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Generalization of An Er's Matrix Method for Computing

In Florentin Smarandache: "Collected Papers", vol. I (second edition).
Ann Arbor (USA): InfoLearnQuest, 2007.

GENERALIZATION OF AN ER'S MATRIX METHOD FOR COMPUTING

Er's matrix method for computing Fibonacci numbers and their sums can be extended to the s-additive sequence:

$$g_{-s+1} = g_{-s+2} = \ldots = g_{-1} = 0, \quad g_0 = 1,$$

and

$$g_n = \sum_{i=1}^{s} g_{n-i} \text{ for } n > 0.$$

For example, if we note $S_n = \sum_{j=1}^{n-1} g_j$, we define two $(s+1)\times(s+1)$ matrixes such that:

$$B_n = \begin{bmatrix} 1 & 0 & 0 & \ldots & 0 & 0 \\ S_n & g_n & g_{n-1} & \cdots & g_{n-s+2} & g_{n-s+1} \\ : & : & : & \ldots & : & : \\ S_{n-s+1} & g_{n-s+1} & g_{n-s} & \cdots & g_{n-2s+3} & g_{n-2s+2} \end{bmatrix},$$

$n \geq 1$, and

$$M = \begin{bmatrix} 1 & 0 & 0 & \ldots & 0 \\ 1 & 1 & 1 & \ldots & 0 \\ : & : & : & \ldots & : \\ 1 & 1 & 0 & \ldots & 1 \\ 1 & 1 & 0 & \ldots & 0 \end{bmatrix},$$

thus, we have analogously:

$$B_{n+1} = M^{n+1}, \; M^{r+c} = M^r \cdot M^c,$$

whence

$$S_{r+c} = S_r + g_r S_c + g_{r-1} S_{c-1} + \ldots + g_{r-s+1} S_{c-s+1},$$

$$g_{r+c} = g_r g_c + g_{r-1} g_{c-1} + \ldots + g_{r-s+1} g_{c-s+1},$$

and for $r = c = n$ it results:

$$S_{2n} = S_n + g_n S_n + g_{n-1} S_{n-1} + \ldots + g_{n-s+1} S_{n-s+1},$$

$$g_{2n} = g_n^2 + g_{n-1}^2 + \ldots + g_{n-s+1}^2;$$

for $r = n$, $c = n-1$, we find:

$$g_{2n-1} = g_n g_{n-1} + g_{n-1} g_{n-2} + \ldots + g_{n-s+1} g_{n-s}, \text{ etc.}$$

$$S_{2n-1} = S_n + g_n S_{n-1} + g_{n-1} S_{n-2} + \ldots + g_{n-s+1} S_{n-s}$$

Whence we can construct a similar algorithm as M. C. Er for computing s-additive numbers and their sums.

REFERENCE:

M. C. Er, Fast Computation of Fibonacci Numbers and Their Sums, J. Inf. Optimization Sci. (Delhi), Vol. 6 (1985), No. 1. pp. 41-47.

[Published in “GAMMA”, Braşov, Anul X, No.. 1-2, October 1987, p. 8]