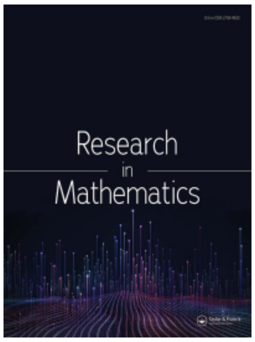

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RESEARCH ARTICLE

Estimation of population mean using generalized neutrosophic exponential estimators

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ABSTRACT
In classical statistics, the data we examine is precise and determinate, resulting in a specific, single value. However, when the data are unclear, ambiguous or in the form of an interval, such as observing daily stock prices or daily temperatures in a city, we cannot rely on classical statistics. In such situations, neutrosophic statistics are far more dependable. In this article, we propose three generalized neutrosophic exponential estimators for the population mean using neutrosophic subsidiary information. The expression for the bias and mean square error of the suggested estimators is computed using a first-order approximation. Then to demonstrate the properties of the suggested neutrosophic estimators, real life neutrosophic data sets based on product sales and marketing in the field of medical are utilized. Additionally, we conducted a simulation study, demonstrating that our proposed estimators outperform the estimators currently described in this article.

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Mean square error (MSE); neutrosophic estimators; percentage relative efficiency (PRE); simulation; subsidiary information

1. Introduction

Within the realm of sampling theory, the fundamental objective of the researchers is to develop estimation methods that can estimate the parameters of the population. This will ultimately lead to the achievement of increased efficiency while minimizing the number of sampling mistakes. It is possible to improve the efficiency of the estimators by enhancing the sample methodology, increasing the amount of the sampling, or making use of the ancillary information that is accessible. There is a known variable that is approximately connected to the variable of interest (study variable), and this known variable is referred to as auxiliary information or subsidiary information. For example, if we know that a forest has N trees and we want to calculate the volume of the trees, we may utilise the diameter of the trees as an auxiliary variable. This approach allows us to get the volume of the trees.

Cochran (1940) was the pioneering researcher to harness auxiliary information for augmenting the accuracy of the ratio estimation method. A ratio estimator is employed when a positive correlation exists between the variable under study and the auxiliary variable. Murthy (1964) introduced the product method of estimation, which acts as a counterpart to the ratio estimator. The product method is applicable when there is a negative correlation between the study variable and the auxiliary variable. Regression-based estimation methods are relevant in both situations. Bahl & Tuteja (1991) advanced an exponential estimation technique by incorporating an exponential function of a secondary variable. Leveraging auxiliary information, Sharma et al. (2013) proposed exponential ratio-product type estimators using second-order approximations. Utilizing two auxiliary variables, Singh & Sharma (2015) introduced a class of exponential ratio estimators for estimating the finite population mean. Singh et al. (2016) proposed exponential-type imputation methods for estimating the population mean when data values are missing completely at random.

In traditional statistics, the data we examine is generally precise, deterministic, and leads to a specific point value. However, when the data is vague, ambiguous, or presented in the form of intervals, conventional statistical methods may prove inadequate. Examples of such scenarios include observing daily stock prices or the temperatures of a city. In these instances, fuzzy statistics offers a more reliable alternative to classical methods. The concept of fuzzy statistics was pioneered by (Zadeh, 1965). A notable drawback of fuzzy statistics is its inability to quantify the degree of indeterminacy in the data. To address this gap, Smarandache (1998) introduced the notion of Neutrosophic statistics. Neutrosophic statistics extends both classical and fuzzy statistics, providing a framework for handling indeterminate data (Vishwakarma & Singh, 2022).

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In recent years, numerous scholars have made significant contributions to the field of neutrosophic statistics. Aslam (2018) developed a novel sampling plan for the process loss function based on the neutrosophic approach. In Aslam (2019), the concept of neutrosophic analysis of variance (NANOVA) was introduced for hypothesis testing of means. Aslam (2021a) designed a control chart for the neutrosophic exponentially weighted moving average (NEWMA) using repetitive sampling, which has proven effective in controlling road traffic crashes (RTC). In Aslam (2021b), solar energy data, represented as intervals, was analyzed by proposing a new Anderson-Darling test within the framework of neutrosophic statistics. Tahir et al. (2021) proposed neutrosophic ratio-type estimators for estimating the population mean using auxiliary variables. Vishwakarma & Singh (2022) introduced a neutrosophic ranked set sampling (NeRSS) method and a generalized estimator for population mean estimation. Kumar et al. (2022) developed a neutrosophic exponential-type estimator for estimating the population mean amidst uncertainty, utilizing neutrosophic study and auxiliary variables. Alomair & Shahzad (2023) proposed neutrosophic Hartley-Ross-type ratio estimators for population mean estimation, even in the presence of outliers and both sensitive and non-sensitive study variables. Singh et al. (2023) introduced neutrosophic regression cum ratio-type estimators for population mean estimation under uncertain conditions, with applications in medical science. To improve population mean estimation, S. K. Yadav & Smarandache (2023), developed a generalized neutrosophic sampling strategy. Yadav & Prasad (2023) devised a factor-type exponential neutrosophic estimator for two-phase survey sampling, while Yadav & Prasad (2024) proposed novel neutrosophic factor-type exponential estimators that leverage auxiliary information to enhance precision. Utilizing auxiliary information in neutrosophic ranked set sampling, Singh & Kumari (2024) introduced neutrosophic estimators to tackle the challenges of estimating population means in neutrosophic data, incorporating demographic data in their analysis. Under neutrosophic stratified sampling, Singh, Patkar, et al. (), proposed a range of product, ratio, and exponential estimators, applying actual climatic data to assess their statistical properties. Additionally, Singh, Kulkarni, et al. (2024) created new regression and ratio estimators under neutrosophic stratified sampling, employing real climate data to analyze the characteristics of the suggested estimators. Singh & Gupta (2025) proposed an innovative estimator combining neutrosophic ratio-cum-product exponential estimators with two auxiliary variables to improve population mean estimation. Furthermore Singh et al. (2025), introduced a neutrosophic almost unbiased estimator, utilizing known neutrosophic auxiliary parameters for estimating the neutrosophic population mean of the primary variable.

In this study, we present three novel generalized neutrosophic estimators. The first two estimators merge regression and exponential methods, resulting in a modified difference-cum-exponential neutrosophic estimator. The third estimator is a modified ratio-cum-exponential estimator. We compare the efficacy of our proposed estimators against existing ones. In Section 4, we explore the properties of our proposed estimators, and in Section 5, we evaluate their performance against established neutrosophic estimators-such as the neutrosophic simple mean estimator, neutrosophic ratio estimator, neutrosophic modified ratio estimator, neutrosophic exponential estimator, neutrosophic improved exponential ratio estimator, and neutrosophic generalized exponential ratio estimator-using both real data sets and simulation studies. The results of our simulations and empirical analysis reveal that the difference-cum-exponential-type neutrosophic estimators and the modified ratio-cum-exponential estimator outperform the current neutrosophic exponential estimators.

2. Notation

In neutrosophic statistics the random variable Z_N is defined as $X_L + Z_L I_N$, where Z_L denotes the determinate part and $Z_L I_L$ denotes the indeterminate part, with $I_N \in [I_L, I_U]$ represents the indeterminate. Aslam (2024). Let us consider a finite population $T_1, T_2, \ldots, T_N$ of size N, from which a neutrosophic random sample of size $n_N \in [n_L, n_U]$ is drawn by the neutrosophic simple random sampling without replacement method. Let $y_N(i)$ is the i_{th} sample observation of neutrosophic data, which is of the form $y_N(i) \in [y_L, y_U]$ and similarly for subsidiary variable $x_N(i)$. Let $\bar{Y}_N(i) \in [\bar{Y}_L, \bar{Y}_U]$ and $\bar{X}_N(i) \in [\bar{X}_L, \bar{X}_U]$ be the population mean of the study variable and subsidiary variable, respectively. Also, let $C_{yN} \in [C_{yL}, C_{yU}]$ and $\rho_{xyN} \in [\rho_{xyL}, \rho_{xyU}]$ are the neutrosophic coefficient of variation and neutrosophic correlation between Y_N and X_N. Moreover, the parameter $\beta_{2(x)N} \in [\beta_{2(x)L}, \beta_{2(x)U}]$ is the neutrosophic subsidiary variable X_N. Let $\bar{e}_{yN} \in [\bar{e}_{yL}, \bar{e}_{yU}]$ and $\bar{e}_{xN} \in [\bar{e}_{xL}, \bar{e}_{xU}]$ are the neutrosophic mean error, where,

$$\bar{e}_{yN} = \bar{y}_{N(i)} - \bar{Y}_N \tag{1}$$

$$\bar{e}_{xN} = \bar{x}_{N(i)} - \bar{X}_N \tag{2}$$

such that

$$E(\bar{e}_{yN}) = E(\bar{e}_{xN}) = 0 \tag{3}$$

$$E(\bar{e}_{yN}^2) = \theta_N \bar{Y}_N^2 C_{yN}^2 \tag{4}$$

$$E(\bar{e}_{xN}^2) = \theta_N \bar{X}_N^2 C_{xN}^2 \tag{5}$$

$$E(\bar{e}_{yN}\bar{e}_{xN}) = \theta_N \bar{X}_N \bar{Y}_N C_{yN} C_{xN} \rho_{xyN} \tag{6}$$

where,
$\theta_N = (\frac{1}{n_N} - \frac{1}{N_N})$, $\theta_N \in [\theta_L, \theta_U]$, $\bar{e}_{yN} \in [\bar{e}_{yL}, \bar{e}_{yU}]$, $\bar{e}_{yN}\bar{e}_{xN} \in [\bar{e}_{yL}\bar{e}_{xL}, \bar{e}_{yU}\bar{e}_{xU}]$, $\bar{e}_{yN}^2 \in [\bar{e}_{yL}^2, \bar{e}_{yU}^2]$, $\bar{e}_{xN}^2 \in [\bar{e}_{xL}^2, \bar{e}_{xU}^2]$, $C_{yN}^2 = \frac{\sigma_{yN}^2}{\bar{Y}_N^2}$, $C_{yN}^2 \in [C_{yL}^2, C_{yU}^2]$, $\sigma_{yN}^2 \in [\sigma_{yL}^2, \sigma_{yU}^2]$, $C_{xN}^2 = \frac{\sigma_{xN}^2}{\bar{X}_N^2}$, $C_{xN}^2 \in [C_{xL}^2, C_{xU}^2]$, $\sigma_{xN}^2 \in [\sigma_{xL}^2, \sigma_{xU}^2]$, $\rho_{xyN} = \frac{\sigma_{xyN}}{\sigma_{xN}\sigma_{yN}}$, $\rho_{xyN} \in [\rho_{xyL}, \rho_{yxU}]$, $\sigma_{xyN} \in [\sigma_{xyL}, \sigma_{xyU}]$.

To evaluate the performance of our proposed estimators, we utilized following existing neutrosophic estimators :

3. Existing estimators

3.1. Neutrosophic simple mean estimator

The Neutrosophic simple mean is $\bar{y}_{nN}$, where $\bar{y}_{nN} \in (\bar{y}_{nL}, \bar{y}_{nU})$ and variance of neutrosophic simple mean estimator is given as

$$V(\bar{y}_N) = f_N \bar{Y}_N^2 C_{yN}^2$$

3.2. Neutrosophic ratio estimator

In the presence of subsidiary information and when the data have indeterminacy (Tahir et al., 2021) introduced a Neutrosophic ratio estimator for estimating the mean of a finite population, which is defined as follows:

$$\bar{y}_{rN} = \frac{\bar{y}_N}{\bar{x}_N}\bar{X}_N \tag{7}$$

where $\bar{y}_{rN} \in [\bar{y}_{rL}, \bar{y}_{rU}]$.

The bias and MSE of $\bar{y}_{rN}$ up to first order of approximation are given by

$$Bias(\bar{y}_{rN}) = \theta_N \bar{Y}_N [C_{xN}^2 - C_{xN} C_{yN} \rho_{xyN}] \tag{8}$$

$$MSE(\bar{y}_{rN}) = \theta_N \bar{Y}_N^2 [C_{yN}^2 + C_{xN}^2 - 2C_{xN} C_{yN} \rho_{xyN}] \tag{9}$$

where,
$Bias(\bar{y}_{rN}) \in [Bias(\bar{y}_{rL}), Bias(\bar{y}_{rU})]$; $MSE(\bar{y}_{rN}) \in [MSE(\bar{y}_{rL}), MSE(\bar{y}_{rU})]$.

3.3. Neutrosophic modified ratio type estimator

Using coefficient of variation as a subsidiary variable (Tahir et al., 2021) proposed a neutrosophic modified ratio type estimator, given as

$$\bar{y}_{MrN} = \bar{y}_N \left[\frac{\bar{X}_N + C_{xN}}{\bar{x}_N + C_{xN}}\right] \tag{10}$$

where, $\bar{y}_{MrN} \in [\bar{y}_{MrL}, \bar{y}_{MrL}]$.

Bias and MSE of $\bar{y}_{MrN}$ up to the first order of approximation are given as

$$Bias(\bar{y}_{MrN}) = \theta_N \bar{Y}_N \left[\left(\frac{\bar{X}_N}{\bar{X}_N + C_{xN}}\right)^2 C_{xN}^2 - \left(\frac{\bar{X}_N}{\bar{X}_N + C_{xN}}\right) C_{xN} C_{yN} \rho_{xyN}\right] \tag{11}$$

$$MSE(\bar{y}_{MrN}) = \theta_N \bar{Y}_N^2 \left[C_{yN}^2 + \left(\frac{\bar{X}_N}{\bar{X}_N + C_{xN}}\right)^2 C_{xN}^2 - 2\left(\frac{\bar{X}_N}{\bar{X}_N + C_{xN}}\right) C_{xN} C_{yN} \rho_{xyN}\right] \tag{12}$$

where, $Bias(\bar{y}_{MrN}) \in [Bias(\bar{y}_{MrL}), Bias(\bar{y}_{MrU})]$ and $MSE(\bar{y}_{MrN}) \in [MSE(\bar{y}_{MrL}), MSE(\bar{y}_{MrU})]$

3.4. Neutrosophic exponential type estimators

A neutrosophic exponential type estimator for estimating population mean suggested by Tahir et al. (2021) is as follows:

$$\bar{y}_{expN} = \bar{y}_N exp\left(\frac{\bar{X}_N - \bar{x}_N}{\bar{X}_N + \bar{x}_N}\right) \tag{13}$$

where, $\bar{y}_{expN} \in [\bar{y}_{expL}, \bar{y}_{expU}]$.

The bias and MSE of $\bar{y}_{expN}$ up to first-order of approximation are given by

$$Bias(\bar{y}_{expN}) = \theta_N \bar{Y}_N \left[\frac{3}{8}C_{xN}^2 - \frac{1}{2}C_{xN} C_{yN} \rho_{xyN}\right] \tag{14}$$

$$MSE(\bar{y}_{expN}) = \theta_N \bar{Y}_N^2 \left[C_{yN}^2 + \frac{1}{4}C_{xN}^2 - C_{xN} C_{yN} \rho_{xyN}\right] \tag{15}$$

where,

$Bias(\bar{y}_{expN}) \in [Bias(\bar{y}_{expL}), Bias(\bar{y}_{expU})]$; $MSE(\bar{y}_{expN}) \in [MSE(\bar{y}_{expL}), MSE(\bar{y}_{expU})]$

3.5. Neutrosophic improved exponential ratio type estimator

A Neutrosophic improved exponential ratio-type estimator developed by (Tahir et al., 2021), given as follows:

$$\bar{y}_{IexpN} = \bar{y}_N exp\left[\frac{(l\bar{X}_N + m) - (l\bar{x}_N + m)}{(l\bar{X}_N + m) + (l\bar{x}_N + m)}\right] \tag{16}$$

where $\bar{y}_{IexpN} \in [\bar{y}_{Iexp_L}, \bar{y}_{Iexp_U}]$ and l & $m(-\infty < l, m < \infty)$ are two real constant and supposed to be estimated.

The bias and MSE of $\bar{y}_{IexpN}$ up to first-order of approximation are given by

$$Bias(\bar{y}_{IexpN}) = \theta_N \bar{Y}_N \left[\frac{3}{8}\left(\frac{\bar{X}_N}{l\bar{X}_N + m}\right)^2 C_{xN}^2 - \frac{1}{2}\left(\frac{\bar{X}_N}{l\bar{X}_N + m}\right) C_{xN} C_{yN} \rho_{xyN}\right] \tag{17}$$

$$MSE(\bar{y}_{IexpN}) = \theta_N \bar{Y}_N^2 \left[C_N^2 + \frac{3}{8}\left(\frac{l\bar{X}_N}{2(l\bar{X}_N + m)}\right)^2 C_{xN}^2 - \frac{1}{2}\left(\frac{2l\bar{X}_N}{2(l\bar{X}_N + m)}\right) C_{xN} C_{yN} \rho_{xyN}\right] \tag{18}$$

where,

$Bias(\bar{y}_{IexpN}) \in [Bias(\bar{y}_{IexpL}), Bias(\bar{y}_{IexpU})]$; $MSE(\bar{y}_{IexpN}) \in [MSE(\bar{y}_{IexpL}), MSE(\bar{y}_{IexpU})]$

3.6. Neutrosophic generalized exponential ratio type estimator

a Neutrosophic generalized exponential ratio-type estimator developed by Tahir et al. (2021), given as follows:

$$\bar{y}_{GexpN} = \bar{y}_N exp\left[\eta\left(\frac{\bar{X}_N^{1/g} - \bar{x}_N^{1/g}}{\bar{X}_N^{1/g} + (a-1)\bar{x}_N^{1/g}}\right)\right] \tag{19}$$

where, $\bar{y}_{GexpN} \in [\bar{y}_{GexpL}, \bar{y}_{GexpU}]$, $\eta(-\infty < \eta < \infty)$ and $g(g > 0)$are two real constants and assumed to be known, and the other constant $a(a > 0)$ is supposed to be estimated.

The bias and MSE of $\bar{y}_{GexpN}$ up to first-order of approximation are given by

$$Bias(\bar{y}_{GexpN}) = \theta_N \bar{Y}_N \left[\frac{\eta C_{xN}^2}{ag^2} - \frac{\eta C_{xN}^2}{a^2 g^2} + \frac{\eta^2 C_{xN}^2}{2a^2 g^2} - \frac{\eta C_{xN} C_{yN} \rho_{xy_N}}{ag}\right] \tag{20}$$

$$MSE(\bar{y}_{GexpN}) = \theta_N \bar{Y}_N^2 \left[C_{yN}^2 + \frac{\eta^2 C_{xN}^2}{a^2 g^2} - \frac{2\eta C_{xN} C_{yN} \rho_{xy_N}}{ag}\right] \tag{21}$$

where, $Bias(\bar{y}_{GexpN}) \in [Bias(\bar{y}_{GexpL}), Bias(\bar{y}_{GexpU})]$ and $MSE(\bar{y}_{GexpN}) \in [MSE(\bar{y}_{GexpL}), MSE(\bar{y}_{GexpU})]$

Minimum $MSE(\bar{y}_{GexpN})$ expression, using optimum value of'a' obtained after differentiating Eq. (21) with respect to'a' and setting it to zero is given as:

$$MSE(\bar{y}_{GexpN})_{min} = \theta_N \bar{Y}_N^2 C_{yN}^2 (1 - \rho_{xyN}^2) \tag{22}$$

$MSE(\bar{y}_{GexpN})_{min}$ is obtained for the optimum value of $\hat{a} = \frac{\eta C_{xN}^2}{g C_{xN} \rho_{xyN}}$

Minimum MSE of estimator $\bar{y}_{GexpN}$ is equal to the MSE of the linear regression estimator.

4. Neutrosophic proposed estimators

In this article, we have proposed three Neutrosophic estimators. Among these, first two estimators are the combination of regression and exponential methods of estimation and the third one is modified exponential ratio estimator. The proposed Neutrosophic estimators are as follows:

4.1. Proposed estimator- I

Inspired by the work of Singh & Singh (2017), we have introduced the following neutrosophic generalized exponential estimator to address the indeterminacy, defined as follows:

$$\bar{y}_{P1_N} = [\alpha_1 \bar{y}_N + \alpha_2 (\bar{X}_N - \bar{x}_N)]\, exp\left[\frac{\bar{X}_N^{\star} - \bar{x}_N^{\star}}{\bar{X}_N^{\star} - \bar{x}_N^{\star}}\right] \tag{23}$$

where, $\bar{x}_N^{\star} = l_N \bar{x}_N + m_N, \bar{X}_N^{\star} = l_N \bar{X}_N + m_N$, here $l_N \neq 0$ and m_N are either real numbers or functions of the known parameters of auxiliary variable X_N such as the coefficient of kurtosis $\beta_2(x_N)$, the correlation coefficient ρ_N of the population, the standard deviation S_{x_N}, and the coefficient of variation C_{x_N}. And α_1 and $\alpha_2(-\infty < \alpha_1, \alpha_2 < \infty)$ are two real constants and are supposed to be estimated, and $\bar{y}_{P1_N} \in [\bar{y}_{P1_L}, \bar{y}_{P1_U}]$.

Using approximations given in Equation (1) and (2), Equation (23) is written as:

$$\begin{aligned}
\bar{y}_{P1_N} &= [\alpha_1(\bar{Y}_N + \bar{e}_{yN}) + \alpha_2\{\bar{X}_N - (\bar{X}_N + \bar{e}_{xN})\}] exp[\frac{l_N\bar{X}_N + m_N - l_N(\bar{X}_N + \bar{e}_{xN}) - m_N}{l_N\bar{X}_N + m_N + l_N(\bar{X}_N + \bar{e}_{xN}) + m_N}] \\
\bar{y}_{P1_N} &= [\alpha_1 \bar{Y}_N + \alpha_1 \bar{e}_{yN} - \alpha_2 \bar{e}_{xN}] exp[-\frac{l_N \bar{e}_{xN}}{2(l_N \bar{X}_N + m_N)}\{1 + \frac{l_N \bar{e}_{xN}}{2(l_N \bar{X}_N + m_N)}\}^{-1}] \\
\bar{y}_{P1_N} &= [\alpha_1 \bar{Y}_N + \alpha_1 \bar{e}_{yN} - \alpha_2 \bar{e}_{xN}] exp[-u_N \bar{e}_{xN}\{1 + u_N \bar{e}_{xN}\}^{-1}] \\
\bar{y}_{P1_N} &= \alpha_1[\bar{Y}_N - u_N \bar{Y}_N \overline{e_{x_N}} + \frac{3}{2}\bar{Y}_N u_N^2 \bar{e}_{xN}^2 + \bar{e}_{yN} - u_N \bar{e}_{xN} \bar{e}_{yN}] \\
&\quad -\alpha_2[\bar{e}_{xN} - u_N \bar{e}_{xN}^2]
\end{aligned}$$

where, $u_N = \frac{\mathrm{l}_N}{2(\mathrm{l}_N\bar{X}_N+m_N)}$

$$\bar{y}_{P1_N} - \bar{Y}_N = \alpha_1\left[\bar{Y}_N - u_N\bar{Y}_N\bar{e}_{xN} + \frac{3}{2}\bar{Y}_N u_N^2\bar{e}_{xN}^2 + \bar{e}_{yN} - u_N\bar{e}_{xN}\bar{e}_{yN}\right] - \alpha_2\left[\bar{e}_{xN} - u_N\bar{e}_{xN}^2\right] - \bar{Y}_N \tag{24}$$

Taking expectation on both side of the Equation (24) we get bias of the proposed estimator $\bar{y}_{P1_N}$

$$Bias(\bar{y}_{P1_N}) = E(\bar{y}_{P1_N} - \bar{Y}_N) = \alpha_1\left[\bar{Y}_N + \frac{3}{2}\bar{Y}_N u_N^2\delta_{x_N}^2 - u_N\delta_{xy_N}\right] + \alpha_2\left[u_N\delta_{x_N}^2\right] - \bar{Y}_N \tag{25}$$

where, $Bias(\bar{y}_{P1_N}) \in [Bias(\bar{y}_{P1_L}), Bias(\bar{y}_{P1_U})]$

For MSE, squaring and taking expectation of Equation (24), we have

$$\begin{aligned}MSE(\bar{y}_{P1_N}) = {} & \alpha_1^2\left[\bar{Y}_N^2 + \delta_{y_N}^2 - 4\bar{Y}_N^2u_N^2\delta_{x_N}^2 - 4\bar{Y}_N u_N^2\delta_{xy_N}\right] \\ & + \alpha_2^2\delta_{x_N}^2 + 2\alpha_1\alpha_2\left[2\bar{Y}_N u_N\delta_{x_N}^2 - \delta_{xy_N}\right] \\ & - 2\alpha_1\left[\bar{Y}_N^2 - \bar{Y}_N u_N\delta xy_N + \frac{3}{2}\bar{Y}_N^2u_N^2\delta_{x_N}^2\right] \\ & - \alpha_2 u_N\bar{Y}_N\delta_{x_N}^2 + \bar{Y}_N^2\end{aligned} \tag{26}$$

where,
$\delta_{x_N} = \theta_N\bar{X}_N^2C_{xN}^2$
$\delta_{y_N} = \theta_N\bar{Y}_N^2C_{yN}^2$
$\delta_{xy_N} = \theta_N\bar{X}_N\bar{Y}_N C_{xN}C_{yN}\rho xy_N$

$$\begin{aligned}MSE(\bar{y}_{P1_N}) = {} & \bar{Y}_N^2 + \alpha_1^2A_{1_N} + \alpha_{1_N}^2B_{1_N} \\ & + 2\alpha_1\alpha_2C_{1_N} - 2\alpha_1D_{1_N} - 2\alpha_2E_{1_N}\end{aligned} \tag{27}$$

where,
$A_{1_N} = \bar{Y}_N^2 + \delta_{y_N}^2 - 4\bar{Y}_N^2u_N^2\delta_{x_N}^2 - 4\bar{Y}_N u_N^2\delta_{xy_N}$
$B_{1_N} = \delta_{x_N}^2$
$C_{1_N} = 2\bar{Y}_N u_N\delta_{x_N}^2 - \delta_{xy_N}$
$D_{1_N} = \bar{Y}_N^2 - \bar{Y}_N u_N\delta xy_N + \frac{3}{2}\bar{Y}_N^2u_N^2\delta_{x_N}^2$
$E_{1_N} = \bar{Y}_N\delta_{x_N}^2$

Differentiating Equation (27) with respect to α_1 and α_2 and equating it to zero, we get

$$\alpha_1A_{1_N} + \alpha_2C_{1_N} = D_{1_N} \tag{28}$$

$$\alpha_2B_{1_N} + \alpha_1C_{1_N} = E_{1_N} \tag{29}$$

Solving Equation (28)and (29), we get the value of α_1 and α_2,

$$\alpha_1 = \frac{C_{1_N}E_{1_N} - B_{1_N}D_{1_N}}{C_{1_N}^2 - A_{1_N}B_{1_N}} \tag{30}$$

$$\alpha_2 = \frac{C_{1_N}D_{1_N} - A_{1_N}E_{1_N}}{C_{1_N}^2 - A_{1_N}B_{1_N}} \tag{31}$$

Substituting these value of α_1 and α_2 in Equation (27) we get the minimum MSE of the estimator $\bar{y}_{P1_N}$,

$$MSE(\bar{y}_{P1_N})_{min} = \bar{Y}_N^2 + \frac{B_{1_N}D_{1_N}^2 - 2C_{1_N}D_{1_N}E_{1_N} + A_{1_N}E_{1_N}^2}{C_{1_N}^2 - A_{1_N}B_{1_N}} \tag{32}$$

where, $MSE(\bar{y}_{P1_N})_{min} \in [MSE(\bar{y}_{P1_L})_{min}, MSE(\bar{y}_{P1_U})_{min}]$

Particular case when $\boldsymbol{\alpha_1+\alpha_2=1}$

Taking $\alpha_1+\alpha_2=1$ in equation (23),estimator $\bar{y}_{P1_N}$ can be written as

$$\bar{y}_{P1_N}^{\star} = \left[\alpha_1\bar{y}_N + (1-\alpha_1)(\bar{X}_N - \bar{x}_N)\right] exp\left[\frac{\bar{X}_N^{\star} - \bar{x}_N^{\star}}{\bar{X}_N^{\star} + \bar{x}_N^{\star}}\right] \tag{33}$$

$$\bar{y}_{P1_N}^{\star} = [\alpha_1\bar{y}_N + (1-\alpha_1)(\bar{X}_N - \bar{x}_N)]exp[\frac{\mathrm{l}_N\bar{X}_N+m_N-\mathrm{l}_N(\bar{X}_N+\bar{e}_{xN})-m_N}{\mathrm{l}_N\bar{X}_N+m_N+\mathrm{l}_N(\bar{X}_N+\bar{e}_{xN})+m_N}] \tag{34}$$

Putting $\alpha_2 = 1-\alpha_1$ in Equation (24) and (27), bias and MSE expression of the estimator $\bar{y}_{P1_N}^{\star}$ upto the first order of approximation is obtained as follows:

$$Bias(\bar{y}_{P1_N}^{\star}) = \alpha_1\left[\bar{Y}_N + \frac{3}{2}\bar{Y}_N u_N^2\delta_{x_N}^2 - u_N\delta_{x_yN} - u_N\delta_{x_N}^2\right] + u_N\delta_{x_N}^2 - \bar{Y}_N \tag{35}$$

where, $Bias(\bar{y}_{P1_N}^{\star}) \in [Bias(\bar{y}_{P1_L}^{\star}), Bias(\bar{y}_{P1_U}^{\star})]$
and

$$\begin{aligned}MSE(\bar{y}_{P1_N}^{\star}) = {} & \bar{Y}_N^2 + (B_{1_N} - 2E_{1_N}) + \alpha_1^2(A_{1_N} + B_{1_N} - 2C_{1_N}) \\ & - 2\alpha_1(B_{1_N} - C_{1_N} + D_{1_N} - E_{1_N})\end{aligned} \tag{36}$$

Differentiating equation(36) with respect to α_1 and equating it to zero we get the optimum value of α_1,as

$$\alpha_1 = \frac{(B_{1_N} - C_{1_N} + D_{1_N} - E_{1_N})}{(A_{1_N} + B_{1_N} - 2C_{1_N})} = \alpha_{opt}(say) \tag{37}$$

Then the resulting minimum MSE of estimator $\bar{y}_{P1}^{\star}$ is

$$MSE(\bar{y}_{P1_N})_{min} = \bar{Y}_N^2 + (B_{1_N} - 2E_{1_N}) - \frac{(B_{1_N} - C_{1_N} + D_{1_N} - E_{1_N})^2}{(A_{1_N} + B_{1_N} - 2C_{1_N})} \tag{38}$$

where, $MSE(\bar{y}_{P1_N}^{\star})_{min} \in [MSE(\bar{y}_{P1_L}^{\star})_{min}, MSE(\bar{y}_{P1_U}^{\star})_{min}]$

4.2. Proposed estimator- II

Inspired by the estimation method presented by Shabbir et al. (2014), we have proposed a modified Neutrosophic difference-cum-exponential estimator to handle indeterminacy, defined as follows:

$$\bar{y}_{P2_N} = \left[k_1\bar{y}_{nN} + k_2(\bar{X}_N - \bar{x}_{nN}) + \frac{\bar{y}_N}{2}\left\{exp\left(\frac{\bar{X}_N - \bar{x}_{nN}}{\bar{X}_N + \bar{x}_{nN}}\right) + exp\left(\frac{\bar{x}_{nN} - \bar{X}_N}{\bar{X}_N + \bar{x}_{nN}}\right)\right\}\right] exp\left[\frac{\bar{X}^\star_N - \bar{x}^\star_N}{\bar{X}^\star_N + \bar{x}^\star_N}\right] \tag{39}$$

where, $\bar{x}^\star_N = l_N\bar{x}_N + m_N$, $\bar{X}^\star_N = l_N\bar{X}_N + m_N$ and k_1 and $k_2(-\infty<k_1, k_2<\infty)$ are two real constant and supposed to be estimated and $\bar{y}_{P2_N} \in [\bar{y}_{P2_L}, \bar{y}_{P2_U}]$.

Using approximation given in Equation (1) and (2), Equation (39) is written as:

$$\begin{aligned}\bar{y}_{P2_N} = {} & [k_1(\overline{Y}_N + \bar{e}_{yN}) + k_2\{\overline{X}_N - (\overline{X}_N + \bar{e}_{xN})\} \\ & + (\frac{\overline{Y}_N + \bar{e}_{yN}}{2})\ \{exp(\frac{\overline{X}_N - (\overline{X}_N + \bar{e}_{xN})}{\overline{X}_N + (\overline{X}_N + \bar{e}_{xN})}) \\ & + exp(\frac{(\overline{X}_N + \bar{e}_{xN}) - \overline{X}_N}{\overline{X}_N + (\overline{X}_N + \bar{e}_{xN})})\}] \\ & exp[\frac{\mathrm{l}_N\overline{X}_N + m_N - \mathrm{l}_N(\overline{X}_N + \bar{e}_{xN}) - m_N}{\mathrm{l}_N\overline{X}_N + m_N + \mathrm{l}_N(\overline{X}_N + \bar{e}_{xN}) + m_N}]\end{aligned} \tag{40}$$

or

$$\begin{aligned}\bar{y}_{P2_N} = {} & [k_1(\overline{Y}_N + \bar{e}_{yN}) - k_2\bar{e}_{xN} + \frac{\overline{Y}_N + \bar{e}_{yN}}{2}\{2 + \frac{\bar{e}^2_{xN}}{4\overline{X}^2_N}\}] \\ & exp[-\frac{\mathrm{l}_N\bar{e}_{xN}}{2(\mathrm{l}_N\overline{X}_N + m_N)}\{1 + \frac{\mathrm{l}_N\bar{e}_{xN}}{2(\mathrm{l}_N\overline{X}_N + m_N)}\}^{-1}] \\ \bar{y}_{P2_N} = {} & [k_1(\overline{Y}_N + \bar{e}_{yN}) - k_2\bar{e}_{xN} + \frac{\overline{Y}_N + \bar{e}_{yN}}{2}\{2 + \frac{\bar{e}^2_{xN}}{4\overline{X}^2_N}\}]exp[-u_N\bar{e}_{xN}\{1 + u_N\bar{e}_{xN}\}^{-1}] \\ \bar{y}_{P2_N} = {} & [k_1(\overline{Y}_N + \bar{e}_{yN}) - k_2\bar{e}_{xN} + \frac{\overline{Y}_N + \bar{e}_{yN}}{2}\{2 + \frac{\bar{e}^2_{xN}}{4\overline{X}^2_N}\}][1 - u_N\bar{e}_{xN} + \frac{3}{2}u^2_N\bar{e}^2_{xN}] \\ \bar{y}_{P2_N} = {} & k_1[\overline{Y}_N + \bar{e}_{yN} - u_N\overline{Y}_N\bar{e}_{xN} - u_N\bar{e}_{xN}\bar{e}_{yN} + \frac{3}{2}\overline{Y}_N u^2_N\bar{e}^2_{xN}] - k_2[\bar{e}_{xN} - u_N\bar{e}^2_{xN}] \\ & + [\overline{Y}_N - u_N\overline{Y}\bar{e}_{xN} + \frac{3}{2}\overline{Y}_N u^2_N\bar{e}_{xN} + \frac{\overline{Y}_N}{8\overline{X}^2_N}\bar{e}^2_{xN}]\end{aligned} \tag{41}$$

or

$$\begin{aligned}\bar{y}_{P2_N} - \bar{Y}_N = {} & k_1\left[\bar{Y}_N + \bar{e}_{yN} - u_N\bar{Y}_N\bar{e}_{xN} - u_N\bar{e}_{xN}\bar{e}_{yN} + \frac{3}{2}\bar{Y}_N u^2_N\bar{e}^2_{xN}\right] \\ & - k_2\left[\bar{e}_{xN} - u_N\bar{e}^2_{xN}\right] \\ & + \left[-u_N\bar{Y}\bar{e}_{xN} + \frac{3}{2}\bar{Y}_N u^2_N\bar{e}_{xN} + \frac{\bar{Y}_N}{8\bar{X}^2_N}\bar{e}^2_{xN}\right]\end{aligned} \tag{42}$$

Now taking expectation of the equation(40) on both sides we get the bias of the proposed estimator $\bar{y}_{P2_N}$

$$\begin{aligned}Bias(\bar{y}_{P2_N}) = {} & k_1\left[\bar{Y}_N - u_N\delta_{xy_N} + \frac{3}{2}\bar{Y}_N u^2_N\delta^2_{x_N}\right] \\ & + k_2\left[u_N\bar{e}^2_{xN}\right] + \left[+\frac{3}{2}\bar{Y}_N u^2_N\bar{e}_{xN} + \frac{\bar{Y}_N}{8\bar{X}^2_N}\bar{e}^2_{xN}\right]\end{aligned} \tag{43}$$

where, $Bias(\bar{y}_{P2_N}) \in [Bias(\bar{y}_{P2_L}), Bias(\bar{y}_{P2_U})]$

Squaring and taking expectation of equation (40), we get the MSE of the proposed estimator $\bar{y}_{P2_N}$

$$\begin{aligned}MSE(\bar{y}_{P2_N}) = {} & k^2_1\left[\bar{Y}^2_N + \delta^2_{y_N} + 4\bar{Y}^2_N u^2_N\delta {x_N}^2 - 4\bar{Y}_N u_N\delta_{xy_N}\right] + k^2_2\delta^2_{x_N} \\ & + 2k_1\left[\delta^2_{y_N} + \frac{5}{2}\bar{Y}^2_N u^2_N\delta {x_N}^2_N - 3\bar{Y}_N u_N\delta_{xy_N} + \frac{\bar{Y}_N}{8\bar{X}^2}\delta^2_{x_N}\right] \\ & - 2k_2\left[\delta_{xy_N} - \bar{Y}_N u_N\delta {x_N}^2\right] \\ & - 2k_1k_2\left[\delta_{xy_N} - u_N\bar{Y}_N\delta^2_{x_N}\right] + \left[\delta^2_{y_N} + u^2_N\bar{Y}^2_N\delta {x_N}^2\right. \\ & \left.-2u_N\bar{Y}_N\delta_{xy_N}\right]\end{aligned} \tag{44}$$

$$MSE(\bar{y}_{P2_N}) = k^2_1A_{2_N} + k^2_2B_{2_N} - 2k_1C_{2_N} - 2k_2D_{2_N} + 2k_1k_2E_{2_N} + F_{2_N} \tag{45}$$

where,

$$\begin{aligned}A_{2_N} &= \bar{Y}^2_N + \delta^2_{y_N} + \frac{\bar{Y}^2_N}{\bar{X}^2_N}\delta {x_N}^2 - \frac{2\bar{Y}_N}{\bar{X}_N}\delta_{yx} \\ B_{2_N} &= \delta^2_{x_N} \\ C_{2_N} &= -\left(\delta^2_{y_N} + \frac{3}{4}\frac{\bar{Y}^2_N}{\bar{X}^2_N}\delta {x_N}^2 - \frac{\bar{Y}_N}{\bar{X}_N}\delta_{xy_N}\right) \\ D_{2_N} &= \delta_{xy_N} - \frac{\bar{Y}_N}{2\bar{X}_N}\delta {x_N}^2 \\ E_{2_N} &= -\delta_{xy_N} \\ F_{2_N} &= \delta^2_{y_N} + \frac{\bar{Y}^2_N}{4\bar{X}^2_N}\delta {x_N}^2 - \frac{\bar{Y}_N}{\bar{X}_N}\delta_{xy_N}\end{aligned}$$

Differentiating Equation (43) with respect to k_1 and k_2 and equating it to zero, we get

$$k_1A_{2_N} + k_2E_{2_N} = C_{2_N} \tag{45}$$

$$k_2B_{2_N} + k_1E_{2_N} = D_{2_N} \tag{46}$$

Solving Equation (44) and (45), we get the value of k_1 and k_2

$$k_1 = \frac{B_{2_N}C_{2_N} - E_{2_N}D_{2_N}}{A_{2_N}B_{2_N} - E^2_{2_N}} \tag{47}$$

$$k_2 = \frac{A_{2_N}D_{2_N} - C_{2_N}E_{2_N}}{A_{2_N}B_{2_N} - E^2_{2_N}} \tag{48}$$

Substituting these value of k_1 and k_2 in Equation (43) we get the minimum MSE of the estimator $\bar{y}_{P2_N}$,

$$MSE(\bar{y}_{P2_N})_{min} = \frac{2C_{2_N}D_{2_N}E_{2_N} - A_{2_N}D^2_{2_N} - B_{2_N}C^2_{2_N}}{A_{2_N}B_{2_N} - E^2_{2_N}} + F_{2_N} \tag{49}$$

where, $MSE(\bar{y}_{P2_N})_{min} \in [MSE(\bar{y}_{P2_L})_{min}, MSE(\bar{y}_{P2_U})_{min}]$

4.3. Proposed estimator- III

Motivated by the work of Koyuncu et al. (2014), we have proposed a modified neutrosophic generalized exponential estimator to address indeterminacy, which is expressed as follows:

$$\bar{y}_{P3_N} = [\gamma_1 \bar{y}_{nN} + \gamma_2]\, exp\left[\frac{\bar{X}_N^{\star} - \bar{x}_N^{\star}}{\bar{X}_N^{\star} + \bar{x}_N^{\star}}\right] \tag{49}$$

where, $\overline{X}^{\star} = \mathrm{l}_N\overline{X}_N + m_N$, $\overline{x}^{\star} = \mathrm{l}_N\overline{x}_N + m_N$, γ_1 and $\gamma_2 (-\infty < \gamma_1, \gamma_2 < \infty)$ are two real constant and supposed to be estimated and $\bar{y}_{P3_N} \in [\bar{y}_{P3_L}, \bar{y}_{P3_U}]$.

Using approximations given in Equation (1) and (2), Equation (49) is framed as:

$$\begin{aligned}
\bar{y}_{P3_N} &= [\gamma_1(\overline{Y}_N + \bar{e}_{yN}) + \gamma_2]exp[\frac{\mathrm{l}_N\overline{X}_N + m_N - \mathrm{l}_N(\overline{X}_N + \bar{e}_{xN}) - m_N}{\mathrm{l}_N\overline{X}_N + m_N + \mathrm{l}_N(\overline{X}_N + \bar{e}_{xN}) + m_N}] \\
\bar{y}_{P3_N} &= [\gamma_1\overline{Y}_N + \gamma_1\bar{e}_{yN} + \gamma_2]exp[-\frac{\mathrm{l}_N\bar{e}_{xN}}{2(\mathrm{l}_N\overline{X}_N + m_N)}\{1 + \frac{\mathrm{l}_N\bar{e}_{xN}}{2\mathrm{l}_N\overline{X}_N + m_N)}\}^{-1}] \\
\bar{y}_{P3_N} &= [\gamma_1\overline{Y}_N + \gamma_1\bar{e}_{yN} + \gamma_2]exp[-u_N\bar{e}_{xN}\{1 + u_N\bar{e}_{xN}\}^{-1}] \\
\bar{y}_{P3_N} &= [\gamma_1\overline{Y}_N + \gamma_1\bar{e}_{yN} + \gamma_2][1 - u_N\bar{e}_{xN} + \frac{3}{2}u_N^2\bar{e}_{xN}^2]
\end{aligned} \tag{50}$$

$$\begin{aligned}
\bar{y}_{P3_N} - \bar{Y}_N = \gamma_1 \Big[\bar{Y}_N &- u_N\bar{Y}_N\bar{e}_{xN} \\
&+ \frac{3}{2}\bar{Y}_N u_N^2\bar{e}_{xN}^2 + \bar{e}_{yN} - u_N\bar{e}_{xN}\bar{e}_{yN}\Big] \\
&+ \gamma_2\left[1 - u_N\bar{e}_{xN} + \frac{3}{2}u_N^2\bar{e}_{xN}^2\right] - \bar{Y}_N
\end{aligned} \tag{50}$$

Taking expectation of Equation (50) on both the sides we get bias of proposed estimator $\bar{y}_{P3_N}$

$$\begin{aligned}
Bias(\bar{y}_{P3_N}) = E(\bar{y}_{P3_N} - \bar{Y}_N) = \gamma_1 &\left[\bar{Y}_N + \frac{3}{2}\bar{Y}_N u_N^2\bar{e}_{xN}^2 - u_N\bar{e}_{xN}\bar{e}_{yN}\right] \\
&+ \gamma_2\left[1 + \frac{3}{2}u_N^2\bar{e}_{xN}^2\right] - \bar{Y}_N
\end{aligned} \tag{51}$$

where, $Bias(\bar{y}_{P3_N}) \in [Bias(\bar{y}_{P3_L}), Bias(\bar{y}_{P3_U})]$

For MSE, squaring and taking expectation of the Equation (50), we have

$$\begin{aligned}
MSE(\bar{y}_{P3_N}) = \gamma_1^2 &\left[\bar{Y}_N^2 + \delta_{y_N}^2 + 4\bar{Y}_N^2 u_N^2 \delta {x_N}^2 - 4u_N\bar{Y}_N\delta_{xy_N}\right] \\
&+ \gamma_2^2\left[1 + 4u_N^2\delta_{x_N}^2\right] \\
&+ 2\gamma_1\gamma_2\left[\bar{Y}_N - 2u_N\delta_{xy_N} + 4\bar{Y}_N u_N^2\delta_{x_N}^2\right] \\
&- 2\gamma_1\left[\bar{Y}_N^2 + \frac{3}{2}\bar{Y}_N^2 u_N^2\delta {x_N}^2 - u_N\bar{Y}_N\delta_{xy_N}\right] \\
&- 2\gamma_2\left[\bar{Y}_N + \frac{3}{2}\bar{Y}_N u_N^2\delta {x_N}^2\right] + \bar{Y}_N^2
\end{aligned} \tag{52}$$

where,
$\delta_{x_N} = \theta_N\bar{X}_N^2 C_{xN}^2$
$\delta_{x_N} = \theta_N\bar{Y}_N^2 C_{yN}^2$

$\delta_{xy_N} = \theta_N\bar{X}_N\bar{Y}_N C_{xN}C_{yN}\rho xyN$

$$MSE(\bar{y}_{P3_N}) = \bar{Y}_N^2 + \gamma_1^2 A_{3_N} + \gamma_2^2 B_{3_N} + 2\gamma_1\gamma_2 C_{3_N} - 2\gamma_1 D_{3_N} - 2\gamma_2 E_{3_N} \tag{53}$$

where,

$$\begin{aligned}
A_{3_N} &= \bar{Y}_N^2 + \delta_{y_N}^2 + 4\bar{Y}_N^2 u_N^2\delta {x_N}^2 - 4u_N\bar{Y}_N\delta_{xy_N} \\
B_{3_N} &= 1 + 4u_N^2\delta_{x_N}^2 \\
C_{3_N} &= \bar{Y}_N - 2u_N\delta_{xy_N} + 4\bar{Y}_N u_N^2\delta_{x_N}^2 \\
D_{3_N} &= \bar{Y}_N^2 + \frac{3}{2}\bar{Y}_N^2 u_N^2\delta {x_N}^2 - u_N\bar{Y}_N\delta_{xy_N} \\
E_{3_N} &= \bar{Y}_N + \frac{3}{2}\bar{Y}_N u_N^2\delta {x_N}^2
\end{aligned}$$

Differentiating Equation (53) with respect to γ_1 and γ_2 and equating it to zero, we get

$$\gamma_1 A_{3_N} + \gamma_2 C_{3_N} = D_{3_N} \tag{54}$$

$$\gamma_2 B_{3_N} + \gamma_1 C_{3_N} = E_{3_N} \tag{55}$$

Solving Equation (54) and (55), we get the value of γ_1 and γ_2

$$\gamma_1 = \frac{B_{3_N}D_{3_N} - C_{3_N}E_{3_N}}{A_{3_N}B_{3_N} - C_{3_N}^2} \tag{56}$$

$$\gamma_2 = \frac{A_{3_N}E_{3_N} - C_{3_N}D_{3_N}}{A_{3_N}B_{3_N} - C_{3_N}^2} \tag{57}$$

Substituting these value of γ_1 and γ_2 in Equation (53) we get the minimum MSE of the estimator $\bar{y}_{P3_N}$,

$$MSE(\bar{y}_{P3_N})_{min} = \bar{Y}_N^2 + \frac{2C_{3_N}D_{3_N}E_{3_N} - B_{3_N}D_{3_N}^2 - A_{3_N}E_{3_N}^2}{A_{3_N}B_{3_N} - C_{3_N}^2} \tag{58}$$

where, $MSE(\bar{y}_{P3_N})_{min} \in [MSE(\bar{y}_{P3_L})_{min}, MSE(\bar{y}_{P3_U})_{min}]$

Particular case when $\gamma_1 + \gamma_2 = 1$

Taking $\gamma_1 + \gamma_2 = 1$ in Equation (49), estimator $\bar{y}_{P3_N}$ can be written as

$$\bar{y}_{P3_N}^{\star} = [\gamma_1\bar{y}_{nN} + (1 - \gamma_1)]\, exp\left[\frac{\bar{X}_N^{\star} - \bar{x}_N^{\star}}{\bar{X}_N^{\star} + \bar{x}_N^{\star}}\right] \tag{59}$$

Putting $\gamma_2 = 1 - \gamma_1$ in Equation (51) and (53), bias and MSE of estimator $\bar{y}_{P3_N}^{\star}$ can be obtained upto the first order of approximation, respectively, as:

$$Bias(\bar{y}_{P3_N}^{\star}) = \gamma_1\left[\bar{Y}_N + \frac{3}{2}\bar{Y}_N u_N^2\bar{e}_{xN}^2 - u_N\bar{e}_{xN}\bar{e}_{yN} - 1 + \frac{3}{2}u_N^2\bar{e}_{xN}^2\right]$$

$$+\left[1+\frac{3}{2}u_N^2\bar{e}_{xN}^2\right]-\bar{Y}_N \tag{60}$$

and

$$MSE(\bar{y}^{\star}_{P3_N}) = \bar{Y}_N^2 + (B_{3_N} - 2E_{3_N}) + \gamma_1^2(A_{3_N} + B_{3_N} - 2C_{3_N}) - 2\gamma_1(B_{3_N} - C_{3_N} + D_{3_N} - E_{3_N}) \tag{61}$$

Differentiating equation(61) with respect to γ_1 and equating it to zero we get the optimum value of γ_1,as

$$\gamma_1 = \frac{(B_{3_N} - C_{3_N} + D_{3_N} - E_{3_N})}{(A_{3_N} + B_{3_N} - 2C_{3_N})} = \gamma_{opt}(say) \tag{62}$$

Then the resulting minimum MSE of estimator $\bar{y}^{\star}_{P3_N}$ is

$$MSE(\bar{y}^{\star}_{P3_N})_{min} = \bar{Y}_N^2 + (B_{3_N} - 2E_{3_N}) - \frac{(B_{3_N} - C_{3_N} + D_{3_N} - E_{3_N})^2}{(A_{3_N} + B_{3_N} - 2C_{3_N})} \tag{63}$$

5. Numerical study

5.1. Empirical study: An application to COVID-19 scenario

To demonstrate the properties of our proposed neutrosophic estimators numerically, we have used two neutrosophic data sets based on product sales and marketing in the field of medical taken from Aleeswari et al. (2023). The data on networking and product sales collected by the medical professionals for a period of two months (1 January–1 March 2021) before and during (1 September – 1 November 2022) the lockdown are taken. In these indeterminate data sets, one set consists of two variables, networking with customers before the pandemic and product sales before the pandemic, and the other set consists of two variables, networking with customers after the pandemic and product sales after the pandemic.

5.1.1 Data Set 1

We are considering the percentage of networking before the pandemic as subsidiary information and the percentage of sales before the pandemic as the study variable. In the same way,

5.1.2 Data Set 2

For the second data set, we are considering the percentage of networking after the pandemic as subsidiary information and the percentage of sales after the pandemic as the study variable. Here, neutrosophic study variable denoted as$Y_N \in [Y_L, Y_U]$ (Y_L is the lowest percentage of sales before/after pandemic) and neutrosophic subsidiary variable denoted as $X_N \in [X_L, X_U]$.The parameters for our study are listed in Table 1.

Table 1. Description of the parameters for the estimation of means under neutrosophic SRSWOR for data set 1 & 2

Parameters	Data Set 1	Data Set 2
N	[30,30]	[30,30]
n	[9,9]	[9,9]
$\bar{Y}_N$	[37.0033,37.3266]	[34.7266,40.040]
$\bar{X}_N$	[34.1600,34.4500]	[34.7266,34.9033]
S_{yN}	[12.3153,12.3233]	[12.2704,26.7004]
S_{xN}	[15.0789,15.0891]	[15.2288,15.2452]
C_{yN}	[0.3328,0.3301]	[0.3533,0.6668]
C_{xN}	[0.4414,0.4380]	[0.4385,0.4367]
ρ_{xy}	[0.8776,0.8775]	[0.8508,0.1462]

Table 2. Description of the parameters for the estimation of means under neutrosophic SRSWOR from data set 3

Parameters	Neutrosophic values	Parameters	Classical values
N_N	[70, 70]	N	70
n_N	[25, 25]	n	25
$\bar{Y}_N$	[71.640, 80.400]	$\bar{Y}$	76.560
$\bar{X}_N$	[98.440, 99.400]	$\bar{X}$	99.160
S_{y_N}	[2.8994, 6.0828]	S_Y	3.1765
S_{x_N}	[0.6506, 1.0801]	S_X	0.6245
C_{y_N}	[0.0405, 0.0757]	C_y	0.0415
C_{x_N}	[0.0066, 0.0109]	C_x	0.0063
ρ_{xy_N}	[0.0212, 0.5644]	ρ_{xy}	0.3730

5.1.3 Data Set 3

This dataset is sourced from Smarandache & Aslam (2023), pages 379–382, and comprises daily records of patients who visited the BHU (Basic Health Unit) and reported gastritis from June 2021 to August 2021. The data is available in both neutrosophic and classical forms. The classical data has been derived by de-neutrosophicating the observations. This dataset is used to demonstrate the comparison between the classical and neutrosophic frameworks. Data contain four variables :

1. Systolic blood pressure
2. Diastolic blood pressure
3. Heartbeat rate and
4. Body temperature

For our study we have considered heart beat rate as a study variable and body temperature as an auxiliary variable. Parameter for our study are given in Table 2.

The MSEs and PREs for the estimators presented in Table 3 were calculated using Data Set 1 and 2 (Table 1), and Table 4 for Data Set 3 (Table 2). The Percent Relative Efficiency (PRE) of the estimators was calculated using the following formula:

$$PRE(ES) = \frac{MSE(\bar{y}_N)}{MSE(ES)} \times 100 \tag{64}$$

Table 3. MSEs and PREs of the existing and proposed estimators under neutrosophic SRSWOR for data set 1 & 2

Estimator	Data Set 1		Data Set 2	
	MSE	PRE	MSE	PRE
$\bar{y}_N$	[10.7848,10.8809]	[100,100]	[11.3698,11.676]	[100,100]
$\bar{y}_{rN}$	[10.8389,10.8579]	[100.3881,99.3264]	[8.7309,8.4136]	[133.7326,135.1352]
$\bar{y}_{MrN}$	[10.7416,10.1076]	[100.4022,101.1103]	[8.6494,8.3363]	[131.451,140.0622]
$\bar{y}_{expN}$	[4.5861,4.6394]	[237.2588,232.4578]	[5.007,4.9254]	[233.1958,230.8397]
$\bar{y}_{lexpN}(l=1,m=1)$	[13.5853,13.6671]	[79.3856,79.6139]	[12.9933,13.1833]	[87.5047,88.5668]
$\bar{y}_{lexpN}(l=1,m=0)$	[13.9111,13.9901]	[78.2173,77.0883]	[13.2317,13.4201]	[88.2426,84.7217]
$\bar{y}_{GexpN}$	[4.5861,4.6394]	[235.1635,234.5308]	[4.9207,4.7955]	[231.0619,243.4807]
$\bar{y}_{P1N}$	**[4.5482,4.6017]**	**[239.2374,234.3669]**	**[4.8835,4.7601]**	**[239.0901,238.8555]**
$\bar{y}^{\star}_{P1N}(\alpha_1+\alpha_2=1)$	[4.5484,4.6017]	[237.1101,236.4533]	[4.9625,4.881]	[229.114,239.2115]
$\bar{y}_{P2N}$	**[4.4002,4.4558]**	**[247.2837,242.0372]**	**[4.7824,4.6631]**	**[244.146,243.8269]**
$\bar{y}_{P3N}$	**[2.6024,2.6119]**	**[418.1157,412.911]**	**[2.151,2.0728]**	**[542.8266,548.5324]**
$\bar{y}^{\star}_{P3N}(\gamma_1+\gamma_2=1)$	[4.547,4.6007]	[237.182,236.5061]	[4.974,4.8951]	[228.5839,238.5262]

Table 4. MSEs and PREs of the existing and proposed estimators under neutrosophic and classical SRSWOR for data set 3

Estimators	Neutrosophic MSE	Classical MSE	Neutrosophic PRE	Classical PRE
$\bar{y}_N$	[0.2161, 0.9514]	0.2595	[100.00, 100.00]	100.00
$\bar{y}_{rN}$	[0.2204, 0.8168]	0.2361	[98.06, 116.48]	109.92
$\bar{y}_{MrN}$	[0.2204, 0.8168]	0.2360	[98.06, 116.49]	109.92
$\bar{y}_{expN}$	[0.2169, 0.8792]	0.2463	[99.68, 108.21]	105.36
$\bar{y}_{lexpN}(l=1,m=1)$	[0.2179, 0.9205]	0.2544	[99.20, 103.39]	102.00
$\bar{y}_{lexpN}(l=1,m=0)$	[0.2180, 0.9202]	0.2544	[99.18, 103.39]	102.01
$\bar{y}_{GexpN}$	[0.2161, 0.6483]	0.2234	[100.04, 146.75]	116.17
$\bar{y}_{P1N}$	[0.2161, 0.6483]	0.2233	[100.05, 146.77]	116.17
$\bar{y}^{\star}_{P1N}(\alpha_1+\alpha_2=1)$	[0.2169, 0.8791]	0.2462	[99.68, 108.23]	105.36
$\bar{y}_{P2N}$	[0.2165, 0.6352]	0.2206	[99.83, 149.78]	117.62
$\bar{y}_{P3N}$	[0.0014, 0.0033]	0.0013	[15004.8, 28454.9]	20166.61
$\bar{y}^{\star}_{P3N}(\gamma_1+\gamma_2=1)$	[0.2169, 0.8791]	0.2462	[99.68, 108.23]	105.36

where, $ES = \bar{y}_N, \bar{y}_{rN}, \bar{y}_{MrN}, \bar{y}_{expN}, \bar{y}_{IexpN}, \bar{y}_{GexpN}, \bar{y}_{P1N}, \bar{y}^{\star}_{P1N}, \bar{y}_{P2N}, \bar{y}_{P3N}, \bar{y}^{\star}_{P3N}$

5.2. Simulation study

For simulation study we have used the concept of Raghav (2023). Following steps are used for the simulation study-

For Neutrosophic data generation

- Generate Neutrosophic auxiliary variable X_N follows neutrosophic normal distribution as $X_N \sim NN(\mu_{xN}, \sigma^2_{xN}); X_N \in (X_L, X_U), \mu_{xN} \in (.2, 1.2), \sigma^2_{xN} \in (1, 1.2)$
- Generate Neutrosophic study variable using the model $Y_N = X_N - 9.8e_N$,where $e_N \sim NN(0, 1)$.

For Classical data generation-

- Generate auxiliary variable X follows normal distribution as, $X \sim N(\mu_x, \sigma^2_x)$ where, $\mu_x = 0.6$ and $\sigma^2_x = 1.2$
- Generate study variable using the model $Y = X - 9.8e$ where $e \sim N(0, 1)$

Table 5. Description of the parameters for the estimation of means under neutrosophic SRSWOR for simulation study

Parameters	Neutrosophic values	Parameters	Neutrosophic values
N_N	[1000, 1000]	S_{yN}	[2.2223, 2.3144]
$\bar{Y}_N$	[0.2223, 1.2343]	C_{yN}	[9.9948, 1.8750]
$\bar{X}_N$	[0.2599, 1.2719]	C_{xN}	[3.8978, 0.9560]
S_{xN}	[1.0134, 1.2161]	ρ_{xyN}	[0.4185, 0.4894]

Table 6. Description of the parameters for the estimation of means under classical SRSWOR for simulation study

Parameters	Classical values	Parameters	Classical values
N	1000	S_y	1.2367
$\bar{Y}$	0.6120	C_y	3.7387
$\bar{X}$	0.6202	C_x	1.9942
S_x	2.2879	ρ_{xy}	0.5286

The parameters for our simulation study are listed in Tables 5 and 6.

Further, we have drawn samples of size $n_N(n_1 = 300, n_2 = 350$ and $n_3 = 400)$ from the population of size $N_N = 1000$ by the method of neutrosophic simple random sampling without replacement. With the help of these samples, we have calculated MSEs of the existing and proposed estimators. The whole process of getting

Table 7. MSEs and PREs from simulation study for $n = 300$

Estimators	Neutrosophic MSE	Classical MSE	Neutrosophic PRE	Classical PRE
$\bar{y}_N$	[0.0115, 0.0125]	0.0122	[100.00, 100.00]	100.00
$\bar{y}_{rN}$	[0.0099, 0.0095]	0.0089	[116.16, 131.58]	137.08
$\bar{y}_{MrN}$	[0.0113, 0.0100]	0.0108	[101.77, 125.00]	112.96
$\bar{y}_{expN}$	[0.0102, 0.0102]	0.0097	[112.75, 122.55]	125.77
$\bar{y}_{lexpN}(l = 1, m = 1)$	[0.0114, 0.0120]	0.0117	[100.88, 104.17]	104.27
$\bar{y}_{lexpN}(l = 1, m = 0)$	[0.0114, 0.0122]	0.0118	[100.88, 102.46]	103.39
$\bar{y}_{GexpN}$	[0.0095, 0.0095]	0.0088	[121.05, 131.58]	138.64
$\bar{y}_{P1N}$	[0.0071, 0.0094]	0.0085	[161.97, 132.98]	143.53
$\bar{y}^{\star}_{P1N}(\alpha_1 + \alpha_2 = 1)$	[0.0073, 0.0101]	0.0093	[157.53, 123.76]	131.18
$\bar{y}_{P2N}$	[0.0070, 0.0094]	0.0085	[164.28, 132.76]	143.53
$\bar{y}_{P3N}$	[0.0004, 0.0006]	0.0006	[2875.0, 2083.33]	2033.33
$\bar{y}^{\star}_{P3N}(\gamma_1 + \gamma_2 = 1)$	[0.0099, 0.0076]	0.0087	[116.16, 164.47]	140.23

Table 8. MSEs and PREs from simulation study for $n = 350$

Estimators	Neutrosophic MSE	Classical MSE	Neutrosophic PRE	Classical PRE
$\bar{y}_N$	[0.0092,0.0099]	0.0097	[100.00,100.00]	100.00
$\bar{y}_{rN}$	[0.0078,0.0076]	0.0071	[117.95,130.26]	136.62
$\bar{y}_{MrN}$	[0.0090,0.0080]	0.0086	[102.22, 123.75]	112.79
$\bar{y}_{expN}$	[0.0081,0.0081]	0.0077	[113.58,122.22]	125.97
$\bar{y}_{lexpN}$(l=1,m=1)	[0.0090,0.0096]	0.0094	[102.22,103.13]	103.19
$\bar{y}_{lexpN}$(l=1,m=0)	[0.0090,0.0097]	0.0094	[102.22,102.06]	103.19
$\bar{y}_{GexpN}$	[0.0075,0.0075]	0.0070	[122.67,132.00]	138.57
$\bar{y}_{P1N}$	[0.0060,0.0075]	0.0068	[153.33,132.00]	142.65
$\bar{y}^{\star}_{P1N}(\alpha_1 + \alpha_2 = 1)$	[0.0062,0.0081]	0.0075	[148.39,122.22]	129.33
$\bar{y}_{P2N}$	[0.0059,0.0075]	0.0068	[155.93,132.00]	142.67
$\bar{y}_{P3N}$	[0.0003,0.0005]	0.0005	[3066.67,1980.00]	1940.00
$\bar{y}^{\star}_{P3N}(\gamma_1 + \gamma_2 = 1)$	[0.0079,0.0064]	0.0071	[116.46,154.69]	136.62

Table 9. MSEs and PREs from simulation study for $n = 400$

Estimators	Neutrosophic MSE	Classical MSE	Neutrosophic PRE	Classical PRE
$\bar{y}_N$	[0.0074, 0.0080]	0.0078	[100.00, 100.00]	100.00
$\bar{y}_{rN}$	[0.0063, 0.0061]	0.0057	[117.46, 131.15]	136.84
$\bar{y}_{MrN}$	[0.0073, 0.0064]	0.0069	[101.37, 125.00]	113.04
$\bar{y}_{expN}$	[0.0065, 0.0066]	0.0062	[113.85, 121.21]	125.81
$\bar{y}_{lexpN}(l = 1, m = 1)$	[0.0073, 0.0077]	0.0076	[101.37, 103.89]	102.63
$\bar{y}_{lexpN}(l = 1, m = 0)$	[0.0073, 0.0078]	0.0076	[101.37, 102.56]	102.63
$\bar{y}_{GexpN}$	[0.0061, 0.0061]	0.0057	[121.31, 131.15]	136.84
$\bar{y}_{P1N}$	[0.0051, 0.0061]	0.0056	[145.10, 131.15]	139.29
$\bar{y}^{\star}_{P1N}(\alpha_1 + \alpha_2 = 1)$	[0.0052, 0.0065]	0.0061	[142.31 ,123.07]	127.87
$\bar{y}_{P2N}$	[0.0052, 0.0061]	0.0055	[148.00, 131.15]	141.82
$\bar{y}_{P3N}$	[0.0003, 0.0004]	0.0004	[2466.67, 2000.00]	1950.00
$\bar{y}^{\star}_{P3N}(\gamma_1 + \gamma_2 = 1)$	[0.0064, 0.0054]	0.0058	[115.63, 148.15]	134.48

MSEs of the estimators through the neutrosophic simple random sampling method is repeated 7000 times.The results of MSEs and PREs for the estimators are shown in Tables 7–9.

6. Result and discussion

Table 3 elucidates the Mean Squared Error (MSE) and the Precision Relative Efficiency (PRE) of both the existing and newly proposed neutrosophic estimators under the neutrosophic Simple Random Sampling Without Replacement (SRSWOR) framework, applied to two real datasets-networking data and product sales data before and after the COVID-19 pandemic, sourced from Aleeswari et al., (2023). Similarly, Table 4 presents the MSE and PRE of the existing and proposed neutrosophic estimators under both classical SRSWOR and neutrosophic SRSWOR, applied to real patient data collected from the BHU (Basic Health Unit), as outlined in Smarandache & Aslam, (2023). The findings presented in Tables 3 and 4 unequivocally demonstrate that the suggested estimators, namely $\bar{y}P1_N$, $\bar{y}P2_N$, and $\bar{y}P3_N$, consistently exhibit the lowest MSEs and the highest PREs when compared to the existing estimators. Specifically, our proposed estimators $\bar{y}P1_N$, $\bar{y}P2_N$, and $\bar{y}P3_N$ markedly outperform the generalized exponential estimator $\bar{y}GexpN$ in terms of efficiency. Among the proposed estimators, $\bar{y}P3_N$ emerges as the most superior, delivering the highest overall performance. Notably, the estimators $\bar{y}P1_N$ and $\bar{y}P3_N$, under the constraints $\alpha_1 + \alpha_2 = 1$ and $\gamma_1 + \gamma_2 = 1$ respectively, exhibit performance characteristics akin to those of the classical estimators $\bar{y}expN$ and $\bar{y}GexpN$.

Tables 7–9 provide further insight into the MSEs and PREs of both the existing and proposed neutrosophic estimators, calculated under neutrosophic SRSWOR and classical SRSWOR frameworks. These results are derived from an extensive simulation study conducted across different sample sizes of 300, 350, and 400. The findings in Tables 7–9 clearly affirm that the proposed estimators, $\bar{y}P1_N$, $\bar{y}P2_N$, and $\bar{y}P3_N$, are the most efficient among all the estimators considered. Of these, $\bar{y}P3_N$ is the most efficient, exhibiting the highest PRE across all sample sizes. Moreover, the results clearly illustrate the variations in performance as sample sizes are adjusted, providing a comprehensive picture of the estimators' behavior under different conditions.

7. Conclusion

In this article, we have propounded three modified Neutrosophic exponential estimators ($\bar{y}_{P1_N}$, $\bar{y}_{P2_N}$ and $\bar{y}_{P3_N}$) for estimating population mean using auxiliary variable under neutrosophic SRSWOR for an interval data and classical SRSWOR for crisp data. Our suggested estimators demonstrate a clear improvement compared to the existing neutrosophic simple mean estimator ($\bar{y}_N$), neutrosophic ratio estimator ($\bar{y}_{rN}$), neutrosophic modified ratio estimator ($\bar{y}_{MrN}$), neutrosophic exponential estimator ($\bar{y}_{expN}$), neutrosophic improved exponential ratio estimator ($\bar{y}_{IexpN}$), neutrosophic generalized exponential ratio estimator ($\bar{y}_{GexpN}$). We found that our suggested estimators acquire the higher efficiency than the existing exponential estimators. Among the suggested estimators, $\bar{y}_{P3_N}$ is the most efficient. This finding is supported both empirically and through a simulation study. Based on the empirical study in Tables 3 and 4 and the simulation study in Tables 7–9, we conclude that our suggested estimators can effectively estimate the mean using auxiliary information for the unclear, ambiguous, indeterminate or in the form of an interval data and also for crisp data.

Disclosure statement

No potential conflict of interest was reported by the author(s).

Author contribution statement

All authors listed have contributed significantly to write this article.

Data availability statement

All data applied is included in the manuscript.

Ethical statement

There are no human/animal subjects in this article therefore an ethics statement is not applicable because this study is applied on already published data.

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