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Converting Some Zero-One Neutrosophic Nonlinear Programming Problems into Zero-One Neutrosophic Linear Programming Problems

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Abstract

The science of operations research is the applied aspect of mathematics and one of the most important modern sciences that is concerned with practical issues and meets the desire and request of decision makers to obtain ideal decisions through the methods it presents that are appropriate for all issues, such as linear programming, nonlinear programming, dynamic programming, integer programming, etc. The basic essence of this science is to build mathematical models consisting of an objective function and constraints. In these models, the objective function is a maximization function or a minimization function for a specific quantity. This quantity depends on a number of decision variables that may be independent of each other or related to each other. Through a set of constraints, we obtain values for these variables by solving the mathematical model that we obtain. Given the great importance of operations research methods, we have in previous research presented a neutrosophic vision for some of these methods, such as neutrosophic linear models, neutrosophic nonlinear models, dynamic programming, neutrosophic programming with binary integers, etc. In this research, we present a neutrosophical study of some of the procedures used to convert some zero-one neutrosophic nonlinear programming problems into zero-one neutrosophic linear programming problems.

Keywords: Neutrosophic Operations Research, Neutrosophic Linear Programming, Neutrosophic Nonlinear Programming, Zero-One Neutrosophic Nonlinear Programming Problems, Zero-One Neutrosophic Linear Programming Problems.

1 | Introduction

To keep pace with scientific development and provide new studies of operations research methods that rely on concepts of logic different from classical logic, and after the emergence of neutrosophic logic, many researchers turned to reformulating many scientific concepts using the concepts of this neutrosophic logic, so we have neutrosophic numbers, neutrosophic groups, neutrosophic probabilities, neutrosophic statistics, neutrosophic differentiation, neutrosophic integration, neutrosophic decision theory, neutrosophic linear programming, neutrosophic nonlinear programming, neutrosophic dynamic programming, neutrosophic simulation, neutrosophic binary integer programming... [1-23], in this research we present some procedures



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used to converting some zero-one neutrosophic nonlinear programming problems into zero-one neutrosophic linear programming problems. We know that nonlinear programming problems are characterized by the presence of a set of terms that implicitly call for the use of nonlinear functions such as: $\ln x, \sin x, e^x, x^n \dots$, nonlinearity also appears as a result of a mutual influence between two or more variables, such as: $x_1 x_2, x_1 \ln x_2, \dots$. If there are such expressions in the objective function, constraints, or both, then the mathematical model is a nonlinear model, and if some or all of the variables are restricted to be one or zero, then we get nonlinear models with binary integers. The mathematical model is a neutrosophic model if it has some indeterminacy. When collecting data that describes a realistic issue and this data is incomplete or contains something ambiguous or may contain some contradictions or may be unclear, we will then be unable to build an accurate traditional model. This will lead us to building an approximate model, in neutrosophic model, the variables in the objective function and constraints, some or all of them, are neutrosophic numbers. The neutrosophic number is a number written in the following standard form, $N = a + bI$, where a and b are real or complex coefficients. a represents the definite part and bI represents the indefinite part (indeterminacy) of the number N , and it is possible that $[\lambda_1, \lambda_2]$ or $\{\lambda_1, \lambda_1\}$ or...otherwise it is any set close to the real value a .

2 | Discussion

Since the aim of this research is to converting some zero-one neutrosophic nonlinear programming problems into zero-one neutrosophic linear programming problems, based on the information contained in references [24-26].

1. From reference [24], we find that what is meant by neutrosophic data are values that are not completely defined, written in the following standard format:

$$N = a + bI$$

where a and b are real or complex coefficients, a represents the definite part and bI the indeterminate part of the number N and it can be $[\lambda_1, \lambda_2]$ or $\{\lambda_1, \lambda_1\}$ or...otherwise it is any set close to the real value a , and the neutrosophic model, the numbers that precede the variables in the objective function and in the constraints are neutrosophic values, as well as the second side of the relationships that represent the constraints, and it is written in the following general formula:

$$Nf = Nf(x_1, x_2, \dots, x_n) \rightarrow (Max) \text{ or } (Min)$$

Constraints:

$$Ng_i(x_1, x_2, \dots, x_n) \begin{pmatrix} \leq \\ \geq \\ = \end{pmatrix} Nb_i ; i = 1, 2, \dots, m$$

$$x_1, x_2, \dots, x_n \geq 0$$

2. In references [25-26] the neutrosophic linear models and the neutrosophic nonlinear models were defined, and we mention that the mathematical model is a nonlinear model if any of the components of the objective function or constraints are nonlinear expressions such as $\ln x, \sin x, e^x, x^n, x_1 x_2, x_1 \ln x_2, \dots$, and the nonlinear expressions may be in both.
3. The Neutrosophic nonlinear programming problem is a zero-one problem if the decision variables x_j are restricted to belonging to the set $\{0,1\}$

Then the neutrosophic mathematical model is written in the following general form:

$$Nf = Nf(x_1, x_2, \dots, x_n) \rightarrow (Max) \text{ or } (Min)$$

Constraints:

$$Ng_i(x_1, x_2, \dots, x_n) \begin{pmatrix} \leq \\ \geq \\ = \end{pmatrix} Nb_i ; i = 1, 2, \dots, m$$

$$x_j \in \{0, 1\} ; j = 1, 2, \dots, n$$

4. From the study reported in the classical reference [27], we find that if the nonlinearity in any zero-one model results from the presence of nonlinearity expressions of the following two forms:

$$x^n, x_1 x_2 x_3 x_4 \dots x_n$$

We can convert this model into zero-one linear model by relying on the validity of the following relation:

- i. For any positive number k and any binary variable we have:

$$x_j \in \{0, 1\} \text{ then } x_j^k = x_j ; j = 1, 2, \dots, n$$

- ii. For binary values of any variables x_r, x_s , the product $x_r x_s$ is always “equal” to 0 or 1, so we replace the product with a new variable, $y_1 = x_r x_s$ achieves the following:

$$y_1 = \begin{cases} 1 & x_r = x_s = 1 \\ 0 & x_r \text{ or } x_s = 0 \end{cases}$$

We achieve this by adding the following two constraints:

$$x_r + x_s - y_1 \leq 1$$

$$-x_r - x_s + 2y_1 \leq 0$$

We note that when $x_r = x_s = 1$ reduces the previous two constraint to $y_1 \geq 1$ and $y_1 \leq 1$ Therefore $y_1 = 1$, when $x_r = 0$ or $x_s = 0$ or $x_r = x_s = 0$, we find that the second constraint forces y_1 to be equal to zero because:

$$-x_r - x_s + 2y_1 \leq 0 \Rightarrow y_1 \leq \frac{x_r + x_s}{2} \Rightarrow y_1 = 0$$

This procedure requires adding an integer variable for every product, and since the time to solve integer programming problems increases proportionally with the number of integer variables, this procedure was replaced by another procedure by (F. Glover and E. Woolsey) that was presented in reference [27] which states the following: Instead of adding an integer variable, a continuous variable is added. The product is replaced by the continuous variable x_{rs} and the following constraints are added:

$$x_r + x_s - x_{rs} \leq 1$$

$$x_{rs} \leq x_r$$

$$x_{rs} \leq x_s$$

$$x_{rs} \geq 0$$

If x_r or $x_s = 0$ or $x_r = x_s = 0$, the constraints impose that x_{rs} be equal to zero. What is taken into account with this procedure is that it adds more constraints than the previous procedure.

The procedure for dealing with the product of two variables can be extended to include the product of any number of variables, if we take the product $x_1 x_2 x_3 x_4 \dots x_n$.

According to the first procedure, we replace the product with the variable y_1 and add the following two constraints:

$$\sum_{j=1}^n x_j - y_1 \leq n - 1$$

$$\begin{aligned}
 -\sum_{j=1}^n x_j - ny_1 &\leq 0 \\
 y_1 &\in \{0,1\} \\
 x_j &\in \{0,1\}
 \end{aligned}$$

According to the second procedure, we replace the product with the non-negative variable y_0 and add the following $n + 1$ constraints:

$$\begin{aligned}
 \sum_{j=1}^n x_j - y_0 &\leq n - 1 \\
 y_0 &\leq x_j \quad ; j = 1, 2, \dots, n
 \end{aligned}$$

2.1 | Example 1

We have the following zero-one neutrosophic nonlinear programming model:

Find:

$$MaxZ = c_{N1}x_1^2 + c_{N2}x_1x_2 - c_{N3}x_3^3$$

Constraints:

$$\begin{aligned}
 -a_{N1}x_1 + a_{N2}x_2 + a_{N3}x_3 &\leq b_N \\
 x_1, x_2, x_3 &\in \{0,1\}
 \end{aligned}$$

Where: $c_{N1}, c_{N2}, c_{N3}, a_{N1}, a_{N2}, a_{N3}, b_N$, neutrosophic values are taken in proportion to the nature of the issue under study.

Using the previous study, this zero-one neutrosophic nonlinear model is converting into a zero-one neutrosophic linear model.

1. Using the following transformations:

- a. For $x_1^2 = x_1$ and also $x_3^3 = x_3$
- b. For the product x_2x_3 , we take instead the variable $y_1 = x_2x_3$ and add the following two constraints:

$$\begin{aligned}
 x_2 + x_3 - y_1 &\leq 1 \\
 -x_2 - x_3 + 2y_1 &\leq 0
 \end{aligned}$$

We obtain the following zero-one neutrosophic linear model:

Find:

$$MaxZ = c_{N1}x_1 + c_{N2}y_1 - c_{N3}x_3$$

Constraints:

$$\begin{aligned}
 -a_{N1}x_1 + a_{N2}x_2 + a_{N3}x_3 &\leq b_N \\
 x_2 + x_3 - y_1 &\leq 1 \\
 -x_2 - x_3 + 2y_1 &\leq 0 \\
 x_1, x_2, x_3 &\in \{0,1\}
 \end{aligned}$$

2. Using the transformation (F. Glover and E. Woolsey):

- a. For $x_1^2 = x_1$ and also $x_3^3 = x_3$
- b. We substitute the product x_2x_3 into the continuous variable x_{23} and add the constraints:

$$x_2 + x_3 - x_{23} \leq 1$$

$$x_{23} \leq x_2$$

$$x_{23} \leq x_3$$

$$x_{23} \geq 0$$

We obtain the following zero-one neutrosophic linear model:

Find:

$$MaxZ = c_{N1}x_1 + c_{N2}x_{23} - c_{N3}x_3$$

Constraints:

$$-a_{N1}x_1 + a_{N2}x_2 + a_{N3}x_3 \leq b_N$$

$$x_2 + x_3 - x_{23} \leq 1$$

$$x_{23} \leq x_2$$

$$x_{23} \leq x_3$$

$$x_{23} \geq 0$$

$$x_1, x_2, x_3 \in \{0,1\}$$

3 | Conclusion and Results

As a continuation of what we presented in previous research of neutrosophic studies on operations research topics, we presented in this research a neutrosophic study specifically for zero-one programming problems, which is procedures that we can follow to convert some neutrosophic zero-one nonlinear models into zero-one neutrosophic linear models., with the aim of taking advantage of the easy-to-apply algorithms that have been developed by scholars in the field of operations research to obtain the optimal solution for zero-one linear models. Restricting the decision variables to be either zero or one helps us convert some nonlinear expressions into linear expressions, and these procedures can be used. In any study that contains binary integer variables.

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Author Contributaion

All authors contributed equally to this work.

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Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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