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Improving industrial automation selection with dynamic exponential distance in neutrosophic group decision-making framework

Amirhossein Nafei[a], Shu-Chuan Chen[a,b], Harish Garg[c], Chien-Yi Huang[a], Florentin Smarandache[d] and Seyed Mohammadtaghi Azimi[e]

[a]Department of Industrial Engineering and Management, National Taipei University of Technology, Taipei, Taiwan; [b]Ming Chi University of Technology, New Taipei, Taiwan; [c]Thapar Institute of Engineering & Technology (Deemed University), Patiala, India; [d]University of New Mexico, Gallup Campus, Gallup, USA; [e]Amirkabir University of Technology, Tehran, Iran

ABSTRACT
In the fast-paced evolution of industrial technology, selecting optimal automation systems is essential for improving operational efficiency. Traditional decision-making frameworks often fall short in addressing the complex interplay of diverse criteria, especially under conditions of uncertainty and imprecision. This research proposes a decision-making methodology that integrates dynamic exponential distance (DED) within the VIKOR (VIseKriterijumska Optimizacija I Kompromisno Resenje) framework, enhanced by neutrosophic logic. This integration provides a more accurate decision-making process by revolutionizing distance calculations with exponential transformations. The DED approach amplifies differences in extreme values, enhancing the ability to distinguish between alternatives. The motivation behind this research is the growing need for a more robust decision-making framework that can better handle complex, uncertain, and contradictory information. The contributions of this research include developing the DED approach to enhance precision in distance calculations within neutrosophic environments using exponential transformations, introducing a comprehensive methodology to address uncertainty and indeterminacy in decision-making scenarios, and demonstrating the method's adaptability to real-world industrial automation challenges by improving discrimination. The comprehensive case study validates the proposed method's effectiveness, showing better discrimination between alternatives and adaptability to dynamic decision contexts. The runtime analysis highlights a balanced trade-off between computational demands and decision-making performance compared to other methods.

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1. Introduction

In the rapidly evolving landscape of industrial technology, the selection of automation systems emerges as a cornerstone in enhancing operational efficiency and maintaining a competitive advantage. The complexity inherent in this selection process is compounded by the diverse and often conflicting criteria that must be evaluated, ranging from system reliability and cost efficiency to integration capability and ongoing support (Fan et al., 2024). Traditional decision-making frameworks often need to address the nuanced interplay of these factors, particularly under conditions of uncertainty and imprecision that characterize real-world industrial environments (Lin, Liu, et al., 2024; Lin, Ma, et al., 2024). The advent of neutrosophic logic (Smarandache, 1999) offers a promising avenue for refining decision-making methodologies. Neutrosophic sets (NSs) can independently encode truth, indeterminacy, and falsity and provide a more flexible and realistic representation of the information typically encountered in industrial settings. However, the challenge of integrating these sets into a robust decision-making process still needs to be solved. Several methodologies stand out when considering decision-making frameworks for evaluating complex systems like industrial automation, each with distinct characteristics. Here is an overview of popular decision-making methods such as the Technique for Order of Preference by Similarity to the Ideal Solution (TOPSIS) (Hwang & Yoon, 1981), Elimination and Choice Expressing Reality (ELECTRE) (Benayoun et al., 1966), and VIseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) (Opricovic, 1979), highlighting the unique advantages of VIKOR and justifying its selection for our analysis.

TOPSIS is based on the idea that the chosen alternative should have the shortest distance from the ideal solution and the longest from the worst

CONTACT Harish Garg harishg58iitr@gmail.com Department of Mathematics, Thapar Institute of Engineering & Technology (Deemed University), Patiala, Punjab 147004, India; Chien-Yi Huang jayhuang@ntut.edu.tw Department of Industrial Engineering and Management, National Taipei University of Technology, Taipei 10608, Taiwan

solution. It effectively uses geometric distances in the criteria space to rank the alternatives. TOPSIS is straightforward and computationally efficient. It provides a clear rationale by considering both the ideal and nadir points, making it intuitively appealing. However, it assumes that criteria are commensurately weighted and often require independent criteria, which may not be feasible in interconnected systems. Additionally, TOPSIS does not inherently handle uncertainty or indeterminacy in data. ELECTRE, which stands for Elimination and Choice Expressing Reality, uses pairwise comparisons to form outranking relationships among alternatives. It is beneficial for handling qualitative data and conflicting criteria. ELECTRE is suitable for handling non-compensatory criteria where trade-offs are not allowed, allows for partial comparability, and manages inconsistent judgements effectively. Nevertheless, the results can be challenging to interpret due to potential incomparability. ELECTRE is also complex to implement and requires setting several thresholds, such as concordance and discordance, which can be subjective. VIKOR ranks and selects alternatives to determine a compromise solution. This method introduces the utility measure for the "group utility" and the individual regret measure for the "individual regret." VIKOR is designed to handle conflicting and non-commensurable (incommensurable) criteria, which is common in industrial decisions. It provides a clear ranking of alternatives and identifies the best compromise solution, making it highly suitable for group decision-making (GDM) scenarios. The computation is relatively simple and produces easy-to-understand outputs, which aid in transparency and stakeholder communication.

Given the complexities and uncertainties inherent in selecting industrial automation systems—where multiple, often conflicting performance criteria must be evaluated—VIKOR emerges as the most appropriate method. The specific reasons for using VIKOR are presented as follows:

1. Compromise solution orientation: VIKOR's methodology is inherently designed to find solutions that best balance the interests of all stakeholders involved, providing a compromise that is often necessary in multifaceted industrial environments.
2. Handling of conflicting criteria: Industrial automation balances performance, cost, reliability, and integration. VIKOR excels in scenarios where these criteria conflict, providing a systematic approach to navigating trade-offs.
3. Adaptability to uncertainty with dynamic exponential distance (DED): By integrating VIKOR with the DED, we enhance its ability to deal with uncertainty and imprecision in criteria assessments—a significant advantage in industrial settings where inputs may be fuzzy or incomplete.
4. Group decision facilitation: VIKOR facilitates decision-making in a group context, where stakeholders may prioritize different criteria. This is particularly valuable in organizational settings where buy-in from various departments is crucial.

Since TOPSIS and ELECTRE have their strengths, VIKOR's suitability for addressing the specific challenges of industrial automation selection—significantly when enhanced with DED—makes it the optimal choice for our study. Its ability to synthesize complex, conflicting criteria into a decision framework that promotes compromise and consensus aligns closely with the needs of modern industrial enterprises.

In the realm of industrial automation, selecting optimal systems requires a careful balance among competing criteria such as system reliability, cost efficiency, integration ease, and technical support. Traditional decision-making frameworks, like TOPSIS and ELECTRE, predominantly use linear distance measures such as Euclidean distance. These methods assume criteria have equal weighting and struggle to effectively handle uncertainty or indeterminacy in data, which limits their ability to capture the complex realities of industrial decision-making where conflicting criteria are prevalent. This methodological shortcoming leaves a significant gap in representing nuanced decision factors under conditions of uncertainty, as traditional distance measures lack the adaptability to differentiate between alternatives in high-stakes industrial settings.

To address these challenges, this study introduces a novel approach integrating DED with the VIKOR framework, leveraging neutrosophic logic to better handle the indeterminacy and imprecision often found in industrial data. The DED method revolutionizes distance calculations by incorporating exponential transformations, which amplify differences in critical decision factors, enabling improved discrimination between closely ranked alternatives. This advancement provides a more robust tool for multi-attribute group decision making (MAGDM), directly addressing the limitations in traditional frameworks and enhancing decision accuracy and reliability, particularly in complex industrial scenarios where uncertain data is prevalent.

From a practical perspective, the lack of a comprehensive methodology that accommodates diverse stakeholder priorities, particularly when handling incomplete or ambiguous information, presents a barrier to optimized decision outcomes and consensus. The DED-enhanced VIKOR framework addresses this gap, allowing for a systematic approach that facilitates GDM while balancing the priorities of multiple stakeholders. Through a real-world case study in industrial automation system selection, this research demonstrates the practical value of the DED-VIKOR approach, showcasing its adaptability to changing decision contexts, its enhanced discrimination capabilities, and its suitability for achieving consensus in multifaceted industrial settings.

1.2. Motivation of this research

In the complex realm of industrial automation, selecting optimal systems is crucial for enhancing operational efficiency and maintaining competitive advantage. Traditional decision-making methods often incorporate distance measures that fail to capture the intricate dynamics of industrial criteria, typically laden with uncertainty and imprecision. Conventional measures like the Euclidean distance assume linear interactions and equal weighting across attributes, which do not adequately reflect the true complexity of decision-making scenarios. These shortcomings necessitate reevaluating distance measures to accommodate better the nuanced relationships and varying significance of different criteria.

Recognizing these challenges motivates us to propose a DED within a MAGDM framework. The DED approach revolutionizes how distances are calculated by incorporating exponential transformations, thus allowing for a more nuanced handling of the degrees of truth, indeterminacy, and falsity in NSs. This method enhances the ability to distinguish between alternatives in a way that aligns more closely with the practical realities of industrial automation systems, where the severity of discrepancies can disproportionately affect outcomes. To use the advantages of DED in decision-making, we are motivated to present a decision-making method that utilizes DED within its framework. This endeavour aims to improve the outcomes of the decision-making process, enhancing the accuracy and reliability of selecting industrial automation systems.

The key advantages of using the DED in a decision-making framework, especially within the context of industrial automation selection, are itemized as follows:

1. Enhanced sensitivity to extremes: DED employs exponential functions amplifying the differences in extreme truth and falsity values. This allows for a more accurate representation of the impact of each criterion, highlighting critical discrepancies that could significantly influence decision outcomes.
2. Adaptive weighting of criteria: The defined parameters in the DED formula can be adjusted to reflect the varying importance of truth, indeterminacy, and falsity in different decision-making contexts. This adaptability makes DED particularly useful in environments where the relevance of criteria may change over time or across scenarios.
3. Improved handling of uncertainty and indeterminacy: Traditional distance measures often struggle to incorporate and handle indeterminacy within the decision-making process effectively. DED, by contrast, explicitly integrates indeterminacy through its mathematical formulation, providing a more robust framework for dealing with uncertain and incomplete information.
4. Better discrimination between options: DED distinguish more clearly between closely ranked alternatives by intensifying the impact of significant differences. This improved discrimination is crucial in decision-making processes where minor differences can lead to divergent outcomes, ensuring that the selected options are the most suitable.
5. Compatibility with neutrosophic data: DED is designed to work with neutrosophic data, characterized by their representation of truth, indeterminacy, and falsity. This compatibility is advantageous in complex decision environments where data may only sometimes be precise or reliable.

1.3. Contribution of this research

This research first introduces the concept of DED. Subsequently, it presents an enhanced version of the VIKOR method, which is extended to incorporate neutrosophic triplets for improved decision-making in complex environments. The contributions of this research are listed as follows:

1. Introduction of DED: This research introduces the DED, a novel distance measure explicitly designed for use in neutrosophic settings where components such as truth, indeterminacy, and falsity need to be quantitatively assessed. By applying exponential transformations, DED enhances sensitivity to variations in these components, particularly accentuating the importance of extremes critical in automation technology decisions.
2. Enhancement of VIKOR method: The integration of DED into the VIKOR method represents a significant advancement in multi-criteria

decision-making (MCDM) and multi-attribute decision-making (MADM). This hybrid approach not only improves the accuracy of determining compromise solutions but also makes the VIKOR method more suitable for environments characterized by high levels of uncertainty and conflicting criteria.
3. Advanced handling of uncertainty and indeterminacy: By utilizing neutrosophic logic to manage uncertainty and indeterminacy in decision-making, the research pushes the boundaries of traditional decision models that often rely on crisp data. This allows for more realistic and reliable decision-making in industrial automation, where input data may only sometimes be precise or entirely specific.
4. Practical application in industrial automation: The research provides a comprehensive framework that can be directly applied to selecting industrial automation systems. This useful application is demonstrated through a case study, showcasing the effectiveness of the proposed method in real-world scenarios and highlighting its potential to improve decision outcomes in complex industrial settings.
5. Facilitation of stakeholder consensus: The adapted VIKOR method, incorporating DED, facilitates greater stakeholder consensus by providing a systematic approach to balance different priorities and minimize regrets. This aspect is precious in industrial contexts where decisions must accommodate diverse objectives and potential trade-offs.

1.4. Structure of the research

The structure of this research paper is outlined as follows: Section 2 presents a review of existing available literature. Section 3 presents the preliminary concepts and foundational theories pertinent to the study. Section 4 elaborates on developing an exponential distance measure tailored for neutrosophic triplets. Section 5 details the algorithm of the extended VIKOR method. Section 6 provides a numerical example to illustrate the efficacy of the proposed method. Section 7 conducts a sensitivity analysis of the proposed method and compares it with existing methods in the literature. The study is concluded in Section 8, where the main findings are summarized, and the research implications are discussed.

2. Literature review

In the literature, several extensions of TOPSIS methods for solving decision-making methods under uncertainty and indeterminacy have been proposed. For instance, Qi (2023) proposed an extended TOPSIS model integrated with the full consistency method (FUCOM) method within a probabilistic hesitant fuzzy context to address the performance evaluation of public charging service quality, a typical MAGDM issue. Jiang and Song (2024) proposed an improved TOPSIS method, dual probabilistic linguistic term—TOPSIS, that integrates the traditional TOPSIS approach with a projection measure designed explicitly for dual probabilistic linguistic term sets. Xu et al. (2024) proposed an innovative method for evaluating the operational management performance of intangible assets in commercial sports events, addressing the challenges posed by a lack of experience in event hosting. Gurmani et al. (2023) proposed a MAGDM model to select the most suitable construction company, considering the complexities of human judgement and the hybrid uncertainty of fuzziness and probability. The critical innovation is introducing a linguistic interval-valued T-spherical fuzzy set framework, which allows decision-makers to provide evaluations in a broader space and better handle vague information. This method integrates aspect-based sentiment analysis, single-valued neutrosophic sets (SVNSs), and an extended TOPSIS approach.

In addition, several extensions of ELECTRE for solving decision-making methods under uncertainty and indeterminacy have been proposed in recent years. Saini et al. (2024) proposed a novel MAGDM model for selecting a female spouse, utilizing the ELECTRE-III method within a Pythagorean neutrosophic environment. This model addresses the complexities and uncertainties of modern partner selection by incorporating multiple criteria and handling conflicting and uncertain information. Kang et al. (2024) proposed a novel hybrid MAGDM methodology for selecting the most suitable wave-energy converter for a power plant, utilizing single-valued neutrosophic probabilistic hesitant fuzzy sets to address uncertainties in data representation. The proposed methodology integrates fuzzy stepwise weighted assessment ratio analysis to assign weights to the selection criteria and fuzzy ELECTRE to evaluate and rank the alternatives. Zhu (2023) presented an adaptive decision model based on deep learning. Akram et al. (2023) proposed an enhanced version of the ELECTRE IV method, incorporating fuzzy set theory to address the water supply problem in Iran. The proposed fuzzy ELECTRE IV method leverages triangular fuzzy numbers to represent linguistic preferences. It uses the outranking principle to evaluate alternatives based on three types of preferences and five dominance relations. J. Luo et al. (2024) presented the deep neural-network-based decision method for financial risk prediction. Udhaya Sankar et al. (2023) proposed a fuzzy ELECTRE multi-criteria decision-making technique to enhance cooperation among mobile nodes in self-configuring, decentralized networks by identifying and mitigating the impact of malicious nodes. This technique addresses uncertainties in node behaviour, utilizing the ELECTRE method to categorize nodes as cooperative or malicious based on positive and negative flow values. Sarwar et al. (2023) proposed a novel approach for failure modes and effects analysis by integrating rough number clouds with the ELECTRE-II method to handle various uncertainties and randomness in the assessment information. This approach utilizes rough integrated clouds, a form of regular cloud evaluations, to address ambiguity and randomness through initial parameters and suppository functions. Talib et al. (2023) presented the decision-making framework for autism patient. Aljanabi (2023) gave the comprehensive analysis of decision-making.

In recent years, various extensions of the VIKOR method have been developed to address decision-making challenges under conditions of uncertainty and indeterminacy. The authors (H. Wang et al. 2023; Z. Wang et al. 2023) presented a novel decision-making method for green and sustainable supplier selection within the context of green supply chain management. This method addresses the MADM problem using type-2 neutrosophic number (T2NN) sets to handle decision-making information. The proposed method combines the TODIM (an acronym in Portuguese for interactive Multi-criteria Decision Making) and VIKOR methods with attribute weights determined by the entropy weight method. X. Luo et al. (2023) suggested a method for selecting sustainable suppliers within a supply chain, addressing the MCDM problem and associated uncertainties. This method integrates the VIKOR technique with SVNSs to effectively handle insufficient information. Abdul et al. (2024) presented a systematic method for determining and ranking the benefits of adopting renewable energy (RE) technologies in remote areas, offering potential solutions to related challenges. The analysis is structured into four levels, beginning with a comprehensive literature review to identify the benefits of RE technologies. Mahmudah et al. (2024) presented a study to determine the most suitable site for constructing nuclear power plants in Indonesia, focusing on socioeconomic factors. They employ two MCDM methods: the fuzzy analytic hierarchy process (FAHP) and fuzzy VIKOR. Dağıstanlı (2024) proposed extending the VIKOR method for MCDM in the interval-valued intuitionistic fuzzy (IVIF) environment, specifically applied to the defence industry. This approach addresses the uncertainty inherent in evaluating high-budget, long-term defence projects by utilizing IVIF methods to incorporate decision-makers preferences comprehensively. Wan et al. (2013) proposed a method for solving MAGDM problems with triangular intuitionistic fuzzy numbers, defining distance measures, calculating attribute and decision-maker weights, and extending the VIKOR method to rank alternatives based on closeness to an ideal solution. The method's effectiveness is demonstrated through a personnel selection case study. Dong et al. (2017) proposed a new Linguistic Hesitant Fuzzy VIKOR (LHF-VIKOR) method for MCDM problems, introducing new distance measures and optimization models to determine criteria weights in linguistically hesitant fuzzy environments. The method's effectiveness is demonstrated through an intelligent transportation system evaluation. For more details, about the MCDM approaches and their diverse applicable, we refer to read the articles (David & Alamoodi, 2023; Mohammed et al., 2024) and their corresponding references.

Apart from that, several researchers have proposed the different kinds of innovative decision-making methods to address complex multi-attribute and multi-criteria group decision-making (MCGDM) challenges under conditions of uncertainty and linguistic hesitation. Nafei, Azizi, et al. (2024) proposed a novel decision-making framework that integrates TOPSIS with Neutrosophic Triplets (NTs) and leverages machine learning, specifically neural networks, to improve accuracy and efficiency in MADM under uncertainty. This approach introduces a frequency analysis-based ranking strategy to better handle indeterminate information. Wan, Zeng, et al. (2024) proposed two new methods for interactive MAGDM using linguistic hesitant fuzzy sets (LHFSs) and comprehensive cloud (CC) power geometric (PG) aggregation operators to support ERP (enterprise resource planning) system selection. They define the CC of LHFS, a distance measure, and introduce CC PG aggregation operators, along with methods to determine both decision-maker and attribute weights. Ye et al. (2024) proposed a MCDM technique based on trigonometric t-norm and t-conorm operational laws for single-valued neutrosophic numbers (SVNNs), tailored for use in inconsistent and indeterminate environments. They develop trigonometric operational laws using tangent, arctangent, cotangent, and inverse cotangent functions and introduce SVNN trigonometric weighted average and geometric aggregation operators with corresponding properties. These operators form the foundation of the proposed MCDM technique, which is validated through a case study on slope treatment scheme selection, demonstrating the method's practicality and effectiveness. Wan, Dong, and Chen (2024a, 2024b) proposed a linguistically hesitant fuzzy MCGDM method for selecting shared power bank suppliers, integrating multi-objective optimization based on a ratio analysis plus the full multiplicative form, best and worst method (BWM), and prospect theory. Key contributions include a new comparison approach for linguistic hesitant fuzzy (LHF), LHF Bonferroni mean operators, and tri-objective optimization for decision-makers' weights. Subjective criteria weights are derived via BWM, objective weights through entropy, and combined using Jenson-Shannon divergence. Garg (2024) introduced the exponential–logarithm SVNS, an extension of the SVNS, which integrates exponential and logarithmic operations to enhance diversity and manage uncertainty in decision-making. A unique attitude parameter allows decision-makers to adapt evaluations to their preferences.

In addition, several innovative decision-making strategies based on the fuzzy set and its extensions have been developed, with a particular focus on addressing real-world decision-making challenges. Imran et al. (2024) proposed the Aczel-Alsina aggregation operator for single-valued neutrosophic hesitant fuzzy sets, combining SVNSs and hesitant fuzzy sets to handle insufficient, unreliable, and vague environments. This operator is more flexible in its t-norm and t-conorm properties, offering adaptability for MADM problems. Wan, Dong, and Chen (2024a, 2024b) proposed a new intuitionistic fuzzy best–worst method for GDM using intuitionistic fuzzy preference relations (IFPRs). The method uses intuitionistic fuzzy values for reference comparisons of criteria and defines additive consistency for IF reference comparisons (IFRCs). A linear goal programming model is developed to calculate optimal priority weights, with a new approach to enhance IFRCs' consistency. Nafei, Huang, et al. (2024) proposed a novel decision-making framework using neutrosophic fuzzy sets (NFSs) to address limitations in handling ambiguity and inconsistency within MADM. The authors introduce an advanced scoring mechanism for NFSs, along with a customized distance measure for identifying inconsistencies in datasets. The framework integrates extensions of TOPSIS and Autocratic methods for group MADM, with frequency-based criteria to achieve clear alternative rankings.

3. Preliminaries and adjustment for consistency

This section concisely overviews critical foundational aspects of NSs, providing the theoretical basis to understand the subsequent development of the proposed adjustment model. It covers essential definitions, properties, and key elements needed for enhancing consistency and achieving consensus in neutrosophic decision-making processes.

Definition 3.1. (Smarandache, 1999) Let us consider a finite universe of goals denoted as X. A NS N in X is indicated by $T_N : X \to 0^-, 1^+, I_N : X \to 0^-, 1^+$, and $F_N : X \to 0^-, 1^+$. Which meets the constraint $0^- \le T_N(x) + I_N(x) + F_N(x) \le 3^+, \forall x \in X$. In this representation, $T_N(x), I_N(x)$, and $F_N(x)$ signify the truth, indeterminacy, and falsity membership functions.

Definition 3.2. (H. Wang et al., 2010) Let X be the universe of discourse. A SVNS N is mapped as $N = \{\langle x, T_N(x), I_N(x), F_N(x)\rangle; x \in X\}$, where $T_N : X \to [0,1]$, $I_N : X \to [0,1]$ and $F_N : X \to [0,1]$ and condition $0 \le T_N(x) + F_N(x) + I_N(x) \le 3, \forall x \in X$ is always satisfied. In the context of the SVNS N, the trinary $(T_N(x), I_N(x), F_N(x))$ is called a NT. For simplicity, this trinary is often represented by the symbol (T, I, F).

Definition 3.3. (Smarandache, 1998) Consider $x = (T_1, I_1, F_1)$ and $y = (T_2, I_2, F_2)$ as two NTs. Following is a description of the mathematical operations between x and y.

$$\text{I.} \quad \lambda x = \left(1 - (1 - T_1)^{\lambda}, I_1^{\lambda}, F_1^{\lambda}\right), \quad \lambda > 0, \tag{1}$$

$$\text{II.} \quad x + y = (T_1 + T_2 - T_1 T_2, I_1 I_2, F_1 F_2), \tag{2}$$

$$\text{III.} \quad x \times y = (T_1 T_2, I_1 + I_2 - I_1 I_2, F_1 + F_2 - F_1 F_2), \tag{3}$$

$$\text{IV.} \quad x^{\lambda} = \left(T_1^{\lambda}, 1 - (1 - I_1)^{\lambda}, 1 - (1 - F_1)^{\lambda}\right), \quad \lambda > 0. \tag{4}$$

$$\text{V.} \quad x/y = \left(\frac{T_1}{T_2}, \frac{I_1 - I_2}{1 - I_2}, \frac{F_1 - F_2}{1 - F_2}\right). \tag{5}$$

Definition 3.4. (Ye, 2017) For any two given NTs $x = (T_1, I_1, F_1)$ and $y = (T_2, I_2, F_2)$, the subtraction operation is defined as:

$$x \ominus y = \left(\frac{T_1 - T_2}{1 - T_2}, \frac{I_1}{I_2}, \frac{F_1}{F_2}\right). \tag{6}$$

Definition 3.4. (Ye, 2014) The complement of a SVNS N is represented by N^c and is calculated as $T_N^c(x) = F_N(x)$, $I_N^c(x) = 1 - I(x)$, and $F_N^c(x) = T_N(x)$ for all elements $x \in X$. Therefore $N^c = \{\langle x, F_N(x), 1 - I_N(x), T_N(x)\rangle; x \in X\}$.

Definition 3.5. (Nafei et al., 2021) Assume that (T, I, F) is a NT. The score function S for ranking the NT (T, I, F) is equal to:

$$S(T, I, F) = \frac{(4 + T - 2I - F)(2 - I)(2 - F)}{5}. \quad (7)$$

Definition 3.6. (Eroğlu & Şahin, 2020) Assume $N = (T_1, I_1, F_1)$ and $M = (T_2, I_2, F_2)$. N is a subset of M, denoted $N \subseteq M$ if $T_1 \leq T_2$, $I_1 \geq I_2$, and $F_1 \geq F_2$. In this case, N is considered a subset of M because N is less true, more indeterminate, and no less false than M.

To elaborate on the advantages of the proposed adjustment model on consistency and consensus, we can emphasize the following aspects to clearly highlight the novelty of the model:

1) Enhanced consistency mechanism:

- The proposed model introduces an iterative consistency adjustment mechanism that recalibrates the NTs to ensure consistency across different decision-makers. This is particularly crucial when dealing with uncertain and indeterminate data.
- Dynamic adjustment of weights: The model allows for the dynamic adjustment of the weights (associated with truth, indeterminacy, and falsity). This flexibility ensures that the model can adjust to varying decision contexts, where different criteria might hold different levels of importance, further improving consistency.

2) Improved consensus-building:

- The adjustment model fosters greater consensus by balancing the judgements of multiple decision-makers through its weighted aggregation approach. The normalization of the NTs ensures that outlier opinions are moderated, while more consistent opinions are reinforced, facilitating a smoother convergence towards a consensus.
- Integration of GDM elements: The model accounts for the varying priorities and perspectives of different stakeholders. By weighting their contributions and adjusting the final aggregation dynamically, the model promotes consensus even in scenarios where opinions are initially divergent.
- Reduction of conflicting opinions: By emphasizing the indeterminacy and falsity components appropriately in the decision-making process, the model reduces the weight of highly conflicting or uncertain opinions, thereby steering the group towards a more cohesive decision.

3) Robustness in uncertain environments:

- The proposed model is particularly effective in handling uncertainty and indeterminacy, which are common challenges in complex decision-making scenarios. By explicitly accounting for the indeterminacy and falsity components, the model provides a more robust framework for decision-making where data may be incomplete or conflicting.
- Improved handling of ambiguous data: The model's ability to integrate indeterminacy directly into the decision-making process allows it to manage uncertain and ambiguous data more effectively than traditional methods, which often rely on crisp inputs.

These advantages demonstrate the novelty of the proposed model by addressing both consistency and consensus in ways that are not effectively handled by existing methodologies. The ability to manage uncertainty, dynamically adjust weights, and normalize triplet components are key features that contribute to the overall improvement of decision-making in complex, indeterminate environments.

4. A new distance measure for NTs

The section introduces a DED measure designed to overcome the limitations of traditional distance measures, such as Hamming and Euclidean distances, particularly in the context of NSs, which involve degrees of truth, indeterminacy, and falsity. The exponential distance measure aims to provide a more nuanced and precise assessment of alternatives in decision-making processes. The following presents the classical distance measures for NTs.

Definition 4.1. (Aydoğdu, 2015) Suppose that $N = \{\langle x, T_N(x), I_N(x), F_N(x)\rangle; x \in X\}$, and $M = \{\langle x, T_M(x), I_M(x), F_M(x)\rangle; x \in X\}$ are two NSs in X. The Hamming distance and Euclidean distance between N and M are defined respectively as follows:

$$d_H(N, M) = \frac{1}{3}\Big(|T_N(x) - T_M(x)| + |I_N(x) - I_M(x)| + |F_N(x) - F_M(x)|\Big). \quad (8)$$

$$d_E(N, M) = \sqrt{\left(\frac{(T_N(x) - T_M(x))^2 + (I_N(x) - I_M(x))^2 + (F_N(x) - F_M(x))^2}{3}\right)}. \quad (9)$$

However, the aforementioned distances are unable to distinguish the distances in some special cases. The following presents some examples to illustrate these limitations.

Example 4.1. Assume that $N = (0.5, 0.2, 0.7)$, $M = (1.0, 0.6, 0.5)$ and $O = (0.9, 0.7, 0.9)$. It is easy to find out $d_H(N,M) = d_H(N,O) = 0.367$, and $d_E(N, M) = d_E(N,O) = 0.3872$.

Example 4.2. Assume that $N = (0.1, 0.2, 0.3)$, $M = (0.5, 0.9, 0.2)$ and $O = (0.2, 0.6, 1.0)$. In this case, $d_H(N, M) = d_H(N, O) = 0.4$, and $d_E(N, M) = d_E(N, O) = 0.4690$.

These examples illustrate the limitations of these traditional measures, which fail to distinguish differences in some exceptional cases. These distance measures yield the same distance for different NSs and are also unable to differentiate effectively between sets.

To overcome these shortcomings and propose an efficient measure with the ability to distinguish distances accurately, we suggest a DED for neutrosophic triplets. The DED approach revolutionizes how distances are calculated by incorporating exponential transformations, thus allowing for a more nuanced handling of the degrees of truth, indeterminacy, and falsity in NSs. The proposed DED is defined as follows:

Definition 4.2. Suppose that $N = \{\langle x, T_N(x), I_N(x), F_N(x)\rangle; x \in X\}$, and $M = \{\langle x, T_M(x), I_M(x), F_M(x)\rangle; x \in X\}$ are two NSs in X. The suggested DED for N and M are defined respectively as follows:

$$d_D(N,M) = \sqrt{\delta^2 \cdot \left(e^{T_N} - e^{T_M}\right)^2 + \lambda \cdot (I_N - I_M)^2 + \mu \cdot \left(e^{-F_N} - e^{-F_M}\right)^2}. \tag{10}$$

The parameters δ, λ and μ are used to balance the importance of truth, indeterminacy, and falsity in the DED framework. These parameters are crucial for adapting the decision-making process to specific contexts, but practical considerations should guide their selection. Below is a more refined explanation with professional examples to illustrate their importance:

1. Contextual importance:
   a. Example 1 (Quality control in manufacturing): In a manufacturing process where accurate detection of defects is paramount, the truth component (δ) holds greater significance. For instance, if the automation system needs to ensure the highest reliability in detecting product defects, then a higher δ should be assigned, emphasizing the importance of truth. On the other hand, in a scenario where the system is newly implemented and data uncertainty is high, the indeterminacy component (λ) should be weighted more heavily. This accounts for the inherent uncertainty in the system's performance during initial stages. Similarly, if the data quality is questionable and prone to errors, the falsity component (μ) could be emphasized to address the risk of erroneous information influencing decisions.
   b. Example 2 (Decision-making in financial risk assessment): In the context of assessing investment risks, a higher λ might be chosen when indeterminacy, or uncertainty about market trends, is of critical concern. Conversely, if the analysis is based on well-established financial models with highly reliable data, the truth component (δ) should be prioritized to reflect the confidence in the data and analysis.
2. Sensitivity analysis:
   a. Example 1 (Industrial automation system selection): In selecting an industrial automation system, sensitivity analysis can be performed to examine how variations in δ, λ, and μ influence the overall decision. For instance, by increasing δ by 10%, the decision-maker can observe whether this change significantly alters the ranking of automation systems. If it does, it suggests that the decision is highly sensitive to the truth component, indicating that the accuracy of the data plays a critical role. Similarly, decreasing λ (indeterminacy) might show whether the system can tolerate uncertain data inputs, which is vital for decisions in volatile environments.
   b. Example 2 (Health care resource allocation): In healthcare, where decision-making involves uncertain patient outcomes and varying reliability of medical data, a higher weight for indeterminacy (λ) may be crucial. Sensitivity analysis could reveal that small changes in λ significantly affect resource allocation decisions, underscoring the need to carefully consider how uncertainty impacts the overall effectiveness of the allocation strategy.

   These examples demonstrate how selecting the appropriate values for δ, λ, and μ is not arbitrary, but depends on the specific decision context and the relative importance of truth, indeterminacy, and falsity in that scenario. Sensitivity analysis further aids in understanding the robustness of the decision model and how the adjustments in these parameters affect the decision outcomes.
3. Normalization: Ensure that the values of δ, λ, and μ are normalized relative to each other to maintain a balanced contribution from each

component. For example, setting $\delta + \lambda + \mu = 1$ can be a useful normalization approach.

Proposition 4.1. The distance measure $d_D(N, M)$ satisfies the following conditions:

I. $0 \leq d_D(N, M)$.
II. $d_D(N, M) = 0$, if and only if $N = M$.
III. $d_D(N, M) = d_D(M, N)$.
IV. If $N \subseteq M \subseteq O$ ($O \in$ SVNSs in X), then $d_D(N, O) \geq d_D(N, M)$, and $d_D(N, O) \geq d_D(M, O)$.

Proof. The properties I, II, and III are straightforward. Thus, it remains necessary to prove statement IV. Based on Definition 3.6, since $N \subseteq M \subseteq O$, if $T_N \leq T_M \leq T_O, I_N \geq I_M \geq I_O$, and $F_N \geq F_M \geq F_O$.

To prove $d_D(N, O) \geq d_D(N, M)$: Firstly, given that e^T is an increasing function and $T_N \leq T_M \leq T_O$, we have $e^{T_N} \leq e^{T_M} \leq e^{T_O}$, and subsequently $(e^{T_N} - e^{T_O})^2 \geq (e^{T_N} - e^{T_M})^2$. Secondly, since $I_N \geq I_M \geq I_O$, $(I_N - I_O)^2 \geq (I_N - I_M)^2$. Thirdly, given that e^{-F} is a decreasing function and $F_N \geq F_M \geq F_O$, we have: $e^{-F_N} \leq e^{-F_M} \leq e^{-F_O}$, and subsequently $(e^{-F_N} - e^{-F_O})^2 \geq (e^{-F_N} - e^{-F_M})^2$.

Combining all for any $n \in N, m \in M$, and $o \in O$ and considering coefficient δ, λ, and μ as positive values, we have: $d_D(N, O) \geq d_D(N, M)$.

To prove $d_D(N, O) \geq d_D(M, O)$: Firstly, given that e^T is an increasing function and $T_N \leq T_M \leq T_O$, we have $e^{T_N} \leq e^{T_M} \leq e^{T_O}$, and subsequently $(e^{T_N} - e^{T_O})^2 \geq (e^{T_M} - e^{T_O})^2$. Secondly, since $I_N \geq I_M \geq I_O$, $(I_N - I_O)^2 \geq (I_M - I_O)^2$. Thirdly, given that e^{-F} is a decreasing function and $F_N \geq F_M \geq F_O$, we have $e^{-F_N} \leq e^{-F_M} \leq e^{-F_O}$, and subsequently $(e^{-F_N} - e^{-F_O})^2 \geq (e^{-F_M} - e^{-F_O})^2$. Combining all for any $n \in N, m \in M$, and $o \in O$ and considering coefficient δ, λ, and μ positive values, we have: $d_D(N, O) \geq d_D(M, O)$.

4.1. Importance of proposed exponential distance measure

By using the exponential distance measure in the same case of numbers (Example 4.1), one has (for $\delta = \mu > \lambda$):

Example 4.3. Assume that $N = (0.5, 0.2, 0.7)$, $M = (1.0, 0.6, 0.5)$ and $O = (0.9, 0.7, 0.9)$. By using the Hamming distance and Euclidean distance, we have: $d_H(N, M) = d_H(N, O) = 0.367$, and $d_E(N, M) = d_E(N, O) = 0.3872$. Also, by using the proposed exponential distance measure, we have: $D_{Ex}(N, M) = 0.4686$, and $D_{Ex}(N, O) = 0.3981$.

Also, by using the exponential distance measure in the same case of numbers (Example 4.2), one has:

Example 4.4. Assume that $N = (0.5, 0.2, 0.7)$, $M = (1.0, 0.6, 0.5)$ and $O = (0.9, 0.7, 0.9)$. By using the Hamming distance and Euclidean distance, we have: $d_H(N, M) = d_H(N, O) = 0.367$, and $d_E(N, M) = d_E(N, O) = 0.3872$. Also, by using the proposed exponential distance measure, we have: $D_{Ex}(N, M) = 0.384$, and $D_{Ex}(N, O) = 0.2997$.

Unlike previous distance criteria, the introduced exponential distance measure can effectively classify and distinguish distances between the provided numbers within the framework of NTs.

The DED measure effectively differentiates between alternatives that traditional measures fail to distinguish. It does so by:

1. Amplifying differences in extreme values through exponential transformations.
2. Providing a balanced contribution from truth, indeterminacy, and falsity components tailored to specific decision-making contexts.
3. Enhancing the sensitivity to component variations, particularly the extremes, is critical in practical decision-making scenarios involving uncertainty and variability.

By adopting the exponential distance measure, decision-makers can achieve more reliable and accurate assessments of alternatives, leading to better-informed decisions. This analysis justifies the preference for the exponential distance measure over traditional linear and quadratic measures in applications involving neutrosophic data, highlighting its effectiveness in handling complex and uncertain information.

The DED measure offers a robust solution for distinguishing distances between NSs, addressing the shortcomings of traditional distance measures. By incorporating exponential transformations and balancing the contributions of truth, indeterminacy, and falsity, the DED measure provides a more precise and reliable assessment framework, crucial for decision-making in environments characterized by high variability and uncertainty.

5. The neutrosophic VIKOR method

This section proposes an extension VIKOR method (Opricovic, 1979) for ordering the alternatives $a_n; n = 1, 2, \ldots, N$ evaluated by DMs $D_l; l = 1, 2, \ldots, L$

based on $\{c_m | m = 1, 2, ..., M\}$. Assume that the decision matrix η_l, which is given by decision-maker D_l is presented as follows:

$$\eta_l = \begin{array}{c} \\ a_1 \\ a_2 \\ \vdots \\ a_N \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & \cdots & c_M \end{array} \\ \begin{bmatrix} (T^l_{11}, I^l_{11}, F^l_{11}) & (T^l_{12}, I^l_{12}, F^l_{12}) & \ldots & (T^l_{1M}, I^l_{1M}, F^l_{1M}) \\ (T^l_{21}, I^l_{21}, F^l_{21}) & (T^l_{22}, I^l_{22}, F^l_{22}) & \ldots & (T^l_{2M}, I^l_{2M}, F^l_{2M}) \\ \vdots & \vdots & \ddots & \vdots \\ (T^l_{n1}, I^l_{n1}, F^l_{n1}) & (T^l_{N2}, I^l_{N2}, F^l_{N2}) & \ldots & (T^l_{NM}, I^l_{NM}, F^l_{NM}) \end{bmatrix} \end{array}. \quad (11)$$

We denote $Z^l_{nm} = \left(T^l_{nm}, I^l_{nm}, F^l_{nm}\right)$. In MAGDM, determining the weights of decision-makers and attributes is crucial for accurate decision outcomes. To address this, we propose using an extended BWM to derive the weights of decision-makers and attributes systematically. Wan, Dong, and Chen (2024a, 2024b) suggested a BWM for GDM with IFPRs. The BWM has received extensive attention because of its fewer pairwise comparisons and maintenance of consistency. Extending it to various fuzzy environments has become a hot research spot (Dong & Wan, 2024). The distinction is that while the weights for decision-makers are crisp values, the weights for attributes are represented by NTs (T, I, F) which capture the truth (T), indeterminacy (I), and falsity (F) of each attribute's importance. The process of using the BWM is presented as follows:

First: Identify best and worst elements:

- For decision-makers: Decision-makers identify the most critical (best) and least critical (worst) decision-makers.
- For attributes: The decision-makers identify the most important (best) and least important (worst) attributes.

Second: Pairwise comparison matrices:

The decision-makers make pairwise comparisons between the best decision-maker and all other decision-makers, and similarly for the worst decision-maker. These comparisons are done using a scale from 1 (equal importance) to 9 (extreme importance).

Let:

- α_{Bl} represent the comparison of the best decision-maker D_b with decision-maker D_l.
- α_{lW} represent the comparison of decision-maker D_l with the worst decision-maker D_w.

We create the vectors based on the comparisons as follows:

$$\alpha_B = (\alpha_{B1}, \alpha_{B2}, ..., \alpha_{BL}), \alpha_W = (\alpha_{1W}, \alpha_{2W}, ..., \alpha_{LW}).$$

The weights of the β_m attributes are represented by NTs. Pairwise comparisons are made between the best attribute and all other attributes, and similarly for the worst attribute.

Let:

- β_{Bm} represent the comparison of the best attribute c_b with attribute c_m.
- β_{mW} represent the comparison of attribute c_m with the worst attribute c_w.

The pairwise comparisons yield vectors with NTs:

$$\beta_B = (\beta_{B1}, \beta_{B2}, ..., \beta_{BM}), \ \beta_W = (\beta_{1W}, \beta_{2W}, ..., \beta_{MW}).$$

In these matrices, each entry β_{Bm} and β_{mW} is a NT (T, I, F) representing the truth, indeterminacy, and falsity in the pairwise comparisons.

Third: Optimization for weight calculation

The optimization problem for decision-makers' crisp weights is formulated to minimize the maximum absolute differences between pairwise comparisons and the derived weights. The optimization problem is defined as:

$$\min_{w} \left[\max_{l} \left| \frac{W_B}{W_l} - \alpha_{Bl} \right|, \max_{l} \left| \frac{W_l}{W_W} - \alpha_{lW} \right| \right]$$

where:

- W_B and W_W represent the weights of the best and worst decision-makers, respectively.
- W_l represents the weight of decision-maker D_l.
- The final solution presents $\boldsymbol{W} = (W_1, W_2, \ldots, W_L)$ is a crisp vector and $\sum_{l=1}^{L} W_l = 1$.

For attributes, the optimization is extended to handle NTs. The goal is to minimize the inconsistency in pairwise comparisons of the attributes while considering their neutrosophic nature. The optimization problem for attribute weights is presented as follows:

$$\min_{v} \left[\max_{m} \left| \frac{v_B}{v_m} - \beta_{Bm} \right|, \max_{m} \left| \frac{v_m}{v_W} - \beta_{mW} \right| \right].$$

where:

- v_B and v_W are the NT weights of the best and worst attributes, respectively.
- v_m represents the NT weight of c_m.

The solution provides the neutrosophic weight vector $\boldsymbol{v} = (v_1, v_2, ..., v_M)$.

The following presents an extension of the VIKOR method for solving the aforementioned problem.

Step 1. Create the aggregated decision matrix (ADM) for considering the opinion of all DMs as follows:

$$ADM = \begin{array}{c} \\ a_1 \\ a_2 \\ \vdots \\ a_N \end{array} \begin{array}{c} \begin{array}{cccc} D_1 & D_2 & \cdots & D_L \end{array} \\ \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1L} \\ A_{21} & A_{22} & \cdots & A_{2L} \\ \vdots & \vdots & \ddots & \vdots \\ A_{N1} & A_{N2} & \cdots & A_{NL} \end{bmatrix} \end{array}. \tag{12}$$

where $A_{nl} = [(Z_{n1}^l \otimes v_1^l) \oplus (Z_{n2}^l \otimes v_2^l) \oplus \cdots \oplus (Z_{nM}^l \otimes v_M^l)]$.

Step 2. By normalizing the NTs in ADM and applying the score function S presented in Equation (7), we can create the score matrix with crisp components as follows:

$$Score(ADM) = \begin{array}{c} \\ a_1 \\ a_2 \\ \vdots \\ a_N \end{array} \begin{array}{c} \begin{array}{cccc} D_1 & D_2 & \cdots & D_L \end{array} \\ \begin{bmatrix} K_{11} & K_{12} & \cdots & K_{1L} \\ K_{21} & K_{22} & \cdots & K_{2L} \\ \vdots & \vdots & \ddots & \vdots \\ K_{N1} & K_{N2} & \cdots & K_{NL} \end{bmatrix} \end{array} \tag{13}$$

Step 3. Obtain the positive ideal solution (I^+) and negative ideal solution (I^-) as follows:

$$I^+ = A_{nl}, \operatorname*{Max}_n (k_{nl}), \quad 1 \le l \le L, \tag{14}$$

$$I^- = A_{nl}, \operatorname*{Min}_n (k_{nl}), \quad 1 \le l \le L. \tag{15}$$

Step 4. Calculate the values of utility measure (U_n) and regret measure (R_n) as follows:

$$U_n = \sum\nolimits_{l=1}^{L} W_l \frac{d(I_l^+, A_{nl})}{d(I_l^+, I_l^-)}, \tag{16}$$

and

$$R_n = \text{Max}\left[W_l \frac{d(I_l^+, A_{nl})}{d(I_l^+, I_l^-)}\right]. \tag{17}$$

where W_l represents the weight of each DM (decision maker).

Step 5. Calculate the values of U^+, U^-, R^+, and R^- as follows:

$$U^+ = \text{Min}(U_n), U^- = \text{Max}(U_n), \quad n = 1, 2, ..., N.$$
$$R^+ = \text{Min}(R_n), R^- = \text{Max}(R_n), \quad n = 1, 2, ..., N. \tag{18}$$

Step 6. Obtain the value of P_n and rank the existing alternatives based on the obtained values of P_n.

$$P_n = \vartheta\left(\frac{U_n - U^+}{U^- - U^+}\right) + (1 - \vartheta)\left(\frac{R_n - R^+}{R^- - R^+}\right) \tag{19}$$

where v represents the weight for the maximum group utility, and $1 - \vartheta$ represents the weight of the individual regret. Typically, $\vartheta = 0.5$. In this case, if $\vartheta > 0.5$, the index of P_n, will tend to a majority agreement. Also, if $\vartheta < 0.5$, the index of P_n indicates a majority negative attitude.

6. Case study related to industrial automation selection

To demonstrate the above-mentioned approach, we illustrate it with a case study involving selecting an industrial automation system. For this, we structured a comprehensive scenario using MAGDM that includes five alternatives, four attributes, and evaluations from three decision-makers. The objective is to select the most appropriate industrial automation system for a manufacturing plant to optimize various performance metrics, including reliability, cost, integration, and support. The considered attributes are listed as follows:

1. System reliability (R): Probability of performing without failure.
2. Cost efficiency (C): Initial costs and long-term operational costs.
3. Ease of integration (I): The ability to integrate with existing systems and processes.
4. Technical support (S): Availability and quality of customer and technical support.

Also, the existing decision-makers are considered as follows:

1. Technical manager (TM): Focuses on system reliability and ease of integration.
2. Financial analyst (FA): Focuses on cost efficiency and long-term value.
3. Operations manager (OM): Prioritizes ease of integration and technical support.

Five alternatives exist in this case study that should be ranked. These alternatives are five different systems as presented as follows:

- System A: High reliability, high cost, moderate integration, excellent support.
- System B: Moderate reliability, low cost, high integration, good support.
- System C: High reliability, moderate cost, excellent integration, moderate support.
- System D: Moderate reliability, moderate cost, moderate integration, excellent support.
- System E: Low reliability, low cost, good integration, poor support.

The evaluating values given by each decision-maker are provided in Table 1.

Then the steps of the proposed algorithms are implemented as below.

Table 1. The evaluating values.

Alternatives		System reliability	Cost efficiency	Ease of integration	Technical support
System A	D_1	(0.4,0.2,0.9)	(0.3,0.4,0.3)	(0.6,0.6,0.6)	(0.3,0.3,0.3)
	D_2	(0.3,1.0,0.1)	(0.5,0.3,0.2)	(0.5,0.6,0.6)	(0.1,0.6,0.8)
	D_3	(0.7,0.4,0.2)	(0.3,0.8,0.6)	(0.5,0.7,0.6)	(1.0,0.9,0.5)
System B	D_1	(0.3,0.1,0.2)	(1.0,1.0,0.2)	(0.0,0.2,0.7)	(0.8,0.0,0.6)
	D_2	(0.3,1.0,0.5)	(1.0,0.4,0.5)	(1.0,0.2,1.0)	(0.0,0.0,0.2)
	D_3	(0.7,0.8,0.4)	(0.8,0.1,0.7)	(0.9,0.5,0.7)	(0.9,0.4,0.7)
System C	D_1	(0.0,0.5,0.6)	(1.0,0.3,0.8)	(0.9,0.9,0.0)	(0.5,1.0,0.4)
	D_2	(0.0,0.4,0.7)	(0.7,0.3,0.8)	(1.0,0.5,0.4)	(0.5,0.8,0.3)
	D_3	(0.6,0.9,0.6)	(0.8,0.3,0.7)	(0.1,1.0,0.0)	(0.5,1.0,0.5)
System D	D_1	(0.8,0.5,0.8)	(0.5,0.1,0.1)	(0.1,0.3,1.0)	(0.6,0.8,0.6)
	D_2	(0.2,0.6,0.6)	(0.8,0.9,0.8)	(1.0,0.6,0.0)	(0.1,0.5,0.7)
	D_3	(0.0,0.8,0.8)	(0.9,0.6,0.4)	(0.9,0.9,0.5)	(0.5,0.1,0.6)
System E	D_1	(0.6,0.8,0.9)	(0.9,0.8,0.1)	(0.4,0.0,0.5)	(0.8,0.1,0.2)
	D_2	(0.7,0.7,1.0)	(0.4,0.5,0.9)	(0.4,0.4,0.4)	(0.3,0.9,0.5)
	D_3	(0.8,0.4,0.3)	(0.8,1.0,0.2)	(0.2,0.7,0.6)	(0.4,0.0,1.0)

Using the BWM, and considering D_3 as the best decision-maker (most important) and D_2 as the worst decision-maker (least important), the weights of decision-makers D_1, D_2, and D_3 are determined to be to 0.2, 0.1, and 0.7, respectively. While, the weights of the attributes are provided in Table 2.

Table 2. Weight of attributes.

Decision-makers	System reliability	Cost efficiency	Ease of integration	Technical support
D_1	(0.2,0.5,0.3)	(0.9,0.6,0.2)	(0.2,0.8,0.1)	(0.9,0.5,0.7)
D_2	(0.3,0.6,0.2)	(0.2,0.4,0.1)	(0.3,0.4,0.7)	(0.7,0.0,0.9)
D_3	(0.2,0.3,1.0)	(0.4,0.7,0.9)	(0.1,0.4,0.6)	(0.9,0.3,0.7)

Step 1. Create the ADM as follows:

$$ADM = \begin{array}{c} \begin{array}{ccc} \quad D_1 \quad & \quad D_2 \quad & \quad D_3 \quad \end{array} \\ \begin{bmatrix} (0.40, 0.41, 0.26) & (0.30, 0.44, 0.06) & (0.28, 0.44, 0.80) \\ (0.90, 0.46, 0.11) & (0.49, 0.33, 0.33) & (0.46, 0.43, 0.85) \\ (0.91, 0.52, 0.06) & (0.39, 0.30, 0.51) & (0.40, 0.73, 0.58) \\ (0.54, 0.38, 0.24) & (0.44, 0.60, 0.39) & (0.41, 0.71, 0.75) \\ (0.84, 0.66, 0.14) & (0.36, 0.39, 0.74) & (0.41, 0.47, 0.77) \end{bmatrix} \end{array}.$$

Step 2. By normalizing the components of ADM and then applying the score function S to the normalized triplets, we can obtain the following matrix with crisp components.

$$Score(ADM) = \begin{array}{c} \begin{array}{ccc} D_1 & D_2 & D_3 \end{array} \\ \begin{bmatrix} 1.9132 & 1.7902 & 1.5494 \\ 2.5380 & 2.0894 & 1.7408 \\ 2.4993 & 1.8770 & 1.5943 \\ 2.1604 & 1.7533 & 1.5912 \\ 2.2171 & 1.6801 & 1.6897 \end{bmatrix} \end{array}$$

Step 3. Obtain the positive ideal solution (I^+) and negative ideal solution (I^-) as follows:

$$I^+ = (0.90, 0.46, 0.11), (0.49, 0.33, 0.33), (0.46, 0.43, 0.85),$$

$$I^- = (0.40, 0.41, 0.26), (0.36, 0.39, 0.74), (0.28, 0.44, 0.80).$$

Step 4. Calculate the values of U_n (utility measure) and R_n (regret measure) for an alternative a_n as follows:

$$U_1 = 1.5902, \ U_2 = 1.3609, \ U_3 = 1.6317,$$
$$U_4 = 1.5887, \ U_5 = 1.4819.$$
$$R_1 = 1.2826, R_2 = 1.1997, R_3 = 1.4353,$$
$$R_4 = 1.3400, R_5 = 1.2254.$$

Step 5. Calculate the values of U^+, U^-, R^+, and R^- by using Equation (18) as

$$U^+ = 1.3609; U^- = 1.6317$$
$$R^+ = 1.1997; R^- = 1.4353$$

Step 6. Obtain the value of P_n by using Equation (19) with $\vartheta = 0.5$ and get

$$P_1 = 0.5993, P_2 = 0.0, P_3 = 1.0, P_4 = 0.7181,$$
$$P_5 = 0.2779.$$

Therefore, the final ranking for existing alternatives is a_2, a_5, a_1, a_4, and a_3.

7. Sensitivity analysis and comparison

This section begins with a detailed sensitivity analysis of the proposed method, evaluating its robustness and reliability under various conditions. A comprehensive comparative analysis is conducted, juxtaposing our method with several other established methodologies. This comparison aims to highlight our approach's strengths and potential advantages, providing a thorough evaluation of its performance in contrast to existing techniques.

7.1. The importance of the weight of decision-makers

To analyse the sensitivity of the proposed method, we have considered different styles of weights of decision-makers and then considered their effect on the ranking of alternatives. Since we have three decision-makers, we have considered seven different modes, as follows:

$$W_1 < W_2 < W_3,$$
$$W_1 < W_3 < W_1,$$
$$W_2 < W_1 < W_3,$$
$$W_2 < W_3 < W_1,$$
$$W_3 < W_2 < W_1,$$
$$W_3 < W_1 < W_2,$$
$$W_1 = W_2 = W_3.$$

The results are provided in Figure 1.

The radar chart in Figure 1 visualizes the performance rankings of five alternatives (a_1 to a_5) based on the VIKOR method. The analysis considers different weight orders (W_1, W_2, and W_3) assigned by decision-makers. By altering the weight order, the impact on the final ranking of each alternative is examined, highlighting the sensitivity and robustness of the alternatives under varying decision-making scenarios.

This study uses the VIKOR method to rank alternatives based on their performance across multiple criteria. The weights (W_1, W_2, and W_3) represent the importance assigned to these criteria by decision-makers. Different permutations of these weights were considered to understand how weight priority changes affect the alternatives' final rankings. The radar chart allows for a comparative visual analysis of how each alternative ranks under different weight permutations. The results are presented in Table 3.

This analysis highlights the sensitivity of alternative rankings to the weights assigned by decision-makers. The radar chart effectively illustrates how different weight orders impact the final rankings, providing insight into the robustness and stability of each alternative. Decision-makers can use this information to understand the implications of their weight preferences and make more informed choices that account for the variability in rankings due to weight changes. Using radar charts to visualize the results of the VIKOR method under different weight permutations provides a comprehensive and clear comparison of alternative performances. This approach underscores the importance of considering multiple weight scenarios in MCDM processes, ultimately contributing to more resilient and well-rounded decision outcomes.

7.2. The impact of various distance measures on alternative rankings

This section delves into the impact of different weighting strategies on the final ranking of alternatives using the VIKOR method integrated with the DED measure. The analysis considers four distinct scenarios, each corresponding to a different configuration of the weighting parameters δ, λ, and μ. The objective is to observe how the five alternatives' rankings (System A

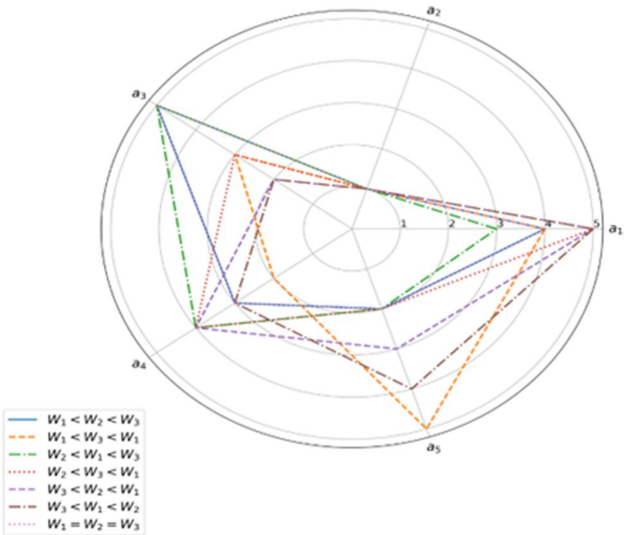

Figure 1. Rankings of alternatives with different weight orders.

Table 3. Comparative visual analysis of alternative ranks under different weight permutations.

Alternative a_1	Exhibits consistently high performance in most scenarios except for when W_3 is highly weighted (purple dashed line). Highest performance is seen when W_1 is given the least weight (blue and orange lines).
Alternative a_2	Performance is variable, with moderate to high rankings when W_2 is given higher weights (green and red lines). Shows a significant drop in performance when W_1 is the highest weight (purple dashed line).
Alternative a_3	Consistently performs well across all weight permutations. Represents a peak performance when W_1 or W_2 are given higher weights.
Alternative a_4	Demonstrates moderate to low performance in most scenarios. Best performance is seen when W_2 is highly weighted (green dash-dot line).
Alternative a_5	Demonstrates variability in performance, with the highest rankings when W_3 is given higher weights (brown dash-dot line). Lowest performance is observed when W_1 is highly weighted.

to System E) vary across these scenarios. The considered scenarios are listed as follows:

1. Scenario 1: $\mu > \delta = \lambda$.
2. Scenario 2: $\delta > \lambda = \mu$.
3. Scenario 3: $\delta > \mu > \lambda$.
4. Scenario 4: $\delta = \mu > \lambda$.

The radar chart in Figure 2 and its results in Table 4 visualize the variations in rankings across the different scenarios, providing a clear depiction of how changes in the weighting parameters impact the final outcomes.

The analysis reveals the following insights:

1. *System B's dominance:*
   - System B consistently ranks first or second across all scenarios, highlighting its robustness and high performance regardless of the weighting strategy. This indicates that System B is a reliable alternative in various decision-making contexts.
2. *System A's variability:*
   - System A exhibits significant variability in its ranking, moving from the bottom to the top depending on the scenario. This sensitivity suggests that System A's performance is highly dependent on the specific configuration of the weighting parameters.
3. *Stability of System D and E:*
   - Systems D and E show stable performance, maintaining their ranks with minor fluctuations. This stability indicates that these systems are less sensitive to changes in the weighting parameters, making them reliable choices in a range of scenarios.
4. *System C's decline:*
   - System C consistently ranks lower in most scenarios, suggesting it is less competitive than the other alternatives under different weighting strategies.

The analysis underscores the importance of selecting appropriate weighting parameters in the VIKOR method, as they significantly influence the ranking of alternatives. Decision-makers must carefully consider the relative importance of truth, indeterminacy, and falsity components in their specific context to ensure that the chosen weighting strategy aligns with their priorities and objectives.

7.3. Analysis of the VIKOR ranking with different threshold values

Figure 3 displays the ranking of five alternatives $(a_1, a_2, a_3, a_4, a_5)$ using the VIKOR method. The x-axis represents different threshold values (ϑ) used in the final step of the VIKOR method, while the y-axis shows the rank of each alternative.

The aforementioned graph can be interpreted as follows:

1. Ranking stability: a_2 (orange line with circles) and a_5 (purple line with stars) consistently maintain their ranks at 1 and 2, respectively, across all threshold values from 0 to 1. This indicates high stability and dominance of a_2 and a_5 regardless of the threshold value applied in the VIKOR method.

Table 4. The factor coefficients in four scenarios.

	Rank			
Alternatives	Scenario 1 $\mu > \delta = \lambda$	Scenario 2 $\delta > \lambda = \mu$	Scenario 3 $\delta > \mu > \lambda$	Scenario 4 $\delta = \mu > \lambda$
System A	5	1	5	3
System B	1	2	1	1
System C	3	5	4	5
System D	4	4	3	4
System E	2	3	2	2

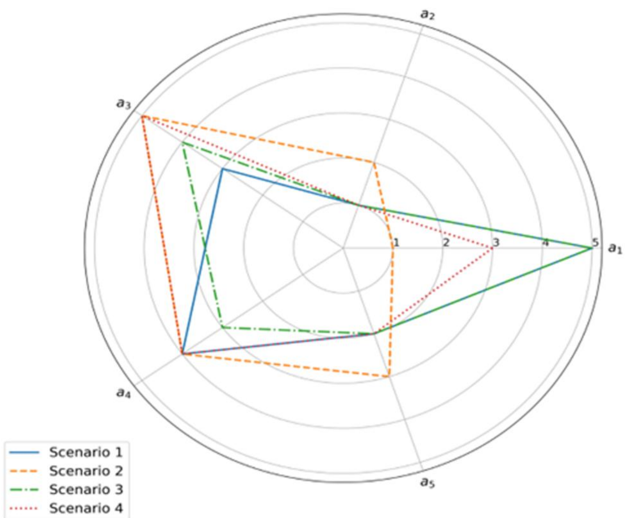

Figure 2. Rankings of alternatives in different scenarios.

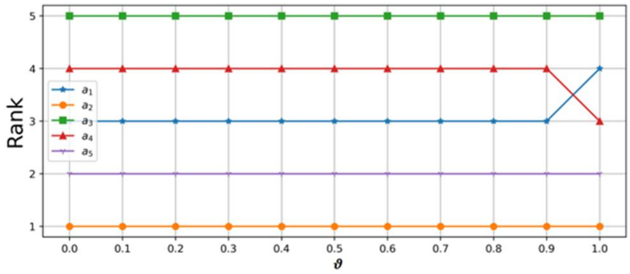

Figure 3. Rankings of alternatives with varying thresholds.

2. Moderate stability: a_1 (blue line with stars) consistently ranks 3 for threshold values from 0 to 0.9. However, at the threshold value of 1.0, a_1 improves its rank to 2. This suggests that a_1's ranking is relatively stable but can improve under certain threshold conditions.
3. Ranking variability: a_3 (green line with squares) consistently maintains the rank 5 across all threshold values from 0 to 1. This indicates a_3 is the least preferred alternative in all scenarios.
4. Significant change: a_4 (red line with triangles) holds the rank 4 for threshold values from 0 to 0.9. At the threshold value of 1.0, a_4's rank drops significantly to 3. This shows that a_4's ranking is sensitive to changes in the threshold value, indicating variability in its performance relative to the other alternatives under different threshold settings.

The threshold value in the VIKOR method plays a crucial role in determining the final ranking of alternatives. While some alternatives show stable rankings across different threshold values, others exhibit sensitivity to threshold changes. Understanding the stability and sensitivity of rankings can help decision-makers better interpret the robustness of their choices under varying decision criteria.

Alternatives a_2 and a_5 can be considered robust choices as their rankings remain consistently high regardless of the threshold value. Alternatives like a_1 and a_4 require more careful consideration, especially under specific threshold conditions that may affect their rankings. Alternative a_3 consistently ranks the lowest, indicating it is the least favourable option among the five alternatives considered. By analysing the impact of different threshold values in the VIKOR method, decision-makers can gain a deeper understanding of the robustness and sensitivity of their rankings, leading to more informed and resilient decision-making processes.

7.4. Comparative analysis

In order to make a comparison between the proposed method and other existing methods, we have compared the run time in our proposed VIKOR method and other methods proposed by Nafei et al. (2021) (TOPSIS-NS), Gulum et al. (2021) (TOPSIS-NS), Nafei et al. (2019) (Autocratic MAGDM-INS), and Lee (2023) (Autocratic MAGDM-INS). The selection of VIKOR, TOPSIS, and Autocratic as benchmarking methods is carefully justified based on several key considerations. First, these methods are among the most widely recognized and validated MCDM approaches, making them ideal benchmarks for evaluating the performance and robustness of new methods. Their extensive use in academic research and industrial decision-making ensures that the benchmarking results are reliable and meaningful for comparison. Additionally, these methods offer complementary methodological characteristics: VIKOR provides compromise solutions in situations with conflicting objectives, making it suitable for multi-objective optimization in complex environments; TOPSIS focuses on evaluating alternatives based on their proximity to an ideal solution, which is effective for ranking options when clear-cut solutions are needed; and the Autocratic approach excels in scenarios where centralized decision-making is required. The selected methods align closely with the study's objectives, particularly in the context of Industry 5.0, which emphasizes human-machine collaboration, sustainability, and efficiency. Each method's ability to handle multi-dimensional decision-making ensures a holistic evaluation of the proposed approach. Finally, by using these three complementary methods, we ensure a comprehensive and robust benchmarking process, allowing us to identify the strengths of the proposed method and how it performs relative to well-established techniques. The results are shown in Figure 4. The x-axis represents the number of alternatives, while the y-axis represents the runtime in seconds.

VIKOR (blue line with stars) demonstrates a steady increase in runtime with the number of alternatives, remaining relatively low compared to other methods. TOPSIS-NS (orange line with triangles) shows a similar runtime trend to VIKOR, with slightly lower runtimes in some cases.

TOPSIS-INS (TOPSIS with Interval Neutrosophic Sets) exhibits a moderate runtime increase, higher than both VIKOR and TOPSIS-NS (TOPSIS with Neutrosophic Sets). AMAGDM-NS (Autocratic Multi-Attribute Group Decision-Making with Neutrosophic Sets) displays a substantial runtime

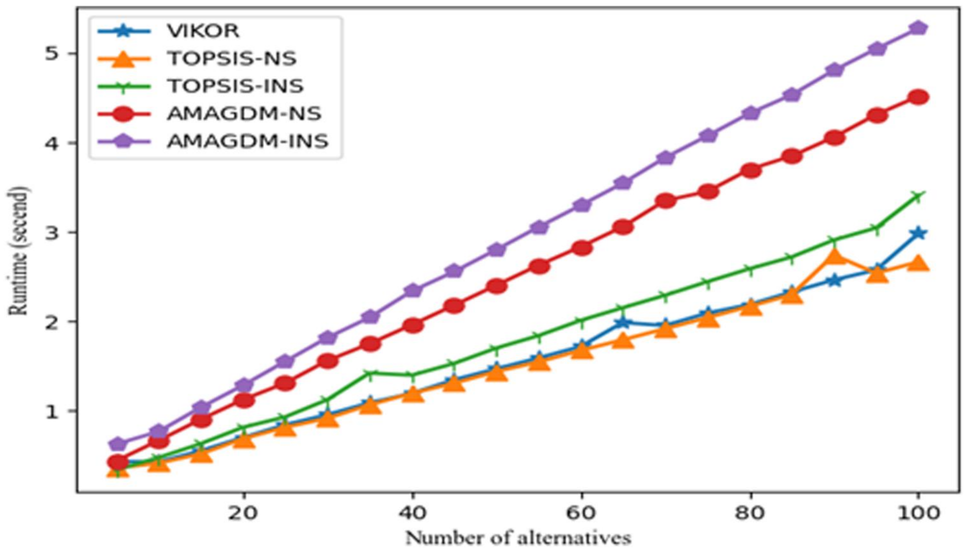

Figure 4. Runtime comparison of different methods.

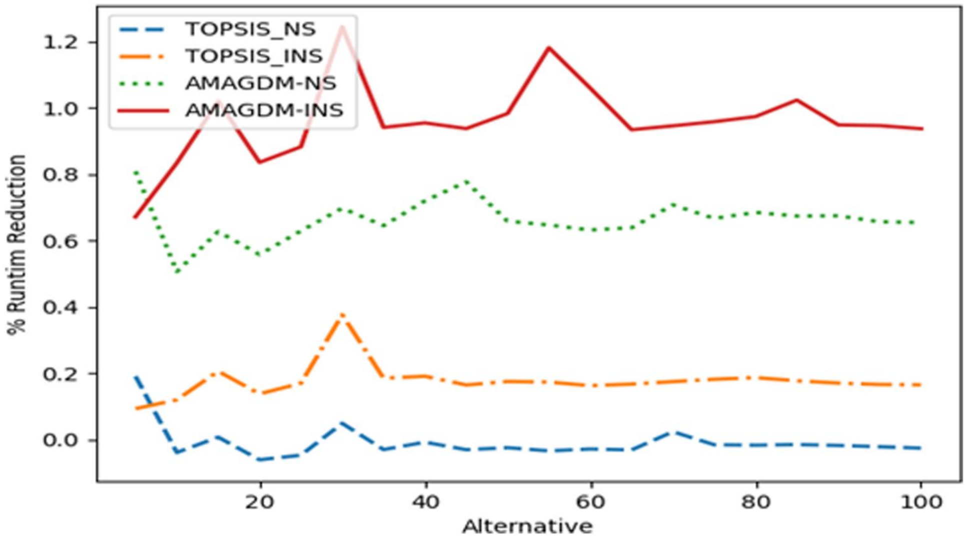

Figure 5. Runtime reduction comparison across methods.

increase, significantly higher than VIKOR and TOPSIS methods. AMAGDM-INS (Autocratic Multi-Attribute Group Decision-Making with Interval Neutrosophic Sets) demonstrates the highest runtime among all methods, indicating the least efficiency.

The comparative analysis highlights VIKOR's efficiency in terms of runtime, especially as the number of alternatives increases. While TOPSIS-NS shows similar or slightly better performance, TOPSIS-INS, AMAGDM-NS, and AMAGDM-INS exhibit significantly higher runtimes, making them less suitable for time-sensitive applications. Decision-makers should consider these findings when selecting MCDM methods, particularly when computational efficiency is critical. The figure demonstrates that VIKOR is the most efficient method in terms of runtime compared to other evaluated MCDM methods. TOPSIS-NS shows similar or slightly better efficiency, while TOPSIS-INS, AMAGDM-NS, and AMGDM-INS are significantly less efficient, with AMAGDM-INS being the least efficient. This analysis underscores the importance of selecting VIKOR for applications requiring high computational efficiency, particularly in complex decision-making environments with many alternatives.

Figure 5 presents a comparative analysis of the runtime performance of various MCDM methods relative to the VIKOR method. In this analysis, we used the proposed VIKOR method as the baseline for comparison. We then calculated different methods' runtime reductions (or increases) relative to VIKOR. Therefore, VIKOR is not displayed in the figure; it serves as the reference point (origin) for the runtime comparisons. The x-axis represents the number of alternatives, while the y-axis represents the percentage runtime reduction compared to VIKOR.

The results are analysed as follows:

1. TOPSIS_NS (blue dashed line): A minimal runtime reduction indicates a comparable time complexity to VIKOR.
2. TOPSIS_INS (orange dash-dot line): A moderate runtime increase indicates worse time complexity than VIKOR, with percentages ranging from approximately 0.1% to 0.4%.
3. AMGDM-NS (green dotted line): Displays a significant runtime increase, showing inefficiency compared to VIKOR, with percentages around 0.5–0.8%.
4. AMGDM-INS (red solid line): Demonstrates the highest runtime increase, indicating it is the least efficient among the compared methods, with percentages around 0.7–1.2%.

The comparative analysis highlights VIKOR's efficiency in terms of runtime. While TOPSIS-NS shows comparable or slightly better performance, TOPSIS-INS, AMAGDM-NS, and AMAGDM-INS exhibit significantly higher runtimes, making them less suitable for time-sensitive applications. Decision-makers should consider these findings when selecting MCDM methods, particularly when computational efficiency is critical.

Figure 6 clearly demonstrates that VIKOR is a more efficient method in terms of runtime compared to other evaluated MCDM methods. TOPSIS-NS shows similar or slightly better efficiency, while TOPSIS-INS, AMAGDM-NS, and AMAGDM-INS are significantly less efficient, with AMAGDM-INS being the least efficient. This analysis underscores the importance of selecting VIKOR for applications requiring high computational efficiency, particularly in complex decision-making environments with many alternatives.

In this context, using an array of inputs, as demonstrated in the case study in Section 6, we thoroughly compared the performance of our suggested methods against various other decision-making methods derived from NSs and their enhancements as proposed by Biswas et al. (2016), Nafei et al. (2019, 2021), Eroğlu and Şahin (2020), Tanaji and Roychowdhury (2024), and Kang et al. (2024). The outcomes of this analysis are displayed in Table 5.

7.5. Multi-criteria method performance review

To strengthen the credibility and convincingness of analysis, the proposed neutrosophic VIKOR method is compared with popular MCDM methods: analytic hierarchy process (AHP), TOPSIS, FAHP, and the

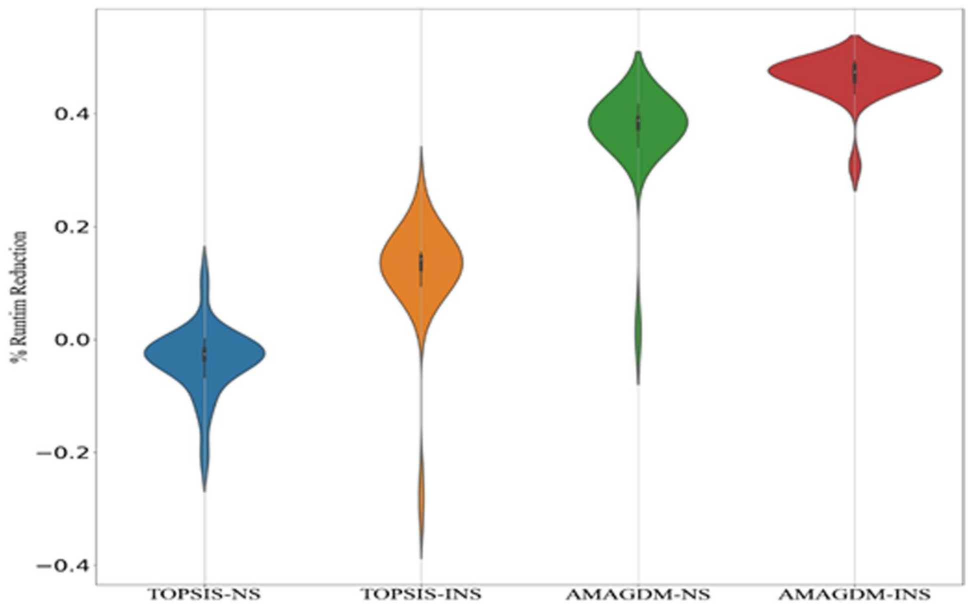

Figure 6. Comparison of runtime reduction percentages across different methods.

Table 5. Comparative analysis.

Methods	The order of alternatives
The proposed VIKOR method	$a_2 > a_5 > a_1 > a_4 > a_3$
TOPSIS method (Biswas et al., 2016)	$a_2 > a_5 > a_4 > a_1 > a_3$
TOPSIS method (Nafei et al., 2021)	$a_2 > a_1 > a_5 > a_4 > a_3$
VIKOR method (Eroğlu & Şahin, 2020)	$a_2 = a_5 > a_1 > a_3 > a_4$
Autocratic method (Nafei et al., 2019)	$a_5 > a_2 > a_4 > a_1 > a_3$
BWM integrated VIKOR (Tanaji & Roychowdhury, 2024)	$a_5 = a_1 > a_2 > a_4 > a_3$
ELECTRE method (Kang, et al., 2024)	$a_2 > a_5 > a_3 > a_1 > a_4$

traditional BWM across four key criteria: consistency, consensus, handling uncertainty, and efficiency.

7.5.1. Consistency

- AHP: High inconsistency due to numerous pairwise comparisons.
- FAHP: Improved consistency over AHP but still complex.
- TOPSIS: Cannot directly address consistency.
- Traditional BWM: Fewer comparisons improve consistency.
- Proposed model: Enhances consistency by reducing comparisons and incorporating neutrosophic logic to handle uncertain or ambiguous information.

Therefore, the proposed model outperforms AHP, FAHP, and TOPSIS in maintaining consistency while improving on traditional BWM with the addition of neutrosophic flexibility.

7.5.2. Consensus

- AHP and FAHP: Difficult to achieve consensus in group settings.
- TOPSIS: Lacks explicit consensus-building mechanisms.
- Traditional BWM: Simpler consensus through best/worst comparisons.
- Proposed model: Fosters better consensus with neutrosophic logic, allowing decision-makers to express partial agreement and handle uncertainty collaboratively.

So, the proposed model excels in consensus-building, outperforming other methods by allowing flexibility in decision-maker inputs.

7.5.3. Handling uncertainty

- AHP and TOPSIS: Cannot handle uncertainty effectively.
- FAHP: Incorporates fuzzy logic, but not as robust as neutrosophic logic.
- Traditional BWM: Limited uncertainty handling.
- Proposed model: Uses NTs to account for truth, indeterminacy, and falsity, providing superior uncertainty management.

Therefore, the proposed model offers the best handling of uncertainty among all methods.

7.5.4. Efficiency

- AHP and FAHP: Inefficient due to numerous comparisons and complexity.
- TOPSIS: Efficient but lacks uncertainty management.
- Traditional BWM: Efficient due to reduced comparisons.
- Proposed model: Maintains efficiency while incorporating neutrosophic logic for greater flexibility.

So, the proposed model is efficient while providing more comprehensive decision-making compared to other methods.

8. Conclusion

This research introduces a novel decision-making methodology integrating DED within the VIKOR framework, leveraging neutrosophic logic to enhance decision accuracy and reliability. The proposed DED approach revolutionizes distance calculation by incorporating exponential transformations, allowing for a more nuanced handling of truth, indeterminacy, and falsity degrees in NSs. This method enhances the ability to distinguish between alternatives by amplifying differences in extreme values, accommodating varying criteria significance, and explicitly integrating uncertainty and indeterminacy. This research contributes by introducing the DED approach, which improves the accuracy of distance calculations in neutrosophic environments through exponential transformations. It also provides a comprehensive methodology for managing uncertainty and indeterminacy in MAGDM contexts, while showcasing the method's ability to effectively address real-world industrial automation challenges by enhancing discrimination between alternatives. The effectiveness of the enhanced VIKOR method combined with DED is demonstrated through a comprehensive case study involving the selection of industrial automation systems. The results highlight improved discrimination between closely ranked alternatives and the method's adaptability to changing decision-making contexts. Additionally, the runtime analysis indicates that while VIKOR is moderately efficient, it offers a balanced trade-off between computational demands and decision-making effectiveness, making it a viable option for practical applications in industrial settings.

Despite these contributions, the research has certain limitations. One notable area for improvement is the assumption of consistent decision-maker preferences and the static nature of the weighting parameters, which might not fully capture real-world decision-making environments' dynamic and evolving nature. Furthermore, the case study is limited to a specific industrial context, which may affect the generalizability of the findings to other sectors. Future research should explore the dynamic adjustment of weighting parameters in response to changing decision contexts and preferences. Additionally, extending the application of the proposed methodology to other domains, such as healthcare or finance, could provide further validation and refinement of the approach.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

Disclosure statement

The authors declare that they have no conflict of interest.

Data availability statement

No data were used to support the study.

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