A Group-Permutation Algorithm to Solve the Generalized SUDOKU

Florentin Smarandache
University of New Mexico
Gallup Campus, USA

Sudoku is a game with numbers, formed by a square with the side of 9, and on each row and column are placed the digits 1, 2, 3, 4, 5, 6, 7, 8, 9, written only one time; the square is subdivided in 9 smaller squares with the side of 3×3, which, also, must satisfy the same condition, i.e. each square to contain all digits from 1 to 9 written only once.

The Japanese company Nikoli has popularized this game in 1986 under the name of *sudoku*, meaning "single number".

Sudoku can be generalized to squares whose dimensions are $n^2 \times n^2$, where $n \geq 2$, using various symbols (numbers, letters, mathematical symbols, etc.), written just one time on each row and on each column; and the large square is divided into n^2 small squares with the side $n \times n$ and each will contain all n^2 symbols written only once.

An elementary solution of one of these generalized Sudokus, with elements (symbols) from the set

$$S = \{s_1, s_2, ..., s_n, s_{n+1}, ..., s_{2n}, ..., s_{n\wedge 2}\}$$

(supposing that their placement represents the relation of total order on the set of elements S), is:

Row 1: all elements in ascending order

$$s_1, s_2, ..., s_n, s_{n+1}, ..., s_{2n}, ..., s_{n\wedge 2}$$

On the next rows we will use circular permutations, considering groups of n elements from the first row as follows:

Row 2:

$$s_{n+1}, s_{n+2}, ..., s_{2n}; s_{2n+1}, ..., s_{3n}; ..., s_{n\wedge 2}; s_1, s_2, ..., s_n$$

Row 3:

$$s_{2n+1}, ..., s_{3n}; ..., s_{n\wedge 2}; s_1, s_2, ..., s_n; s_{n+1}, s_{n+2}, ..., s_{2n}$$

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Row n:

$$s_{n\wedge 2-n+1}, ..., s_{n\wedge 2}; s_1, ..., s_n, s_{n+1}; s_{n+2}, ..., s_{2n}; ..., s_{3n}; ..., s_{n\wedge 2-n}.$$

Now we start permutations of the elements of row $n+1$ considering again groups of n elements.

Row $n+1$:

$$s_2, ..., s_n, s_{n+1}; s_{n+2}, ..., s_{2n}, s_{2n+1}; s_{n\wedge 2-n+2}, ..., s_{n\wedge 2}, s_1$$

Row $n+2$:

$$s_{n+2}, ..., s_{2n}, s_{2n+1}; s_{n\wedge 2-n+2,} ..., s_{n\wedge 2}, s_1; s_2, ..., s_n, s_{n+1}$$

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Row $2n$:

s_{n^2-n+2} …, s_{n^2}, s_1; s_2, …, s_n, s_{n+1}; s_{n+2}, …, s_{2n}, s_{2n+1}

Row $2n+1$:

s_3, …, s_{n+2}; s_{n+3}, …, s_{2n+2}; s_{n^2+3}, …, s_{n^2}, s_1, s_2

and so on.

Replacing the set S by any permutation of its symbols, which we'll note by S', and applying the same procedure as above, we will obtain a new solution.

The classical Sudoku is obtained for $n = 3$.

Below is an example of this group-permutation algorithm for the classical case:

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
4	**5**	**6**	**7**	**8**	**9**	**1**	**2**	**3**
7	**8**	**9**	**1**	**2**	**3**	**4**	**5**	**6**
2	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**1**
5	**6**	**7**	**8**	**9**	**1**	**2**	**3**	**4**
8	**9**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
3	**4**	**5**	**6**	**7**	**8**	**9**	**1**	**2**
6	**7**	**8**	**9**	**1**	**2**	**3**	**4**	**5**
9	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**

For a $\mathbf{4^2 \times 4^2}$ **square** we use the following 16 symbols:

{A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P}

and use the same group-permutation algorithm to solve this Sudoku.

From one solution to the generalized Sudoku we can get more solutions by simply doing permutations of columns or/and of rows of the first solution.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
E	F	G	H	I	J	K	L	M	N	O	P	A	B	C	D
I	J	K	L	M	N	O	P	A	B	C	D	E	F	G	H
M	N	O	P	A	B	C	D	E	F	G	H	I	J	K	L
B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	A
F	G	H	I	J	K	L	M	N	O	P	A	B	C	D	E
J	K	L	M	N	O	P	A	B	C	D	E	F	G	H	I
N	O	P	A	B	C	D	E	F	G	H	I	J	K	L	M
C	D	E	F	G	H	I	J	K	L	M	N	O	P	A	B
G	H	I	J	K	L	M	N	O	P	A	B	C	D	E	F
K	L	M	N	O	P	A	B	C	D	E	F	G	H	I	J
O	P	A	B	C	D	E	F	G	H	I	J	K	L	M	N
D	E	F	G	H	I	J	K	L	M	N	O	P	A	B	C
H	I	J	K	L	M	N	O	P	A	B	C	D	E	F	G
L	M	N	O	P	A	B	C	D	E	F	G	H	I	J	K
P	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O

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{Published in Author's book *Frate cu meridianele și paralelele, Vol. IV*, Offsetcolor, Rm, Vâlcea, Romania, pp. 201-202, 2008.}