

Double Indeterminacy - Neutrosophic study of an Approximation Techniques Used to Find Random Variables

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Abstract

The main interest in statistical analysis is to generate a series of random variables that follow the probability distribution in which the system under study operates. In almost all simulation tests, we need to generate random variables that follow a distribution, a distribution that adequately describes and represents the physical process involved in the experiment at That point. During the experiment, it may be necessary to simulate a real and perform the process of generating a random variable from a distribution many times depending on the complexity of the model to be simulated in order to obtain more accurate simulation results. In previous research, we presented a neutrosophical study of the process of generating random numbers and some techniques that can be used to convert these random numbers into variables. Randomness follows the probability distributions according to which the system to be simulated operates. These techniques were specific to probability distributions defined by a probability density function that is easy to deal with in terms of finding the cumulative distribution function and the inverse function of the cumulative distribution function or by calculating the values of this function at a certain value, and in reality, we encounter Many systems operate according to these distributions, which requires techniques other than the techniques presented. Therefore, in this research we will present a neutrosophical study of the approximation technique for generating random variables that follow probability distributions known as a complex probability density function. We will apply this study to find random variables that follow the distribution. Standard natural

Keywords: Generating neutrosophic random numbers; generating neutrosophic random variables; approximation technique for generating neutrosophic random variables; standard normal distribution.

1. Introduction:

In light of the great development witnessed by our contemporary world, the complexity of systems, and the large material and non-material losses that can result from operating any system without prior study, there must be a scientific method that enables us to know the results that we can obtain when operating systems and helps us reduce these losses. The simulation process was the modern tool through which we could predict the results that we could obtain through the operation of these systems over time. It became the modern-day tool that helps us study many systems that could not have been studied or predict the results that we could obtain. Through the operation of these systems over time, the simulation process depends on generating a series of random numbers subject to a regular probability distribution over the field [0, 1], then converting these random numbers into random variables that follow the probability distribution in which the system to be simulated operates. Since the process Simulation depends on the data that is collected and is subject to change and is not fixed over the length of time during which the system operates. Therefore, we must prepare a study that takes into account all the conditions and changes that the operating environment of the system to be simulated may undergo. In other words, a study based on the concepts of neutrosophic logic. Logic, which studies ideas and concepts that are neither true nor false, but in between, which means (neutrality, indeterminacy, ambiguity, contradiction, etc.), is expressed in three dimensions: truth T, neutrality I, and error F, each of which has degrees. This new vision of concepts has prompted many researchers and scholars to reformulate many scientific concepts according to the concepts of this logic [1-12]. As a complementary study to these studies and given the importance of the science of operations research, we have presented and reformulated many methods of operations research using the concepts of this logic [12-15], in order to obtain more accurate simulation results, we presented a neutrosophical study of the process of generating random numbers and some techniques that can be used to transform these random numbers into random variables that follow the probability distributions according to which the system to be simulated operates [16-23], and these techniques were special With probability distributions known as a probability density function, it is easy to deal with in terms of finding the cumulative distribution function and its inverse function. Also, calculating the value of this function at a certain value is easy. It should be noted here that there are many systems that operate according to probability distributions known as a complex probability density function that requires the process of generating random variables depends on special techniques. In this research, we present a neutrosophical study of the approximation technique for generating random variables that follow probability distributions defined by a complex probability density function. As an example, we will apply this technique to generate random variables that follow the standard normal distribution.

2. **Discussion:**

The process of neutrosophic simulation depends on generating a series of random numbers that follow a uniform distribution over the field [0,1], then converting these random numbers into neutrosophic random numbers using the information contained in the research [16], to generate random variables that follow the probability distribution according to which the system operates. What is to be simulated? If the probability density function of the distribution is easy, we use the appropriate method from among the methods presented in research [16-23]. It should be noted here that there are many systems that operate according to probability distributions defined by a complex probability density function that requires the process of generating random variables. Tracing it to special techniques, we present in this research a neutrosophical study of the approximation technique for generating random variables that follow probability distributions defined by a complex probability density function,

2.1. The basic idea of approximation techniques: [24,25]

Approximation techniques are related to the inverse transformation technique, and are used when the probability distribution is known as a complex probability density function. In approximation methods, we approximate one of three different quantities:

- Follow the probability density of the distribution to be simulated.
- Follow the cumulative distribution of the distribution to be simulated.
- The inverse transformation of the cumulative distribution function of the distribution to be simulated.

Approximation methods are used when the cumulative distribution function cannot be obtained from the given probability density function, or when it is impossible to obtain the opposite transformation even if F(x) exists. When this is true, an approximation must be made to the probability density function to facilitate the derivation of the function. Cumulative distribution, or an approximation must be made to F(x) directly. Once the cumulative distribution function of the approximated probability density function is obtained, the inverse transformation can be applied and the required random variables can be generated.

2.2. In the classic study, this technique was applied to generate random variables that follow the standard normal distribution, and the following relationship was obtained:

$$z = \frac{1}{k} ln \left[\frac{1 + R_1}{1 - R_1} \right] \quad (1)$$

To generate random variables that follow the general normal distribution $N(\mu, \sigma^2)$, we use the following transformation:

$$x = \sigma z + \mu \Longrightarrow z = \frac{x - \mu}{\sigma}$$

Substituting into relationship (1) we get the following relation:

$$\Rightarrow x = \frac{\sigma}{k} ln \left[\frac{1+R_1}{1-R_1} \right] + \mu \quad (2)$$

Example 1:

Using the approximation technique, find a random variable that follows a normal distribution defined by the parameters $\mu = 15$ and $\sigma = 2$.

the solution:

Using the mean square method, we generate a random number that follows a uniform distribution in the range [0,1].

The mean square method is given by the following relation:

$$R_{i+1} = Mid[R_i^2]; i = 0, 1, 2, 3, ...$$
 (3)

Where *Mid* symbolizes the middle four ranks of R_i^2 , and R_i is chosen, i.e., a fractional random number consisting of four ranks (called a seed) that does not contain a zero in any of its four ranks, and let $R_0 = 0.1234$, we get: $R_1 = 0.5227$

Applying relation (3) we get:

$$x_1 = \frac{\sigma}{k} \ln\left[\frac{1+R_1}{1-R_1}\right] + \mu = 2.\sqrt{\frac{\pi}{8}} \cdot \ln\left[\frac{1+0.5227}{1-0.5227}\right] + 15$$
$$x_1 = 16.4541$$

2.3. Neutrosophic study to generate random variables that follow the standard normal distribution using the approximation technique:

Based on the study mentioned in references [24,25] for the quadrature technique for generating random variables that follow the standard normal distribution, and the study mentioned in reference [2] to define neutrosophic probability distributions, the study mentioned in reference [16] for generating neutrosophic random numbers, and the study mentioned in the two references. [17,18] The neutrosophic study of the inverse transformation method. We can present the following neutrosophic formulation of the approximation technique used to generate random variables that follow the standard normal distribution, and then we use the appropriate transformation to formulate the appropriate relationship to generate random variables that follow the general normal distribution:

To generate random variables that follow the standard normal distribution defined by the following probability density function:

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} - \infty \le z \le +\infty$$

• One of the most important approximations of the quantity $e^{-\frac{z^2}{2}}$ is the Kahn approximation, which is given by the following relation:

$$e^{-\frac{z^2}{2}}\approx \frac{2e^{-kz}}{(1+e^{-kz})^2} ~~; z>0$$

in order to

$$k = \sqrt{\frac{8}{\pi}}$$

• We calculate the standard cumulative distribution function from the following approximate relation:

$$F(z) = \int_0^z \frac{1}{\sqrt{2\pi}} \cdot \frac{2e^{-kt}}{(1+e^{-kt})^2} dt = \left[\frac{2}{1+e^{-kx}}\right] - 1$$

• We generate a series of random numbers that follow a uniform distribution over the domain [0,1] and let it be: [24,25]

$$R_1, R_2, ..., R_n$$

• We convert these random numbers into neutrosophic random numbers [16] and we get:

$$R_{1N}, R_{2N}, \dots, R_{nN}$$

• We use the inverse transformation [17,18] as follows:

$$F(z) = \left[\frac{2}{1+e^{-kz}}\right] - 1 = R_{iN} \Longrightarrow \frac{2}{1+e^{-kz}} = 1 + R_{iN}$$
$$\implies 1 + e^{-kz} = \frac{2}{1+R_{iN}} \Longrightarrow e^{-kz} = \frac{2}{1+R_{iN}} - 1$$

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$$\Rightarrow e^{-kz} = \frac{1 + R_{iN}}{1 - R_{iN}}$$
$$z_{iN} = \frac{1}{k} ln \left[\frac{1 + R_{iN}}{1 - R_{iN}} \right] \quad (4)$$

When we want to generate neutrosophic random variables that follow the general neutrosophic normal distribution $N(\mu_N, \sigma_N)$, we use the following transformation:

We get the following relation:

$$\Rightarrow x_{iN} = \frac{\sigma_N}{k} ln \left[\frac{1 + R_{iN}}{1 - R_{iN}} \right] + \mu_N \quad (5)$$

 $\Rightarrow x_{iN} = \sigma_N z_{iN} + \mu_N$

Neutrosophic random variables are obtained according to the following conditions:

The first case: Neutrosophic random numbers and probability distribution in the classical form

The second case: classical random numbers and the probability distribution is in the neutrosophic form.

The third case: neutrosophic random numbers and probability distribution in neutrosophic form.

Example 2:

Using the approximation technique, find a random variable that follows the normal distribution defined by the parameters $\mu_N \in [15, 17]$ and $\sigma_N \in \{2, 3, 4\}$

the solution:

From the data, we notice that the probability distribution is in the neutrosophic form, and here we can apply either the second case or the third case.

For this, we follow the following steps:

- Using the mean square method, we generate a random number that follows a uniform distribution in the range [0,1].
- The mean square method is given by the following relation:

$$R_{i+1} = Mid[R_i^2]$$
; $i = 0, 1, 2, 3, ...$

Where *Mid* symbolizes the middle four levels of R_i^2 , and R_i is chosen, i.e., a fractional random number composed of four places (called a seed) that does not contain a zero in any of its four places, and let $R_0 = 0.1234$, we get $R_1 =$ 0.5227

We transform this random number into a neutrosophic random number and here we distinguish three forms of the field [0,1] with a margin of indeterminacy, in the three forms we have $\varepsilon \in [0, n]$ and 0 < n < 1

The first form: $[0 + \varepsilon, 1]$ indeterminacy at the minimum of the field. We substitute in the

relation:

$$R_{iN} = \frac{R_i - \varepsilon}{1 - \varepsilon}$$

The second form: $[0, 1 + \varepsilon]$ The indeterminacy at the upper limit of the range, we substitute

into the relation:

$$R_{iN} = \frac{R_i}{1+\varepsilon}$$

The third form: $[0 + \varepsilon, 1 + \varepsilon]$ The indeterminacy in the upper and lower limits of the field,

we substitute in the relation:

$$R_{iN} = R_i - \varepsilon$$

We take $\varepsilon \in [0,0.02]$ then we obtain the appropriate neutrosophic random numbers for each of the three forms as follows:

From the first form we get the following neutrosophic random number:

$$R_{1N} = \frac{R_1 - \varepsilon}{1 - \varepsilon} = \frac{0.5227 - [0, 0.02]}{1 - [0, 0.02]} = \frac{[0.5027, 0.5227]}{[1, 0.98]} \in [0.5027, 0.5334]$$

From the second figure we get the following two neutrosophic random numbers:

$$R_{1N} = \frac{R_1}{1+\varepsilon} = \frac{0.5227}{1+[0,0.02]} = \frac{0.5227}{[1,1.02]} \in [0.5125, 0.5227]$$

From the third figure we get the following two neutrosophic random numbers:

$$R_{1N} = R_1 - \varepsilon = 0.5227 - [0, 0.02] \in [0.5027, 0.5227]$$

To generate neutrosophic random variables that follow the normal distribution defined by $\mu_N \in [15, 17]$ and $\sigma_N \in \{2, 3, 4\}$, we note that the probability distribution is given in the neutrosophic form, meaning that we obtain random variables that follow it from the second case or from the third case.

According to the second case:

the random numbers are classical and the probability distribution is in the neutrosophic form: The data of the problem becomes:

That is, $R_1 = 0.5227$, and the distribution is defined by $\mu_N \in [15,17]$ and $\sigma_N \in \{2,3,4\}$. Substituting into relation (5) we get:

$$x_{1N} = \frac{\sigma_N}{k} ln \left[\frac{1+R_1}{1-R_1} \right] + \mu_N$$
$$x_{1N} = \{2,3,4\}. \sqrt{\frac{\pi}{8}} ln \left[\frac{1+0.5227}{1-0.5227} \right] + [15,17]$$

For $\sigma = 2$:

$$\begin{aligned} x_{1N} &= 2. \sqrt{\frac{\pi}{8}} \cdot ln \left[\frac{1.5227}{0.4773} \right] + [15,17] \\ & \left[\frac{1.5227}{0.4773} \right] = 3.1902 \\ & \ln (3.1902) = 1.1601 \\ & \sqrt{\frac{\pi}{8}} = 0.3927 \\ & 2. \sqrt{\frac{\pi}{8}} = 0.7854 \\ 2. \sqrt{\frac{\pi}{8}} \cdot ln \left[\frac{1.5227}{0.4773} \right] = 0.7854 * 1.1601 = 0.9111 \\ & x_{1N} = 2. \sqrt{\frac{\pi}{8}} \cdot ln \left[\frac{1.5227}{0.4773} \right] + [15,17] \\ & x_{1N} = 0.9111 + [15,17] \in [15.9111,17.9111] \\ & x_{1N} \in [15.9111,17.9111] \end{aligned}$$

For $\sigma = 3$:

$$\begin{aligned} x_{1N} &= 1.1781 + [15,17] \in [16.1781,18.1781] \\ x_{1N} &\in [16.1781,18.1781] \end{aligned}$$

For $\sigma = 4$:

$$\begin{aligned} x_{1N} &= 1.5708 + [15,17] \in [16.5708, 18.5708] \\ x_{1N} &\in [16.5708, 18.5708] \end{aligned}$$
 Thus, for any $R_1 = 0.5227, \, \mu_N \in [15,17] \text{ and } \sigma_N \in \{2,3,4\} \text{ then:} \\ x_{1N} \in \{[15.9111, 17.9111], [16.1781, 18.1781], [16.5708, 18.5708]\} \end{aligned}$

We note that we have obtained a set consisting of three elements that are fields, and this means the multiple choices that the random variable that follows the normal distribution can take, which provides us with highly accurate simulation results due to the double indeterminacy that we obtained.

According to the third case, the random numbers are neutrosophic, i.e.:

From the first form we have: $R_{1N} \in [0.5027, 0.5334]$

The data of the problem becomes:

Using the neutrosophic random number $R_{1N} \in [0.5027, 0.5334]$ which follows a uniform distribution in the range $[0 + \varepsilon, 1]$ where $\varepsilon \in [0, 0.02]$, find a random variable that follows the normal distribution defined by $\mu_N \in [15, 17]$ and $\sigma_N \in \{2,3,4\}$

We substitute into the following relation:

$$x_{1N} = \frac{\sigma_N}{k} ln \left[\frac{1+R_1}{1-R_1} \right] + \mu_N$$

We get the neutrosophic random variable:

$$x_{1N} \in \{[16.3859, 18.4887], [17.0788, 19.2329], [17.7718, 19.9773]\}$$

From the second figure we have: $R_{1N} \in [0.5125, 0.5227]$

The data of the problem becomes:

Using the neutrosophic random number $R_{1N} \in [0.5125, 0.5227]$ which follows a uniform distribution in the range $[0, 1 + \varepsilon]$ where $\varepsilon \in [0, 0.02]$, find a random variable that follows the normal distribution defined by $\mu_N \in [15, 17]$ and $\sigma_N \in \{2, 3, 4\}$ In the same way as before, we obtain the neutrosophic random variable:

 $x_{1N} \in \{[16.3859, 18.4540], [17.0788, 19.1809], [17.7718, 19.9079]\}$

From the third form we have: $R_{1N} \in [0.5027, 0.5227]$ The data of the problem becomes:

Using the neutrosophic random number $R_{1N} \in [0.5027, 0.5227]$ which follows a uniform distribution in the domain $[0 + \varepsilon, 1 + \varepsilon]$ where $\varepsilon \in [0, 0.02]$, find a random variable that follows the normal distribution defined by $\mu_N \in [15, 17]$ and

$$\sigma_N \in \{2,3,4\}$$

In the same way as before, we obtain the neutrosophic random variable:

 $x_{1N} \in \{[16.3859, 18.4540], [17.0788, 19.7334], [17.7718, 19.9079]\}$

Comparison between the classical study and the neutrosophic study:

1- In the classical study, example (1), the data are specific values

The random number $R_1 = 0.5227$ The normal distribution is defined by the parameters $\mu = 15$ and $\sigma = 2$.

We obtained a specific value for the required random variable:

$$x_1 = 16.4541$$

2- In the neutrosophic study:

The second case:

the random number $R_1 = 0.5227$. The normal distribution is defined by $\mu_N \in [15,17]$ and $\sigma_N \in \{2,3,4\}$ we obtain a nitrosophic random variable (double indeterminacy):

 $x_{1N} \in \{[15.9111, 17.9111], [16.1781, 18.1781], [16.5708, 18.5708]\}$

The third case:

the neutrosophic random number normal distribution defined by $\mu_N \in [15,17]$ and $\sigma_N \in \{2,3,4\}$, according to the three forms of the neutrosophic random number, we obtain a neutrosophic random variable (double indeterminacy) which is:

According to the first figure:

 $x_{1N} \in \{[16.3859, 18.4887], [17.0788, 19.2329], [17.7718, 19.9773]\}$ According to the second figure:

 $x_{1N} \in \{[16.3859, 18.4540], [17.0788, 19.1809], [17.7718, 19.9079]\}$ According to the third figure:

 $x_{1N} \in \{[16.3859, 18.4540], [17.0788, 19.7334], [17.7718, 19.9079]\}$

Conclusion and Results: 3.

From the above, we note that to obtain random variables that follow the normal distribution using the approximation technique, we use this technique with respect to the standard normal distribution, and then we use the transformation relationship defined between the normal distribution and the standard normal distribution to obtain the relationship through which we obtain random variables that follow the general normal distribution, and here When we take the data in the studied system as classical values, we get a single value for the random variable, and this value is appropriate and gives appropriate simulation results for working conditions that are completely similar to the conditions in which the data was collected, and any change may cause an unexpected loss. However, when we take the data as neutrosophic values, the random variable belongs the minimum range corresponds to the worst working conditions that the system to be simulated can experience, and the highest range corresponds to the best working conditions that the system to be simulated can experience. Therefore, simulating systems using the concepts of neutrosophic logic provides us with more accurate results than the results provided by the classic study.

References

Florentin Smarandache, Maissam Jdid, On Overview of Neutrosophic and Plithogenic Theories and [1] Applications, Doi :https://doi.org/10.54216/PAMDA.020102

F. Smarandache. Introduction to Neutrosophic statistics, Sitech & Education Publishing, 2014. [2]

Faisal Al-Sharqi, Ashraf Al-Quran, and Zahari Md. Rodzi , Multi-Attribute Group Decision-Making [3] Based on Aggregation Operator and Score Function of Bipolar Neutrosophic Hypersoft Environment,

Neutrosophic Sets and Systems, Vol. 61, 2023, pp. 465-492. DOI: 10.5281/zenodo.10428642

Mountajab Alhasan, Malath F Alaswad. A Study of the Movement of a Neutrosophic Material Point in [4] the Neutrosophic Plane by Using a Neutrosophic AH-Isometry, Journal Prospects for Applied Mathematics and Data Analysis, Doi: https://doi.org/10.54216/PAMDA.020101, Vol 2, No 1, 2023.

Mona Gharib, Florentin Smarandache, Mona Mohamed, CSsEv: Modelling QoS Metrics in Tree Soft [5] Toward Cloud Services Evaluator based on Uncertainty Environment, Doi

:https://doi.org/10.54216/IJNS.230204, Vol,23, No,2

Said Broumi, S. krishna Prabha, Fermatean Neutrosophic Matrices and Their Basic [6]

Operations, Neutrosophic Sets and Systems, Vol. 58, 2023, pp. 572-595. DOI: 10.5281/zenodo.8404536 Mohamed Abdel-Basset, Uncertainty-infused Representation Learning Using Neutrosophic-based [7] Transformer Network, Neutrosophic Sets and Systems, Vol. 58, 2023, pp. 644-

657. DOI: 10.5281/zenodo.10028758

Florentin Smarandache, Introduction to Neutrosophic Genetics Doi: [8] https://doi.org/10.54216/IJNS.130103, Vol 13, No, 1.

Maissam Jdid, Hla Hasan, The State of Risk and Optimum Decision According to Neutrosophic Rules, [9] Doi :https://doi.org/10.54216/IJNS.200107, Volume 20, Issue 1

S. Loganayaki, N. Rajeswari, K. Kirupa, S. Broumi, Optimization of triangular neutrosophic based [10] economic order quantity model under preservation technology and power demand with shortages, Vol. 64 (2024): Neutrosophic Sets and Systems

M. Anandhkumar, H. Prathab, S. M. Chithra, A. S. Prakaash, A. Bobin, Secondary K-Range [11] Symmetric Neutrosophic Fuzzy Matrices, Volume 23, Issue 4, Doi :https://doi.org/10.54216/IJNS.230402

Mohammed Alshikho, Maissam Jdid, Said Broumi, A Study of a Support Vector Machine Algorithm [12] with an Orthogonal Legendre Kernel According to Neutrosophic logic and Inverse Lagrangian Interpolation, Doi : https://doi.org/10.54216/JNFS.050105

Florentin Smarandache, Maissam Jdid, Research on the topics of neutrosophic operations research, [13] https://fs.unm.edu/NeutrosophicOperationsResearch.pdf, Biblio Publishing, Volume (1),2023

Maissam Jdid, Neutrosophic linear models and algorithms to find their optimal solution, Biblio [14] Publishing, ISBN, 978_1 _59973_778_2, USA, 2023 https://fs.unm.edu/NeutrosophicLinearModels-ar.pdf

Maissam Jdid, NEUTROSOPHIC TRANSPORT AND ASSIGNMENT ISSUES, Publisher: Global [15] Knowledge's, ISBN, 978_1_59973_770_6, USA, 2023 https://fs.unm.edu/NeutrosophicOptimization-ar.pdf, Maissam Jdid, Rafif Alhabib and A. A. Salama, Fundamentals of Neutrosophical Simulation for [16]

Generating Random Numbers Associated with Uniform Probability Distribution, Neutrosophic Sets and Systems, Vol. 49, 2022, pp. 92-102 DOI: 10.5281/zenodo.6426375

Maissam Jdid, A. Salama, Using the Inverse Transformation Method to Generate Random Variables [17] that follow the Neutrosophic Uniform Probability Distribution, Doi: https://doi.org/10.54216/JNFS.060202

Maissam Jdid, Rafif Alhabib and A. A. Salama, The Basics of Neutrosophic Simulation for Converting [18] Random Numbers Associated with a Uniform Probability Distribution into Random Variables Follow an Exponential Distribution, Neutrosophic Sets and Systems, Vol. 53, 2023, pp. 358-366. DOI: 10.5281/zenodo.7536049

[19] Maissam Jdid, Said Broumi, Neutrosophical Rejection and Acceptance Method for the Generation of Random Variables, Neutrosophic Sets and Systems, Vol. 56, 2023, pp. 153-166. DOI: 10.5281/zenodo.8194749 [20] Maissam Jdid and Nada A. Nabeeh, Generating Random Variables that follow the Beta Distribution Using the Neutrosophic Acceptance-Rejection Method, Neutrosophic Sets and Systems, Vol. 58, 2023, pp. 139-148. DOI: 10.5281/zenodo.8404445

[21] Maissam Jdid, Florentin Smarandache, and Khalifa Al Shaqsi, Generating Neutrosophic Random Variables Based Gamma Distribution, Plithogenic Logic and Computation Vol. 1 (2024), 16-24, DOI: https://doi.org/10.61356/j.plc.2024.18760, https://sciencesforce.com/index.php/plc/article/view/87

[22] Maissam Jdid, Florentin Smarandache, Generating Neutrosophic Random Variables Following the Poisson Distribution Using the Composition Method (The Mixed Method of Inverse Transformation Method and Rejection Method), Neutrosophic Sets and Systems, Vol. 64, 2024, pp. 132-140. DOI: 10.5281/zenodo.10703089

Maissam Jdid, Florentin Smarandache, The Box and Muller Technique for Generating Neutrosophic [23] Random Variables Follow a Normal Distribution, Volume 23, Issue 4, Doi: https://doi.org/10.54216/IJNS.230406

Alali. Ibrahim Muhammad, Operations Research. Tishreen University Publications, 2004. [24]

Bukajh JS -. Mualla, W... and others-Operations Research Book translated into Arabic - The Arab [25] Center for Arabization, Translation, Authoring and Publishing -Damascus ,1998.