

**IMPROVEMENT OF WEBER'S AND FECHNER'S LAWS  
ON SENSATIONS AND STIMULI**

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**Abstract.**

In this paper one uses a mathematical modeling of psychological processes and one improves the Weber's Law and Fechner's Law on sensations and stimuli.

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**Introduction.**

According to the neutrosophic theory, between an <idea> (= spiritual) and an <object> (= material) there are infinitely many states.

Then, how can we mix an <idea> with an <object> and obtain something in between: s% spiritual and m% material (s+m = 100)? [kind of chemical alloy].

Or, as Boethius, a founder of scholasticism, urged to "join faith to reason" in order to reconcile the Christian judgement with the rational judgement.

**Fechner's Law Improvement:**

For example <mind> and <body> co-exist. Gustav Theodor Fechner, who inaugurated the experimental psychology, obsessed with this problem, advanced the theory that every object is both mental and physical (psychophysics). Fechner's Law,  $S = k \log R$ , with S the sensation, R the stimulus, and k a constant, which is derived from Weber's Law,  $\Delta R / R = k$ , with  $\Delta R$  the increment of stimulus just detectable, should be improved, because the function  $\log R$  is indefinitely increasing as  $R \rightarrow \infty$ , to

$$S(R) = k \frac{\ln R}{\ln RM}, \text{ for } R \text{ in } [R_m, RM],$$

and

$$S(R) = 0, \text{ for } R \text{ in } [0, R_m) \text{ union with } (RM, \infty),$$

where k is a positive constant depending on three parameters: individual being, type of sensation, and the kind of stimulus, and  $R_m$ ,  $RM$  represent the minimum and maximum stimulus magnitude respectively perceptible by the subject, the second one bringing about the death of sensation. Fechner's "functional relation", as well as later psychologists'

power law  $R = kS^n$ , with  $n$  depending on the kind of stimulus, were upper unbounded, while the beings are surely limited in perception.

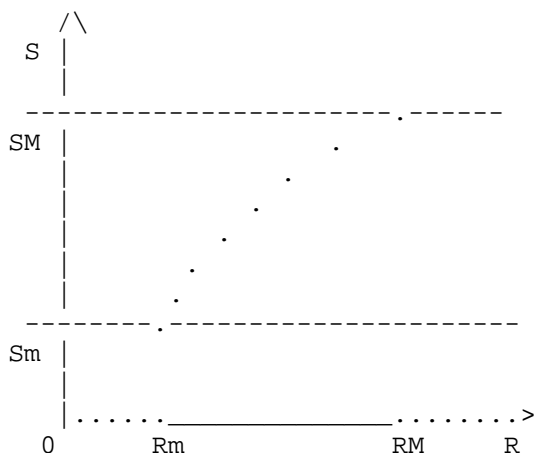
$S: [0, \hat{i}) \rightarrow \{0\}$  union with  $[S_m, S_M]$ , with  $S_m, S_M$  the minimum and maximum perceptible sensation respectively.  
Of course  $R_m > 1$ ,  $S(R_m) = S_m$ , and  $S(R_M) = S_M = k$ .

$\ln$ , increasing faster, replaces  $\log$  because the sensation is more rapidly increasing at the beginning, and later going on much slower.

At  $R = R_M$ ,  $S$  attains its maximum, beyond whom it becomes flat again, falling to zero.

The beings have a low and high threshold respectively, a range where they may feel a sensation.

**Graph of Fechner's Law Improvement:**



For example in acoustics: a sound is not heard at the beginning and, if it constantly keeps enlarging its intensity, at a given moment we hear it, and for a while its loudness increases in our ears, until the number of decibels - getting bigger than our possibility of hearing - breaks our eardrums... We would not hear anything anymore, our sensation died...

Now, if at a given moment  $t_0$  the stimulus  $R$  remains constant equal to  $R_0$  (between the conscious limits of the being, for a long period of time  $t$ ), and the sensation  $S(R_0) = c$ , then we get the following formulas:

In the case when the stimulus is not physically or physiologically damaging the individual being:  
 $S_{dec}(t) = c \log_{1/e}(t+1/e) = -c \ln(t+1/e)$ ,  
 for  $0 \leq t \leq \exp(-S_m/c)-1/e$ , and 0 otherwise;  
 which is a decreasing function;

In the case when the stimulus is hurting the individual being:  
 $S_{inc}(t) = c \ln(t+e)$ , for  $0 \leq t \leq \exp(S_M/c)-e$ , and 0 otherwise;  
 which is an increasing function until the sensation reaches its upper bound;

where  $c$ , as a constant, depends on individual being, type of

sensation, and kind of stimulus.

**Examples:**

a) If a prisoner feels a constant smell in his closed room for days and days, isolated from the exterior, and he doesn't go outside to change the environment, he starts to feel it less and less and after a critical moment he becomes inured to the smell and do not feel it anymore -  
thus the sensation disappears under the low perceptible limit.

b) If a water drop licks constantly, at the same interval of time, with the same intensity, on the head of a prisoner tied to a pillar, the prisoner after a while will feel the water drop heavier and heavier, will mentally get ill and out of his mind, and will even physically die -  
therefore again disappears the sensation, but above the high limit. See how one can kill someone with a... water drop!

c) If one permanently plays the same song for days and days to a person enclosed in a room without any other noise from outside, that person will be driven crazy, even psychologically die, and the sensation will disappear.

**Weber's Law Improvement.**

Weber's Law can be improved to  $\Delta R / \ln R = k$ , with R defined on  $[R_m, RM]$ , where k is a constant depending on individual being, type of sensation, and kind of stimulus, due to the fact that the relative threshold  $\Delta R$  increases slower with respect to R.

**References:**

- [1] Reber, Arthur S., "Dictionary of Psychology", Penguin Group, "Fechner's Law", p. 271; "Weber's Law", p. 827; 1985.
- [2] Smarandache, Florentin, "Neutrosophy. / Neutrosophic Probability, Set, and Logic", American Research Press, Rehoboth, 105 p., 1998;  
<http://www.gallup.unm.edu/~smarandache/NeutroSo.txt>.