Econometric Analysis on Efficiency of Estimator

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ABSTRACT

This paper investigates the efficiency of an alternative to ratio estimator under the super population model with uncorrelated errors and a gamma-distributed auxiliary variable. Comparisons with usual ratio and unbiased estimators are also made.

Key words: Bias, Mean Square Error, Ratio Estimator Super Population.

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1. INTRODUCTION

It is well known that the ratio method of estimation occupies an important place in sample surveys. When the study variate y and the auxiliary variate x is positively (high) correlated, the ratio method of estimation is quite effective in estimating the population mean of the study variate y utilizing the information on auxiliary variate x.

Consider a finite population with N units and let x_i and y_i denote the values for two positively correlated variates x and y respectively for the ith unit in this population, i=1,2,…,N. Assume that the population mean $\overline{X}$ of x is known. Let $\bar{x}$ and $\bar{y}$ be the sample means of x and y respectively based on a simple random sample of size n (n < N) units drawn without replacement scheme. Then the classical ratio estimator for $\bar{Y}$ is defined by

$$\bar{y}_r = \bar{y}(\bar{X} / \bar{x}) \qquad (1.1)$$

The bias and mean square error (MSE) of $\bar{y}_r$ are, up to second order moments,

$$B(\bar{y}_r) = \lambda\left(R S^2{}_x - S_{yx}\right)/\bar{X} \qquad (1.2)$$

$$M(\bar{y}_r) = \lambda\left(S_y^2 + R^2 S_x^2 - 2R S_{yx}\right), \qquad (1.3)$$

where $\lambda = (N-n)/(nN)$,

$R = \bar{Y}/\bar{X}$, $S_y^2 = (N-1)^{-1}\sum_{i=1}^{N}\left(y_i - \bar{Y}\right)^2$, $s_x^2 = (N-1)^{-1} \sum_{i=1}^{N} (x_i - \bar{X})^2$,

and $S_{yx} = (N-1)^{-1} \sum_{i=1}^{N} (y_i - \bar{Y})(x_i - \bar{X})$.

It is clear from (1.3) that $M(\bar{y}_r)$ will be minimum when

$$R = S_{yx}/S_x^2 = \beta, \qquad (1.4)$$

where β is the regression coefficient of y on x. Also for R = β,

the bias of $\bar{y}_r$ in (1.2) is zero. That is, $\bar{y}_r$ is almost unbiased for $\bar{Y}$.

Let E ($\bar{y}|\bar{x}$) $= \alpha + \beta\ \bar{x}$ be the line of regression of $\bar{y}$ on $\bar{x}$, where E denotes averaging over all possible sample design simple random sampling without replacement (SRSWOR).Then $\beta = S_{yx}/S_x^2$ and $\bar{Y} = \alpha + \beta\ \bar{X}$ so that, in general ,

$$R = (\alpha / \bar{X}) + \beta \qquad (1.5)$$

It is obvious from (1.4) and (1.5) that any transformation that brings the ratio of population means closer to β will be helpful in reducing the mean square error (MSE) as well as the bias of the ratio estimator $\bar{y}_r$. This led Srivenkataramana and Tracy (1986) to suggest an alternative to ratio estimator $\bar{y}_r$ as

$$\bar{y}_a = \bar{z}\left(\bar{X} / \bar{x}\right) + A = \bar{y}_r - A\left\{\left(\bar{X} / \bar{x}\right) - 1\right\} \qquad (1.6)$$

which is based on the transformation

$$\bar{z} = \bar{y} - A, \qquad (1.7)$$

where $E(\bar{z}) = \bar{Z} (= \bar{Y} - A)$ and A is a suitably chosen scalar.

In this paper exact expressions of bias and MSE of $\bar{y}_a$ are worked out under a super population model and compared with the usual ratio estimator.

2. THE SUPER POPULATION MODEL

Following Durbin (1959) and Rao (1968) it is assumed that the finite population under consideration is itself a random sample from a super population and the relation between x and y is of the form:

$$y_i = \alpha + \beta \, x_i + u_i \; ; \quad (i = 1,2,\ldots,N)$$

where α and β are unknown real constants; u_i's are uncorrelated random errors with conditional (given x_i) expectations

$$E\left(u_i \middle| x_i\right) = 0$$

$$E\left(u_i^2 \middle| x_i\right) = \delta \, x_i^g$$

(i=1,2,….,N), $o \langle \delta \langle \infty$, $o \le g \le 2$ and x_i are independently identically

distributed (i.i.d.) with a common gamma density

$$G(\theta) = e^{-x} x^{\theta-1} / \Gamma\theta \, , \; x \rangle o, \; 2 \langle \theta \langle \infty \quad . \qquad (2.1)$$

We will write E_x to denote expectation operator with respect to the common distribution of x_i (i=1,2,3,…,N) and E_x E_c, as the over all expectation operator for the model. We denote a design by p and the design expectation E_p, for instance, see Chaudhuri and Adhikary (1983,89) and Shah and Gupta (1987). Let 's' denote a simple random sample of N distinict labels chosen without replacement out of i=1,2,3……N. Then

$$X(=N \; \bar{X}) = \sum_{i \in s} x_i + \sum_{i \notin s} x_i$$

Following Rao and Webster (1966) we will utilize the distributional properties of x_j / x_i , $\sum_{i\in s} x_i$, $\sum_{i\notin s} x_i$, $\sum_{i\in s} x_i \ / \ \sum_{i\notin s} x_i$ in our subsequent derivations.

3. THE BIAS AND MEAN SQUARE ERROR

The estimator $\bar{y}_a$ in (1.6) can be written as

$$\bar{y}_a = \left[(1/n)\left(\sum_{i\in s} y_i\right) \frac{\left(n\sum_{i=1}^{N} x_i\right)}{\left(N\sum_{i\in s} x_i\right)} - A\left\{ \frac{\left(n\sum_{i=1}^{N} x_i\right)}{\left(N\sum_{i\in s} x_i\right)} - 1 \right\} \right] \tag{3.1}$$

based on a simple random sample of n distinct labels chosen without replacement out of i = 1,2,…,N.

The bias

$$B = E_p \left(\bar{y}_a - \bar{Y} \right) \tag{3.2}$$

of $\bar{y}_a$ has model expectation $E_m(B)$ which works out as follows:

$$E_m \left(B\left(\bar{y}_a \right) \right) = E_p E_x E_c \left[\left\{ \alpha + \beta(1/n)\left(\sum_{i\in s} x_i \right) + \bar{u} \right\} \frac{n\sum_{i=1}^{N} x_i}{n\sum_{i\in s} x_i} \right.$$

$$- \ A \left\{ \frac{n\left(\sum_{i=1}^{N} x_i \right) - 1}{N\left(\sum_{i\in s} x_i \right)} \right\} -$$

$$- \ E_x \ E_c \ (\alpha + \beta \ \bar{x} + \bar{U})$$

$=E_pE_xE_c$

$$\left[\alpha\left(n\sum_{i=1}^{N} x_i / N\sum_{i\in s} x_i \right) + \beta(1/N)\left(\sum_{i=1}^{N} x_i \right) + \left(\sum_{i\in s} u_i \right)\left(\sum_{i=1}^{N} x_i / N\sum_{i\in s} x_i \right) - A\left\{ \left(\left(n\sum_{i=1}^{N} x_i \right) \Big/ \left(N\sum_{i\in s} x_i \right) \right) - 1 \right\} \right]$$

$$- \ E_x \, E_c \ (\alpha + \beta \ \bar{X})$$

$$= \ E_p \, E_x \left[\alpha\left(n\sum_{i=1}^{N} x_i \right) / \left(N\sum_{i\in s} x_i \right) + \beta\bar{X} - A\left\{ \left(n\sum_{i=1}^{N} x_i \right) / \left(N\sum_{i\in s} x_i \right) - 1 \right\} \right] - \alpha - \beta \ E_x \ \left(\bar{X} \right)$$

$$= E_x \left[\alpha(n/N)\left(1+\sum_{i\notin s} x_i / \sum_{i\in s} x_i\right) - A\left\{(n/N)\left(1+\sum_{i\notin s} x_i / \sum_{i\in s} x_i\right) - 1\right\}\right] - \alpha$$

$$= \alpha(n/N)\{1+(N-n)\theta/(n\theta-1)\}$$

$$-A \{(n/N)(1+(N-n)\theta/(n\theta-1)) - 1\} - \alpha$$

$$= \alpha[(n/N-1) + \{n(N-n)\theta/N(n\theta-1)\}]$$

$$-A [-(N-n)/N + \{(N-n)n\theta/N(n\theta-1)\}]$$

$$= (N-n)(\alpha - A)/N(n\theta-1) \qquad (3.3)$$

For SRSWOR sampling scheme , the mean square error

$$M(\bar{y}_a) = E_p (\bar{y}_a - \bar{Y})^2 \qquad (3.4)$$

of $\bar{y}_a$ has the following formula for model expectations

E_m ($M(\bar{y}_a)$) :

$$E_m(M(\bar{y}_a)) = \left[E_m(M(\bar{y}_r)) + (N-n)(Nn\theta + 2N - 2n)(A^2 - 2A\alpha)/N^2(n\theta-1)(n\theta-2)\right] \qquad (3.5)$$

where

$$M(\bar{y}_r) = E_p(\bar{y}_r - \bar{Y})^2 \qquad (3.6)$$

is the MSE of $\bar{y}_r$ under SRSWOR scheme has the model expectation

$$E_m(M(\bar{y}_r)) = \{(N-n)/N^2\}$$
$$\left[\left\{\frac{(Nn\theta+2N-2n)\alpha^2}{(n\theta-1)(n\theta-2)}\right\} + \frac{\delta\left\{(n\theta+g-1)(n\theta+g-2) + n\theta(N\theta - n\theta + 1)\right\}}{(n\theta+g-1)(n\theta+g-2)} \frac{\Gamma(\theta+g)}{\Gamma\theta}\right] \qquad (3.7)$$

$[See, Rao(1968, p.439)]$

Further, we note that for SRSWOR sampling scheme, the bias

$$B(\bar{y}_r) = E_p(\bar{y}_r - \bar{Y}) \quad (3.8)$$

of usual ratio estimator has the model expectation

$$\mathrm{E_m}(B(\bar{y}_r)) = (N-n)\alpha \,/\, (n\theta - 1) \quad (3.9)$$

We note from (3.3) and (3.9) that

$$|E_m(B(\bar{y}_a))| \;\langle\; |E_m\;(B(\bar{y}_r))|$$

if

$$|(\alpha - A)| \;\langle\; |\alpha|$$

or if

$$(\alpha - A)^2 \langle \alpha^2$$

or if

$$o \langle A \langle 2\alpha \quad (3.10)$$

Further we have from (3.5) that

$$\mathrm{E_m}(M(\bar{y}_a)) - E_m(M(\bar{y}_r)) < o$$

if

$$(A^2 - 2A\alpha) < o$$

or if

$$o \langle A \langle 2\alpha \quad (3.11)$$

which is the same as in (3.10).
Thus we state the following theorem:
Theorem 3.1 : The estimator $\bar{y}_a$ is less biased as well as more efficient than usual ratio estimator $\bar{y}_r$ if

$$o \langle A \langle 2\alpha \qquad (\alpha \neq o)$$

i . e . when A lies between o and 2α .
Therefore , when intercept term $\alpha(\neq o)$ in the model (2.1) is sizable , there will be sufficient flexibility in picking A.

It is to be noted that for $\alpha = o, \bar{y}_r$ is unbiased and efficient than $\bar{y}_a$.
The minimization of (3.5) with respect to A leads to

$$A = \alpha = A_{opt} \text{ (say)} \quad (3.12)$$

Substitution of (3.12) in (3.5) yields the minimum value of

$E_m\left(M\left(\bar{y}_a\right)\right) as$

$$\text{min. E}_\text{m}\left(M\left(\bar{y}_a\right)\right)=\frac{(N-1)}{N^2}\frac{\delta\left[(n\theta+g-1)(n\theta+g-2)+n\theta(N\theta-n\theta+1)\right]}{(n\theta+g-1)(n\theta+g-2)}\frac{\Gamma(\theta+g)}{\Gamma\theta} \quad (3.13)$$

which equals to $E_m\left(M\left(\bar{y}_r\right)\right)$ $when \alpha = o.$

It is interesting to note that when A $=\alpha, \bar{y}_a$ is unbiased and attained its minimum average MSE in model (2.1).

In practice the value of α will have to be assessed, at the estimation stage, to be used as A. To assess α, we may use scatter diagram of y versus x for data from a pilot study, or a part of the data from the actual study and judge the y-intercept of the best fitting line.

From (3.7) and (3.13) we have

$$E_m\left(M\left(\bar{y}_r\right)\right)-\min .E_m\left(M\left(\bar{y}_a\right)\right)=\left\{(N-n)(Nn\theta+2N-2n)\alpha^2\right\}/\left\{N^2(n\theta-1)(n\theta-2)\right\}$$
$$\rangle \quad o \quad (3.14)$$

which shows that $\bar{y}_a$ is more efficient than ratio estimator when A $=\alpha$ is known exactly. For $\alpha = o$

$$\text{min.E}_\text{m}\ \left(M\left(\bar{y}_a\right)\right)=E_m\left(M\left(\bar{y}_r\right)\right) \quad (3.15)$$

For SRSWOR , the variance

$$\text{V}\left(\bar{y}\right)=E_p\left(\bar{y}-\bar{Y}\right)^2 \quad (3.16)$$

of usual unbiased estimator has the model expectation:

$$E_m\left(V\left(\bar{y}\right)\right)=(N-n)\left[\beta^2\theta+\left\{\delta\Gamma(\theta+g)/\Gamma\theta\right\}\right]/nN \quad (3.17)$$

The expressions of $E_m\left(M\left(\bar{y}_a\right)\right)$ and $E_m\left(V\left(\bar{y}\right)\right)$ are not easy task to compare algebraically. Therefore in order to facilitate the comparison, denoting

$E_1=100E_m\left(V\left(\bar{y}\right)\right)/E_m\left(M\left(\bar{y}_a\right)\right)$ and $E_2=100E_m\left(V\left(\bar{y}_r\right)\right)/E_m\left(M\left(\bar{y}_a\right)\right)$,

we present below in tables 1,2,3, the values of the relative efficiencies of

$\bar{y}_a$ with respect to $\bar{y}$ and $\bar{y}_r$ for a few combination of the parametric values under the model (2.1). Values are given for N = 60 , $\delta=2.0, \theta=8, \alpha=0.5$, 1.0, 1.5, $\beta=0.5, 1.0, 1.5$ and g = 0.0, 0.5,1.0,1.5,2.0.

The ranges of A, for $\bar{y}_a$ to be better than $\bar{y}_r$ for given $\alpha = 0.5, 1.0, 1.5$ are respectively (0,1), (0,2), (0,3). This clearly indicates that as the size of α increases the range of A for $\bar{y}_a$ to be better than $\bar{y}_r$ increases i.e. flexibility of choosing A increases.

We have made the following observations from the tables 1,2 and 3 :

(i) As g increases both E_1 and E_2 decrease. When n increases E_1 increases while E_2 decreases.

(ii) As α increases (i.e. if the intercept term α departs from origin in positive direction) relative efficiency of $\bar{y}_a$ with respect to $\bar{y}$ decreases while E_2 increases.

(iii) As β increases E_1 increases for fixed g while E_2 is unaffected.

(iv) The maximum gain in efficiency is observed over $\bar{y}$ as well as over $\bar{y}_r$ if A coincide with the value of α. Finally, the estimator $\bar{y}_a$ is to be preferred when the intercept term α departs substantially from origin.

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Table 1: Relative efficiencies of $\bar{y}_a$ with respect to $\bar{y}$ and $\bar{y}_\Gamma$

$\alpha = 0.5$							
g	β	n = 10					
		E_1			E_2		
		A			A		
		0.30	0.60	0.90	0.30	0.60	0.90
	0.5	192.86	193.23	191.40	101.34	101.54	100.57
0.0	1.0	482.16	483.16	478.09	101.34	101.54	100.57
	1.5	964.32	966.17	956.98	101.34	101.54	100.57
	0.5	132.67	132.77	132.30	100.49	100.56	100.21
0.5	1.0	237.82	237.99	237.16	100.49	100.56	100.21
	1.5	413.08	413.36	411.93	100.49	100.56	100.21
	0.5	111.06	111.08	110.95	10.17	100.19	100.07
1.0	1.0	148.08	148.11	147.93	10.17	100.19	100.07
	1.5	209.78	209.83	209.57	10.17	100.19	100.07
	0.5	103.99	104.00	103.96	100.06	100.07	100.03
1.5	1.0	116.64	116.65	116.60	100.06	100.07	100.03
	1.5	137.71	137.72	137.66	100.06	100.07	100.03
	0.5	102.23	102.23	102.22	100.02	100.02	100.01
2.0	1.0	106.43	106.43	106.42	100.02	100.02	100.01
	1.5	113.43	113.43	113.42	100.02	100.02	100.01

$\alpha = 0.5$							
g	β	n = 20					
		E_1			E_2		
		A			A		
		0.30	0.60	0.90	0.30	0.60	0.90
	0.5	196.58	196.96	195.11	103.33	101.52	100.56
0.0	1.0	491.46	492.39	487.77	103.33	101.52	100.56
	1.5	982.92	984.39	975.53	103.33	101.52	100.56
	0.5	134.37	134.46	134.46	100.48	100.55	100.20
0.5	1.0	240.86	241.02	240.02	100.48	100.55	100.20
	1.5	418.35	418.63	417.20	100.48	100.55	100.20

1.0	0.5	111.76	111.79	111.65	100.17	100.19	100.07
	1.0	149.01	149.05	148.87	100.17	100.19	100.07
	1.5	211.10	211.16	210.90	100.17	100.19	100.07
1.5	0.5	104.00	104.00	103.96	100.06	100.07	100.02
	1.0	116.64	116.65	116.60	100.06	100.07	100.02
	1.5	137.71	137.73	137.67	100.06	100.07	100.02
2.0	0.5	101.60	101.60	101.58	100.02	100.02	100.01
	1.0	105.77	105.77	105.76	100.02	100.02	100.01
	1.5	112.73	112.73	112.73	100.02	100.02	100.01

Table 2: Relative efficiencies of $\bar{y}_a$ with respect to $\bar{y}$ and $\bar{y}_r$

$\alpha = 1.0$									
g	β	n = 10							
		E_1 A				E_2 A			
		0.50	1.0	1.50	1.90	0.50	1.0	1.50	1.90
0.0	0.5	190.31	193.36	190.31	183.82	104.73	106.41	104.73	101.16
	1.0	475.78	483.40	475.78	459.55	104.73	106.41	104.73	101.16
	1.5	951.55	966.79	951.55	919.10	104.73	106.41	104.73	101.16
0.5	0.5	132.03	132.80	132.03	130.34	101.73	102.32	101.73	100.43
	1.0	236.67	238.05	236.67	233.65	101.73	102.32	101.73	100.43
	1.5	411.07	413.46	411.07	405.82	101.73	102.32	101.73	100.43
1.0	0.5	110.87	111.09	110.87	110.36	100.61	100.82	100.61	100.15
	1.0	147.82	148.12	147.82	147.15	100.61	100.82	100.61	100.15
	1.5	209.42	209.84	209.42	208.46	100.61	100.82	100.61	100.15
1.5	0.5	103.93	104.00	103.93	103.77	100.21	100.28	100.21	100.05
	1.0	116.57	116.65	116.57	116.39	100.21	100.28	100.21	100.05
	1.5	137.63	137.73	137.63	137.41	100.21	100.28	100.21	100.05
2.0	0.5	102.21	102.23	102.21	102.15	100.67	100.09	100.07	100.01
	1.0	106.41	106.43	106.41	106.3	100.67	100.09	100.07	100.01
	1.5	113.41	113.43	113.41	113.35	100.67	100.09	100.07	100.01

$\alpha = 1.0$									
g	β	n = 20							
		E_1 A				E_2 A			
		0.50	1.0	1.50	1.90	0.50	1.0	1.50	1.90
0.0	0.5	194.01	197.08	194.01	187.47	104.67	106.33	104.67	101.14
	1.0	485.03	492.70	485.03	468.68	104.67	106.33	104.67	101.14
	1.5	970.06	985.40	970.06	937.36	104.67	106.33	104.67	101.14
0.5	0.5	133.73	134.49	133.73	132.05	101.70	102.28	101.70	100.08
	1.0	239.71	241.08	239.71	236.71	101.70	102.28	101.70	100.08
	1.5	416.35	418.73	416.35	411.13	101.70	102.28	101.70	100.08
	0.5	111.07	111.08	111.07	111.08	100.60	100.80	100.60	100.15

1.0	1.0	148.77	149.06	148.77	148.11	100.60	100.80	100.60	100.15
	1.5	210.75	211.17	210.75	209.82	100.60	100.80	100.60	100.15
	0.5	103.94	104.01	103.94	103.78	100.20	100.27	100.20	100.05
1.5	1.0	116.57	116.65	116.57	116.40	100.20	100.27	100.20	100.05
	1.5	137.64	137.73	137.64	137.42	100.20	100.27	100.20	100.05
	0.5	101.58	101.60	101.58	101.52	100.07	100.09	100.07	100.01
2.0	1.0	105.75	105.77	105.75	105.70	100.07	100.09	100.07	100.01
	1.5	112.71	112.73	112.71	112.65	100.07	100.09	100.07	100.01

Table 3: Relative efficiencies of $\bar{y}_a$ with respect to $\bar{y}$ and $\bar{y}_r$

$\alpha = 1.5$											
g	β	n = 10									
		E_1 A					E_2 A				
		0.60	1.20	1.80	2.40	2.90	0.60	1.20	1.80	2.40	2.90
	0.5	183.82	192.25	192.25	183.82	171.79	108.77	113.76	113.76	108.77	101.65
0.0	1.0	459.55	480.62	480.62	459.55	429.47	108.77	113.76	113.76	108.77	101.65
	1.5	919.10	961.25	961.25	919.10	858.94	108.77	113.76	113.76	108.77	101.65
	0.5	130.34	132.52	132.52	130.34	127.01	103.29	105.01	105.01	103.29	100.64
0.5	1.0	233.64	237.55	237.55	233.65	227.67	103.29	105.01	105.01	103.29	100.64
	1.5	405.82	412.60	412.60	405.82	395.44	103.29	105.01	105.01	103.29	100.64
	0.5	110.36	111.01	111.01	110.36	109.34	101.17	101.77	101.77	101.17	100.23
1.0	1.0	147.15	148.02	148.02	147.15	147.79	101.17	101.77	101.77	101.17	100.23
	1.5	208.46	209.69	209.69	208.46	206.53	101.17	101.77	101.77	101.17	100.23
	0.5	103.77	103.98	103.98	103.77	103.44	100.40	100.60	100.60	100.40	100.08
1.5	1.0	116.39	116.62	116.62	116.39	116.01	100.40	100.60	100.60	100.40	100.08
	1.5	137.41	137.69	137.69	137.41	139.68	100.40	100.60	100.60	100.40	100.08
	0.5	102.15	102.22	102.22	102.15	102.04	100.13	100.20	100.20	100.13	100.03
2.0	1.0	106.35	106.42	106.42	106.35	106.24	100.13	100.20	100.20	100.13	100.03
	1.5	113.35	113.42	113.42	113.35	113.23	100.13	100.20	100.20	100.13	100.03

$\alpha = 1.5$											
G	β	n = 20									
		E_1 A					E_2				
		0.60	1.20	1.80	2.40	2.90	0.60	1.20	1.80	2.40	2.90
	0.5	187.47	196.97	195.97	187.47	175.33	108.67	113.59	113.59	108.67	101.63
0.0	1.0	468.68	489.91	489.91	468.68	438.34	108.67	113.59	113.59	108.67	101.63
	1.5	937.36	979.83	979.83	937.36	876.67	108.67	113.59	113.59	108.67	101.63
	0.5	132.05	134.21	134.21	132.05	128.73	103.23	104.92	104.92	103.23	100.63
0.5	1.0	236.70	240.58	240.58	236.70	230.76	103.23	104.92	104.92	103.23	100.63
	1.5	411.13	417.87	417.87	411.13	400.80	103.23	104.92	104.92	103.23	100.63

1.0	0.5	111.08	111.72	111.72	111.08	110.08	101.14	101.72	101.72	101.14	100.23
	1.0	148.11	148.96	148.96	148.11	146.77	101.14	101.72	101.72	101.14	100.23
	1.5	209.82	211.02	211.02	209.82	207.92	101.14	101.72	101.72	101.14	100.23
1.5	0.5	103.78	103.98	103.98	103.78	103.46	100.39	100.58	100.58	100.39	100.08
	1.0	116.40	116.62	116.62	116.40	116.40	100.39	100.58	100.58	100.39	100.08
	1.5	137.43	137.70	137.70	137.43	137.00	100.39	100.58	100.58	100.39	100.08
2.0	0.5	101.53	101.59	101.59	101.53	101.42	100.13	100.19	100.19	100.03	100.03
	1.0	105.70	105.77	105.77	105.70	105.59	100.13	100.19	100.19	100.03	100.03
	1.5	112.65	112.72	112.72	112.65	112.54	100.13	100.19	100.19	100.03	100.03