Complex Neutrosophic Soft Set

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Abstract—**In this paper, we propose the complex neutrosophic soft set model, which is a hybrid of complex fuzzy sets, neutrosophic sets and soft sets. The basic set theoretic operations and some concepts related to the structure of this model are introduced, and illustrated. An example related to a decision making problem involving uncertain and subjective information is presented, to demonstrate the utility of this model.**

Keywords—complex fuzzy sets; soft sets; complex neutrosophic sets; complex neutrosophic soft sets; decision making

I. INTRODUCTION

The neutrosophic set model (NS) proposed by Smarandache [1, 2] is a powerful tool to deal with incomplete, indeterminate and inconsistent information in the real world. It is a generalization of the theory of fuzzy sets [3], intuitionistic fuzzy sets [4, 5], interval-valued fuzzy sets [6] and interval-valued intuitionistic fuzzy sets [7]. A neutrosophic set is characterized by a truth-membership degree (*t*), an indeterminacy-membership degree (*i*) and a falsity-membership degree (*f*), all defined independently, and all of which lie in the real standard or nonstandard unit interval $]^-0, 1^+[$. Since this interval is difficult to be used in real-life situations, Wang et al. [8] introduced single-valued neutrosophic sets (SVNSs) whose functions of truth, indeterminacy and falsity all lie in [0, 1]. Neutrosophic sets and its extensions such as single valued neutrosophic sets, interval neutrosophic sets, and intuitionistic neutrosophic sets have been applied in a wide variety of fields including decision making, computer science, engineering, and medicine [1-2, 8-27, 36-37]. The study of complex fuzzy sets were initiated by Ramot et al. [28]. Among the well-known complex fuzzy based models in literature are complex intuitionistic fuzzy sets (CIFSs) [29, 30], complex vague soft sets (CVSSs) [31, 32] and complex intuitionistic fuzzy soft sets (CIFSSs) [33].These models have been used to represent the uncertainty and periodicity aspects of an object simultaneously, in a single set. The complex-valued membership and non-membership functions in these models have the potential to be used to represent uncertainty in instances such as the wave function in quantum mechanics, impedance in electrical engineering, the changes in meteorological activities, and time-periodic decision making problems.

Recently, Ali and Smarandache [34] developed a hybrid model of complex fuzzy sets and neutrosophic sets, called complex neutrosophic sets. This model has the capability of handling the different aspects of uncertainty, such as incompleteness, indeterminacy and inconsistency, whilst simultaneously handling the periodicity aspect of the objects, all in a single set. The complex neutrosophic set is defined by complex-valued truth, indeterminacy and falsity membership functions. The complex-valued truth membership function consists of a truth amplitude term (truth membership) and a phase term which represents the periodicity of the object. Similarly, the complex-valued indeterminacy and falsity membership functions consists of an indeterminacy amplitude and a phase term, and a falsity amplitude and a phase term, respectively. The complex neutrosophic set is a generalized framework of all the other existing models in literature.

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However, as the CNS model is an extension of ordinary fuzzy sets, it lacks adequate parameterization qualities. Adequate parameterization refers to the ability of a model to define the parameters in a more comprehensive manner, without any restrictions. Soft set theory works by defining the initial description of the parameters in an approximate manner, and allows for any form of parameterization that is preferred by the users. This includes using words and sentences, real numbers, functions and mappings, among others to describe the parameters. The absence of any restrictions on the approximate description in soft set theory makes it very convenient to be used and easily applicable in practice. The adequate parameterization capabilities of soft set theory and the lack of such capabilities in the existing CNS model served as the motivation to introduce the CNSS model in this paper. This is achieved by defining the complex neutrosophic set in a soft set setting.

The rest of the paper is organized as follows. In section 2, we present an overview of some basic definitions and properties which serves as the background to our work in this paper. In section 3, the main definition of the CNSS and some related concepts are presented. In section 4, the basic set theoretic operations for this model are defined. The utility of this model is demonstrated by applying it in a decision making problem in section 5. Concluding remarks are given in section 6.

II. PRELIMINARIES

In this section, we recapitulate some important concepts related to neutrosophic sets (NSs), and complex neutrosophic sets (CNSs). We refer the readers to [1, 8, 10, 34] for further details pertaining to these models.

Let X be a space of points (objects) with generic elements in X denoted by x.

Definition 1. *[1]* A neutrosophic set A is an object having the form $A=\{\langle x,T_A(x),I_A(x),F_A(x)\rangle: x\in X\}$, where the functions $T,I,F: X\to]^-0,1^+[$, denote the truth, indeterminacy, and falsity membership functions, respectively, of the element $x\in X$ with respect to set A. These membership functions must satisfy the condition $^-0\le T_A(x)+ I_A(x)+ F_A(x)\le 3^+$. (1)

The functions $T_A(x), I_A(x)$, and $F_A(x)$ are real standard or nonstandard subsets of the interval $]^-0,1^+[$. However, these intervals make it difficult to apply NSs to practical problems, and this led to the introduction of a single-valued neutrosophic set (SVNS) in [12]. This model is a special case of NSs and is better suited to handle real-life problems and applications.

Definition 2. *[8]* An SVNS A is a neutrosophic set that is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$, where $T_A(x), I_A(x), F_A(x)\in[0,1]$. A SVNS A can thus be written as

$$A=\{\langle x,T_A(x),I_A(x),F_A(x)\rangle: x\in X\} \quad (2)$$

Definition 3. *[8]* The complement of a neutrosophic set A, denoted by A^c, is as defined below for all $x\in X$:

$$T_A^c(x)=F_A(x), I_A^c(x)=1-I_A(x), F_A^c(x)=T_A(x).$$

Definition 4. *[10]* Let U be an initial set and E be a set of parameters. Let $P(U)$ denote the power set of U, and let $A\to E$. A pair (F,A) is called a soft set over U, where F is a mapping given by $F: A\to P(U)$. In other words, a soft set is a parameterized family of subsets of the set U. Every $F(e)$, where $e\in E$, from this family may be considered as the set of e-elements of the soft set (F,A).

Definition 5. *[34]* A complex neutrosophic set A defined on a universe of discourse X, is characterized by a truth membership function $T_A(x)$, an indeterminacy membership function $I_A(x)$, and a falsity membership function $F_A(x)$ that assigns a complex-valued grade for each of these membership function in A for any $x\in X$. The values of $T_A(x), I_A(x)$ and $F_A(x)$, and their sum may assume any values within a unit circle in the complex plane, and is of the form $T_A(x)=p_A(x)e^{i\mu_A(x)}, I_A(x)=q_A(x)e^{i\nu_A(x)}$, and $F_A(x)=r_A(x)e^{i\omega_A(x)}$. All the amplitude and phase terms are real-valued and $p_A(x), q_A(x), r_A(x)\in[0,1]$, whereas $\mu_A(x), \nu_A(x), \omega_A(x)\in(0,2\pi]$, such that the condition

$$0\le p_A(x)+q_A(x)+r_A(x)\le 3 \quad (3)$$

is satisfied. A complex neutrosophic set A can thus be represented in set form as:

$$A=\{\langle x,T_A(x)=a_T, I_A(x)=a_I, F_A(x)=a_F\rangle: x\in X\},$$

where $T_A: X\to\{a_T: a_T\in C, |a_T|\le 1\}$, $I_A: X\to\{a_I: a_I\in C, |a_I|\le 1\}$, $F_A: X\to\{a_F: a_F\in C, |a_F|\le 1\}$, and also

$$|T_A(x)+I_A(x)+F_A(x)|\le 3. \quad (4)$$

The interval $(0,2\pi]$ is chosen for the phase term to be in line with the original definition of a complex fuzzy set in which the amplitude terms lie in an interval of $(0,1)$, and the phase terms lie in an interval of $(0,2\pi]$.

Remark: In the definition above, i denotes the imaginary number $i=\sqrt{-1}$ and it is this imaginary number i that makes the CNS have complex-valued membership grades. The term $e^{i\theta}$ denotes the exponential form of a complex number and represents $e^{i\theta}=\cos\theta+i\sin\theta$.

Definition 6. *[34]* Let $A=\{\langle x,T_A(x),I_A(x),F_A(x)\rangle: x\in X\}$ be a complex neutrosophic set over X. Then the complement of A, denoted by A^c, is defined as:

$$A^c=\{\langle x,T_A^c(x),I_A^c(x),F_A^c(x)\rangle: x\in X\},$$

where $T_A^c(x)=r_A(x)e^{i(2\pi-\mu_A(x))}$,

$I_A^c(x)=\left(1-q_A(x)\right)e^{i(2\pi-\nu_A(x))}$, and

$F_A^c(x)=p_A(x)e^{i(2\pi-\omega_A(x))}$.

III. COMPLEX NEUTROSOPHIC SOFT SETS

In this section, we introduce the complex neutrosophic soft set (CNSS) model which is a hybrid of the CNS and soft set models. The formal definition of this model as well as some concepts related to this model are as given below:

Definition 7. Let U be universal set, E be a set of parameters under consideration, $A \subseteq E$, and ψ_A be a complex neutrosophic set over U for all $x \in U$. Then a complex neutrosophic soft set $\chi_A$over U is defined as a mapping $\chi_A: E \to CN(U)$, where $CN(U)$ denotes the set of complex neutrosophic sets in U, and $\Psi_A(x) = \emptyset$ if $x \notin A$. Here $\Psi_A(x)$ is called a complex neutrosophic approximate function of χ_A and the values of $\Psi_A(x)$ is called the x -elements of the CNSS for all $x \in U$. Thus, χ_A can be represented by the set of ordered pairs of the following form:

$$\chi_A = \{(x, \Psi_A(x)): x \in E, \Psi_A(x) \in CN(U)\},$$

where $\Psi_A(x) = (p_A(x)e^{i\mu_A(x)}, q_A(x)e^{i\nu_A(x)}, r_A(x)e^{i\omega_A(x)})$, p_A, q_A, r_A are real-valued and lie in [0,1], and $\mu_A, \nu_A, \omega_A \in (0, 2\pi]$. This is done to ensure that the definition of the CNSS model is line with the original structure of the complex fuzzy set, on which the CNSS model is based on.

Example 1. Let U be a set of developing countries in the Southeast Asian (SEA) region, that are under consideration, E be a set of parameters that describe a country's economic indicators, and $A = \{e_1, e_2, e_3, e_4\} \subseteq E$, where these sets are as defined below:

$U = \{u_1 = \text{Republic of Philippines}, u_2 = \text{Vietnam}, u_3 = \text{Myanmar}, u_4 = \text{Indonesia}\}$,

$E = \{e_1 = \text{inflation rate}, e_2 = \text{population growth}, e_3 = \text{GDP growth rate}, e_4 = \text{unemployment rate}, e_5 = \text{export volume}\}$.

The CNS $\Psi_A(e_1), \Psi_A(e_2), \Psi_A(e_3)$ and $\Psi_A(e_4)$ are defined as:

$$\psi_A(e_1) = \left\{ \frac{\left(0.6\,e^{j0.8\pi}, 0.3e^{j\frac{3\pi}{4}}, 0.5e^{j0.3\pi}\right)}{u_1}, \frac{\left(0.7\,e^{j0\pi}, 0.2e^{j0.9\pi}, 0.1e^{j\frac{2\pi}{3}}\right)}{u_2}, \frac{\left(0.9\,e^{j0.1\pi}, 0.4e^{j\pi}, 0.7e^{j0.7\pi}\right)}{u_3}, \frac{\left(0.3\,e^{j0.4\pi}, 0.2e^{j0.6\pi}, 0.7e^{j0.5\pi}\right)}{u_4} \right\},$$

$$\psi_A(e_2) = \left\{ \frac{\left(0.2\,e^{j0.2\pi}, 0.5e^{j\frac{3\pi}{4}}, 0.6e^{j0.3\pi}\right)}{u_1}, \frac{\left(0.4\,e^{j0.3\pi}, 0.4e^{j0.4\pi}, 0.2e^{j\frac{2\pi}{3}}\right)}{u_2}, \frac{\left(0.3\,e^{j0.1\pi}, 0.2e^{j0.1\pi}, 0.3e^{j0.7\pi}\right)}{u_3}, \frac{\left(0.1\,e^{j0.4\pi}, 0.5e^{j1.2\pi}, 0.3e^{j0.1\pi}\right)}{u_4} \right\},$$

$$\psi_A(e_3) = \left\{ \frac{\left(0.4.e^{j0.4\pi}, 0.1.e^{j\frac{\pi}{4}}, 0.2.e^{j0.1\pi}\right)}{u_1}, \frac{\left(0.3\,e^{j0.2\pi}, 0.3e^{j0.4\pi}, 0.2e^{j\frac{\pi}{3}}\right)}{u_2}, \frac{\left(0.2\,e^{j0.1\pi}, 0.4.e^{j0.5\pi}, 0.5e^{j0.2\pi}\right)}{u_3}, \frac{\left(0.5\,e^{j0.2\pi}, 0.4e^{j2\pi}, 0.6e^{j0.1\pi}\right)}{u_4} \right\},$$

and

$$\psi_A(e_4) = \left\{ \frac{\left(0.3\,e^{j0.2\pi}, 0.5e^{j\frac{\pi}{4}}, 0.5e^{j0.1\pi}\right)}{u_1}, \frac{\left(0.1\,e^{j0\pi}, 0.6e^{j0.4\pi}, 0.4e^{j\frac{\pi}{3}}\right)}{u_2}, \frac{\left(0.1\,e^{j0.1\pi}, 0.2e^{j0.2\pi}, 0.4e^{j0.2\pi}\right)}{u_3}, \frac{\left(0.2\,e^{j0.2\pi}, 0.5e^{j0.3\pi}, 0.3e^{j0.1\pi}\right)}{u_4} \right\}.$$

Then the complex neutrosophic soft set χ_A can be written as a collection of CNSs of the form:

$$\chi_A = \{\Psi_A(e_1), \Psi_A(e_2), \Psi_A(e_3), \Psi_A(e_4)\}.$$

Definition 8. Let χ_A and χ_B be two CNSSs over a universe U. Then we have the following:

(i) χ_A is said to be an empty CNSS, denoted by $\chi_{A_\emptyset}$, if $\Psi_A(x) = \emptyset$, for all $x \in U$;

(ii) χ_A is said to be an absolute CNSS, denoted by χ_{A_U}, if $\Psi_A(x) = U$ for all $x \in U$;

(iii) χ_A is said to be a CNS-subset of χ_B, denoted by $\chi_A \subseteq \chi_B$, if for all $x \in U$, $\Psi_A(e) \subseteq \Psi_B(e)$, that is the following conditions are satisfied:

$$p_A(e) \le p_B(e), q_A(e) \le q_B(e), r_A(e) \le r_B(e),$$

and $\mu_A(e) \le \mu_B(e), \nu_A(e) \le \nu_B(e), \omega_A(e) \le \omega_B(e)$.

(iv) χ_A is said to be equal to χ_B, denoted by $\chi_A = \chi_B$, if for all $x \in U$ the following conditions are satisfied:

$$p_A(e) = p_B(e), q_A(e) = q_B(e), r_A(e) = r_B(e),$$

and $\mu_A(e) = \mu_B(e), \nu_A(e) = \nu_B(e), \omega_A(e) = \omega_B(e)$.

Proposition 1. Let $\chi_A \in CN(U)$. Then the following hold:

(i) $(\chi_A^c)^c = \chi_A$;

(ii) $\chi_{A_\emptyset}^c = \chi_{A_U}$.

Proof. The proofs are straightforward from Definition 8.

IV. OPERATIONS ON COMPLEX NEUTROSOPHIC SOFT SETS

In this section we define the basic set theoretic operations on CNSSs, namely the complement, union and intersection.

Let χ_A and χ_B be two CNSSs over a universe U.

Definition 9. The complement of χ_A, denoted by χ_A^c, is a CNSS defined by $\chi_A^c = \{(x, \psi_A^c(x)): x \in U\}$, where $\psi_A^c(x)$ is the complex neutrosophic complement of ψ_A.

Example 2. Consider Example 1. The complement of χ_A is given by $\chi_A^c = \{\psi_A^c(e_1), \psi_A^c(e_2), \psi_A^c(e_3), \psi_A^c(e_4)\}$. For the sake of brevity, we only give the complement for $\psi_A^c(e_1)$ below:

$$\psi_A^c(e_1) = \left\{ \frac{\left(0.5\mathrm{e}^{j1.2\pi}, 0.7e^{j\frac{5\pi}{4}}, 0.6e^{j1.7\pi}\right)}{u_1}, \frac{\left(0.1\mathrm{e}^{j2\pi}, 0.8e^{j1.1\pi}, 0.7e^{j\frac{4\pi}{3}}\right)}{u_2}, \frac{\left(0.7\mathrm{e}^{j1.9\pi}, 0.6e^{j\pi}, 0.9e^{j1.3\pi}\right)}{u_3}, \frac{\left(0.7\mathrm{e}^{j1.6\pi}, 0.8e^{j1.4\pi}, 0.3e^{j1.5\pi}\right)}{u_4} \right\}.$$

The complements for the rest of the CNSs can be found in a similar manner.

Definition10. The union of χ_A and χ_B, denoted by $\chi_A \,\breve{\cup}\, \chi_B$, is defined as:

$$\chi_C = \chi_A \,\breve{\cup}\, \chi_B = \{(x, \psi_A(x) \,\breve{\cup}\, \psi_B(x)) : x \in U\},$$

$$\chi_C(e) = \begin{cases} (x, \psi_A(x)) & \text{if } e \in A - B, \\ (x, \psi_B(x)) & \text{if } e \in B - A, \\ (x, \psi_A(x) \,\breve{\cup}\, \psi_B(x)) & \text{if } e \in A \cap B, \end{cases}$$

where $C = A \cup B, x \in U$, and

$$\psi_A(x) \,\breve{\cup}\, \psi_B(x) = \begin{Bmatrix} (p_A(x) \vee p_B(x))e^{i(\mu_A(x) \cup \mu_B(x))} \\ (q_A(x) \wedge q_B(x))e^{i(\nu_A(x) \cup \nu_B(x))} \\ (r_A(x) \wedge r_B(x))e^{i(\omega_A(x) \cup \omega_B(x))} \end{Bmatrix},$$

where $\vee$ and $\wedge$ denote the maximum and minimum operators respectively, whereas the phase terms of the truth, indeterminacy and falsity functions lie in the interval $(0, 2\pi]$, and can be calculated using any one of the following operators:

(i) Sum:
$\mu_{A\cup B}(x) = \mu_A(x) + \mu_B(x), \nu_{A\cup B}(x) = \nu_A(x) + \nu_B(x)$, and $\omega_{A\cup B}(x) = \omega_A(x) + \omega_B(x)$.

(ii) Max:
$\mu_{A\cup B}(x) = \max(\mu_A(x), \mu_B(x)), \nu_{A\cup B}(x) = \max(\nu_A(x), \nu_B(x))$, and $\omega_{A\cup B}(x) = \max(\omega_A(x), \omega_B(x))$.

(iii) Min:
$\mu_{A\cap B}(x) = \min(\mu_A(x), \mu_B(x)), \nu_{A\cap B}(x) = \min(\nu_A(x), \nu_B(x))$, and $\omega_{A\cap B}(x) = \min(\omega_A(x), \omega_B(x))$.

(iv) "The game of winner, neutral, and loser":

$$\mu_{A\cup B}(x) = \begin{cases} \mu_A(x) & \text{if } p_A(x) > p_B(x) \\ \mu_B(x) & \text{if } p_B(x) > p_A(x) \end{cases},$$

$$\nu_{A\cup B}(x) = \begin{cases} \nu_A(x) & \text{if } q_A(x) < q_B(x) \\ \nu_B(x) & \text{if } q_B(x) < q_A(x) \end{cases}, \text{ and}$$

$$\omega_{A\cup B}(x) = \begin{cases} \omega_A(x) & \text{if } r_A(x) < r_B(x) \\ \omega_B(x) & \text{if } r_B(x) < r_A(x) \end{cases}.$$

All of the operators presented above are straightforward generalizations of the corresponding operators that were originally defined in [28]. The intersection between CNSSs are defined in a similar manner in Definition 11.

Definition 11. The intersection of χ_A and χ_B, denoted by $\chi_A \,\breve{\cap}\, \chi_B$, is defined as:

$$\chi_D = \chi_A \,\breve{\cap}\, \chi_B = \{(x, \psi_A(x) \,\breve{\cap}\, \psi_B(x)) : x \in U\},$$

$$\chi_D(e) = \begin{cases} (x, \psi_A(x)) & \text{if } e \in A - B, \\ (x, \psi_B(x)) & \text{if } e \in B - A, \\ (x, \psi_A(x) \,\breve{\cap}\, \psi_B(x)) & \text{if } e \in A \cap B, \end{cases}$$

where $D = A \cup B, x \in U$, and

$$\psi_A(x) \,\breve{\cap}\, \psi_B(x) = \begin{Bmatrix} (p_A(x) \wedge p_B(x))e^{i(\mu_A(x) \cup \mu_B(x))} \\ (q_A(x) \vee q_B(x))e^{i(\nu_A(x) \cup \nu_B(x))} \\ (r_A(x) \vee r_B(x))e^{i(\omega_A(x) \cup \omega_B(x))} \end{Bmatrix},$$

where $\vee$ and $\wedge$ denote the maximum and minimum operators respectively, whereas the phase terms of the truth, indeterminacy and falsity functions lie in the interval $(0, 2\pi]$, and can be calculated using any one of the following operators that were defined in Definition 10.

V. APPLICATION OF THE CNSS MODEL IN A DECISION MAKING PROBLEM

In Example 1, we presented an example related to the economic indicators of four countries. In this section, we use the same information to determine which one of the four countries that are studied has the strongest economic indicators. To achieve this, a modified algorithm and an accompanying score function is presented in Definition 12 and 13. This algorithm and score function are an adaptation of the corresponding concepts introduced in [35], which was then made compatible with the structure of the CNSS model. The steps involved in the decision making process, in the context of this example, until a final decision is reached, is as given below.

Definition 12. A comparison matrix is a matrix whose rows consists of the elements of the universal set $U = \{u_1, u_2, \ldots, u_m\}$, whereas the columns consists of the corresponding parameters $E = \{e_1, e_2, \ldots, e_n\}$ that are being considered in the problem. The entries of this matrix are c_{ij}, such that

$$c_{ij} = (\alpha_{amp} + \beta_{amp} - \gamma_{amp}) + (\alpha_{phase} + \beta_{phase} - \gamma_{phase}),$$

where the components of this formula are as defined below for all $b_k \in U$, such that $b_i \neq b_k$:

α_{amp} = the number of times the value of the amplitude term of $T_{b_i}(e_j) \geq T_{b_k}(e_j)$,

β_{amp} = the number of times the value of the amplitude term of $I_{b_i}(e_j) \geq I_{b_k}(e_j)$,

γ_{amp} = the number of times the value of the amplitude term of $F_{b_i}(e_j) \geq F_{b_k}(e_j)$,

and

α_{phase} = the number of times the value of the phase term of $T_{b_i}(e_j) \geq T_{b_k}(e_j)$,
β_{phase} = the number of times the value of the phase term of $I_{b_i}(e_j) \geq I_{b_k}(e_j)$,
γ_{phase} = the number of times the value of the phase term of $F_{b_i}(e_j) \geq F_{b_k}(e_j)$.

Definition 13. The score of an element u_i can be calculated by the score function S_i which is defined as $S_i = \sum_j c_{ij}$.

Next, we apply the algorithm and score function in a decision making problem. The steps are as given below:

Step 1: Define a CNSS

Construct a CNSS for the problem that is being studied, which includes the elements $u_i(i = 1, 2, \dots, m)$, and the set of parameters $e_j(j = 1, 2, \dots, n)$, that are being considered.

In the context of this example, the universal set U, set of parameters A, and the CNSS χ_A that were defined in Example 1 will be used.

Step 2: Construct and compute the comparison matrix

A comparison matrix is constructed, and the values of c_{ij} for each element u_i and the corresponding parameter e_j is calculated using the formula given in Definition 12. For this example, the comparison matrix is given in Table 1.

Table 1.Comparison matrix for χ_A

U	e_1	e_2	e_3	e_4
u_1	6	3	3	5
u_2	3	5	1	2
u_3	4	-3	1	-1
u_4	-1	7	5	8

Remark: In this example, the phase terms denotes the time taken for any change in the economic indicators to affect the performance of the economy. The magnitude of these phase terms would indicate the economic sectors that has the most influence on the economy and by extension, the sectors that the economy is dependent on. Therefore, the closer the phase term is to 0, the smaller it is, whereas the closer the phase term is to 2π, the larger it is. For example, phase terms of $\frac{3\pi}{4}$ is larger than the phase terms of $\frac{\pi}{3}$ and $\frac{\pi}{2}$. As such, the values of α_{phase}, β_{phase} and γ_{phase} was by computing the number of times the value of the phase term of element b_{ij} exceeds the value of the phase term of element b_{kj}.

Step 3: Calculate the score function

Compute the scores S_i for each element $u_i(i = 1, 2, \dots, m)$ using Definition 13. The score values obtained are given in Table 2.

Table 2. Score function for χ_A

U	S_i
u_1	17
u_2	11
u_3	1
u_4	19

Step 4: Conclusion and discussion

The values of the score function are compared and the element with the maximum score will be chosen as the optimal alternative. In the event that there are more than one element with the maximum score, any of the elements may be chosen as the optimal alternative.

In the context of this example, $\max_{u_i \in U}\{S_i\} = u_4$. As such, it can be concluded that country u_4 i.e. Indonesia is the country with the strongest economic indicators, followed closely by the Republic of Phillipines and Vietnam, whereas Myanmar is identified as the country with the weakest and slowest growing economy, among the four South East Asian countries that were considered.

VI. CONCLUSION

In this paper, we introduced the complex neutrosophic soft set model which is a hybrid between the complex neutrosophic set and soft set models. This model has a more generalized framework than the fuzzy soft set, neutrosophic set, complex fuzzy set models and their respective generalizations. The basic set theoretic operations were defined. The CNSS model was then applied in a decision making problem involving to demonstrate its utility in representing the uncertainty and indeterminacy that exists when dealing with uncertain and subjective data.

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