enables us to establish mathematics on continuity flows \vec{G}^L on Banach space \mathcal{B} over topological graphs \vec{G} .

3.1. Banach Flow Space

Let \mathcal{G} be a graph family closed under the union operation of graph, \mathcal{B} be a Banach space over field \mathcal{F} and denoted by $\mathcal{G}_{\mathcal{B}}$ all continuity flows \vec{G}^L with $\vec{G} \in \mathcal{G}$, $L : V(\vec{G}) \cup E(\vec{G}) \rightarrow \mathcal{B}$. Then, we have

Theorem 3.1([25],[32]) *If \mathcal{G} is a closed family of graphs under the union operation and \mathcal{B} a linear space $(\mathcal{B}; +, \cdot)$, then, all continuity flows $(\mathcal{G}_{\mathcal{B}}; +, \cdot)$ is a linear space, and furthermore, a commutative ring if \mathcal{B} is a commutative ring $(\mathcal{B}; +, \cdot)$ over a field \mathcal{F} .*

Assume all end-operators are continuous linear operators in \mathcal{A} and define the norm of a continuity flow \vec{G}^L by

$$\left\| \vec{G}^L \right\| = \sum_{(v,u) \in E(\vec{G})} \left\| L^{A_{vu}^+}(v, u) \right\|, \quad (3.6)$$

where $\|\cdot\|$ is the norm on Banach space \mathcal{B} . Then, we can verify the non-negative, homogeneity and the triangle inequality hold with $\mathcal{G}_{\mathcal{B}}$ and the non-negative, conjugacy and the linearity if \mathcal{B} is further a Hilbert space, i.e.,

Theorem 3.2([25],[32]) *If \mathcal{G} is a closed family of graphs under the union operation and \mathcal{B} a Banach space $(\mathcal{B}; +, \cdot)$, then, $\mathcal{G}_{\mathcal{B}}$ with linear operators A_{vu}^+ , A_{uv}^+ for $\forall (v, u) \in E\left(\bigcup_{G \in \mathcal{G}} \vec{G}\right)$ is a Banach space, and furthermore, $\mathcal{G}_{\mathcal{B}}$ is a Hilbert space if \mathcal{B} is a Hilbert space.*

3.2. G-Isomorphic Operators on Banach Flow Space

Definition 3.3([29]) *A mapping $f : G_1^{L_1} \rightarrow G_2^{L_2}$ is a G-isomorphic operator between continuity flows $G_1^{L_1}$ and $G_2^{L_2}$ if*

(1) *there is an isomorphism $\varphi : \widehat{G} \rightarrow \widehat{G}$ with $\widehat{G} \supset G_1, G_2$ in graph;*

(2) *for $\forall(v, u) \in E(G_1)$ there is $L_2 = f \circ \varphi \circ L_1$ but for $\forall(v, u) \in E(G_2 \setminus G_1)$, $f : \mathbf{0} \rightarrow L_2(v, u)$ and for $\forall(v, u) \in E(G_1 \setminus G_2)$ and $\forall(v, u) \in E(\widehat{G} \setminus (G_1 \cup G_2))$, $f : L(v, u) \rightarrow \mathbf{0}$.*

A G-isomorphic operator $f : \mathcal{G}_{\mathcal{B}} \rightarrow \mathcal{G}_{\mathcal{B}'}$ is linear if

$$f\left(\lambda G_1^{L_1} + \mu G_2^{L_2}\right) = \lambda f\left(G_1^{L_1}\right) + \mu f\left(G_2^{L_2}\right),$$

Is continuous if

$$\left\|G_1^{L_1} - G_0^{L_0}\right\| < \delta(\varepsilon) \Rightarrow \left\|f\left(G_1^{L_1}\right) - f\left(G_0^{L_0}\right)\right\| < \varepsilon$$

and is bounded if there exists a constant $\xi \in [0, \infty)$ such that

$$\left\|f\left(G_1^{L_1}\right) - f\left(G_2^{L_2}\right)\right\| \leq \xi \left\|G_1^{L_1} - G_2^{L_2}\right\|, \quad \xi \in [0, 1)$$

Then, we have

Theorem 3.4(Fixed Flow Theorem, [25],[29],[32]) *For a continuous G -isomorphic contractor $f : \mathcal{G}_{\mathcal{B}} \rightarrow \mathcal{G}_{\mathcal{B}'}$ there is only one continuity flow $G^L \in \mathcal{G}_{\mathcal{B}}$ such that $f(G^L) = G^L$.*

Theorem 3.5(Banach Inverse Theorem, [25],[29],[32]) *A G -isomorphic linear operator $f : \mathcal{G}_{\mathcal{B}} \rightarrow \mathcal{G}_{\mathcal{B}'}$ is continuous if and only if it is bounded and furthermore, if f is 1 – 1 then the inverse operator f^{-1} of f is also a G -isomorphic continuous operator.*

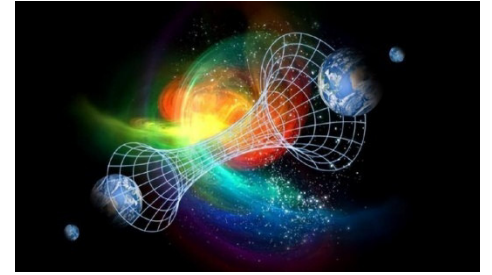
Theorem 3.6(Closed Graph Theorem, [25],[29],[32]) *If $\mathbf{T} : \mathcal{G}_{\mathcal{B}_1} \rightarrow \mathcal{G}_{\mathcal{B}_2}$ is a closed linear operator with Banach spaces $\mathcal{B}_1, \mathcal{B}_2$, then \mathbf{T} is continuous.*

Particularly, a G -isomorphic linear operator $f : \mathcal{G}_{\mathcal{B}} \rightarrow \mathbb{R}$ or \mathbb{C} is called a *flow functional*, which can be applied to generalize the Hahn-Banach theorem to Banach flow space $\mathcal{G}_{\mathcal{B}}$.

Theorem 3.7(Hahn-Banach Theorem, [25],[29],[32]) *Let $\mathcal{H}_{\mathcal{B}}$ be a subspace of Banach flow space $\mathcal{G}_{\mathcal{B}}$ and let $F : \mathcal{H}_{\mathcal{B}} \rightarrow \mathbb{C}$ be a continuous linear flow functional on $\mathcal{H}_{\mathcal{B}}$. Then, there is a continuous linear flow functional $\tilde{F} : \mathcal{G}_{\mathcal{B}} \rightarrow \mathbb{C}$ satisfies the conditions that if $\vec{G}^L \in \mathcal{H}_{\mathcal{B}}$ then $\tilde{F}(\vec{G}^L) = F(\vec{G}^L)$ and $\|\tilde{F}\| = \|F\|$. Particularly, if $\mathbf{0} \neq \vec{G}_0^{L_0} \in \mathcal{G}_{\mathcal{B}}$, there is a continuous linear flow functional F such that $\|F\| = 1$ and $\|F(\vec{G}_0^{L_0})\| = \|\vec{G}_0^{L_0}\|$.*

[25]Linfan Mao, Harmonic flow's dynamics on animals in microscopic level with balance recovery, *International J.Math. Combin.*, Vol.1(2019), 1-44.

[29]Linfan Mao, Dynamic network with e-index applications, *International J.Math.Combin.*, Vol.4 (2020), 1-35.



>>Quantum Mechanics

Theorem 3.8(Fréchet-Riesz Theorem, [25],[29],[32]) *Let $f : \mathcal{G}_{\mathcal{B}} \rightarrow \mathbb{C}$ be a continuous linear flow functional. For any continuous flow $G^L \in \mathcal{G}_{\mathcal{B}}$, there uniquely exists a continuous flow of $\hat{G}^L \in \mathcal{G}_{\mathcal{B}}$ holding with $f(\vec{G}^L) = \langle \vec{G}^L, \hat{G}^L \rangle$.*

[32]Linfan Mao, *Combinatorial Theory on the Universe*, Global Knowledge-Publishing House, 2023.

What is the important role of Hahn-Banach theorem in Banach flow space $\mathcal{G}_{\mathcal{B}}$?

Answer: It concludes that the existence of Hermitian operator H in a quantum Q with quark structure \vec{G}^L or a matter T such as the proton and neutron consist of quanta over a topological graph \vec{G}^L , i.e., the correctness of the three assumptions in quantum mechanics with topological structure.

3.3. Integral and Differential on Continuity Flow

Let the isomorphism $\varphi = \text{id}_G$, i.e., the identity mapping of topological graph \vec{G} and let \mathcal{B} be a Hilbert space and particularly, a function field on variable \mathbf{x} , Then, a G -isomorphic operator f is determined ([29]) by equation

$$L_2(v, u) = f \circ L_1(v, u), \quad \forall (v, u) \in E(\vec{G}) \quad (3.11)$$

which is equivalent to

$$f\left(\vec{G}^L[\mathbf{x}]\right) = \vec{G}^{f(L[\mathbf{x}])}. \quad (3.12)$$

Thereby, we can define the power

$$\vec{G}^{aL}[\mathbf{x}] = \vec{G}^{aL[\mathbf{x}]}, \quad a^{G^L}[\mathbf{x}] = \vec{G}^{a^{L[\mathbf{x}]}} \quad (3.13)$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + \cdots, \quad (3.14)$$

$$e^{tA} = \mathbf{I} + \frac{tA}{1!} + \frac{t^2 A^2}{2!} + \cdots + \frac{t^n A^n}{n!} + \cdots, \quad (3.15)$$

$$e^{G^L[\mathbf{x}]} = \mathbf{I} + \frac{\vec{G}^L[\mathbf{x}]}{1!} + \frac{\vec{G}^{2L}[\mathbf{x}]}{2!} + \cdots + \frac{\vec{G}^{nL}[\mathbf{x}]}{n!} + \cdots, \quad (3.16)$$

The variation on continuity flows

$\vec{G}^L[\mathbf{x}]$, namely assume a G -isomorphic mapping $\mathcal{L} : (v, u) \in E(\vec{G}) \rightarrow \mathcal{L}[\mathcal{L}(t, \mathbf{x}(t), \dot{\mathbf{x}}(t))(v, u)]$ is differentiable and respectively define the action $J[\vec{G}^{\mathcal{L}}[t]]$ and variation $\delta J[\vec{G}^{\mathcal{L}}[t]]$ on a continuity flow $\vec{G}^{\mathcal{L}}[t]$ by

$$J[\vec{G}^{\mathcal{L}}[t]] = \left| \int_{t_1}^{t_2} \vec{G}^{\mathcal{L}}[\mathcal{L}(t, \mathbf{x}(t), \dot{\mathbf{x}}(t))] dt \right|, \quad \delta J[\vec{G}^{\mathcal{L}}[t]] = \left| \delta \int_{t_1}^{t_2} \vec{G}^{\mathcal{L}}[\mathcal{L}(t, \mathbf{q}(t), \dot{\mathbf{q}}(t))] dt \right|, \quad (3.18)$$

where the variation $\delta : \mathcal{G}_{\mathcal{B}} \rightarrow \mathcal{G}_{\mathcal{B}}$ is a G -isomorphic operator. And so, the Euler-Lagrange equations

$$\frac{\partial \vec{G}^{\mathcal{L}}}{\partial q_i} - \frac{d}{dt} \frac{\partial \vec{G}^{\mathcal{L}}}{\partial \dot{q}_i} = \mathbf{0}, \quad 1 \leq i \leq n. \quad (3.19)$$

on continuity flow $\vec{G}^{\mathcal{L}}[t]$ can be established by the least action principle $\delta J[\vec{G}^{\mathcal{L}}[t]](v, u) = 0$ for $\forall (v, u) \in E(\vec{G}^{\mathcal{L}}[t])$ with the properties of norm in Banach flow space $\mathcal{G}_{\mathcal{B}}$.

4. An Interesting Example

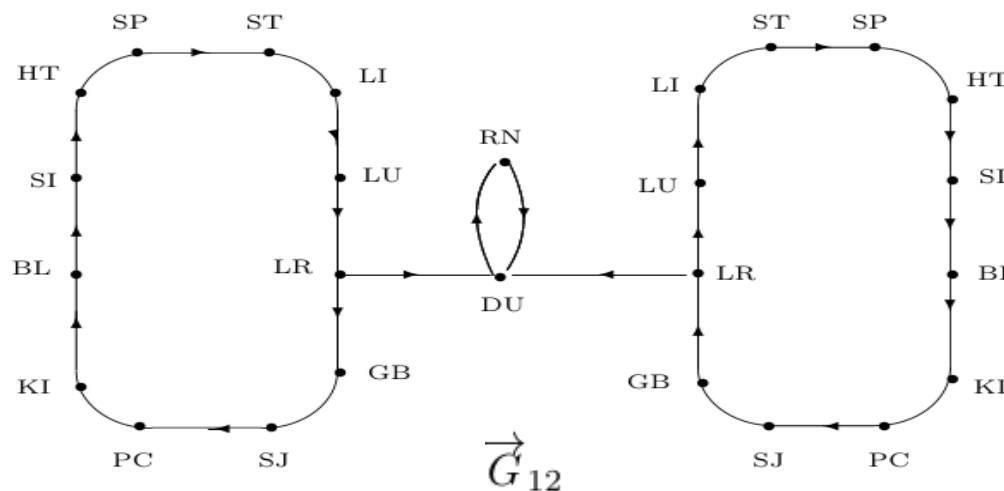
Although we are all human but it is very hard to answer what a human is unless by behavioral characteristics. A more useful definition on human is by the pair {Y-, Y+} of Yin and Yang with meridians running the vital energy on body in traditional Chinese medicine, including 12 meridians, i.e., the lung meridian of hand-Taiyin (LU), heart meridian of hand-Shaoyin (HT), pericardium meridian of hand-Jueyin (PC), the spleen meridian of foot-Taiyin (SP), kidney meridian of foot-Shaoyin (KI), liver meridian of foot-Jueyin (LR), large intestine meridian of hand-Yangming (LI), small intestine meridian of hand-Taiyang (SI), Sanjiao (triple burner) meridian of hand-Shaoyang (SJ), stomach meridian of foot-Yangming (ST), bladder meridian of foot Taiyang (BL), gallbladder meridian of foot-Shaoyang (GB), Ren meridian, Du meridian and 671 acupoints in next page

How to modeling the running of a living body of human? The answer is by the running of vital energy on 12 meridians with Ren and Du meridians in traditional Chinese medicine.

Furthermore, how to modeling the running of vital energy on 12 meridians with Ren and Du meridians? The answer is by a G-flow \vec{G}_{12}^L defined by

$$V(\vec{G}_{12}) = \{\text{All acupoints } v \text{ on 12 meridians with Ren and Du meridians}\},$$

$$E(\vec{G}_{12}) = \{\text{All segments } (v, u) \text{ connecting adjacent points on 12 meridians with Ren and Du meridians with orientation of Ying Qi running}\}$$



with $L : (v, u) \rightarrow \{\Psi^-(v, u), \Psi^+(v, u)\},$

$$L : (v, u) \rightarrow \Psi^-(v, u)$$

or $L : (v, u) \rightarrow \Psi^+(v, u)$

Where, the Ying Qi Ψ^- and Wei Qi Ψ^+ both are the vital energy with constant distributing c on each meridian of human body, running in accordance with the law of conservation of energy, namely $\vec{G}_{12}^{L_c}$ is a G-flow of, continuity flow, characterized by

$$\left\{ \begin{array}{l} \frac{\partial \vec{G}_{12}^{\Psi^-}}{\partial x_i} - \frac{d}{dt} \frac{\partial \vec{G}_{12}^{\Psi^-}}{\partial \dot{x}_i} = \mathbf{0} \\ \frac{\partial \vec{G}_{12}^{\Psi^+}}{\partial x_i} - \frac{d}{dt} \frac{\partial \vec{G}_{12}^{\Psi^+}}{\partial \dot{x}_i} = \mathbf{0} \\ \vec{G}_{12}^{\Psi^-} + \vec{G}_{12}^{\Psi^+} = \vec{G}_{12}^{L_c} \end{array} \right.$$

Where, the 1st, 2nd equations are Euler-Lagrange equations, the 3rd equation is the balance equation of Ying Qi and Wei Qi.

5. Conclusion

The reality of thing T existing in universe should be a combinatorial one in the eyes of human because of the cognitive limitation and particularly, the mathematical reality. A new mathematics should be established for the recognition on reality of thing by combinatorics, which is a natural way for holding on the reality of thing , crossing the cognitive gap from the local to the whole, including both of the macroscopic and microscopic such as the system of celestial body, particle moving or living evolution, and the digital economy devolving of international or domestic trade, namely the evolution of all system, no matter it is harmonious or self-organized.



Thanks for your Attention !

