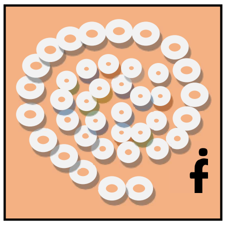

International Journal of Mathematical Analysis and Modelling

(Formerly Journal of the Nigerian Society for Mathematical Biology)

Volume 8 Issue 1, 2025

ISSN (Print): 2682 - 5694
ISSN (Online): 2682 – 5708
www.tnsmb.org

International Journal of Mathematical Analysis and Modelling (2025) 8(1):14-21

Differential BCI Algebras

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Abstract

In this paper, the notion of differential BCI algebras is introduced. The concept of partial ordering in BCI algebras is used to investigate some properties of differential BCI algebras. Necessary and sufficient conditions for the differentiability of BCI algebras are obtained. Moreover, necessary and sufficient conditions for an algebra of type (2,0) to be a differentiable BCI are also presented.

Keywords and phrases: Differential BCI algebras; palindromic; smarandachely.

1 Introduction

BCI algebras were introduced by Imai and Iseki in [1]. In [2], Iseki introduced the notion of BCK algebras which is a generalization of BCI algebras. These two algebras originated from two different sourses. One of the motivations is based on set theory. In set theory, there are three most elementary and fundamental operations. They are union, intersection and set difference. If we consider these operations and their properties, then as a generalization of them, we have the notion of Boolean algebras. If we take both union and intersection, then as a general algebra, the notion of distributive lattices is obtained. Moreover, if we consider union or intersection alone, we have the notion of upper semilattices or lower semilattices. However, the set difference together with its properties had not been considered systematically before the works of Imai and Iseki.

Another motivation is from propositional calculi. There are some systems which contain only the implicational functor among logical functors, such as the system of positive implicational calculus, the system of weak positive implicational calculus, BCK- systems and BCI- systems. Undoubtedly, there are common properties among these systems. It is well known that there are close relationships between the notion of set difference in set theory and implication functor in logical systems. Some questions were therefore raised. What are the most essential and fundamental properties of these relationships? Can there be a formulation of a general algebra from the above consideration? How would an axiom system be obtained that establishes a theory of the general algebra. It was while answering these pertinent questions that the notion of BCI algebras was birthed.

Several generalizations of BCI algebras have been studied. For instance, the notion of BCH algebras was introduced in [3]. In [4], d algebras were studied. In [5], the notion of BE algebras was introduced. Ideals and upper sets in BE algebras were investigated in [6] and [7]. Pre- commutative algebras were studied in [8]. Fenyves algebras were studied in [9], [10] and [11]. In [12], Q algebras were introduced. Homomorphisms of Q algebras were studied in [13].

Recently, it has been shown in [14] that BCI algebras and their generalizations have diverse applications in coding theory. Motivated by this, more research interest has been given to the study of BCI algebras and their generalizations.. Obic algebras were introduced in [15]. In [16], torian algebras were studied. It was shown that the class of torian algebras is a wider class than the class of obic algebras. Ideals of torian algebras were investigated in [17]. The dual and nuclei of ideals as well as congruences developed on ideals of torian algebras were studied. In [18], right distributive torian algebras were studied. Isomorphism Theorems of torian algebras were studied in [19].

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In this paper, the notion of differential BCI algebras is introduced. The concept of partial ordering in BCI algebras is used to investigate some properties of differential BCI algebras. Necessary and sufficient conditions for the differentiability of BCI algebras are obtained. Moreover, necessary and sufficient conditions for an algebra of type (2,0) to be a differentiable BCI are also presented.

2 Preliminaries

In this section, some basic concepts necessary for proper understanding of this paper are discussed.

Definition 2.1. *[1]. An algebra $(X;*,0)$; where X is a non-empty set, $*$ a binary operation defined on X, and 0 a constant element of X is called a BCI algebra if the following hold for all $x, y, z \in X$:*

1. $((x*y)*(x*z))*(z*y) = 0$
2. $(x*(x*y))*y = 0$
3. $x*x = 0$
4. $x*y = 0,\ y*x = 0 \Rightarrow x = y$
5. $x*0 = x$

Define a binary relation $\leq$ on a BCI algebra $(X;*,0)$ by $x \leq y$ if and only if $x*y = 0$. Then $(X;\leq)$ is a partially ordered set.

Definition 2.2. *[1]. A BCI algebra $(X;*,0)$ which satisfies $0*x = 0$ for all $x \in X$ is called a BCK algebra.*

Let $(X;*,0)$ be a BCI algebra. Then $(x*y)*z = (x*z)*y$ for all $x, y, z \in X$.
We shall denote a BCI algebra by X unless there is the need to emphasize its binary operation and the constant element.

3 Differential BCI Algebras

In this section, we introduce the notion of differential BCI algebras and investigate some of their properties. Necessary and sufficient conditions for the differentiability of BCI algebras are discussed. Moreover, necessary and sufficient conditions for an algebra of type (2,0) to be a differentiable BCI are also presented.

Definition 3.1. *Let X be a BCI algebra, and let $x, y \in X$. Define $x*y^k = ((x*y)*y)*...)*y$.
(y appears k times); where k is a natural number.*

Remark 3.2. It follows therefore that $x*(x*y)^k = (x*(x*y)*(x*y)*...)*(x*y)$.
($(x*y)$ appears k times).

Definition 3.3. *Let X be a BCI algebra, and let $\mathbb{N}^*$ be the set of non-negative integers. Let $x, y \in X,\ i, j \in \mathbb{N}^*$. Define the following:*

1. $D^{0,0}(x,y) = x*(x*y)$
2. $D^{i,j}(x,y) = (D^{0,0}(x,y)*(x*y)^i)*(y*x)^j$; *or equivalently,* $D^{i,j}(x,y) = (x*(x*y)^{i+1})*(y*x)^j$
3. $D^{i,j}(y,x) = (y*(y*x)^{i+1})*(x*y)^j$
4. $D^{i+1,j}(x,y) = D^{i,j}(x,y)*(x*y)$
5. $D^{i,j+1}(x,y) = D^{i,j}(x,y)*(y*x)$

Definition 3.4. *A BCI algebra X is said to be differentiable if there exist $k, l, i, j \in \mathbb{N}^*$ such that*

$$D^{k,l}(x, y) = D^{i,j}(y, x) \tag{1}$$

or equivalently,

$$(x * (x * y)^{k+1} * (y * x)^l = (y * (y * x)^{i+1}) * (x * y)^j \tag{2}$$

Expression (2) is called the derivative of X, while expression (1) is called the differential form of X.

Remark 3.5. Let X be a BCI algebra. If X is differentiable as in Definition 3.3, then X is of order $(k, l) \times (i, j)$.

We show in the following proposition that the order of a differentiable BCI algebra is symmetric.

Proposition 3.6. *Let X be a differentiable BCI algebra. Then X is of order $(k, l) \times (i, j)$ if and only if X is of order $(i, j) \times (k, l)$.*

Proof. This is obtained by the replacement of x with y in the respective derivative of X.

Proposition 3.7. *Let X be a differentiable BCI algebra of order $(k, l) \times (i, j)$. Then X is also of order $(k + n, l) \times (i, j + n)$ for any natural number n.*

Proof. Let X be a differentiable BCI algebra of order $(k, l) \times (i, j)$. Then $D^{k,l}(x, y) = D^{i,j}(y, x) \Rightarrow$

$$(x * (x * y)^{k+1}) * (y * x)^l = (y * (y * x)^{i+1}) * (x * y)^j \tag{3}$$

Now, 'multiplying' both sides of expression (3) on the right by $x * y$ (n times) and writing the resulting expression in differential form gives $D^{k+n,l}(x, y) = D^{i,j+n}(y, x)$ as required.

Proposition 3.8. *Let X be a differentiable BCI algebra of order $(k, l) \times (i, j)$. Then X is also of order $(k, l + n) \times (i + n, j)$ for any natural number n.*

Proof. This is obtained by a similar argument in the proof of Proposition 3.2.

Definition 3.9. *A BCI algebra X is said to be palindromic if $x * (x * y) = y * (y * x)$ for all $x, y \in X$*

Proposition 3.10. *A BCI algebra X is palindromic if and only if it is differentiable of order $(0, 0) \times (0, 0)$.*

Proof. Let X be a BCI algebra. Suppose X is differentiable of order $(0, 0) \times (0, 0)$. Then $x*(x*y) \Leftrightarrow D^{0,0}(x, y) = D^{0,0}(y, x) \Leftrightarrow y * (y * x)$ as required.

The following lemma is straightforward from definition.

Lemma 3.11. *Let X be a BCI algebra. Then $(x * (x * y)) * y = 0$ for all $x, y \in X$.*

Proposition 3.12. *Let X be a BCI algebra. Then for all natural numbers k the following hold:*

1. $D^{k-1,k}(x, y) \geq D^{k,k}(y, x)$
2. $D^{k,k}(y, x) \geq D^{k,k+1}(x, y)$
3. $D^{k-1,k}(x, y) \geq D^{k,k+1}$

Proof.

1. By Lemma 3.1, we have
$$x \geq y * (y * x) \tag{4}$$
Now 'multiply' both sides of expression (4) on the right by $x * y$ (k times) to get
$$x * (x * y)^k \geq (y * (y*)) * (x * y)^k \tag{5}$$
'Multiply' both sides of expression (5) on the right by $y * x$ (k times) to get
$$(x * (x * y)^k) * (y * x)^k \geq ((y * (y * x)) * (x * y)^k) * (y * x)^k \tag{6}$$
Expression (6) is the same as
$$(x * (x * y)^k) * (y * x)^k \geq (y * (y * x)^{k+1}) * (x * y)^k \tag{7}$$
Writing expression (7) in differential form gives $D^{k-1,k}(x, y) \geq D^{k,k}(y, x)$ as required.
2. This holds by performing similar operations in item (1) to the result of Lemma 3.1.
3. This follows from item (1) and item (2) since $(X, \leq)$ is a partially ordered set.

We therefore have the following corollary.

Corollary 3.13. *Let X be a BCI algebra. Then* $D^{0,1}(x, y) \geq D^{1,2}(x, y) \geq D^{2,3}(x, y) \geq ...$

Proposition 3.14. *Let X be a BCI algebra of length k; (k a natural number). Then X is differentiable of order* $(k - 1, k) \times (k, k)$.

Proof. Let X be a BCI algebra of length k. Then there are at most $k + 1$ terms in any connected chain in X. By Corollary 3.1, there exists $n \in \mathbb{N}^*$ $(n < k)$ such that for any $x, y \in X$, we have
$$D^{n,n+1}(x, y) = D^{n+1,n+2}(x, y)... \tag{8}$$
Now, 'multiply' both sides of expression (8) on the right by $x * y$ $((k - 1) - n$ times) and by $y * x$ $((k - 1) - n$ times) in turn, to get
$D^{k-1,k}(x, y) = D^{k,k+1}(x, y)$. Now, by Proposition 3.5 (1) and (2), we have
$D^{k-1,k}(x, y) \geq D^{k,k}(y, x) \geq D^{k,k+1}(x, y)$. Hence, $D^{k-1,k}(x, y) = D^{k,k}(y, x)$ as required.

Example 3.15. Let $X = \{0, 1, 2\}$. Define a binary operation $*$ on X by
$$x * y = \begin{cases} x, \ x \neq y \\ 0, \ x = y \end{cases}$$

Then $(X; *, 0)$ is a BCK algebra. Now, let $m, n, k, l \in \mathbb{N}^*$. Notice that
$(x * (x * y)^{m+1}) * (y * x)^n$
$= (x * x^{m+1}) * y^n = (0 * x^m) * y^n = 0$. Thus, $D^{m,n}(x, y) = 0$.
Similar argument gives $D^{k,l}(y, x) = 0$. Hence, $D^{m,n}(x, y) = D^{k,l}(y, x)$. Therefore, X is differentiable of order $(m, n) \times (k, l)$.

Remark 3.16. Let $(G, \cdot)$ be an abelian group with e as the identity element. Define a binary operation $*$ on G by $x * y = x \cdot y^{-1}$ for all $x, y \in G$. Then $(G; *, e)$ is a BCI algebra. This BCI algebra is called the BCI algebra induced by the group $(G, \cdot)$. We may replace the identity element e of the group by 0 without loss of generality.

Proposition 3.17. *Let $(G; *, 0)$ be the BCI algebra induced by the group $(G, \cdot)$. Then* $D^{i,j}(x, y) * D^{m,n}(y, x) = (x^{-1} \cdot y)^{i-j+m-n+1}$ *for all* $x, y \in G,\ i, j, m, n \in \mathbb{N}^*$.

Proof. Since $(G, \cdot)$ is an abelian group and $x * y = x \cdot y^{-1}$ for all $x, y \in G$, we have $x * y^k = x \cdot y^{-k}$ for all $k \in \mathbb{N}*$.
Now, $(x * (x * y)^{i+1}) * (y * x)^j$
$= x \cdot (x \cdot y^{-1})^{-i-1} \cdot (y \cdot x^{-1})^{-j}$
$= (x^{-1})^{i-j} \cdot y^{i-j+1}$.
That is, $D^{i,j}(x, y) = (x^{-1})^{i-j} \cdot y^{i-j+1}$.
We also have $D^{m,n}(y, x) = (y^{-1})^{m-n} \cdot x^{m-n+1}$
That is, $D^{m,n}(y, x) = (x^{-1})^{-(m-n+1)} \cdot y^{-(m-n)}$.
Hence, $D^{i,j}(x, y) * D^{m,n}(y, x) = x^{-1} \cdot y)^{i-j+m-n+1}$ as required.

Definition 3.18. *Let $(X; *, 0)$ be a BCI algebra. If there exists an $x \in X$ such that $0 * x \neq 0$, then X is said to be a smarandachely BCI algebra.*

Proposition 3.19. *Let X be a smarandachely BCI algebra induced by the group $(G, \cdot)$ such that X is differentiable of order $(i, j) \times (m, n)$. Then $i - j + m - n + 1 \neq \pm 1$.*

Proof. Suppose $i - j + m - n + 1 = \pm$. Since X is smarandachely, the set $\overline{X} = \{x \in X : 0 * x = 0\}$ is non-zero. Let $x \in \overline{X}$. Then $D^{i,j}(0, x) * D^{m,n}(x, 0) = (0^{-1} \cdot x)^{i-j+m-n+1} = x \pm 1$.
Also, since X is of order $(i, j) \times (m, n)$, we have
$D^{i,j}(0, x) * D^{m,n}(x, 0) = 0$. Hence, $x^{\pm 1} = 0$ and $x = 0$; a contradiction to $x \neq 0$. Therefore, $i - j + m - n + 1 \neq \pm 1$ as required.

Proposition 3.20. *Let X be a smarandachely BCI algebra. If $i - j + m - n + 1 \neq \pm 1$, then X is differentiable of order $(i, j) \times (m, n)$.*

Proof. Let us denote $\mid i - j + m - n + 1 \mid= k$. Then $k \neq 1$. If $k = 0$, then the result holds. Now, suppose $k > 0$. Let $(G; *, 0)$ be the BCI algebra induced by the cyclic group $(G; \cdot, 0)$ of order k. Then we have
$D^{i,j}(x, y) * D^{m,n}(y, x) = (x^{-1} \cdot y)^{i-j+m-n+1} = (x^{-1} \cdot y)^{\pm k} = 0$.
Therefore, $D^{i,j}(x, y) = D^{m,n}(y, x)$ as required.

By Proposition 3.8 and Proposition 3.9, we have the following theorem

Theorem 3.21. *A smarandachely BCI algebra X is differentiable of order $(i, j) \times (m, n)$ if and only if $i - j + m - n + 1 \neq \pm 1$.*

Theorem 3.22. *Let $(X; *, 0)$ be an algebra of type (2,0). Then X is differentiable of order $(i, j) \times (m, n)$ if and only if the following hold for all $x, y, z \in X$:*

1. *$((x * y) * (x * z)) * (z * y) = 0$;*
2. *$x * 0 = x$;*
3. *$D^{i,j}(x, y) = D^{m,n}(y, x)$ for some $i, j, m, n \in \mathbb{N}^*$.*

Proof. Suppose $(X; *, 0)$ is an algebra of type (2,0) which satisfies items (1),(2) and (3) hold. We only need to show that $x * y = 0$ and $y * x = 0 \Rightarrow x = y$.
By item (2), $x * 0^k = x$ for some natural number k. Suppose $x * y = 0$ and $y * x = 0$, then
$(x * (x * y)^{i+1}) * (y * x)^j = (x * 0^{i+1}) * 0^j = x$.
Thus, $D^{i,j}(x, y) = x$. Similar argument gives $D^{m,n}(y, x) = y$. Therefore, $x = y$ as required. Therefore, $(X; *, 0)$ is a differentiable BCI algebra of order $(i, j) \times (m, n)$.
The converse is obvious.

Theorem 3.23. *An algebra $(X; *, 0)$ of type (2,0) is differentiable of order $(i, j) \times (m, n)$ if and only if the following hold for all $x, y, z, u \in X$:*

1. *$u * (((x * y) * (x * z)) * (z * y)) = u$;*

2. $D^{i,j}(x * y) = D^{m,n}(y * x) * 0$

Proof. Suppose $(X; *, 0)$ is an algebra of type (2,0) which satisfies items (1) and (2). Let $p = (((0 * 0) * (0 * 0)) * (0 * 0))$. Then by item (1), we have $p * (((0 * 0) * (0 * 0)) * (0 * 0)) = p$. Thus, $p * p = p$. Using item (1) again, we have

$$u * (((p * p) * (p * p)) * (p * p)) = u \tag{9}$$

Now, repeated application of $p * p = p$ to expression (9) gives $u * p = u$ for all $u \in X$. Substitute p for y and z in the left hand side of item (1) to get $u(((x * p) * (x * p)) * (p * p))$ $= u * ((x * x) * p) = u * (x * x) = u$, and so by induction, for every natural number k we have

$$u * (x * x)^k = u \tag{10}$$

From this, we have that $D^{i,j}(0, 0) = (0 * (0 * 0)^{i+1}) * (0 * 0)^j$ $= 0 * (0 * 0)^j = 0$. Similar argument gives $D^{m,n}(0, 0) = 0$. Therefore item (2) implies $0 * 0 = 0$. Replacing x with 0 and k with 1 in expression (10), we have $u * (0 * 0) = u$. Hence, for all $u \in X$, we have

$$u * 0 = u \tag{11}$$

Combining expression (11) with item (2), we have

$$D^{i,j}(x, y) = D^{m,n}(y, x) \tag{12}$$

and so by Theorem 3.2, it remains to show that $((x * y) * (x * z)) * (z * y) = 0$ for all $x, y, z \in X$. Now, replacing u with $u * u$; x with u and k with $i + 1$ in expression (10), and using expression (11), we get

$$(u * u) * ((u * u) * 0)^{i+1} = u * u \tag{13}$$

Substituting 0 for u; u for x and n for k in expression (10), we have

$$0 * (u * u)^n = 0 \tag{14}$$

In particular, if $n = 1$, we have

$$0 * (u * u) = 0 \tag{15}$$

Now, expressions (13) and (15) together with expression (11) give
$D^{i,j}(u * u, 0) = ((u * u) * ((u * u) * 0)^{i+j}) * (0 * (u * u))^j$
$= (u * u) * 0^j = u * u$
Expressions (14) and (15) together with expression (11) give
$D^{m,n}(0, u * u) = (0 * (0 * (u * u)^{m+1}) * ((u * u) * 0)^n$
$= (0 * 0^{m+1}) * (u * u)^n = 0 * (u * u)^n = 0.$
Hence, expression (12) implies $u * u = 0$ for every $u \in X$. Now, substituting u with $((x * y) * (x * z)) * (z * y)$ in item (1), it follows that $((x * y) * (x * z)) * (z * y) = 0$ as required. Therefore, $(X; *, 0)$ is a differentiable BCI algebra of order $(i, j) \times (m, n)$.
The converse is obvious.

Theorem 3.24. *Let X be a BCK algebra such that $x * (x * (x * y))^{i+1} = x * y^{i+1}$ for all $i \in \mathbb{N}^*$. If X is differentiable of order $(i, j) \times (m, n)$, then the following hold:*

1. $x * y^{i+1} = x * y^{n+1}$;
2. $x * y^{j+1} = x * y^{m+1}$;
3. *X is also of order $(i, j) \times (j, n)$*

Proof.

1. Since X is BCK, we have $(x*y)*x=0$ for all $x,y\in X$. Also, since $x*(x*(x*y))^{i+1}=x*y^{i+1}$ for all $i\in\mathbb{N}^*$, we have
   $D^{i,j}(x,x*y)=(x*(x*(x*y))^{i+1})*((x*y)*x)^j=x*y^{i+1}$.
   Similarly, $D^{m,n}(x*y,x)=x*y^{n+1}$. Now, since X is of order $(i,j)\times(m,n)$, we have $D^{i,j}(x,x*y)=D^{m,n}(x*y,x)$. Hence, $x*y^{i+1}=x*y^{n+1}$ as required.

2. By Proposition 3.1, X is of order $(m,n)\times(i,j)$. Therefore, by item (1) above, the result follows.

3. Since X is of order $(i,j)\times(m,n)$, we have $(x*(x*y)^{i+1})*(y*x)^j$
   $=(y*(y*x)^{m+1})*(x*y)^n$.
   Also, by item (2) above, we have $y*(y*x)^{j+1}=y*(y*x)^{m+1}$. Hence,
   $(x*(x*y)^{i+1})*(y*x)^j=(y*(y*x)^{j+1}*(x*y)^n$. Therefore, X is of order $(i,j)\times(j,n)$ as required.

Theorem 3.25. *Let X be a differentiable BCK algebra such that $x*(x*(x*y))^{i+1}=x*y^{i+1}$ for all $i\in\mathbb{N}^*$ and $x\geq x*y\geq x*y^2\geq\ldots\geq x*y^n$ for all $x,y\in X$. Suppose X is of order $(i,j)\times(m,n)$ with $i=min\{i,j,m,n\}$, then the following hold:*

1. *If $i=j$, then X is also of order $(i,i)\times(i,i)$;*

2. *If $i<j$ and $i<n$, then X is also of order $(i,i+1)\times(i,i+1)$;*

3. *If $i<j$ and $i=m=n$, then X is also of order $(i,i)\times(i,i)$;*

4. *If $i<j,\ i=n$ and $i<m<j$, then X is also of order $(i,m+1)\times(m,i)$;*

5. *If $i<j,\ i=n$ and $j\leq m$, then X is also of order $(i,j)\times(j,i)$.*

Proof. Since X is of order $(i,j)\times(m,n)$, then

$$(x*(x*y)^{i+1}*(y*x)^j=(y*(y*x)^{m+1})*(x*y)^n \tag{16}$$

Also, since $i=min\{i,j,m,n\}$, then $i\leq m<m+1$ and $i<n$.

1. If $i=j$, then expression (16) can be written as
   $(x*(x*y)^{i+1}*(y*x)^i=(y*(y*x)^{m+1})*(x*y)^n$. Since $i\leq m$ and $i\leq n$, then
   $(y*(y*x)^{m+1})*(x*y)^n\leq(y*(y*x)^{i+1})*(x*y)^i$, and so

   $$(x*(x*y)^{i+1})*(y*x)^i\leq(y*(y*x)^{i+1})*(x*y)^i \tag{17}$$

   Exchanging x and y in expression (17), the opposite inequality holds. Therefore
   $(x*(x*y)^{i+1})*(y*x)^i=(y*(y*x)^{i+1})*(x*y)^i$ as required.

2. By Theorem 3.4(1), we have $u*v^{i+1}=u*v^{n+1}$ for all $u,v\in X$. Since $i<n$, we have $u*v^{i+1}=u*v^k$ whenever $k>i$. Since u and v are arbitrary and $i<j$, the left side of expression (16) is the same as $(x*(x*y)^{i+1})*(y*x)^{i+1}$ and its right side is the same as $(y*(y*x)^{i+1})*(x*y)^{i+1}$. Hence,
   $(x*(x*y)^{i+1})*(y*x)^{i+1}=(y*(y*x)^{i+1}*(x*y)^{i+1}$ as required.

3. Since $i=m=n$, X is of order $(i,j)\times(i,i)$. Then the result follows from Proposition 3.1 and item (1) above.

4. By Theorem 3.4(2), we have $u*v^{j+1}=u*v^{m+1}$ for any $u,v\in X$. Since $m<j$, we have $u*v^j=u*v^{m+1}$. Since u and v are arbitrary, the left side of expression (16) becomes $(x*(x*y)^{i+1})*(y*x)^{m+1}$. Also, since $i=n$, the right side of expression (16) becomes $(y*(y*x)^{m+1}*(x*y)^i$. Hence,
   $(x*(x*y)^{i+1}*(y*x)^{m+1}=(y*(y*x)^{m+1})*(x*y)^i$ as required.

5. Since $i=n$, X is of order $(i,j)\times(m,i)$. Therefore the result follows from Theorem 3.4(iii).

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