Threat assessment of a possible Vehicle-Borne Improvised Explosive Device using DSmT

Jean Dezert
French Aerospace Lab.
ONERA/DTIM/SIF
29 Av. de la Div. Leclerc
92320 Châtillon, France.
jean.dezert@onera.fr

Florentin Smarandache
ENSIETA
E^3I^2-EA3876 Laboratory
2 rue François Verny
29806 Brest Cedex 9, France.
Florentin.Smarandache@ensieta.fr

Abstract – *This paper presents the solution about the threat of a VBIED (Vehicle-Borne Improvised Explosive Device) obtained with the DSmT (Dezert-Smarandache Theory). This problem has been proposed recently to the authors by Simon Maskell and John Lavery as a typical illustrative example to try to compare the different approaches for dealing with uncertainty for decision-making support. The purpose of this paper is to show in details how a solid justified solution can be obtained from DSmT approach and its fusion rules thanks to a proper modeling of the belief functions involved in this problem.*

Keywords: Information fusion, DSmT, Threat assessment, Decison-making support.

1 The VBIED problem

- **Concern**: VBIED (Vehicle-Borne Improvised Explosive Device) attack on an administrative building B
- **Prior information**: We consider an Individual A under surveillance due to previous unstable behavior who drives customized white Toyota (WT) vehicle.
- **Observation done at time** $t - 10$ min: From a video sensor on road that leads to building B 10 min ago, one has observed a White Toyota 200m from the building B traveling in normal traffic flow toward building B. We consider the following two sources of information based on this video observation available at time $t - 10$ min:
  - **Source 1**: An Analyst 1 with 10 years experience analyses the video and concludes that individual A is now probably near building B.
  - **Source 2**: An Automatic Number Plate Recognition (ANPR) system analyzing same video outputs 30% probability that the vehicle is individual A's white Toyota.
- **Observation done at time** $t - 5$ min: From a video sensor on road 15km from building B 5 min ago one gets a video that indicates a white Toyota with some resemblance to individual A's white Toyota. We consider the following thrid source of information based on this video observation available at time $t - 5$ min:
  - **Source 3**: An Analyst 2 (new in post) analyses this video and concludes that it is improbable that individual A is near building B.
- **Question 1**: Should building B be evacuated?
- **Question 2**: Is experience (Analyst 1) more valuable than physics (the ANPR system) combined with inexperience (Analyst 2)? How do we model that?

NOTE: Deception (e.g., individual A using different car, false number plates, etc.) and biasing (on the part of the analysts) are often a part of reality, but they are not part of this example.

2 Modeling the VBIED problem

Before applying DSmT fusion techniques to solve this VIEB problem it is important to model the problem in the framework of belief functions.

2.1 Marginal frames with their models

The marginal frames involved in this problem are:

- Frame related with individuals:

$$\Theta_1 = \{A = \text{Suspicious person}, \bar{A} = \text{not } A\}$$

- Frame related with the vehicle:

$$\Theta_2 = \{V = \text{White Toyota Vehicle}, \bar{V} = \text{not } V\}$$

- Frame related with a position w.r.t the given building B:

$$\Theta_3 = \{B = \text{near building}, \bar{B} = \text{not } B\}$$

The underlying models of marginal frames are based on the following very reasonable assumptions:

- **Assumption 1**: We assume naturally $A \cap \bar{A} = \emptyset$ there is no quantum living people (to avoid Shrödinger cat paradox) !!! If working only with the frame of people Θ_P, the marginal bba's must be then be defined on the power-set

$$2^{\Theta_1} = \{\emptyset_1, A, \bar{A}, A \cup \bar{A}\}$$

- **Assumption 2**: We assume also that $V \cap \bar{V} = \emptyset$ so that the marginal bba (if needed) must be defined on the power-set

$$2^{\Theta_2} = \{\emptyset_2, V, \bar{V}, V \cup \bar{V}\}$$

- **Assumption 3**: We assume also that $L \cap \bar{L} = \emptyset$ so that the marginal bba (if needed) must be defined on the power-set

$$2^{\Theta_3} = \{\emptyset_3, L, \bar{L}, L \cup \bar{L}\}$$

  This modeling is disputable since the notion of closeness/"near" is not clearly defined and we could prefer to work on

$$D^{\Theta_3} = \{\emptyset_3, B \cap \bar{B}, B, \bar{B}, B \cup \bar{B}\}$$

The emptyset elements have been indexed by the index of the frame they are referring to for notation convenience and avoiding confusion.

2.2 Joint frame and its model

Since we need to work with all aspects of available information, we need to define a common joint frame to express all what we have from different sources of information. The easiest way for defining the joint frame, denoted Θ, is to consider the classical Cartesian (cross) product space and to work with propositions (a Lindenbaum-Tarski algebra of propositions) since one has a correspondence between sets and propositions [5, 6], i.e.

$$\Theta = \Theta_1 \times \Theta_2 \times \Theta_3$$

which consists of the following 8 triplets elements

$$\begin{aligned}\Theta = \{&\theta_1 = (\bar{A}, \bar{V}, \bar{B}), \theta_2 = (A, \bar{V}, \bar{B}),\\ &\theta_3 = (\bar{A}, V, \bar{B}), \theta_4 = (A, V, \bar{B}),\\ &\theta_5 = (\bar{A}, \bar{V}, B), \theta_6 = (A, \bar{V}, B),\\ &\theta_7 = (\bar{A}, V, B), \theta_8 = (A, V, B)\}\end{aligned}$$

We define the union $\cup$, intersection $\cap$ as componentwise operators in the following way:

$$(x_1, x_2, x_3) \cup (y_1, y_2, y_3) \triangleq (x_1 \cup y_1, x_2 \cup y_2, x_3 \cup y_3)$$

$$(x_1, x_2, x_3) \cap (y_1, y_2, y_3) \triangleq (x_1 \cap y_1, x_2 \cap y_2, x_3 \cap y_3)$$

The complement $\bar{X}$ of X is defined in the usual way by

$$\bar{X} = \overline{(x_1, x_2, x_3)} \triangleq I_t \setminus \{X\}$$

where I_t is the total ignorance (i.e. the whole space of solutions) which corresponds to the maximal element defined by $I_t = (I_{t1}, I_{t2}, I_{t3})$, where I_{ti} is the maximal (ignorance) of Θ_i, $i = 1, 2, 3$. The minimum element (absolute empty proposition) is $\emptyset_a = (\emptyset_1, \emptyset_2, \emptyset_3)$, where $\emptyset_i$ is the minimum element (empty proposition) of Θ_i. We also define a relative minimum element in $S^{\Theta_1 \times \Theta_2 \times \Theta_3}$ as follows: $\emptyset_r = (x, y, z)$, where at least one of the components x, y, or z is a minimal element in its respective frame Θ_i. A general relative minimum element $\emptyset_{gr}$ is defined as the union/join of all relative minima (including the absolute minimum element). Similarly to the relative and general relative minimum we can define a relative maximum and a general relative maximum, where the empty set in the above definitions is replaced by the total ignorance. Whence the super-power set $(S^{\Theta}, \cap, \cup, {}^{-}, \emptyset_a, I_t)$ is equivalent to Lindenbaum-Tarski algebra of propositions.

For example, if we consider $\Theta_1 = \{x_1, x_2\}$ and $\Theta_2 = \{y_1, y_2\}$ satisfying both Shafer's model, then $\Theta = \Theta_1 \times \Theta_2 = \{(x_1, y_1), (x_1, y_2), (x_2, y_1), (x_2, y_2)\}$, and one has:

$$\begin{aligned}\emptyset_a &= (\emptyset_1, \emptyset_2)\\ \emptyset_{r1} &= (\emptyset_1, y_1)\\ \emptyset_{r2} &= (\emptyset_1, y_2)\\ \emptyset_{r3} &= (\emptyset_1, y_1 \cup y_2)\\ \emptyset_{r4} &= (x_1, \emptyset_2)\\ \emptyset_{r5} &= (x_2, \emptyset_2)\\ \emptyset_{r6} &= (x_1 \cup x_2, \emptyset_2)\end{aligned}$$

and thus

$$\emptyset_{gr} = \emptyset_a \cup \emptyset_{r1} \cup \emptyset_{r2} \cup \ldots \cup \emptyset_{r6}$$

Based on definition of joint frame Θ with operations on its elements, we need to choose its underlying model (Shafers, free or hybrid model) to define its fusion sapce where the bba's will be defined on. According to the defintion of absolute and relative minimal elements, we then assume for the given VIEB problem that Θ satisfies Shafer's model, i.e. all (triplets) elements $\theta_i \in \Theta$ are exclusive, so that the bba's of sources will be defined on the classical power-set 2^{Θ}.

2.3 Choice of bba's of sources

Let's define first the bba of each source without regard to what could be their reliability and importance in the fusion process. Reliability and importance will be examined in details in next section.

- **Bba related with source 0** (prior information): The prior information states that the suspect A drives a white Toyota, and nothing is state about the prior information with respect to his location, so that we must consider the bba's representing the prior information as

$$\begin{aligned} m_0(\theta_4 \cup \theta_8) &= m_0((A, V, \bar{B}) \cup (A, V, B)) \\ &= m_0((A, V, B \cup \bar{B})) \\ &= 1 \end{aligned}$$

- **Bba related with source 1** (Analyst 1 with 10 years experience): The source 1 reports that the suspect A is probably now near the building B. This source however doesn't report explicitly that the suspect A is still with its white Toyota car or not. So the fair way to model this report when working on Θ is to commit a high mass of belief to the element $\theta_6 \cup \theta_8$, that is

$$\begin{aligned} m_1(\theta_6 \cup \theta_8) &= m_1((A, \bar{V}, B) \cup (A, V, B)) \\ &= m_1((A, V \cup \bar{V}, B)) \\ &= 0.75 \end{aligned}$$

and to commit the uncommitted mass to I_t based on the principle of minimum of specificity, so that

$$m_1(\theta_6 \cup \theta_8) = 0.75 \quad \text{and} \quad m_1(I_t) = 0.25$$

- **Bba related with source 2** (ANPR system): The source 3 reports 30% probability that the vehicle is individual A's wite Toyota. Nothing is reported on the position information. The information provided by this source corresponds actually to incomplete probabilistic information. Indeed, when working on $\Theta_1 \times \Theta_2$, what we only know is that $P\{(A, V)\} = 0.3$ and $P\{(\bar{A}, V) \cup (A, \bar{V}) \cup (\bar{A}, \bar{V})\} = 0.7$ (from additivity axiom of probability theory) and thus the bba $m_2(.)$ we must choose on $\Theta_1 \times \Theta_2 \times \Theta_3$ has to be compatible with this incomplete probabilistic information, i.e. the projection $m'_2(.) \triangleq m_2^{\downarrow \Theta_1 \times \Theta_2}(.)$ of $m_2(.)$ on $\Theta_1 \times \Theta_2$ must satisfy the following constraints on belief and plausibility functions

$$Bel'((A, V)) = 0.3$$

$$Bel'((\bar{A}, V) \cup (A, \bar{V}) \cup (\bar{A}, \bar{V})) = 0.7$$

and also

$$Pl'((A, V)) = 0.3$$

$$Pl'((\bar{A}, V) \cup (A, \bar{V}) \cup (\bar{A}, \bar{V})) = 0.7$$

because belief and plausibility correspond to lower and upper bounds of probability measure [5]. So it is easy to verify that the following bba $m'_2(.)$ satisfy these constraints because the elements of the frame $\Theta_1 \times \Theta_2$ are exclusive:

$$m'_2((A, V)) = 0.3$$

$$m'_2((\bar{A}, V) \cup (A, \bar{V}) \cup (\bar{A}, \bar{V})) = 0.7$$

We can then extend $m'_2(.)$ into $\Theta_1 \times \Theta_2 \times \Theta_3$ using the minimum specificity principle (i.e. take the vacuous extension of $m'_2(.)$) to get the bba $m_2(.)$ that we need to solve the VIEB problem. That is $m_2(.) = m_2'^{\uparrow \Theta_1 \times \Theta_2 \times \Theta_3}(.)$ with

$$m_2((A, V, B \cup \bar{B})) = 0.3$$

$$m_2((\bar{A}, V, B \cup \bar{B}) \cup (A, \bar{V}, B \cup \bar{B}) \cup (\bar{A}, \bar{V}, B \cup \bar{B})) = 0.7$$

or equivalently

$$m_2(\theta_4 \cup \theta_8) = 0.3$$

$$m_2(\theta_1 \cup \theta_2 \cup \theta_3 \cup \theta_5 \cup \theta_6 \cup \theta_7) = 0.7$$

- **Bba related with source 3** (Analyst 3 with no experience): The source 3 reports that it is improbable that the suspect A is near the building B. This source however doesn't report explicitly that the suspect A is still with its white Toyota car or not. So the fair way to model this report when working on Θ is to commit a low mass of belief to the element $\theta_6 \cup \theta_8$, that is

$$\begin{aligned} m_3(\theta_6 \cup \theta_8) &= m_3((A, \bar{V}, B) \cup (A, V, B)) \\ &= m_3((A, V \cup \bar{V}, B)) \\ &= 0.25 \end{aligned}$$

and to commit the uncommitted mass to I_t based on the principle of minimum of specificity, so that

$$m_3(\theta_6 \cup \theta_8) = 0.25 \quad \text{and} \quad m_3(I_t) = 0.75$$

2.4 Reliability of sources

Let's identify what is known about the reliability of sources and information:

- **Reliability of prior information**: it is (implicitly) supposed that the prior information is 100% reliable that is "Suspect A drives a white Toyota" which corresponds to the element $(A, V, B) \cup (A, V, \bar{B})$. So we can take the reliability factor of prior information as $\alpha_0 = 1$. If one considers the priori information highly reliable (but not totally reliable) then one could take $\alpha_0 = 0.9$ so that $m_0(.)$ would be

$$m_0(\theta_4 \cup \theta_8) = 0.9 \quad \text{and} \quad m_0(I_t) = 0.1$$

- **Reliability of source 1**: One knows that Analyst # 1 has 10 years experience, so we must consider him/her having a good reliability (say greater than 75%) or to be less precise we can just assign to him a qualitative reliability factor with minimal number of labels in $\{L_1 = \text{not good}, L_2 = \text{good}\}$. Here we should choose $\alpha_1 = L_2$. As first approximation, we can consider $\alpha_1 = 1$.
- **Reliability of source 2**: No information about the reliability of ANPR system is explicitly given. We may consider that if such device is used it is because it is also considered as a valuable tool and thus we assume it has a good reliability too, that is $\alpha_2 = 1$. If we are prudent we should consider the reliability factor of this source as totally unknown and thus we should take it as very imprecise with $\alpha_2 = [0, 1]$ (or qualitatively as $\alpha_2 = [L_0, L_3]$). If we are more optimistic and consider ANPR system as reliable enough, we could take α_2 a bit more precise with $\alpha_2 = [0.75, 1]$ (i.e. $\alpha_2 \geq 0.75$) or just qualitatively as $\alpha_2 = L_2$.
- **Reliability of source 3**: It is said explicitly that Analyst 2 is new in post, which means that Analyst 2 has no great experience and it can be inferred logically that it is less reliable than Analyst 1 so that we must choose $\alpha_3 < \alpha_1$. But we can also have a very young brillant analyst who perform very well too with respect to the older Analyst 1. So to be more cautious/prudent, we should also consider the case of unknown reliability factor α_3 by taking qualitatively $\alpha_3 = [L_0, L_3]$ or quantitatively by taking α_3 as a very imprecise value that is $\alpha_3 = [0, 1]$.

2.5 Importance of sources

Not that much is explicitly said about the importance of the sources of information in the VIEB problem statement, but the fact that Analyst 1 has ten years experience and Analyst 2 is new in post, so that it seems logical to choose as importance factor $\beta_1 > \beta_3$. The importances discounting factors have been introduced and presented by the authors in [2, 7]. As a prudent attitude we could choose also $\beta_0 = [0, 1] = [L_0, L_3]$ and $\beta_2 = [0, 1] = [L_0, L_3]$ (vey imprecise values). If we consider that the prior information and the source 2 (ANPR) have the same importance, we could just take $\beta_0 = \beta_1 = 1$ to make derivations easier and adopt a more optimistic point of view[1].

3 Solution of VIEB problem

We apply PCR5 and PCR6 fusion rules developed originally in the DSmT framework to get the solution of the VIEB problem. PCR5 has been developed by the authors in [6], Vol.2, and PCR6 is a variant of PCR5 proposed by Arnaud Martin and Christophe Osswald in [3]. Several codes for using PCR5 and PCR6 have been proposed in the literature for example in [3, 1, 7] and are available to the authors upon request.

Two cases are explored depending on the taking into account or not of the reliability and the importance of sources in the fusion process. To simplify the presentation of the results we denote the focal elements involved in this VIEB problem as:

$$f_1 \triangleq \theta_4 \cup \theta_8$$
$$f_2 \triangleq \theta_6 \cup \theta_8$$
$$f_3 \triangleq \theta_1 \cup \theta_2 \cup \theta_3 \cup \theta_5 \cup \theta_6 \cup \theta_7 = \overline{\theta_4 \cup \theta_8}$$
$$f_4 \triangleq I_t = \theta_1 \cup \theta_2 \cup \theta_3 \cup \theta_4 \cup \theta_5 \cup \theta_6 \cup \theta_7 \cup \theta_8$$

Only these focal elements are involved in inputs of the problem and we recall the two questions that we must answer:

Question 1 (Q1): Should building B be evacuated?

The question 1 must be answered by analyzing the level of belief and plausibility committed in the propositions supporting B through the fusion process.

Question 2 (Q2): Is experience (Analyst 1) more valuable than physics (the ANPR system) combined with inexperience (Analyst 2)? How do we model that?

The question 2 must be answered by analyzing and comparing the results of the fusion $m_1 \oplus m_3$ (or eventually $m_0 \oplus m_1 \oplus m_3$) with respect to m_2 only (resp. $m_0 \oplus m_2$).

3.1 Without reliability and importance

We provide here the solutions of the VIEB problem with direct PCR5 and PCR6 fusion of the sources for different qualitative inputs summarized in the tables below. We also present the result of DSmP probabilistic transformation [6] (Vol.3, Chap. 3) of resulting bba's to get and approximate probability measure of elements of Θ. No importance and reliability discounting has been applied since in this section, we consider that all sources have same importances and same reliabilities.

Example 1: We take the bba's described in section 2.3, that is

focal element	$m_0(.)$	$m_1(.)$	$m_2(.)$	$m_3(.)$
$\theta_4 \cup \theta_8$	1	0	0.3	0
$\theta_6 \cup \theta_8$	0	0.75	0	0.25
$\overline{\theta_4 \cup \theta_8}$	0	0	0.7	0
I_t	0	0.25	0	0.75

Table 1: Quantitative inputs of VIEB problem.

[1] Of course the importance discounting factors can also be chosen arbitrarily from exogenous information upon the desiderata of the fusion system designer. This question is out of the scope of this paper.

focal element	$m_{PCR5}(.)$	$m_{PCR6}(.)$
$\theta_1 \cup \theta_2 \cup \theta_3 \cup \theta_5 \cup \theta_6 \cup \theta_7$	0.19740	0.16811
θ_8	0.24375	0.24375
$\theta_4 \cup \theta_8$	0.33826	0.29641
$\theta_6 \cup \theta_8$	0.11029	0.14587
I_t	0.11029	0.14587

Table 2: Results of $m_0 \oplus m_1 \oplus m_2 \oplus m_3$ for Table 1.

Singletons	$DSmP_{\epsilon,PCR5}(.)$	$DSmP_{\epsilon,PCR6}(.)$
θ_1	0.0333	0.0286
θ_2	0.0333	0.0286
θ_3	0.0333	0.0286
θ_4	0.0018	0.0018
θ_5	0.0333	0.0286
θ_6	0.0338	0.0292
θ_7	0.0333	0.0286
θ_8	0.7977	0.8260

Table 3: $DSmP_\epsilon$ of $m_0 \oplus m_1 \oplus m_2 \oplus m_3$ for Table 1.

From fusion result of Table 2, one gets when considering θ_8 only

- with PCR5: $\Delta(\theta_8) = \Delta(\bar{\theta}_8) = 0.5588$

$$P(\theta_8) \in [0.24375, 0.8026]$$

$$P(\bar{\theta}_8) \in [0.1974, 0.7562]$$

- with PCR6: $\Delta(\theta_8) = \Delta(\bar{\theta}_8) = 0.5881$

$$P(\theta_8) \in [0.24375, 0.8319]$$

$$P(\bar{\theta}_8) \in [0.1681, 0.7562]$$

where $\Delta(X) = Pl(X) - Bel(X)$ is the imprecision related to $P(X)$.

If we are only interested by the fact that there is potentially a dangerous White Toyota vehicle near the building B, without regards to the presence or not of the suspicious individual A (he/she may have left the car parked with explosive in it near the building). Then we are concerned only in the evaluation of the proposition supporting B and V which is $\theta_7 \cup \theta_8$. Therefore from the fusion result of Table 2, one gets

- with PCR5: $\Delta(\theta_7 \cup \theta_8) = \Delta(\overline{\theta_7 \cup \theta_8}) = 0.7562$

$$P(\theta_7 \cup \theta_8) \in [0.24375, 1]$$

$$P(\overline{\theta_7 \cup \theta_8}) \in [0, 0.7562]$$

- with PCR6: $\Delta(\theta_7 \cup \theta_8) = \Delta(\overline{\theta_7 \cup \theta_8}) = 0.7562$

$$P(\theta_7 \cup \theta_8) \in [0.24375, 1]$$

$$P(\overline{\theta_7 \cup \theta_8}) \in [0, 0.7562]$$

If one approximates the bba's into probabilistic measures with DSmP transformation[2], one gets results with $\epsilon = 0.001$ presented in Table 3, more specifically one gets very high probabilities in θ_8 or in $\theta_7 \cup \theta_8$ because

$$DSmP_{\epsilon,PCR5}(\theta_8) = 0.7977$$

$$DSmP_{\epsilon,PCR6}(\theta_8) = 0.8260$$

$$DSmP_{\epsilon,PCR5}(\theta_7 \cup \theta_8) = 0.8310$$

$$DSmP_{\epsilon,PCR6}(\theta_7 \cup \theta_8) = 0.8546$$

- **Answer to Q1:** One sees that the result provided by PCR6 is less precise than with PCR5, but PCR6 provides higher upper bound on $P(\theta_8)$ than with PCR5. Based on these very imprecise results, it is very difficult to take the right decision without decision-making error because the sources of information are highly uncertain and conflicting, but the analysis of lower and upper bounds show that the most reasonable answer to the question based either on max of credibility or max of plausibility is to evacuate the building B since $Bel(\theta_8) > Bel(\bar{\theta}_8)$ and also $Pl(\theta_8) > Pl(\bar{\theta}_8)$. Same conclusion must be drawn when considering the element $theta_7 \cup \theta_8$. The same conclusion also is drawn (more easier) based on DSmP values. In summary, the answer to Q1 is : **Evacuation of the building B.**

In order to answer to the second question (Q2), let's compute the fusion results of the fusion $m_0 \oplus m_2$ and $m_0 \oplus m_1 \oplus m_3$ using inputs given in Table 1. The fusion results with corresponding DSmP are given in the Tables 4-7.

focal element	$m_{PCR5}(.)$	$m_{PCR6}(.)$
$\theta_1 \cup \theta_2 \cup \theta_3 \cup \theta_5 \cup \theta_6 \cup \theta_7$	0.28824	0.28824
$\theta_4 \cup \theta_8$	0.71176	0.71176

Table 4: Result of $m_0 \oplus m_2$.

Singletons	$DSmP_{\epsilon,PCR5}(.)$	$DSmP_{\epsilon,PCR6}(.)$
θ_1	0.0480	0.0480
θ_2	0.0480	0.0480
θ_3	0.0480	0.0480
θ_4	0.3559	0.3559
θ_5	0.0480	0.0480
θ_6	0.0480	0.0480
θ_7	0.0480	0.0480
θ_8	0.3559	0.3559

Table 5: $DSmP_\epsilon$ of $m_0 \oplus m_2$.

[2] DSmP transformation has been introduced and justified in details by the authors in the book [6] (Vol.3, Chap. 3) freely downloadable from the web with many examples, and therefore it will not be presented here.

Based on $m_0 \oplus m_2$ fusion result, one gets a large imprecision on $P(\theta_8)$ since $\Delta_{02}(\theta_8) = \Delta_{02}(\bar{\theta}_8) = 0.71176$ with

$$P(\theta_8) \in [0, 0.71176]$$
$$P(\bar{\theta}_8) \in [0.28824, 1]$$

and total imprecision when considering $\theta_7 \cup \theta_8$ since

$$P(\theta_7 \cup \theta_8) \in [0, 1]$$
$$P(\overline{\theta_7 \cup \theta_8}) \in [0, 1]$$

Based on max of Bel or max of Pl criteria[3] the decision using $m_0 \oplus m_2$ (i.e. with prior information m_0 and ANPR system m_2) should be to NOT evacuate the building B. Same decision would be taken based on DSmP values.

focal element	$m_{PCR5}(.)$	$m_{PCR6}(.)$
θ_8	0.8125	0.8125
$\theta_4 \cup \theta_8$	0.1875	0.1875

Table 6: Result of $m_0 \oplus m_1 \oplus m_3$.

Singletons	$DSmP_{\epsilon,PCR5}(.)$	$DSmP_{\epsilon,PCR6}(.)$
θ_1	0	0
θ_2	0	0
θ_3	0	0
θ_4	0.0002	0.0002
θ_5	0	0
θ_6	0	0
θ_7	0	0
θ_8	0.9998	0.9998

Table 7: $DSmP_\epsilon$ of $m_0 \oplus m_1 \oplus m_3$.

Based on $m_0 \oplus m_1 \oplus m_3$ fusion result, one gets $\Delta_{013}(\theta_8) = \Delta_{013}(\bar{\theta}_8) = 0.1875$ and

$$P(\theta_8) \in [0.8125, 1]$$
$$P(\bar{\theta}_8) \in [0, 0.1875]$$

and also

$$P(\theta_7 \cup \theta_8) \in [0.8125, 1]$$
$$P(\overline{\theta_7 \cup \theta_8}) \in [0, 0.1875]$$

Based on max of Bel or max of Pl criteria, the decision using $m_0 \oplus m_1 \oplus m_3$ (i.e. with prior information m_0 and both analysts) should be to evacuate the building B. Same decision would be taken based on DSmP values. It is worth to note that the precision on the result obtained with $m_0 \oplus m_1 \oplus m_3$ is much better than with $m_0 \oplus m_2$ since $\Delta_{013}(\theta_8) < \Delta_{02}(\theta_8)$, or $\Delta_{013}(\theta_7 \cup \theta_8) < \Delta_{02}(\theta_7 \cup \theta_8)$. Moreover it is easy to verify that $m_0 \oplus m_1 \oplus m_3$ fusion system is more informative than $m_0 \oplus m_2$ fusion system because Shannon entropy of DSmP of $m_0 \oplus m_2$ is much bigger than Shannon entropy of DSmP of $m_0 \oplus m_1 \oplus m_3$.

[3]We consider here only θ_8 for decision-making since it is not possible to take any rational decision from $\theta_7 \cup \theta_8$ because of the full imprecision range of $P(\theta_7 \cup \theta_8)$.

- **Answer to Q2:** Since the information obtained by the fusion $m_0 \oplus m_2$ is less informative and less precise than the information obtained with the fusion $m_0 \oplus m_1 \oplus m_3$, it is better to choose and to trust the fusion system $m_0 \oplus m_1 \oplus m_3$ rather than $m_0 \oplus m_2$. Based on this choice, the final decision will be to evacuate the building B which is consistent with answer to question Q1.

Example 2: Let's modify a bit the previous Table 1 and take higher belief for sources 1 and 3 as

focal element	$m_0(.)$	$m_1(.)$	$m_2(.)$	$m_3(.)$
$\theta_4 \cup \theta_8$	1	0	0.3	0
$\theta_6 \cup \theta_8$	0	0.9	0	0.1
$\overline{\theta_4 \cup \theta_8}$	0	0	0.7	0
I_t	0	0.1	0	0.9

Table 8: Quantitative inputs of VIEB problem.

The results of the fusion $m_0 \oplus m_1 \oplus m_2 \oplus m_3$ using PCR5 and PCR6 and the corresponding DSmP values are given in tables 9-10.

focal element	$m_{PCR5}(.)$	$m_{PCR6}(.)$
$\theta_1 \cup \theta_2 \cup \theta_3 \cup \theta_5 \cup \theta_6 \cup \theta_7$	0.16525	0.14865
θ_8	0.27300	0.27300
$\theta_4 \cup \theta_8$	0.26308	0.23935
$\theta_6 \cup \theta_8$	0.14934	0.16950
I_t	0.14934	0.16950

Table 9: Results of $m_0 \oplus m_1 \oplus m_2 \oplus m_3$ for Table 8.

Singletons	$DSmP_{\epsilon,PCR5}(.)$	$DSmP_{\epsilon,PCR6}(.)$
θ_1	0.0281	0.0254
θ_2	0.0281	0.0254
θ_3	0.0281	0.0254
θ_4	0.0015	0.0015
θ_5	0.0281	0.0254
θ_6	0.0286	0.0260
θ_7	0.0281	0.0254
θ_8	0.8295	0.8456

Table 10: $DSmP_\epsilon$ of $m_0 \oplus m_1 \oplus m_2 \oplus m_3$ for Table 8.

From fusion result of Table 9, one gets when considering θ_8 only

- with PCR5: $\Delta(\theta_8) = \Delta(\bar{\theta}_8) = 0.5617$

$$P(\theta_8) \in [0.27300, 0.8347]$$
$$P(\bar{\theta}_8) \in [0.1653, 0.7270]$$

- with PCR6: $\Delta(\theta_8) = \Delta(\bar{\theta}_8) = 0.5784$

$$P(\theta_8) \in [0.27300, 0.8514]$$
$$P(\bar{\theta}_8) \in [0.1486, 0.7270]$$

and when considering $\theta_7 \cup \theta_8$

- with PCR5: $\Delta(\theta_7 \cup \theta_8) = \Delta(\overline{\theta_7 \cup \theta_8}) = 0.7270$

$$P(\theta_7 \cup \theta_8) \in [0.27300, 1]$$

$$P(\overline{\theta_7 \cup \theta_8}) \in [0, 0.7270]$$

- with PCR6: $\Delta(\theta_7 \cup \theta_8) = \Delta(\overline{\theta_7 \cup \theta_8}) = 0.7270$

$$P(\theta_7 \cup \theta_8) \in [0.27300, 1]$$

$$P(\overline{\theta_7 \cup \theta_8}) \in [0, 0.7270]$$

One gets also the following DSmP values

$$DSmP_{\epsilon,PCR5}(\theta_8) = 0.8295$$

$$DSmP_{\epsilon,PCR6}(\theta_8) = 0.8456$$

$$DSmP_{\epsilon,PCR5}(\theta_7 \cup \theta_8) = 0.8576$$

$$DSmP_{\epsilon,PCR6}(\theta_7 \cup \theta_8) = 0.8710$$

- **Answer to Q1:** Using an analysis similar to the one done for Example 1, based on max of credibility or max of plausibility criteria, or by considering the DSmP values of θ_8 or $\theta_7 \cup \theta_8$, the decision to take is : **Evacuate the building** B.

In order to answer to the second question (Q2) for this Example 2, let's compute the fusion results of the fusion $m_0 \oplus m_2$ and $m_0 \oplus m_1 \oplus m_3$ using inputs given in Table 8. Since the inputs m_0 and m_2 are the same as those in Example 1, the $m_0 \oplus m_2$ fusion results with corresponding DSmP are those already given in Tables 4-5. Only the fusion $m_0 \oplus m_1 \oplus m_3$ must be derived with the new bba's m_1 and m_3 chosen for this Example 2. The $m_0 \oplus m_1 \oplus m_3$ fusion results obtained with PCR5 and PCR6, and the corresponding DSmP values are shown in Tables 11-12. According to these results, one gets $\Delta_{013}(\theta_8) = \Delta_{013}(\bar{\theta}_8) = 0.09$ and

$$P(\theta_8) \in [0.91, 1], \quad P(\bar{\theta}_8) \in [0, 0.09]$$

and also

$$P(\theta_7 \cup \theta_8) \in [0.91, 1], \quad P(\overline{\theta_7 \cup \theta_8}) \in [0, 0.09]$$

focal element	$m_{PCR5}(.)$	$m_{PCR6}(.)$
θ_8	0.91	0.91
$\theta_4 \cup \theta_8$	0.09	0.09

Table 11: Result of $m_0 \oplus m_1 \oplus m_3$.

Based on max of Bel or max of Pl criteria, the decision using $m_0 \oplus m_1 \oplus m_3$ (i.e. with prior information m_0 and both analysts) should be to evacuate the building B. Same decision would be taken based on DSmP values. It is worth to note that the precision on the result obtained with $m_0 \oplus m_1 \oplus m_3$ is much better than with $m_0 \oplus m_2$ since $\Delta_{013}(\theta_8) < \Delta_{02}(\theta_8)$, or $\Delta_{013}(\theta_7 \cup \theta_8) < \Delta_{02}(\theta_7 \cup \theta_8)$. Moreover it is easy to verify that $m_0 \oplus m_1 \oplus m_3$ fusion system is more informative than $m_0 \oplus m_2$ fusion system because Shannon entropy of DSmP of $m_0 \oplus m_2$ is much bigger than Shannon entropy of DSmP of $m_0 \oplus m_1 \oplus m_3$.

Singletons	$DSmP_{\epsilon,PCR5}(.)$	$DSmP_{\epsilon,PCR6}(.)$
θ_1	0	0
θ_2	0	0
θ_3	0	0
θ_4	0.0001	0.0001
θ_5	0	0
θ_6	0	0
θ_7	0	0
θ_8	0.9999	0.9999

Table 12: $DSmP_\epsilon$ of $m_0 \oplus m_1 \oplus m_3$.

- **Answer to Q2:** Since the information obtained by the fusion $m_0 \oplus m_2$ is less informative and less precise than the information obtained with the fusion $m_0 \oplus m_1 \oplus m_3$, it is better to choose and to trust the fusion system $m_0 \oplus m_1 \oplus m_3$ rather than $m_0 \oplus m_2$. Based on this choice, the final decision will be to evacuate the building B which is consistent with answer to question Q1.

3.2 Impact of prior information

To see the impact of the quality/reliability of prior information on the result, let's modify the input $m_0(.)$ in previous Tables 1 and 8 and consider now a very uncertain prior source.

Example 3: We consider the very uncertain prior source of information $m_0(\theta_4 \cup \theta_8) = 0.1$ and $m_0(I_t) = 0.9$. The results for the modified inputs Table 13 ate given in Tables 14 and 15.

focal element	$m_0(.)$	$m_1(.)$	$m_2(.)$	$m_3(.)$
$\theta_4 \cup \theta_8$	0.1	0	0.3	0
$\theta_6 \cup \theta_8$	0	0.75	0	0.25
$\overline{\theta_4 \cup \theta_8}$	0	0	0.7	0
I_t	0.9	0.25	0	0.75

Table 13: Quantitative inputs of VIEB problem.

From the fusion result of Table 14, one gets when considering θ_8 only

- with PCR5: $\Delta(\theta_8) = \Delta(\bar{\theta}_8) = 0.0934$

$$P(\theta_8) \in [0.24375, 0.33710]$$

$$P(\bar{\theta}_8) \in [0.66290, 0.75620]$$

focal element	$m_{PCR5}(.)$	$m_{PCR6}(.)$
θ_6	0.511870	0.511870
$\theta_1 \cup \theta_2 \cup \theta_3 \cup \theta_5 \cup \theta_6 \cup \theta_7$	0.151070	0.142670
θ_8	0.243750	0.243750
$\theta_4 \cup \theta_8$	0.060957	0.059757
$\theta_6 \cup \theta_8$	0.016173	0.020973
I_t	0.016173	0.020973

Table 14: Result of $m_0 \oplus m_1 \oplus m_2 \oplus m_3$ for Table 13.

Singletons	$DSmP_{\epsilon,PCR5}(.)$	$DSmP_{\epsilon,PCR6}(.)$
θ_1	0.0003	0.0003
θ_2	0.0003	0.0003
θ_3	0.0003	0.0003
θ_4	0.0003	0.0003
θ_5	0.0003	0.0003
θ_6	0.6833	0.6815
θ_7	0.0003	0.0003
θ_8	0.3149	0.3168

Table 15: $DSmP_\epsilon$ of $m_0 \oplus m_1 \oplus m_2 \oplus m_3$ for Table 13.

- with PCR6: $\Delta(\theta_8) = \Delta(\bar{\theta}_8) = 0.1017$

$$P(\theta_8) \in [0.24375, 0.34550]$$

$$P(\bar{\theta}_8) \in [0.65450, 0.75620]$$

and when considering $\theta_7 \cup \theta_8$

- with PCR5: $\Delta(\theta_7 \cup \theta_8) = \Delta(\overline{\theta_7 \cup \theta_8}) = 0.2443$

$$P(\theta_7 \cup \theta_8) \in [0.24375, 0.48810]$$

$$P(\overline{\theta_7 \cup \theta_8}) \in [0.51190, 0.75620]$$

- with PCR6: $\Delta(\theta_7 \cup \theta_8) = \Delta(\overline{\theta_7 \cup \theta_8}) = 0.2443$

$$P(\theta_7 \cup \theta_8) \in [0.24375, 0.48810]$$

$$P(\overline{\theta_7 \cup \theta_8}) \in [0.51190, 0.75620]$$

Using DSmP transformation, one gets a low probability in θ_8 or in $\theta_7 \cup \theta_8$ because

$$DSmP_{\epsilon,PCR5}(\theta_8) = 0.3149$$

$$DSmP_{\epsilon,PCR6}(\theta_8) = 0.3168$$

$$DSmP_{\epsilon,PCR5}(\theta_7 \cup \theta_8) = 0.3152$$

$$DSmP_{\epsilon,PCR6}(\theta_7 \cup \theta_8) = 0.3171$$

- **Answer to Q1:** Based on these results, one sees that the decision to take is to NOT evacuate the building B since one gets a low probability in θ_8 or in $\theta_7 \cup \theta_8$. So there is a strong impact of prior information on the final decision since without strong prior information supporting $\theta_4 \cup \theta_8$ we have to conclude to the non evacuation of building B based either on the max of credibility, the max of plausibility or the max of DSmP considering both cases θ_8 or $\theta_7 \cup \theta_8$ for decision-making. The decision to take in this case is to: **NOT Evacuate the building** B.

Let's examine the results of fusion systems $m_0 \oplus m_2$ and $m_0 \oplus m_1 \oplus m_3$ given in Tables 16-12.

focal element	$m_{PCR5}(.)$	$m_{PCR6}(.)$
$\theta_1 \cup \theta_2 \cup \theta_3 \cup \theta_5 \cup \theta_6 \cup \theta_7$	0.69125	0.69125
$\theta_4 \cup \theta_8$	0.30875	0.30875

Table 16: Result of $m_0 \oplus m_2$.

Singletons	$DSmP_{\epsilon,PCR5}(.)$	$DSmP_{\epsilon,PCR6}(.)$
θ_1	0.1152	0.1152
θ_2	0.1152	0.1152
θ_3	0.1152	0.1152
θ_4	0.1544	0.1544
θ_5	0.1152	0.1152
θ_6	0.1152	0.1152
θ_7	0.1152	0.1152
θ_8	0.1544	0.1544

Table 17: $DSmP_\epsilon$ of $m_0 \oplus m_2$.

focal element	$m_{PCR5}(.)$	$m_{PCR6}(.)$
θ_8	0.08125	0.08125
$\theta_4 \cup \theta_8$	0.01875	0.01875
$\theta_6 \cup \theta_8$	0.73125	0.73125
I_t	0.16875	0.16875

Table 18: Result of $m_0 \oplus m_1 \oplus m_3$.

Singletons	$DSmP_{\epsilon,PCR5}(.)$	$DSmP_{\epsilon,PCR6}(.)$
θ_1	0.0019	0.0019
θ_2	0.0019	0.0019
θ_3	0.0019	0.0019
θ_4	0.0021	0.0021
θ_5	0.0019	0.0019
θ_6	0.0107	0.0107
θ_7	0.0019	0.0019
θ_8	0.9778	0.9778

Table 19: $DSmP_\epsilon$ of $m_0 \oplus m_1 \oplus m_3$.

Based on $m_0 \oplus m_2$ fusion result, one gets a substantial imprecision on $P(\theta_8)$ since $\Delta_{02}(\theta_8) = \Delta_{02}(\bar{\theta}_8) = 0.30875$ with

$$P(\theta_8) \in [0, 0.30875]$$

$$P(\bar{\theta}_8) \in [0.6912, 1]$$

and total imprecision when considering $\theta_7 \cup \theta_8$ since

$$P(\theta_7 \cup \theta_8) \in [0, 1]$$

$$P(\overline{\theta_7 \cup \theta_8}) \in [0, 1]$$

Based on max of Bel or max of Pl criteria on θ_8 the decision using $m_0 \oplus m_2$ (i.e. with uncertain prior information m_0 and ANPR system m_2) should be to NOT evacuate the building B. Same conclusion can be drawn using DSmP, since $DSmP(\bar{\theta}_8) = 0.8456$ is much bigger than $DSmP(\theta_8) = 0.1544$. Based on $m_0 \oplus m_1 \oplus m_3$ fusion result, one gets $\Delta_{013}(\theta_8) = \Delta_{013}(\bar{\theta}_8) = 0.9187$ and

$$P(\theta_8) \in [0.08125, 1]$$

$$P(\bar{\theta}_8) \in [0, 0.9187]$$

and also

$$P(\theta_7 \cup \theta_8) \in [0.08125, 1]$$

$$P(\overline{\theta_7 \cup \theta_8}) \in [0, 0.9187]$$

Based on max of Bel or max of Pl criteria, the decision using $m_0 \oplus m_1 \oplus m_3$ must be evacuate the building B. Same decision is drawn when using DSmP results according to Table 19. With this uncertain prior information, it is worth to note that the precision on the result obtained with $m_0 \oplus m_1 \oplus m_3$ is less than with $m_0 \oplus m_2$ since $\Delta_{013}(\theta_8) > \Delta_{02}(\theta_8)$, or $\Delta_{013}(\theta_7 \cup \theta_8) > \Delta_{02}(\theta_7 \cup \theta_8)$. However, it is easy to verify that $m_0 \oplus m_1 \oplus m_3$ fusion system is more informative than $m_0 \oplus m_2$ fusion system because Shannon entropy of DSmP of $m_0 \oplus m_2$ is much bigger than Shannon entropy of DSmP of $m_0 \oplus m_1 \oplus m_3$.

- **Answer to Q2:** The answer of question Q2 is not easy because it depends on the criterion we choose. Based on precision criterion, it is better to trust $m_0 \oplus m_2$ fusion system since $\Delta_{02}(\theta_8) = \Delta_{02}(\bar{\theta}_8) = 0.30875$ whereas $\Delta_{013}(\theta_8) = \Delta_{013}(\bar{\theta}_8) = 0.9187$. In such case, one should NOT evacuate the building B. If we consider that is better to trust result of $m_0 \oplus m_1 \oplus m_3$ fusion system because it is more informative than $m_0 \oplus m_2$ then the decision should be to evacuate the building B. So the main open question is what solution to choose for selecting either $m_0 \oplus m_2$ or $m_0 \oplus m_1 \oplus m_3$ fusion system ? In authors opinion, in such case it seems better to base our choice on the precision level of information one has really in hands, rather using entropy measure (or its dual measure defined as the Probabilistic Information Content (PIC) in [9]) related with some probabilistic transformation of bba into approximate subjective probability measure. Using approximate probabilities (DSmP,BetP or whatever) always introduces some ad-hocity in the approximation which is not good. So based on precision criterion, we suggest in that case to select $m_0 \oplus m_2$ fusion system instead of $m_0 \oplus m_1 \oplus m_3$ and to decide finally to NOT evacuate building B.

Example 4: Let's modify a bit the previous input Table 13 and take higher belief for sources 1 and 3 as Therefore, one gets when considering θ_8 only

- with PCR5: $\Delta(\theta_8) = \Delta(\bar{\theta}_8) = 0.0713$

$$P(\theta_8) \in [0.2730, 0.3443]$$

$$P(\bar{\theta}_8) \in [0.6557, 0.7270]$$

focal element	$m_0(.)$	$m_1(.)$	$m_2(.)$	$m_3(.)$
$\theta_4 \cup \theta_8$	0.1	0	0.3	0
$\theta_6 \cup \theta_8$	0	0.9	0	0.1
$\overline{\theta_4 \cup \theta_8}$	0	0	0.7	0
I_t	0.9	0.1	0	0.9

Table 20: Quantitative inputs of VIEB problem.

focal element	$m_{PCR5}(.)$	$m_{PCR6}(.)$
θ_6	0.573300	0.573300
$\theta_1 \cup \theta_2 \cup \theta_3 \cup \theta_5 \cup \theta_6 \cup \theta_7$	0.082365	0.077355
θ_8	0.273000	0.273000
$\theta_4 \cup \theta_8$	0.030666	0.029951
$\theta_6 \cup \theta_8$	0.020334	0.023197
I_t	0.020334	0.023197

Table 21: Result of $m_0 \oplus m_1 \oplus m_2 \oplus m_3$ for Table 20.

Singletons	$DSmP_{\epsilon,PCR5}(.)$	$DSmP_{\epsilon,PCR6}(.)$
θ_1	0.0002	0.0002
θ_2	0.0002	0.0002
θ_3	0.0002	0.0002
θ_4	0.0001	0.0001
θ_5	0.0002	0.0002
θ_6	0.6824	0.6813
θ_7	0.0002	0.0002
θ_8	0.3166	0.3178

Table 22: $DSmP_\epsilon$ of $m_0 \oplus m_1 \oplus m_2 \oplus m_3$ for Table 20.

- with PCR6: $\Delta(\theta_8) = \Delta(\bar{\theta}_8) = 0.0763$

$$P(\theta_8) \in [0.2730, 0.3493]$$

$$P(\bar{\theta}_8) \in [0.6507, 0.7270]$$

and when considering $\theta_7 \cup \theta_8$

- with PCR5: $\Delta(\theta_7 \cup \theta_8) = \Delta(\overline{\theta_7 \cup \theta_8}) = 0.1537$

$$P(\theta_7 \cup \theta_8) \in [0.2730, 0.4267]$$

$$P(\overline{\theta_7 \cup \theta_8}) \in [0.5733, 0.7270]$$

- with PCR6: $\Delta(\theta_7 \cup \theta_8) = \Delta(\overline{\theta_7 \cup \theta_8}) = 0.1537$

$$P(\theta_7 \cup \theta_8) \in [0.2730, 0.4267]$$

$$P(\overline{\theta_7 \cup \theta_8}) \in [0.5733, 0.7270]$$

Based on DSmP transformation, one gets a low probability in θ_8 or in $\theta_7 \cup \theta_8$ because

$$DSmP_{\epsilon,PCR5}(\theta_8) = 0.3166$$

$$DSmP_{\epsilon,PCR6}(\theta_8) = 0.3178$$

$$DSmP_{\epsilon,PCR5}(\theta_7 \cup \theta_8) = 0.3168$$

$$DSmP_{\epsilon,PCR6}(\theta_7 \cup \theta_8) = 0.3180$$

- **Answer to Q1:** Based on these results, one sees that the decision based either on the max of credibility, the max of plausibility or the max of DSmP considering both cases θ_8 or $\theta_7 \cup \theta_8$ is to: **NOT Evacuate the building** B.

Let's examine the results of fusion systems $m_0 \oplus m_2$ and $m_0 \oplus m_1 \oplus m_3$ corresponding to the input Table 20. Naturally, one gets same results for the fusion $m_0 \oplus m_2$ as in Example 3 and for the fusion $m_0 \oplus m_1 \oplus m_3$ one gets:

focal element	$m_{PCR5}(.)$	$m_{PCR6}(.)$
θ_8	0.091	0.091
$\theta_4 \cup \theta_8$	0.009	0.009
$\theta_6 \cup \theta_8$	0.819	0.819
I_t	0.081	0.081

Table 23: Result of $m_0 \oplus m_1 \oplus m_3$.

Singletons	$DSmP_{\epsilon,PCR5}(.)$	$DSmP_{\epsilon,PCR6}(.)$
θ_1	0.0008	0.0008
θ_2	0.0008	0.0008
θ_3	0.0008	0.0008
θ_4	0.0009	0.0009
θ_5	0.0008	0.0008
θ_6	0.0096	0.0096
θ_7	0.0008	0.0008
θ_8	0.9854	0.9854

Table 24: $DSmP_\epsilon$ of $m_0 \oplus m_1 \oplus m_3$.

As in Example 3, based on $m_0 \oplus m_2$ fusion result, one gets $\Delta_{02}(\theta_8) = \Delta_{02}(\bar{\theta}_8) = 0.30875$ with

$$P(\theta_8) \in [0, 0.30875]$$

$$P(\bar{\theta}_8) \in [0.6912, 1]$$

and total imprecision when considering $\theta_7 \cup \theta_8$ since

$$P(\theta_7 \cup \theta_8) \in [0, 1]$$

$$P(\overline{\theta_7 \cup \theta_8}) \in [0, 1]$$

Based on $m_0 \oplus m_1 \oplus m_3$ fusion result, one gets $\Delta_{013}(\theta_8) = \Delta_{013}(\bar{\theta}_8) = 0.9090$ and

$$P(\theta_8) \in [0.091, 1]$$

$$P(\bar{\theta}_8) \in [0, 0.9090]$$

and also

$$P(\theta_7 \cup \theta_8) \in [0.091, 1]$$

$$P(\overline{\theta_7 \cup \theta_8}) \in [0, 0.9090]$$

Based on max of Bel or max of Pl criteria on θ_8, the decision using $m_0 \oplus m_1 \oplus m_3$ must be the evacuation of the building B. Same decision must be drawn when using DSmP results according to Table 24. With this uncertain prior information, it is worth to note that the precision on the result obtained with $m_0 \oplus m_1 \oplus m_3$ is less than with $m_0 \oplus m_2$ since $\Delta_{013}(\theta_8) > \Delta_{02}(\theta_8)$, or $\Delta_{013}(\theta_7 \cup \theta_8) > \Delta_{02}(\theta_7 \cup \theta_8)$. However, it is easy to verify that $m_0 \oplus m_1 \oplus m_3$ fusion system is more informative than $m_0 \oplus m_2$ fusion system because Shannon entropy of DSmP of $m_0 \oplus m_2$ is much bigger than Shannon entropy of DSmP of $m_0 \oplus m_1 \oplus m_3$.

- **Answer to Q2:** Similar remarks and conclusions to those given in Example 3 held also for Example 4, i.e. it is better to trust the most precise fusion system $m_0 \oplus m_2$ and to decide to NOT evacuate the building B if one has in hands such highly uncertain prior information m_0.

3.3 Impact of no prior information

Example 5: Let's examine the result of the fusion process if one doesn't include[4] the prior information $m_0(.)$ and if we combine directly only the three sources $m_1 \oplus m_2 \oplus m_3$ altogether with PCR5 or PCR6.

focal element	$m_1(.)$	$m_2(.)$	$m_3(.)$
$\theta_4 \cup \theta_8$	0	0.3	0
$\theta_6 \cup \theta_8$	0.75	0	0.25
$\overline{\theta_4 \cup \theta_8}$	0	0.7	0
I_t	0.25	0	0.75

Table 25: Quantitative inputs of VIEB problem.

focal element	$m_{PCR5}(.)$	$m_{PCR6}(.)$
θ_6	0.56875	0.56875
$\theta_1 \cup \theta_2 \cup \theta_3 \cup \theta_5 \cup \theta_6 \cup \theta_7$	0.13125	0.13125
θ_8	0.24375	0.24375
$\theta_4 \cup \theta_8$	0.05625	0.05625

Table 26: Result of $m_1 \oplus m_2 \oplus m_3$ for Table 25.

Singletons	$DSmP_{\epsilon,PCR5}(.)$	$DSmP_{\epsilon,PCR6}(.)$
θ_1	0.0002	0.0002
θ_2	0.0002	0.0002
θ_3	0.0002	0.0002
θ_4	0.0002	0.0002
θ_5	0.0002	0.0002
θ_6	0.6989	0.6989
θ_7	0.0002	0.0002
θ_8	0.2998	0.2998

Table 27: $DSmP_\epsilon$ of $m_1 \oplus m_2 \oplus m_3$ for Table 25.

- **Answer to Q1:** The result presented in Table 26 is obviously the same as the one we obtain by combining the sources $m_0 \oplus m_1 \oplus m_2 \oplus m_3$ altogether when taking $m_0(.)$ as the vacuous belief assignment, i.e. when $m_0(I_t) = 1$. Based on results of

[4] Or equivalently we can take m_0 as the vacuous bba corresponding to $m_0(I_t) = 1$ and to the fully ignorant prior source.

Tables 26-27 the decision based on max of belief, max of plausibility on either θ_8 or $\theta_7 \cup \theta_8$ will be to NOT evacuate building B. Same conclusions is obtained when analyzing values of DSmP of θ_8 or $\theta_7 \cup \theta_8$. The decision to take in this case is to: **NOT Evacuate the building** B. So we see the strong impact of the miss of prior information in the decision-making support process (by comparison between Example 1 and this example).

Let's compare now the source m_2 with respect to the $m_1 \oplus m_3$ fusion system when no prior information m_0 is used. Naturally, there is no need to fusion m_2 since we consider it alone and one has $\Delta_2(\theta_8) = \Delta_2(\bar{\theta}_8) = 0.3$ with

$$P(\theta_8) \in [0, 0.3]$$

$$P(\bar{\theta}_8) \in [0.7, 1]$$

and we get a total imprecision when considering $\theta_7 \cup \theta_8$ since

$$P(\theta_7 \cup \theta_8) \in [0, 1]$$

$$P(\overline{\theta_7 \cup \theta_8}) \in [0, 1]$$

and its corresponding DSmP is given by

Singletons	$DSmP_{\epsilon,PCR5}(.)$
θ_1	0.1167
θ_2	0.1167
θ_3	0.1167
θ_4	0.1500
θ_5	0.1167
θ_6	0.1167
θ_7	0.1167
θ_8	0.1500

Table 28: $DSmP_\epsilon$ of m_2.

Based on max of Bel or max of Pl criteria on θ_8 the decision using m_2 (ANPR system alone) should be to NOT evacuate the building B. Same conclusion can be drawn using DSmP (see, since $DSmP(\bar{\theta}_8) = 0.85$ is much bigger than $DSmP(\theta_8) = 0.15$.

Now if we combine m_1 with m_3 using PCR5 or PCR6 we get[5] results given in Tables 29 and 30.

focal element	$m_{PCR5}(.)$	$m_{PCR6}(.)$
$\theta_6 \cup \theta_8$	0.8125	0.8125
I_t	0.1875	0.1875

Table 29: Result of $m_1 \oplus m_3$.

Whence $\Delta_{013}(\theta_8) = \Delta_{013}(\bar{\theta}_8) = 1$ and

$$P(\theta_8) \in [0, 1]$$

$$P(\bar{\theta}_8) \in [0, 1]$$

[5]Note that for two sources, PCR6 equals PCR5 [?], Vol.2.

Singletons	$DSmP_{\epsilon,PCR5}(.)$	$DSmP_{\epsilon,PCR6}(.)$
θ_1	0.0234	0.0234
θ_2	0.0234	0.0234
θ_3	0.0234	0.0234
θ_4	0.0234	0.0234
θ_5	0.0234	0.0234
θ_6	0.4297	0.4297
θ_7	0.0234	0.0234
θ_8	0.4297	0.4297

Table 30: $DSmP_\epsilon$ of $m_1 \oplus m_3$.

and also

$$P(\theta_7 \cup \theta_8) \in [0, 1]$$

$$P(\overline{\theta_7 \cup \theta_8}) \in [0, 1]$$

Based on max of Bel or max of Pl criteria on θ_8, the decision using $m_1 \oplus m_3$ must be the evacuation of the building B. Same decision must be drawn when using DSmP results according to Table 30.

- **Answer to Q2:** Similar remarks and conclusions to those given in Example 3 held also for Example 5, i.e. it is better to trust the most precise source, i.e. the APNR system m_2 and to decide to NOT evacuate the building B if one has no prior information at all rather than using the fusion of sources m_1 and m_3.

Example 6: It can be easily verified that the same analysis, remarks and conclusions for Q1 and Q2 as for Example 5 also hold when considering the sources m_1 and m_3 corresponding to the following input Table

focal element	$m_1(.)$	$m_2(.)$	$m_3(.)$
$\theta_4 \cup \theta_8$	0	0.3	0
$\theta_6 \cup \theta_8$	0.90	0	0.10
$\overline{\theta_4 \cup \theta_8}$	0	0.7	0
I_t	0.10	0	0.90

Table 31: Quantitative inputs of VIEB problem.

3.4 Impact of reliability of sources

The reliability of sources (when known) can be easily taken into using Shafer's classical discounting technique [5], p. 252, which consists in multiplying the masses of focal elements by the reliability factor α, and transferring all the remaining discounted mass to the full ignorance Θ. When $\alpha < 1$, such very simple reliability discounting technique discounts all focal elements with the same factor α and it increases the non specificity of the discounted sources since the mass committed to the full ignorance always increases. When $\alpha = 1$, no reliability discounting occurs (the bba is kept unchanged). Mathematically, Shafer's discounting technique for taking into account the reliability factor $\alpha \in [0, 1]$ of a given source with a bba $m(.)$ and a frame Θ is defined by:

$$\begin{cases} m_\alpha(X) = \alpha \cdot m(X), & \text{for } X \neq \Theta \\ m_\alpha(\Theta) = \alpha \cdot m(\Theta) + (1-\alpha) \end{cases} \tag{1}$$

Example 7: Let's consider back the inputs of Table 20. The impact of strong unreliability of prior information m_0 has already been analyzed in Examples 3 and 4 by considering actually $\alpha_0 = 0.1$. Here we analyze the impact of reliabilities of sources m_0, m_1, m_2 and m_3 according presentation done in section 2.4 and we choose the following set of reliability factors $\alpha_0 = 0.9$, $\alpha_1 = 0.75$, $\alpha_2 = 0.75$ and $\alpha_3 = 0.25$. These values have been chosen arbitrarily but they reflect the fact that one has a very good confidence in our prior information, a good confidence in sources 1 and 2, and a low confidence in source 3. Let's examine the change in the fusion result of sources. Applying reliability discounting technique [5], the new inputs corresponding to the discounted bba's by (1) are given in Table 32.

focal element	$m_{\alpha_0}(.)$	$m_{\alpha_1}(.)$	$m_{\alpha_2}(.)$	$m_{\alpha_3}(.)$
$\theta_4 \cup \theta_8$	0.90	0	0.2250	0
$\theta_6 \cup \theta_8$	0	0.5625	0	0.0625
$\overline{\theta_4 \cup \theta_8}$	0	0	0.5250	0
I_t	0.10	0.4375	0.2500	0.9375

Table 32: Discounted inputs with $\alpha_0 = 0.9$, $\alpha_1 = 0.75$, $\alpha_2 = 0.75$ and $\alpha_3 = 0.25$.

focal element	$m_{PCR5}(.)$	$m_{PCR6}(.)$
θ_6	0.030967	0.030967
$\overline{\theta_4 \cup \theta_8}$	0.13119	0.11037
θ_8	0.26543	0.26543
$\theta_4 \cup \theta_8$	0.37256	0.33686
$\theta_6 \cup \theta_8$	0.063483	0.068147
I_t	0.13637	0.18822

Table 33: Result of $m_0 \oplus m_1 \oplus m_2 \oplus m_3$ for Table 32.

Singletons	$DSmP_{\epsilon,PCR5}(.)$	$DSmP_{\epsilon,PCR6}(.)$
θ_1	0.0040	0.0036
θ_2	0.0040	0.0036
θ_3	0.0040	0.0036
θ_4	0.0018	0.0019
θ_5	0.0040	0.0036
θ_6	0.1655	0.1535
θ_7	0.0040	0.0036
θ_8	0.8126	0.8266

Table 34: $DSmP_\epsilon$ of $m_0 \oplus m_1 \oplus m_2 \oplus m_3$ for Table 32.

From the fusion result of Table 33, one gets when considering θ_8 only

- with PCR5: $\Delta(\theta_8) = \Delta(\bar{\theta}_8) = 0.5724$

$$P(\theta_8) \in [0.26543, 0.8378]$$

$$P(\bar{\theta}_8) \in [0.1622, 0.7346]$$

- with PCR6: $\Delta(\theta_8) = \Delta(\bar{\theta}_8) = 0.5933$

$$P(\theta_8) \in [0.26543, 0.8587]$$

$$P(\bar{\theta}_8) \in [0.1413, 0.7346]$$

and when considering $\theta_7 \cup \theta_8$

- with PCR5: $\Delta(\theta_7 \cup \theta_8) = \Delta(\overline{\theta_7 \cup \theta_8}) = 0.73457$

$$P(\theta_7 \cup \theta_8) \in [0.26543, 1]$$

$$P(\overline{\theta_7 \cup \theta_8}) \in [0, 0.73457]$$

- with PCR6: $\Delta(\theta_7 \cup \theta_8) = \Delta(\overline{\theta_7 \cup \theta_8}) = 0.73457$

$$P(\theta_7 \cup \theta_8) \in [0.26543, 1]$$

$$P(\overline{\theta_7 \cup \theta_8}) \in [0, 0.73457]$$

Using DSmP transformation, one gets a low probability in θ_8 or in $\theta_7 \cup \theta_8$ because

$$DSmP_{\epsilon,PCR5}(\theta_8) = 0.8126$$

$$DSmP_{\epsilon,PCR6}(\theta_8) = 0.8266$$

$$DSmP_{\epsilon,PCR5}(\theta_7 \cup \theta_8) = 0.8166$$

$$DSmP_{\epsilon,PCR6}(\theta_7 \cup \theta_8) = 0.8302$$

- **Answer to Q1:** Based on these results, one sees that the decision to take is to evacuate the building B since one gets a high probability in θ_8 or in $\theta_7 \cup \theta_8$. So there is a little impact of reliability discounting on the final decision with respect to Example 1. It is however worth to note that introducing reliability discounting increases the non specificity of information since now I_t is a new focal element of m_{α_0} and of m_{α_2} and in the final result we get the new focal element θ_6 appearing with PCR5 or PCR6 fusion rules. This θ_6 focal element doesn't exist in Example 1 when no reliability discounting is used. The decision to take in this case is to: **Evacuate the building** B.

To answer to the question Q2 for this Example 7, let's compute the fusion results of the fusion $m_0 \oplus m_2$ and $m_0 \oplus m_1 \oplus m_3$ using inputs given in Table 32. The fusion results with corresponding DSmP are given in the Tables 35-38.

focal element	$m_{PCR5}(.)$	$m_{PCR6}(.)$
$\theta_4 \cup \theta_8$	0.74842	0.74842
$\overline{\theta_4 \cup \theta_8}$	0.22658	0.22658
I_t	0.025	0.025

Table 35: Result of $m_0 \oplus m_2$.

Singletons	$DSmP_{\epsilon,PCR5}(.)$	$DSmP_{\epsilon,PCR6}(.)$
θ_1	0.0409	0.0409
θ_2	0.0409	0.0409
θ_3	0.0409	0.0409
θ_4	0.3773	0.3773
θ_5	0.0409	0.0409
θ_6	0.0409	0.0409
θ_7	0.0409	0.0409
θ_8	0.3773	0.3773

Table 36: $DSmP_\epsilon$ of $m_0 \oplus m_2$.

Based on $m_0 \oplus m_2$ fusion result, one gets a large imprecision[6] on $P(\theta_8)$ since $\Delta_{02}(\theta_8) = \Delta_{02}(\bar{\theta}_8) = 0.7734$ with

$$P(\theta_8) \in [0, 0.7734]$$

$$P(\bar{\theta}_8) \in [0.2266, 1]$$

and total imprecision when considering $\theta_7 \cup \theta_8$ since

$$P(\theta_7 \cup \theta_8) \in [0, 1]$$

$$P(\overline{\theta_7 \cup \theta_8}) \in [0, 1]$$

Based on max of bel or max of Pl criteria the decision using $m_0 \oplus m_2$ (i.e. with discounted sources m_0 and ANPR system m_2) should be to NOT evacuate the building B. Same decision would be taken based on DSmP values.

Let's examine now the result of the $m_0 \oplus m_1 \oplus m_3$ fusion given in Tables 37 and 38.

focal element	$m_{PCR5}(.)$	$m_{PCR6}(.)$
θ_8	0.53086	0.53086
$\theta_4 \cup \theta_8$	0.36914	0.36914
$\theta_6 \cup \theta_8$	0.058984	0.058984
I_t	0.041016	0.041016

Table 37: Result of $m_0 \oplus m_1 \oplus m_3$.

Singletons	$DSmP_{\epsilon,PCR5}(.)$	$DSmP_{\epsilon,PCR6}(.)$
θ_1	0.0001	0.0001
θ_2	0.0001	0.0001
θ_3	0.0001	0.0001
θ_4	0.0008	0.0008
θ_5	0.0001	0.0001
θ_6	0.0002	0.0002
θ_7	0.0001	0.0001
θ_8	0.9987	0.9987

Table 38: $DSmP_\epsilon$ of $m_0 \oplus m_1 \oplus m_3$.

One sees clearly the impact of reliability discounting on the specificity of information provided by the fusion of sources. Indeed when using the discounting of sources (mainly because we introduce I_t as focal element for m_0) one gets now 4 focal elements whereas we did get only two focal elements when no discounting was used (see Table 7). Based on $m_0 \oplus m_1 \oplus m_3$ fusion result, one gets $\Delta_{013}(\theta_8) = \Delta_{013}(\bar{\theta}_8) = 0.4691$ and

$$P(\theta_8) \in [0.53086, 1]$$

$$P(\bar{\theta}_8) \in [0, 0.4691]$$

and also

$$P(\theta_7 \cup \theta_8) \in [0.53086, 1]$$

$$P(\overline{\theta_7 \cup \theta_8}) \in [0, 0.4691]$$

Based on max of Bel or max of Pl, the decision using $m_0 \oplus m_1 \oplus m_3$ should be to evacuate the building B. Same decision would be taken based on DSmP values. It is worth to note that the precision on the result obtained with $m_0 \oplus m_1 \oplus m_3$ is much better than with $m_0 \oplus m_2$ since $\Delta_{013}(\theta_8) < \Delta_{02}(\theta_8)$, or $\Delta_{013}(\theta_7 \cup \theta_8) < \Delta_{02}(\theta_7 \cup \theta_8)$. Moreover it is easy to verify that $m_0 \oplus m_1 \oplus m_3$ fusion system is more informative than $m_0 \oplus m_2$ fusion system because Shannon entropy of DSmP of $m_0 \oplus m_2$ is much bigger than Shannon entropy of DSmP of $m_0 \oplus m_1 \oplus m_3$.

- **Answer to Q2:** Since the information obtained by the fusion $m_0 \oplus m_2$ is less informative and less precise than the information obtained with the fusion $m_0 \oplus m_1 \oplus m_3$, it is better to choose and to trust the fusion system $m_0 \oplus m_1 \oplus m_3$ rather than $m_0 \oplus m_2$. Based on this choice, the final decision will be to evacuate the building B.

3.5 Impact of importance of sources

The importance discounting technique has been proposed recently by the authors in [7] and consists in discounting the masses of focal elements by a factor $\beta \in [0, 1]$ and in transferring the remaining mass to empty set, i.e.

$$\begin{cases} m_\beta(X) = \beta \cdot m(X), & \text{for } X \neq \emptyset \\ m_\beta(\emptyset) = \beta \cdot m(\emptyset) + (1 - \beta) \end{cases} \tag{2}$$

It has been proved in [7] that such importance discounting technique preserves the specificity of the information and that Dempster's rule of combination doesn't respond to such new interesting discounting technique specially useful and crucial in multicriteria decision-making support. Let's examine the impact of the importance of the sources in the fusion process for final decision-making through the next very simple illustrating example.

Example 8: To evaluate this we consider at first the same inputs as in Table 1 and we consider that source 1 (Analyst 1 with 10 years experience) is

[6] This imprecision is larger than in Example 1 which is normal because one has degraded the information of both prior and the source m_2.

much more important than source 3 (Analyst 2 with no experience). To reflect the difference between importance of this sources we consider the following relative importance factors $\beta_1 = 0.8$ and $\beta_3 = 0.2$. We also assume that source 0 (prior information) and source 2 (ANPR system) have the same maximal importance, i.e. $\beta_0 = \beta_2 = 1$, i.e $\beta = (\beta_0, \beta_1, \beta_2, \beta_3) = (1, 0.8, 1, 0.2)$. These values have been chosen arbitrarily but they do reflect the fact that sources m_0 and m_2 have same importance in the fusion process, and that sources m_1 and m_3 may have less importance in the fusion process taking into the fact that m_3 is considered as less important than m_1. Let's examine the change in the fusion result of sources in this example with respect to what we get in Example 1.

In applying importance discounting technique [7], the new inputs corresponding to the discounted (unormalized) bba's by (2) are given in Table 39.

focal element	$m_{\beta_0}(.)$	$m_{\beta_1}(.)$	$m_{\beta_2}(.)$	$m_{\beta_3}(.)$
f_0	0	0.2	0	0.8
$\theta_4 \cup \theta_8$	1	0	0.70	0
$\theta_6 \cup \theta_8$	0	0.6	0	0.05
$\overline{\theta_4 \cup \theta_8}$	0	0	0.30	0
I_t	0	0.2	0	0.15

Table 39: Discounted inputs with $\beta_0 = 1$, $\beta_1 = 0.8$, $\beta_2 = 1$ and $\beta_3 = 0.2$.

where the focal element f_0 denotes the absolute empty set $\emptyset_a$.

focal element	$m_{PCR5,\beta}(.)$	$m_{PCR6,\beta}(.)$
$\emptyset_a$	0.25015	0.24585
$\theta_1 \cup \theta_2 \cup \theta_3 \cup \theta_5 \cup \theta_6 \cup \theta_7$	0.19425	0.17757
θ_8	0.039	0.039
$\theta_4 \cup \theta_8$	0.36008	0.39587
$\theta_6 \cup \theta_8$	0.13228	0.11873
I_t	0.024234	0.022985

Table 40: Result of non normalized $m_0 \oplus m_1 \oplus m_2 \oplus m_3$ for Table 39.

After normalization, one finally gets the following result

focal element	$m_{PCR5}(.)$	$m_{PCR6}(.)$
$\theta_1 \cup \theta_2 \cup \theta_3 \cup \theta_5 \cup \theta_6 \cup \theta_7$	0.25905	0.23545
θ_8	0.052011	0.051714
$\theta_4 \cup \theta_8$	0.48021	0.52492
$\theta_6 \cup \theta_8$	0.17641	0.15743
I_t	0.032318	0.030479

Table 41: Result of normalized $m_0 \oplus m_1 \oplus m_2 \oplus m_3$ for Table 39.

As we can see, the importance discounting doesn't degrade the specificity of sources since no mass is committed to partial ignorances, and it doesn't also increase the number of focal elements of the resulting bba contrariwise to the reliability discounting approach. Indeed in Table 41 one gets only 5 focal elements whereas one gets 6 focal elements with reliability discounting as shown in Table 33 of Example 7. The corresponding DSmP values of bba's given in Table 41 are summarized in Table 42.

Singletons	$DSmP_{\epsilon,PCR5}(.)$	$DSmP_{\epsilon,PCR6}(.)$
θ_1	0.0437	0.0398
θ_2	0.0437	0.0398
θ_3	0.0437	0.0398
θ_4	0.0094	0.0103
θ_5	0.0437	0.0398
θ_6	0.0470	0.0427
θ_7	0.0437	0.0398
θ_8	0.7250	0.7483

Table 42: $DSmP_\epsilon$ of $m_0 \oplus m_1 \oplus m_2 \oplus m_3$ for Table 39.

From the fusion result of Table 41, one gets when considering θ_8 only

- with PCR5: $\Delta(\theta_8) = \Delta(\bar{\theta}_8) = 0.6889$

$$P(\theta_8) \in [0.052011, 0.7409]$$

$$P(\bar{\theta}_8) \in [0.2591, 0.9480]$$

- with PCR6: $\Delta(\theta_8) = \Delta(\bar{\theta}_8) = 0.7129$

$$P(\theta_8) \in [0.051714, 0.7646]$$

$$P(\bar{\theta}_8) \in [0.2354, 0.9483]$$

and when considering $\theta_7 \cup \theta_8$

- with PCR5: $\Delta(\theta_7 \cup \theta_8) = \Delta(\overline{\theta_7 \cup \theta_8}) = 0.9480$

$$P(\theta_7 \cup \theta_8) \in [0.052011, 1]$$

$$P(\overline{\theta_7 \cup \theta_8}) \in [0, 0.9480]$$

- with PCR6: $\Delta(\theta_7 \cup \theta_8) = \Delta(\overline{\theta_7 \cup \theta_8}) = 0.9483$

$$P(\theta_7 \cup \theta_8) \in [0.051714, 1]$$

$$P(\overline{\theta_7 \cup \theta_8}) \in [0, 0.9483]$$

Using DSmP transformation, one gets a high probability in θ_8 or in $\theta_7 \cup \theta_8$ because

$$DSmP_{\epsilon,PCR5}(\theta_8) = 0.7250$$

$$DSmP_{\epsilon,PCR6}(\theta_8) = 0.7483$$

$$DSmP_{\epsilon,PCR5}(\theta_7 \cup \theta_8) = 0.7687$$

$$DSmP_{\epsilon,PCR6}(\theta_7 \cup \theta_8) = 0.7881$$

- **Answer to Q1:** Based on these results, one sees that the decision to take based either on max of Bel or max of Pl of $\theta_7 \cup \theta_8$, or also based on DSmP, is to evacuate the building B since one gets a high probability in θ_8 or in $\theta_7 \cup \theta_8$. However, if one uses the max of Bel or max of Pl of θ_8 only as decision criteria, then the decision to take is to NOT evacuate the building B which looks a bit as a counter-intuitive decision for this case !!!

To answer to the question Q2 for this Example 8, let's compute the fusion results of the fusion $m_0 \oplus m_2$ and $m_0 \oplus m_1 \oplus m_3$ using inputs given in Table 39. Because one has considered $\beta_0 = \beta_2 = 1$ one does not actually discount sources m_0 and m_2 and therefore the $m_0 \oplus m_2$ fusion results are already given in Tables 4 and 5 of Example 1. Therefore based on max of Bel or max of Pl criteria on θ_8 the decision using $m_0 \oplus m_2$ is to NOT evacuate the building B since $P(\theta_8) \in [0, 0.71176]$ and $P(\bar{\theta}_8) \in [0.28824, 1]$ and $\Delta_{02}(\theta_8) = 0.71176$. Same decision would be taken based on DSmP values with the $m_0 \oplus m_2$ fusion sub-system. Let's now compute the fusion $m_0 \oplus m_1 \oplus m_3$ with the importance discounted sources m_1 and m_3. The fusion results are given in Tables 43 and 44.

focal element	$m_{PCR5}(.)$	$m_{PCR6}(.)$
θ_8	0.17382	0.18842
$\theta_4 \cup \theta_8$	0.63934	0.60906
$\theta_6 \cup \theta_8$	0.16099	0.1745
I_t	0.025851	0.028021

Table 43: Result of $m_0 \oplus m_1 \oplus m_3$.

Singletons	$DSmP_{\epsilon,PCR5}(.)$	$DSmP_{\epsilon,PCR6}(.)$
θ_1	0.0001	0.0001
θ_2	0.0001	0.0001
θ_3	0.0001	0.0001
θ_4	0.0038	0.0033
θ_5	0.0001	0.0001
θ_6	0.0011	0.0011
θ_7	0.0001	0.0001
θ_8	0.9945	0.9949

Table 44: $DSmP_\epsilon$ of $m_0 \oplus m_1 \oplus m_3$.

Based on $m_0 \oplus m_1 \oplus m_3$ fusion result, one gets

- with PCR5 $\Delta_{013}(\theta_8) = \Delta_{013}(\bar{\theta}_8) = 0.8262$ and

$$P(\theta_8) \in [0.17382, 1]$$

$$P(\bar{\theta}_8) \in [0, 0.8262]$$

- with PCR6 $\Delta_{013}(\theta_8) = \Delta_{013}(\bar{\theta}_8) = 0.8116$ and

$$P(\theta_8) \in [0.18842, 1]$$

$$P(\bar{\theta}_8) \in [0, 0.8116]$$

and also

- with PCR5 $\Delta_{013}(\theta_7 \cup \theta_8) = \Delta_{013}(\overline{\theta_7 \cup \theta_8}) = 0.8262$ and

$$P(\theta_7 \cup \theta_8) \in [0.17382, 1]$$

$$P(\overline{\theta_7 \cup \theta_8}) \in [0, 0.8262]$$

- with PCR6 $\Delta_{013}(\theta_7 \cup \theta_8) = \Delta_{013}(\overline{\theta_7 \cup \theta_8}) = 0.8116$ and

$$P(\theta_7 \cup \theta_8) \in [0.18842, 1]$$

$$P(\overline{\theta_7 \cup \theta_8}) \in [0, 0.8116]$$

Based on max of Bel or max of Pl, the decision using θ_8 or on $\theta_7 \cup \theta_8$ for the $m_0 \oplus m_1 \oplus m_3$ fusion sub-system should be to evacuate the building B. Same decision must be taken based on DSmP values. It is worth to note that the precision on the result obtained with $m_0 \oplus m_2$ is much better than with $m_0 \oplus m_1 \oplus m_3$ since $\Delta_{02}(\theta_8) < \Delta_{013}(\theta_8)$. But when considering $\theta_7 \cup \theta_8$ one has $\Delta_{013}(\theta_7 \cup \theta_8) = 0.8262$ (or 0.8116 with PCR6) $< \Delta_{02}(\theta_7 \cup \theta_8) = 1$, which means that $m_0 \oplus m_1 \oplus m_3$ fusion sub-system is more precise than $m_0 \oplus m_2$ fusion sub-system. From the probabilistic information content (PIC) based on DSmP transformation, the $m_0 \oplus m_1 \oplus m_3$ fusion system is more informative than $m_0 \oplus m_2$ fusion system because Shannon entropy of DSmP of $m_0 \oplus m_2$ is much bigger than Shannon entropy of DSmP of $m_0 \oplus m_1 \oplus m_3$.

- **Answer to Q2:** The analysis of both fusion subsystems $m_0 \oplus m_2$ and $m_0 \oplus m_1 \oplus m_3$ shows that the $m_0 \oplus m_2$ system must be chosen in only the precision on θ_8 is concerned and the decision will be to NOT evacuate the building B. If $\theta_7 \cup \theta_8$ is concerned is is prefreable to choose the system $m_0 \oplus m_1 \oplus m_3$ instead because it provides a better precision and PIC than $m_0 \oplus m_2$ and in that case the decion will be to evcuate the building B. But globally one sees that both subsystems provides very imprecise probability measures which make the decision-making very risky whatever the chosen fusion sub-system is.

Example 8 BIS: To evaluate this we consider at first the same inputs as in Table 1 and we consider that source 1 (Analyst 1 with 10 years experience) is much more important than source 3 (Analyst 2 with no experience). To reflect the difference between importance of this sources we consider the following relative importance factors $\beta_1 = 0.9$ and $\beta_3 = 0.5$. We also assume that source 0 (prior information) and source 2 (ANPR system) have the same maximal importance, i.e. $\beta_0 = \beta_2 = 1$, i.e $\beta = (\beta_0, \beta_1, \beta_2, \beta_3) = (1, 0.9, 1, 0.5)$. These values have been chosen arbitrarily but they do reflect the fact that sources m_0 and m_2 have same importance in the fusion process, and that sources m_1 and m_3 may have less importance in the fusion process taking into the fact that m_3 is considered as less important than m_1. Let's examine the change in the fusion result of sources in this example with respect to what we get in Example 1.

In applying importance discounting technique [7], the new inputs corresponding to the discounted (unormalized) bba's by (2) are given in Table 45.

focal element	$m_{\beta_0}(.)$	$m_{\beta_1}(.)$	$m_{\beta_2}(.)$	$m_{\beta_3}(.)$
$\emptyset$	0	0.1	0	0.5
$\theta_4 \cup \theta_8$	1	0	0.70	0
$\theta_6 \cup \theta_8$	0	0.675	0	0.125
$\overline{\theta_4 \cup \theta_8}$	0	0	0.30	0
I_t	0	0.225	0	0.375

Table 45: Discounted inputs with $\beta_0 = 1$, $\beta_1 = 0.9$, $\beta_2 = 1$ and $\beta_3 = 0.5$.

focal element	$m_{PCR5,\beta}(.)$	$m_{PCR6,\beta}(.)$
$\emptyset$	0.11508	0.10252
θ_8	0.10969	0.10969
$\theta_4 \cup \theta_8$	0.36664	0.39168
$\theta_6 \cup \theta_8$	0.15075	0.14755
$\overline{\theta_4 \cup \theta_8}$	0.20405	0.18887
I_t	0.05380	0.05969

Table 46: Result of non normalized $m_0 \oplus m_1 \oplus m_2 \oplus m_3$ for Table 45.

After normalization, one finally gets the following result

focal element	$m_{PCR5}(.)$	$m_{PCR6}(.)$
θ_8	0.12395	0.12222
$\theta_4 \cup \theta_8$	0.41432	0.43642
$\theta_6 \cup \theta_8$	0.17035	0.16440
$\overline{\theta_4 \cup \theta_8}$	0.23058	0.21045
I_t	0.060797	0.066508

Table 47: Result of normalized $m_0 \oplus m_1 \oplus m_2 \oplus m_3$ for Table 45.

As we can see, the importance discounting doesn't degrade the specificity of sources since no mass is committed to partial ignorances, and it doesn't also increase the number of focal elements of the resulting bba contrariwise to the reliability discounting approach. Indeed in Table 41 one gets only 5 focal elements whereas one gets 6 focal elements with reliability discounting as shown in Table 33 of Example 7. The corresponding DSmP values of bba's given in Table 47 are summarized in Table 48.

From the fusion result of Table 47, one gets when considering θ_8 only

- with PCR5: $\Delta(\theta_8) = \Delta(\bar{\theta}_8) = 0.6454$

$$P(\theta_8) \in [0.12395, 0.7694]$$

Singletons	$DSmP_{\epsilon,PCR5}(.)$	$DSmP_{\epsilon,PCR6}(.)$
θ_1	0.0389	0.0356
θ_2	0.0389	0.0356
θ_3	0.0389	0.0356
θ_4	0.0038	0.0040
θ_5	0.0389	0.0356
θ_6	0.0402	0.0369
θ_7	0.0389	0.0356
θ_8	0.7615	0.7811

Table 48: $DSmP_\epsilon$ of $m_0 \oplus m_1 \oplus m_2 \oplus m_3$ for Table 45.

$$P(\bar{\theta}_8) \in [0.2306, 0.8760]$$

- with PCR6: $\Delta(\theta_8) = \Delta(\bar{\theta}_8) = 0.6673$

$$P(\theta_8) \in [0.12222, 0.7895]$$

$$P(\bar{\theta}_8) \in [0.2105, 0.8778]$$

and when considering $\theta_7 \cup \theta_8$

- with PCR5: $\Delta(\theta_7 \cup \theta_8) = \Delta(\overline{\theta_7 \cup \theta_8}) = 0.8760$

$$P(\theta_7 \cup \theta_8) \in [0.12395, 1]$$

$$P(\overline{\theta_7 \cup \theta_8}) \in [0, 0.8760]$$

- with PCR6: $\Delta(\theta_7 \cup \theta_8) = \Delta(\overline{\theta_7 \cup \theta_8}) = 0.8778$

$$P(\theta_7 \cup \theta_8) \in [0.12222, 1]$$

$$P(\overline{\theta_7 \cup \theta_8}) \in [0, 0.8778]$$

Using DSmP transformation, one gets a high probability in θ_8 or in $\theta_7 \cup \theta_8$ because

$$DSmP_{\epsilon,PCR5}(\theta_8) = 0.7615$$

$$DSmP_{\epsilon,PCR6}(\theta_8) = 0.7811$$

$$DSmP_{\epsilon,PCR5}(\theta_7 \cup \theta_8) = 0.8004$$

$$DSmP_{\epsilon,PCR6}(\theta_7 \cup \theta_8) = 0.8167$$

- **Answer to Q1:** Based on these results, one sees that the decision to take based either on max of Bel or max of Pl of $\theta_7 \cup \theta_8$, or also based on DSmP, is to evacuate the building B since one gets a high probability in θ_8 or in $\theta_7 \cup \theta_8$. However, if one uses the max of Bel or max of Pl of θ_8 as decision criteria, then the decision to take is to NOT evacuate the building B which looks a bit as a counter-intuitive decision for this case !!!

If one works directly with non normalized bba given in Table 46, one would get unnormalized imprecise probabilities $P^*(.)$

- with PCR5: $\Delta^*(\theta_8) = \Delta^*(\bar{\theta}_8) = 0.5712$

$$P^*(\theta_8) \in [0.10969, 0.6809]$$

$$P^*(\bar{\theta}_8) \in [0.2040, 0.7752]$$

- with PCR6: $\Delta^*(\theta_8) = \Delta^*(\bar{\theta}_8) = 0.5989$

$$P^*(\theta_8) \in [0.10969, 0.7086]$$

$$P^*(\bar{\theta}_8) \in [0.1889, 0.7878]$$

SAME PROBLEM OCCURS !!! even whe working with non normalized bba, based on Max of Bel or Max of Pl for θ_8 only we wil decide to NOT evacuate the building B. This is VERY counter-intuitive and not good to promote importance discounting technique ....

Example 8 TER: To evaluate this we consider at first the same inputs as in Table 1 and we consider that source 1 (Analyst 1 with 10 years experience) is much more important than source 3 (Analyst 2 with no experience). To reflect the difference between importance of these sources we consider the following relative importance factors $\beta_1 = 1$ and $\beta_3 = 0.5$. We also assume that source 0 (prior information) and source 2 (ANPR system) have the same maximal importance, i.e. $\beta_0 = \beta_2 = 1$, i.e $\beta = (\beta_0, \beta_1, \beta_2, \beta_3) = (1, 1, 1, 0.5)$. These values have been chosen arbitrarily but they do reflect the fact that sources m_0 and m_2 have same importance in the fusion process, and that sources m_1 and m_3 may have less importance in the fusion process taking into the fact that m_3 is considered as less important than m_1. Let's examine the change in the fusion result of sources in this example with respect to what we get in Example 1.

In applying importance discounting technique [7], the new inputs corresponding to the discounted (unormalized) bba's by (2) are given in Table 49.

focal element	$m_{\beta_0}(.)$	$m_{\beta_1}(.)$	$m_{\beta_2}(.)$	$m_{\beta_3}(.)$
$\emptyset$	0	0	0	0.5
$\theta_4 \cup \theta_8$	1	0	0.70	0
$\theta_6 \cup \theta_8$	0	0.75	0	0.125
$\overline{\theta_4 \cup \theta_8}$	0	0	0.30	0
I_t	0	0.25	0	0.375

Table 49: Discounted inputs with $\beta_0 = 1$, $\beta_1 = 1$, $\beta_2 = 1$ and $\beta_3 = 0.5$.

focal element	$m_{PCR5,\beta}(.)$	$m_{PCR6,\beta}(.)$
$\emptyset$	0.1165	0.093554
θ_8	0.12187	0.12187
$\theta_4 \cup \theta_8$	0.33871	0.3679
$\theta_6 \cup \theta_8$	0.17819	0.17571
$\overline{\theta_4 \cup \theta_8}$	0.19467	0.18105
I_t	0.050056	0.059912

Table 50: Result of non normalized $m_0 \oplus m_1 \oplus m_2 \oplus m_3$ for Table 49.

After normalization, one finally gets the following result

focal element	$m_{PCR5}(.)$	$m_{PCR6}(.)$
θ_8	0.13795	0.13445
$\theta_4 \cup \theta_8$	0.38338	0.40587
$\theta_6 \cup \theta_8$	0.20168	0.19385
$\overline{\theta_4 \cup \theta_8}$	0.22034	0.19973
I_t	0.056657	0.066096

Table 51: Result of normalized $m_0 \oplus m_1 \oplus m_2 \oplus m_3$ for Table 49.

As we can see, the importance discounting doesn't degrade the specificity of sources since no mass is committed to partial ignorances, and it doesn't also increase the number of focal elements of the resulting bba contrariwise to the reliability discounting approach. Indeed in Table 41 one gets only 5 focal elements whereas one gets 6 focal elements with reliability discounting as shown in Table 33 of Example 7. The corresponding DSmP values of bba's given in Table 51 are summarized in Table 52.

Singletons	$DSmP_{\epsilon,PCR5}(.)$	$DSmP_{\epsilon,PCR6}(.)$
θ_1	0.0371	0.0338
θ_2	0.0371	0.0338
θ_3	0.0371	0.0338
θ_4	0.0031	0.0034
θ_5	0.0371	0.0338
θ_6	0.0386	0.0352
θ_7	0.0371	0.0338
θ_8	0.7728	0.7926

Table 52: $DSmP_\epsilon$ of $m_0 \oplus m_1 \oplus m_2 \oplus m_3$ for Table 49.

From the fusion result of Table 51, one gets when considering θ_8 only

- with PCR5: $\Delta(\theta_8) = \Delta(\bar{\theta}_8) = 0.6417$

$$P(\theta_8) \in [0.13795, 0.7797]$$

$$P(\bar{\theta}_8) \in [0.2203, 0.8620]$$

- with PCR6: $\Delta(\theta_8) = \Delta(\bar{\theta}_8) = 0.6659$

$$P(\theta_8) \in [0.13445, 0.8003]$$

$$P(\bar{\theta}_8) \in [0.1997, 0.8656]$$

and when considering $\theta_7 \cup \theta_8$

- with PCR5: $\Delta(\theta_7 \cup \theta_8) = \Delta(\overline{\theta_7 \cup \theta_8}) = 0.8620$

$$P(\theta_7 \cup \theta_8) \in [0.13795, 1]$$

$$P(\overline{\theta_7 \cup \theta_8}) \in [0, 0.8620]$$

- with PCR6: $\Delta(\theta_7 \cup \theta_8) = \Delta(\overline{\theta_7 \cup \theta_8}) = 0.8656$

$$P(\theta_7 \cup \theta_8) \in [0.13445, 1]$$

$$P(\overline{\theta_7 \cup \theta_8}) \in [0, 0.8656]$$

Using DSmP transformation, one gets a high probability in θ_8 or in $\theta_7 \cup \theta_8$ because

$$DSmP_{\epsilon,PCR5}(\theta_8) = 0.7728$$

$$DSmP_{\epsilon,PCR6}(\theta_8) = 0.7811$$

$$DSmP_{\epsilon,PCR5}(\theta_7 \cup \theta_8) = 0.7926$$

$$DSmP_{\epsilon,PCR6}(\theta_7 \cup \theta_8) = 0.8264$$

- **Answer to Q1:** Based on these results, one sees that the decision to take based either on max of Bel or max of Pl of $\theta_7 \cup \theta_8$, or also based on DSmP, is to evacuate the building B since one gets a high probability in θ_8 or in $\theta_7 \cup \theta_8$. However, if one uses the max of Bel or max of Pl of θ_8 as decision criteria, then the decision to take is to NOT evacuate the building B which looks a bit as a counter-intuitive decision for this case !!!

SAME PROBLEM OCCURS !!! even whe working with non normalized bba, based on Max of Bel or Max of Pl for θ_8 only we wil decide to NOT evacuate the building B. This is VERY counter-intuitive and not good to promote importance discounting technique ....

3.6 Using imprecise bba's

Let's examine the fusion result when dealing directly with imprecise bba's. We just consider here a simple imprecise example which consider both inputs of Examples 1 and 2 to generate imprecise bba's inputs.

Example 9: We consider the imprecise bba's according to input Table 53.

focal element	$m_0(.)$	$m_1(.)$	$m_2(.)$	$m_3(.)$
$f_1 = \theta_4 \cup \theta_8$	1	0	0.3	0
$f_2 = \overline{\theta_6 \cup \theta_8}$	0	[0.75,0.9]	0	[0.10,0.25]
$f_3 = \overline{\theta_4 \cup \theta_8}$	0	0	0.7	0
I_t	0	[0.1,0.25]	0	[0.75,0.9]

Table 53: Imprecise quantitative inputs for VIEB problem.

Applying the conjunctive rule, we have $1 \times 2 \times 2 \times 2 = 8$ products to compute which are listed below:

- Product $\pi_1 = 1 \boxdot [0.75, 0.90] \boxdot 0.3 \boxdot [0.10, 0.25]$. Using operators on sets defined in [6], Vol.1, Chap. 6, one gets $\pi_1 = [0.75, 0.90] \boxdot [0.03, 0.075] = [0.0225, 0.0675]$ which is committed to $f_1 \cap f_2 = \theta_8$.
- Product $\pi_2 = 1 \boxdot [0.75, 0.90] \boxdot 0.3 \boxdot [0.75, 0.90]$ is equal to $[0.75, 0.90] \boxdot [0.225, 0.27] = [0.16875, 0.243]$ which is also committed to $f_1 \cap f_2 = \theta_8$.
- Product $\pi_3 = 1 \boxdot [0.75, 0.90] \boxdot 0.7 \boxdot [0.10, 0.25] = [0.0525, 0.1575]$ corresponds to the imprecise mass of $f_1 \cap f_2 \cap f_3 = \emptyset$ which will be redistributed back to f_1, f_2 and f_3 according to PCR6.
- Product $\pi_4 = 1 \boxdot [0.75, 0.90] \boxdot 0.7 \boxdot [0.75, 0.90] = [0.39375, 0.567]$ corresponds to the imprecise mass of $f_1 \cap f_2 \cap f_3 \cap I_t = \emptyset$ which will be redistributed back to f_1, f_2, f_3 and I_t according to PCR6.
- Product $\pi_5 = 1 \boxdot [0.10, 0.25] \boxdot 0.3 \boxdot [0.10, 0.25] = [0.003, 0.01875]$ is committed to $f_1 \cap f_2 = \theta_8$.
- Product $\pi_6 = 1 \boxdot [0.10, 0.25] \boxdot 0.3 \boxdot [0.75, 0.90] = [0.0225, 0.0675]$ is committed to f_1.
- Product $\pi_7 = 1 \boxdot [0.10, 0.25] \boxdot 0.7 \boxdot [0.10, 0.25] = [0.007, 0.04375]$ corresponds to the imprecise mass of $f_1 \cap I_t \cap f_3 \cap f_2 = \emptyset$ which will be redistributed back to f_1, f_2, f_3 and I_t according to PCR6.
- Product $\pi_8 = 1 \boxdot [0.10, 0.25] \boxdot 0.7 \boxdot [0.75, 0.90] = [0.0525, 0.1575]$ corresponds to the imprecise mass of $f_1 \cap I_t \cap f_3 \cap I_t = \emptyset$ which will be redistributed back to f_1, f_3 and I_t according to PCR6.

We now redistribute the imprecise masses π_3, π_4, π_7 and π_8 associated with the empty set using PCR6 principle. Lets' compute the proportions of π_3, π_4, π_7 and π_8 committed to each focal element involved in the conflict they are associated with.

- The product $\pi_3 = [0.0525, 0.1575]$ is distributed to f_1, f_2 and f_3 according to PCR6 as follows

$$\begin{aligned}\frac{x_{f_1,\pi_3}}{1} &= \frac{y_{f_2,\pi_3}}{[0.75,0.90] \boxplus [0.10,0.25]} = \frac{z_{f_3,\pi_3}}{0.7}\\ &= \frac{\pi_3}{1 \boxplus [0.75,0.90] \boxplus [0.10,0.25] \boxplus 0.7}\\ &= \frac{[0.0525,0.1575]}{1.7 \boxplus [0.85,1.15]} = \frac{[0.0525,0.1575]}{[2.55,2.85]}\\ &= [\frac{0.0525}{2.85}, \frac{0.1575}{2.55}]\\ &= [0.018421, 0.061765]\end{aligned}$$

whence

$$\begin{aligned}x_{f_1,\pi_3} &= 1 \boxdot [0.018421, 0.061765]\\ &= [0.018421, 0.061765]\\ y_{f_2,\pi_3} &= [0.85, 1.15] \boxdot [0.018421, 0.061765]\\ &= [0.015658, 0.071029]\\ z_{f_3,\pi_3} &= 0.7 \boxdot [0.018421, 0.061765]\\ &= [0.012895, 0.043236]\end{aligned}$$

- The product $\pi_4 = [0.39375, 0.567]$ is distributed to

f_1, f_2, f_3 and I_t according to PCR6 as follows

$$\begin{aligned}\frac{x_{f_1,\pi_4}}{1} &= \frac{y_{f_2,\pi_4}}{[0.75,0.90]} = \frac{z_{f_3,\pi_4}}{0.7} = \frac{w_{I_t,\pi_4}}{[0.75,0.90]}\\ &= \frac{\pi_4}{1 \boxplus [0.75,0.90] \boxplus 0.7 \boxplus [0.75,0.90]}\\ &= \frac{[0.39375,0.567]}{[3.2,3.5]}\\ &= [0.1125,0.177188]\end{aligned}$$

whence

$$\begin{aligned}x_{f_1,\pi_4} &= 1 \boxdot [0.1125,0.177188]\\ &= [0.1125,0.177188]\\ y_{f_2,\pi_4} &= [0.75,0.90] \boxdot [0.1125,0.177188]\\ &= [0.084375,0.159469]\\ z_{f_3,\pi_4} &= 0.7 \boxdot [0.1125,0.177188]\\ &= [0.07875,0.124031]\\ w_{I_t,\pi_4} &= [0.75,0.90] \boxdot [0.1125,0.177188]\\ &= [0.084375,0.159469]\end{aligned}$$

- The product $\pi_7 = [0.007, 0.04375]$ is distributed to f_1, f_2, f_3 and I_t according to PCR6 as follows

$$\begin{aligned}\frac{x_{f_1,\pi_7}}{1} &= \frac{y_{f_2,\pi_7}}{[0.10,0.25]} = \frac{z_{f_3,\pi_7}}{0.7} = \frac{w_{I_t,\pi_7}}{[0.10,0.25]}\\ &= \frac{\pi_7}{1 \boxplus [0.10,0.25] \boxplus 0.7 \boxplus [0.10,0.25]}\\ &= \frac{[0.007,0.04375]}{[1.9,2.2]}\\ &= [0.003182,0.023026]\end{aligned}$$

whence

$$\begin{aligned}x_{f_1,\pi_7} &= 1 \boxdot [0.003182,0.023026]\\ &= [0.003182,0.023026]\\ y_{f_2,\pi_7} &= [0.10,0.25] \boxdot [0.003182,0.023026]\\ &= [0.000318,0.005757]\\ z_{f_3,\pi_7} &= 0.7 \boxdot [0.003182,0.023026]\\ &= [0.002227,0.016118]\\ w_{I_t,\pi_7} &= [0.10,0.25] \boxdot [0.003182,0.023026]\\ &= [0.000318,0.005757]\end{aligned}$$

- The product $\pi_8 = [0.0525, 0.1575]$ is distributed to f_1, f_3 and I_t according to PCR6 as follows

$$\begin{aligned}\frac{x_{f_1,\pi_8}}{1} &= \frac{z_{f_3,\pi_8}}{0.7} = \frac{w_{I_t,\pi_8}}{[0.10,0.25] \boxplus [0.75,0.90]}\\ &= \frac{\pi_8}{1 \boxplus 0.7 \boxplus [0.10,0.25] \boxplus [0.75,0.90]}\\ &= \frac{[0.0525,0.1575]}{[2.55,2.85]}\\ &= [0.018421,0.061765]\end{aligned}$$

whence

$$\begin{aligned}x_{f_1,\pi_8} &= 1 \boxdot [0.018421,0.061765]\\ &= [0.018421,0.061765]\\ z_{f_3,\pi_8} &= 0.7 \boxdot [0.018421,0.061765]\\ &= [0.012895,0.043235]\\ w_{I_t,\pi_8} &= [0.85,1.15] \boxdot [0.018421,0.061765]\\ &= [0.015658,0.071029]\end{aligned}$$

Summing the results, we get for $m_0 \oplus m_1 \oplus m_2 \oplus m_3$ with PCR6 the following imprecise m_{PCR6} bba:

$$\begin{aligned}m_{PCR6}(\theta_8) &= \pi_1 \boxplus \pi_2 \boxplus \pi_5\\ &= [0.19425,0.32925]\\ m_{PCR6}(f_1) &= x_{f_1,\pi_3} \boxplus x_{f_1,\pi_4} \boxplus x_{f_1,\pi_7} \boxplus x_{f_1,\pi_8}\\ &= [0.152524,0.323743]\\ m_{PCR6}(f_2) &= y_{f_2,\pi_3} \boxplus y_{f_2,\pi_4} \boxplus y_{f_2,\pi_7}\\ &= [0.100351,0.236255]\\ m_{PCR6}(f_3) &= z_{f_3,\pi_3} \boxplus z_{f_3,\pi_4} \boxplus z_{f_3,\pi_7} \boxplus z_{f_3,\pi_8}\\ &= [0.106767,0.226620]\\ m_{PCR6}(I_t) &= w_{I_t,\pi_4} \boxplus w_{I_t,\pi_7} \boxplus w_{I_t,\pi_8}\\ &= [0.100351,0.236255]\end{aligned}$$

where $\boxplus$ and $\boxdot$ operators (i.e. the addition and multiplication of imprecise values), and other operators on sets, were defined in [6], Vol. 1, p127–130 by

$$S_1 \boxplus S_2 = \{x | x = s_1 + s_2, s_1 \in S_1, s_2 \in S_2\}$$

$$S_1 \boxdot S_2 = \{x | x = s_1 \cdot s_2, s_1 \in S_1, s_2 \in S_2\}$$

with

$$\inf(S_1 \boxplus S_2) = \inf(S_1) + \inf(S_2)$$
$$\sup(S_1 \boxplus S_2) = \sup(S_1) + \sup(S_2)$$

and

$$\inf(S_1 \boxdot S_2) = \inf(S_1) \cdot \inf(S_2)$$
$$\sup(S_1 \boxdot S_2) = \sup(S_1) \cdot \sup(S_2)$$

We have summarized the results in Table 54. The left column of this table corresponds to the imprecise values of m_{PCR6} based on exact calculus with operators on sets (i.e. the exact calculus with imprecision). The right column of this table (m^{approx}_{PCR6}) corresponds to the result obtained with non exact calculus based on results given in Examples 1 and 2 in right columns of Tables 2 and 9. This is what we call approximate results since they are not based on exact calculus with operators on sets. One shows an important differences between results in left and right columns which can make an impact on final decision process when working with imprecise bba's and we suggest to always use exact imprecise calculus (more complicated) instead of approximate calculus (more easier) in order to get the real imprecision on bba values. Same approach can be

focal element	$m_{PCR6}(.)$	$m_{PCR6}^{approx}(.)$
θ_8	[0.194250,0.329250]	[0.24375,0.27300]
$f_1 = \theta_4 \cup \theta_8$	[0.152524,0.323743]	[0.23935,0.29641]
$f_2 = \theta_6 \cup \theta_8$	[0.100351,0.236255]	[0.14587, 0.16950]
$f_3 = \overline{\theta_4 \cup \theta_8}$	[0.106767,0.226620]	[0.14865,0.16811]
I_t	[0.100351,0.236255]	[0.14587, 0.16950]

Table 54: Results of $m_0 \oplus m_1 \oplus m_2 \oplus m_3$ for Table 53.

done for combining imprecise bba's with PCR5 (not reported in this paper).

Based on results on left column of Table 54, one can easily compute the imprecise Bel and Pl values also which are

$$\begin{aligned} Bel(\theta_8) &= [0.194250, 0.329250] \\ Pl(\theta_8) &= [0.547476, 1.125502] \equiv [0.547476, 1] \end{aligned}$$

and

$$\begin{aligned} Bel(\theta_7 \cup \theta_8) &= [0.194250, 0.329250] \\ Pl(\theta_7 \cup \theta_8) &= [0.654243, 1.352123] \equiv [0.654243, 1] \end{aligned}$$

Therefore, one gets the following imprecision ranges for probabilities

$$P(\theta_8) \in [0.194250, 1]$$

$$P(\bar{\theta}_8) \in [0, 0.805750]$$

and

$$\begin{aligned} P(\theta_7 \cup \theta_8) &\in [0.194250, 1] \\ P(\overline{\theta_7 \cup \theta_8}) &\in [0, 0.805750] \end{aligned}$$

Based on max of Bel or max of Pl criteria, one sees that the decision will be to evacuate the building B.

Let's compute now the imprecise DSmP values for $\epsilon = 0.001$. The focal element $f_1 = \theta_4 \cup \theta_8$ is redistributed back to θ_4 and θ_8 directly proportionally to their corresponding masses and cardinalities

$$\begin{aligned} \frac{x_{\theta_4}}{0 \boxplus 0.001} &= \frac{y'_{\theta_8}}{[0.194250, 0.329250] \boxplus 0.001} \\ &= \frac{m_{PCR6}(\theta_4 \cup \theta_8)}{0.002 \boxplus [0.194250, 0.329250]} \\ &= \frac{[0.152524, 0.323743]}{[0.196250, 0.331250]} \\ &= [\frac{0.152524}{0.331250}, \frac{0.323743}{0.196250}] \\ &= [0.46045, 1.64965] \end{aligned}$$

whence

$$\begin{aligned} x_{\theta_4} &= 0.001 \boxdot [0.46045, 1.64965] \\ &= [0.000460, 0.001650] \\ y'_{\theta_8} &= [0.195250, 0.330250] \boxdot [0.46045, 1.64965] \\ &= [0.089903, 0.544797] \end{aligned}$$

The focal element $f_2 = \theta_6 \cup \theta_8$ is redistributed back to θ_6 and θ_8 directly proportionally to their corresponding masses and cardinalities

$$\begin{aligned} \frac{z_{\theta_6}}{0 \boxplus 0.001} &= \frac{y''_{\theta_8}}{[0.194250, 0.329250] \boxplus 0.001} \\ &= \frac{m_{PCR6}(\theta_6 \cup \theta_8)}{0.002 \boxplus [0.194250, 0.329250]} \\ &= \frac{[0.100351, 0.236255]}{[0.196250, 0.331250]} \\ &= [0.302943, 1.20385] \end{aligned}$$

whence

$$\begin{aligned} z_{\theta_6} &= 0.001 \boxdot [0.302943, 1.20385] \\ &= [0.000303, 0.001204] \\ y''_{\theta_8} &= [0.195250, 0.330250] \boxdot [0.302943, 1.20385] \\ &= [0.0591496, 0.3975714] \end{aligned}$$

The focal element $f_3 = \overline{\theta_4 \cup \theta_8}$ which is also equal to $\theta_1 \cup \theta_2 \cup \theta_3 \cup \theta_5 \cup \theta_6 \cup \theta_7$ is redistributed back to θ_1, θ_2, θ_3, θ_5, θ_6 and θ_7 directly proportionally to their corresponding masses and cardinalities

$$\begin{aligned} \frac{w_{\theta_1}}{0 \boxplus 0.001} &= \frac{w_{\theta_2}}{0 \boxplus 0.001} = \frac{w_{\theta_3}}{0 \boxplus 0.001} = \frac{w_{\theta_5}}{0 \boxplus 0.001} \\ &= \frac{w_{\theta_6}}{0 \boxplus 0.001} = \frac{w_{\theta_7}}{0 \boxplus 0.001} \\ &= \frac{m_{PCR6}(\overline{\theta_4 \cup \theta_8})}{0.006} \\ &= \frac{[0.106767, 0.226620]}{0.006} \\ &= [17.7945, 37.770] \end{aligned}$$

Since all are equal, we get

$$\begin{aligned} w_{\theta_1} &= w_{\theta_2} = w_{\theta_3} = w_{\theta_5} = w_{\theta_6} = w_{\theta_7} \\ &= 0.001 \boxdot [17.7945, 37.770] \\ &= [0.0177945, 0.03777] \end{aligned}$$

The total ignorance $I_t = \theta_1 \cup \theta_2 \cup \theta_3 \cup \theta_4 \cup \theta_5 \cup \theta_6 \cup \theta_7 \cup \theta_8$ is redistributed back to all eight elements of the frame Θ directly proportionally to their corresponding masses and cardinalities

$$\begin{aligned} \frac{v_{\theta_1}}{0 \boxplus 0.001} &= \frac{v_{\theta_2}}{0 \boxplus 0.001} = \frac{v_{\theta_3}}{0 \boxplus 0.001} = \frac{v_{\theta_4}}{0 \boxplus 0.001} \\ &= \frac{v_{\theta_5}}{0 \boxplus 0.001} = \frac{v_{\theta_6}}{0 \boxplus 0.001} = \frac{v_{\theta_7}}{0 \boxplus 0.001} \\ &= \frac{v_{\theta_8}}{[0.194250, 0.329250] \boxplus 0.001} \\ &= \frac{m_{PCR6}(I_t)}{[0.194250, 0.329250] \boxplus 0.008} \\ &= \frac{[0.100351, 0.236255]}{[0.202550, 0.337250]} \\ &= [0.297557, 1.16813] \end{aligned}$$

whence

$$\begin{aligned}
v_{\theta_1} &= v_{\theta_2} = v_{\theta_3} = v_{\theta_4} = v_{\theta_5} = v_{\theta_6} = v_{\theta_7} \\
&= 0.001 \boxdot [0.297557, 1.16813] \\
&= [0.000298, 0.001168] \\
v_{\theta_8} &= [0.195250, 0.330250] \boxdot [0.297557, 1.16813] \\
&= [0.058098, 0.385775]
\end{aligned}$$

The imprecise DSmP probabilities are computed by

$$\begin{aligned}
DSmP_{\epsilon,PCR6}(\theta_1) &= w_{\theta_1} \boxplus v_{\theta_1} \\
DSmP_{\epsilon,PCR6}(\theta_2) &= w_{\theta_2} \boxplus v_{\theta_2} \\
DSmP_{\epsilon,PCR6}(\theta_3) &= w_{\theta_3} \boxplus v_{\theta_3} \\
DSmP_{\epsilon,PCR6}(\theta_4) &= x_{\theta_4} \boxplus v_{\theta_4} \\
DSmP_{\epsilon,PCR6}(\theta_5) &= w_{\theta_5} \boxplus v_{\theta_5} \\
DSmP_{\epsilon,PCR6}(\theta_6) &= z_{\theta_6} \boxplus w_{\theta_6} \boxplus v_{\theta_6} \\
DSmP_{\epsilon,PCR6}(\theta_7) &= w_{\theta_7} \boxplus v_{\theta_7} \\
DSmP_{\epsilon,PCR6}(\theta_8) &= y'_{\theta_8} \boxplus y''_{\theta_8} \boxplus v_{\theta_8}
\end{aligned}$$

which are summarized[7] in Table 55 below.

Singletons	$DSmP_{\epsilon,PCR6}(.)$
θ_1	[0.0181,0.0389]
θ_2	[0.0181,0.0389]
θ_3	[0.0181,0.0389]
θ_4	[0.0008,0.0028]
θ_5	[0.0181,0.0389]
θ_6	[0.0184 0.0402]
θ_7	[0.0181,0.0389]
θ_8	[0.2072,1]

Table 55: Imprecise $DSmP_\epsilon$ of $m_0 \oplus m_1 \oplus m_2 \oplus m_3$ for Table 53.

- **Answer to Q1:** As we have shown, it is possible to fuse imprecise bba's with PCR6, and PCR5 too (see [6], Vol. 2) to get an imprecise result for decision-making support under uncertainty and imprecision. It is also possible to compute the exact imprecise values of DSmP if necessary. According to our analysis and our results, and using either the max of Bel, the max of Pl of the max of DSmP criterion, the decision will be to evacuate the building B. Of course, a similar analysis can be done to answer to Q2 as well when working with imprecise bba's.

4 Qualitative approach

In this section we just showhow the fusion and decision-making can be done using qualitative information expressed with labels. In our previous examples the quantitative baa's have been defined ad-hoc in satisfying some reasonable modeling and using minimal assumption compatible with what is given in the statement of the VIEB problem. The numerical values can be slightly changed (as we have shown in Examples 1 and 2, or in Examples 3 or 4) or evan can be taken as imprecise as in Example 9, but they still need to be remain coherent with sources reports in order to obtain what we consider as pertinent and motived answers to questions Q1 and Q2.

In this section we show how to solve the problem using qualitative information using labels. We investigate the possibility to work either with a minimal set of labels $\{L_1 = Low, L_2 = High\}$ (i.e. with $m = 2$ labels) or a more refined set consisting in 3 labels $\{L_1 = Low, L_2 = Medium, L_3 = High\}$ (i.e. with $m = 3$ labels). Each set is extended with minimal L_0 and maximal L_{m+1} labels as follows (see [6], Vol.3, Chap. 2 for examples and details)

$$\mathcal{L}_2 = \{L_0 \equiv 0, L_1 = Low, L_2 = High, L_3 \equiv 1\}$$

and

$$\mathcal{L}_3 = \{L_0 \equiv 0, L_1 = Low, L_2 = Medium, L_3 = High, L_4 \equiv 1\}$$

To simplify the presentation, we only present the results when combining directly the sources altogether and considering that they have all the same maximal reliability and importance in the fusion process. In other words, we just consider the qualitative counterpart of Example 1 only.

4.1 Fusion of sources using $\mathcal{L}_2$

Example 10: When using $\mathcal{L}_2$, the qualitative inputs[8] characterizing the VIEB problem must be chosen according yo Table 56.

focal element	$qm_0(.)$	$qm_1(.)$	$qm_2(.)$	$qm_3(.)$
$\theta_4 \cup \theta_8$	L_3	L_0	L_1	L_0
$\theta_6 \cup \theta_8$	L_0	L_2	L_0	L_1
$\overline{\theta_4 \cup \theta_8}$	L_0	L_0	L_2	L_0
I_t	L_0	L_1	L_0	L_2

Table 56: Qualitative inputs using $\mathcal{L}_2$.

Using DSm field and linear algebra of refined labels based on equidistant labels assumption, one gets the following mapping between labels and numbers $L_0 \equiv 0$, $L_1 \equiv 1/3$, $L_2 \equiv 2/3$ and $L_3 \equiv 1$ and therefore, the Table 56 is equivalent to the quantitative inputs table 57 (which are close to the numerical values taken in Example 1).
Applying PCR5 and PCR6 fusion rules, one gets the results given in Table 58 for quantitative and approximate qualitative bba's.

[7] Actually for θ_8, one gets with exact calculus of imprecision $DSmP_{\epsilon,PCR6}(\theta_8) = [0.2072, 1.3281]$, buf since a probability cannot be greater than 1, the upper bound of imprecision interval has been set to 1.

[8] When dealing with qualitative information, we prefix the notations with 'q' letter, for example quantitative bba $m(.)$ becomes qualitative bba $qm(.)$, etc.

focal element	$m_0(.)$	$m_1(.)$	$m_2(.)$	$m_3(.)$
$\theta_4 \cup \theta_8$	1	0	1/3	0
$\theta_6 \cup \theta_8$	0	2/3	0	1/3
$\overline{\theta_4 \cup \theta_8}$	0	0	2/3	0
I_t	0	1/3	0	2/3

Table 57: Corresponding quantitative inputs.

focal element	$m_{PCR5} \approx qm_{PCR5}$	$qm_{PCR6} \approx qm_{PCR6}$
θ_8	$0.25926 \approx L_1$	$0.25926 \approx L_1$
$\theta_4 \cup \theta_8$	$0.36145 \approx L_1$	$0.3157 \approx L_1$
$\theta_6 \cup \theta_8$	$0.093855 \approx L_0$	$0.13198 \approx L_0$
$\overline{\theta_4 \cup \theta_8}$	$0.19158 \approx L_1$	$0.16108 \approx L_0$
I_t	$0.093855 \approx L_0$	$0.13198 \approx L_0$

Table 58: Results of $m_0 \oplus m_1 \oplus m_2 \oplus m_3$ for Table 56.

One sees that the crude approximation of numerical values to their closest corresponding labels in $\mathcal{L}_2$ can yield to unnormalized qualitative bba. For example, $qm_{PCR6}(.)$ is not normalized since the sum of labels of focal elements in the right column of Table 58 is $L_1 + L_1 + L_0 + L_0 + L_0 = L_2 \neq L_3$. To preserve the normalization of qbba result it is better to work with refined labels as suggested in [6], Vol.3, Chap. 2. Using refined labels, one will get now better approximation as shown in the Table 59.

focal element	$m_{PCR5} \approx qm_{PCR5}$	$m_{PCR6} \approx qm_{PCR6}$
θ_8	$0.25926 \approx L_{0.79}$	$0.25926 \approx L_{0.79}$
$\theta_4 \cup \theta_8$	$0.36145 \approx L_{1.08}$	$0.31570 \approx L_{0.95}$
$\theta_6 \cup \theta_8$	$0.09385 \approx L_{0.28}$	$0.13198 \approx L_{0.39}$
$\overline{\theta_4 \cup \theta_8}$	$0.19158 \approx L_{0.57}$	$0.16108 \approx L_{0.48}$
I_t	$0.09385 \approx L_{0.28}$	$0.13198 \approx L_{0.39}$

Table 59: Results of $m_0 \oplus m_1 \oplus m_2 \oplus m_3$ for Table 56.

It can be easily verified that the qbba's based on refined label approximations are now (qualitatively) normalized (because the sum of refined labels of each column is equal to L_3).

The results of qDSmP based on refined and crude approximations are given in Table 60.

Singletons	$qDSmP_{\epsilon,PCR5}(.)$	$qDSmP_{\epsilon,PCR6}(.)$
θ_1	$0.0323 \approx L_{0.10} \approx L_0$	$0.0273 \approx L_{0.08} \approx L_0$
θ_2	$0.0323 \approx L_{0.10} \approx L_0$	$0.0273 \approx L_{0.08} \approx L_0$
θ_3	$0.0323 \approx L_{0.10} \approx L_0$	$0.0273 \approx L_{0.08} \approx L_0$
θ_4	$0.0017 \approx L_{0.00} \approx L_0$	$0.0017 \approx L_{0.01} \approx L_0$
θ_5	$0.0323 \approx L_{0.10} \approx L_0$	$0.0274 \approx L_{0.08} \approx L_0$
θ_6	$0.0326 \approx L_{0.10} \approx L_0$	$0.0279 \approx L_{0.08} \approx L_0$
θ_7	$0.0323 \approx L_{0.10} \approx L_0$	$0.0273 \approx L_{0.08} \approx L_0$
θ_8	$0.8042 \approx L_{2.40} \approx L_2$	$0.8338 \approx L_{2.51} \approx L_3$

Table 60: Results of $qDSmP_\epsilon$ for Table 56.

Answer to Q1 using crude approximation: Based on these qualitative results, one sees that using crude approximation (i.e. using only labels in $\mathcal{L}_2$) one gets[9]

- with qPCR5

$$qP(\theta_8) \in [L_1, L_2]$$
$$qP(\bar{\theta}_8) \in [L_1, L_2]$$

and

$$qP(\theta_7 \cup \theta_8) \in [L_1, L_3]$$
$$qP(\overline{\theta_7 \cup \theta_8}) \in [L_0, L_2]$$

- with qPCR6

$$qP(\theta_8) \in [L_1, L_2]$$
$$qP(\bar{\theta}_8) \in [L_1, L_2]$$

and

$$qP(\theta_7 \cup \theta_8) \in [L_1, L_2]$$
$$qP(\overline{\theta_7 \cup \theta_8}) \in [L_1, L_2]$$

These results show that is is almost impossible to answer fairly to question Q1 using the max of Bel or the max of Pl criteria based on such very inaccurate qualitative bba's using this crude approximation. However it is possible and easy to answer to Q1 using qualitative DSmP value. According to Table 60, the final decision must be to **evacuate the building** B. This decision is the same as the decision taken in the Example 1 which has the quantitative input Table 1 close to Table 57.

Answer to Q1 using refined approximation: Using the refined approximation using refined labels which is more accurate, one gets

- with qPCR5

$$qP(\theta_8) \in [L_{0.79}, L_{2.43}]$$
$$qP(\bar{\theta}_8) \in [L_{0.57}, L_{2.21}]$$

and

$$qP(\theta_7 \cup \theta_8) \in [L_{0.79}, L_3]$$
$$qP(\overline{\theta_7 \cup \theta_8}) \in [L_0, L_{2.21}]$$

- with qPCR6

$$qP(\theta_8) \in [L_{0.79}, L_{2.52}]$$
$$qP(\bar{\theta}_8) \in [L_{0.48}, L_{2.21}]$$

and

$$qP(\theta_7 \cup \theta_8) \in [L_{0.79}, L_3]$$
$$qP(\overline{\theta_7 \cup \theta_8}) \in [L_0, L_{2.21}]$$

[9] The derivations of $qBel(\bar{X})$ and $qPl(\bar{X})$ were obtained using qualitative extension of Dempster's formulas [5], i.e. with $qBel(\bar{X}) = L_m - qPl(X)$ and $qPl(\bar{X}) = L_m - qBel(X)$. These results are valid only if the qbba is normalized, but are used here even when using non normalized qbba as crude approximation.

One sees that accuracy of the result obtained using refined labels allows us to take the decision more easily. Indeed, using the refined approximation, it is possible here to take the decision based on the max of Bel, or on the max of Pl and whatever the criterion used, the answer to question Q1 is: **Evacuation of the building** B. Of course, the same answer can also be drawn from the analysis of qDSmP values as well when considering refined labels in Table 60.

4.2 Fusion of sources using $\mathcal{L}_3$

Here we propose to go further in our analysis and to use a bit more refined set of labels defined by $\mathcal{L}_3$. We need to adapt the qualitative inputs of the VIEB problem in order to work with $\mathcal{L}_3$.

Example 11: We propose to solve the VIEB problem for the following qualitative inputs which reflects well what is stated by the sources when using labels belonging to $\mathcal{L}_3$.

focal element	$qm_0(.)$	$qm_1(.)$	$qm_2(.)$	$qm_3(.)$
$\theta_4 \cup \theta_8$	L_4	L_0	L_1	L_0
$\theta_6 \cup \theta_8$	L_0	L_3	L_0	L_1
$\overline{\theta_4 \cup \theta_8}$	L_0	L_0	L_3	L_0
I_t	L_0	L_1	L_0	L_3

Table 61: Qualitative inputs based on $\mathcal{L}_3$.

Based on the equidistant labels assumption, one gets the following mapping between labels and numbers $L_0 \equiv 0$, $L_1 \equiv 1/4$, $L_2 \equiv 2/4$, $L_3 \equiv 3/4$ and $L_4 = 1$ and therefore, the Table 61 is equivalent to the quantitative inputs table 62 (which are more close to the numerical values taken in Example 1 than the inputs chosen in Table 57 for Example 10).

focal element	$m_0(.)$	$m_1(.)$	$m_2(.)$	$m_3(.)$
$\theta_4 \cup \theta_8$	1	0	0.25	0
$\theta_6 \cup \theta_8$	0	0.75	0	0.25
$\overline{\theta_4 \cup \theta_8}$	0	0	0.75	0
I_t	0	0.25	0	0.75

Table 62: Corresponding quantitative inputs.

Applying PCR5 and PCR6 fusion rules, one gets the results given in Table 63 for quantitative and approximate qualitative bba's using refined and crude approximations of labels.

foc. elem.	$m_{PCR5} \approx qm_{PCR5}$	$m_{PCR6} \approx qm_{PCR6}$
θ_8	$0.20312 \approx L_{0.81} \approx L_1$	$0.20312 \approx L_{0.81} \approx L_1$
$\theta_4 \cup \theta_8$	$0.34269 \approx L_{1.37} \approx L_1$	$0.29979 \approx L_{1.21} \approx L_1$
$\theta_6 \cup \theta_8$	$0.11617 \approx L_{0.47} \approx L_0$	$0.15370 \approx L_{0.61} \approx L_1$
$\overline{\theta_4 \cup \theta_8}$	$0.22185 \approx L_{0.88} \approx L_1$	$0.18969 \approx L_{0.76} \approx L_1$
I_t	$0.11617 \approx L_{0.47} \approx L_0$	$0.15370 \approx L_{0.61} \approx L_1$

Table 63: Results of $m_0 \oplus m_1 \oplus m_2 \oplus m_3$ for Table 61.

It can be easily verified that the qbba's based on refined label approximations are (qualitatively) normalized because the sum of refined labels of each column is equal to L_4. Using crude approximation when working only with labels in $\mathcal{L}_3$ we get non normalized qbba's. The results of qDSmP based on refined and crude approximations are given in Table 64.

Singletons	$qDSmP_{\epsilon,PCR5}(.)$	$qDSmP_{\epsilon,PCR6}(.)$
θ_1	$0.0375 \approx L_{0.15} \approx L_0$	$0.0323 \approx L_{0.13} \approx L_0$
θ_2	$0.0375 \approx L_{0.15} \approx L_0$	$0.0323 \approx L_{0.13} \approx L_0$
θ_3	$0.0375 \approx L_{0.15} \approx L_0$	$0.0323 \approx L_{0.13} \approx L_0$
θ_4	$0.0022 \approx L_{0.01} \approx L_0$	$0.0022 \approx L_{0.01} \approx L_0$
θ_5	$0.0375 \approx L_{0.15} \approx L_0$	$0.0323 \approx L_{0.13} \approx L_0$
θ_6	$0.0381 \approx L_{0.15} \approx L_0$	$0.0331 \approx L_{0.13} \approx L_0$
θ_7	$0.0375 \approx L_{0.15} \approx L_0$	$0.0323 \approx L_{0.13} \approx L_0$
θ_8	$0.7722 \approx L_{3.09} \approx L_3$	$0.8032 \approx L_{3.21} \approx L_3$

Table 64: Results obtained with $qDSmP_\epsilon$ for Table 61.

Ones sees that the use of refined labels allows to obtain normalized qualitative probabilities. This is not possible to get normalized qualitative probabilities when using only crude approximations with labels in $\mathcal{L}_3$ for this example.

Answer to Q1 using crude approximation: Based on these qualitative results, one sees that using crude approximation (i.e. using only labels in $\mathcal{L}_3$) one gets

- with qPCR5

$$qP(\theta_8) \in [L_1, L_2]$$

$$qP(\bar{\theta}_8) \in [L_2, L_3]$$

  and

$$qP(\theta_7 \cup \theta_8) \in [L_1, L_3]$$
$$qP(\overline{\theta_7 \cup \theta_8}) \in [L_1, L_3]$$

- with qPCR6

$$qP(\theta_8) \in [L_1, L_4]$$

$$qP(\bar{\theta}_8) \in [L_0, L_3]$$

  and

$$qP(\theta_7 \cup \theta_8) \in [L_1, L_5] \equiv [L_1, L_4]$$
$$qP(\overline{\theta_7 \cup \theta_8}) \in [L_0, L_3]$$

These results show that with the max of Bel or max of Pl criteria, one will take the decision to evacuate the building B more easily when working with the refined set $\mathcal{L}_3$ than when working with $\mathcal{L}_2$. This decision is however mistaken by fact that crude approximation is used. Indeed, one sees that with PCR5 and considering θ_8 as supporting hypothesis for decision, one will have to decide to NOT evacuate the building B.

According to Table 64, the final decision to take using DSmP with crude approximation, must be to evacuate the building B.

Answer to Q1 using refined approximation: Using the refined approximation using refined labels which is more accurate, one gets

- with qPCR5

$$qP(\theta_8) \in [L_{0.81}, L_{3.12}]$$

$$qP(\bar{\theta}_8) \in [L_{0.88}, L_{3.19}]$$

and

$$qP(\theta_7 \cup \theta_8) \in [L_{0.81}, L_4]$$

$$qP(\overline{\theta_7 \cup \theta_8}) \in [L_0, L_{3.19}]$$

- with qPCR6

$$qP(\theta_8) \in [L_{0.81}, L_{3.24}]$$

$$qP(\bar{\theta}_8) \in [L_{0.76}, L_{3.19}]$$

and

$$qP(\theta_7 \cup \theta_8) \in [L_{0.81}, L_4]$$

$$qP(\overline{\theta_7 \cup \theta_8}) \in [L_0, L_{3.19}]$$

One sees that based on PCR6 and using the max of Bel, or the max of Pl criteria, the decision will be to evacuate the building B. Same decision is drawn with PCR5 when considering $\theta_7 \cup \theta_8$ as decision support hypothesis. However if we use PCR and we consider only θ_8 as decision support hypothesis one will conclude to NOT evacuate the building B because $Bel(\bar{\theta}_8) = L_{0.88} > Bel(\theta_8) = L_{0.81}$ and also $Pl(\bar{\theta}_8) = L_{3.19} > Pl(\theta_8) = L_{3.12}$ whereas PCR6 using decision support hypothesis θ_8 will conclude to the evacuation of the building B. So this example shows a strong difference between the behavior of PRC6 w.r.t. PCR5 for decision-making support. The two results are highly conflicting here.

However according to Table 64, the final decision to take using DSmP with refined approximation with PCR5 and PCR6, must be to evacuate the building B.

5 Sequential fusion of sources

6 Conclusions

To be completed ...

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