Local-Neutrosophic Logic and Local-Neutrosophic Sets: Incorporating Locality with Applications

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1 author:

Takaaki Fujita

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Takaaki Fujita [1] *

[1] Independent Researcher, Shinjuku, Shinjuku-ku, Tokyo, Japan.

Abstract

The study of uncertainty has been a significant area of research, with concepts such as fuzzy sets [87], fuzzy graphs [51], and neutrosophic sets [58] receiving extensive attention. In Neutrosophic Logic, indeterminacy often arises from real-world complexities.

This paper explores the concept of locality as a key factor in determining indeterminacy, building upon the framework introduced by F. Smarandache in [73]. Locality refers to processes constrained within a specific region, where an object or system is directly influenced by its immediate surroundings. In contrast, nonlocality involves effects that transcend spatial or temporal boundaries, where changes in one location have direct implications for another.

This paper introduces the concepts of Local-Neutrosophic Logic and Local-Neutrosophic Set by integrating the notion of locality into Neutrosophic Logic. It provides their mathematical definitions and examines potential applications.

Keywords: Neutrosophic Logic, Neutrosophic Set, Fuzzy Logic, Locality

1 Short Introduction

1.1 Uncertain Logic

Uncertainty is an inherent characteristic of real-world events and is often modeled using mathematical frameworks. In the realm of logic (cf. [12, 81]), several approaches have been developed to address uncertainty, including Fuzzy Logic [87,89,90], Neutrosophic Logic [58,62,68], and Plithogenic Logic [67,77]. For instance, Neutrosophic Logic expands upon classical logic by incorporating three dimensions: truth, indeterminacy, and falsity. This framework allows for the simultaneous handling of uncertainty and contradictions, making it a versatile tool for modeling complex systems.

These uncertain logics have been further generalized to other mathematical concepts, such as sets [59, 75] and graphs [20, 22, 23, 25, 26]. This has led to a proliferation of studies that parallel the development of logical systems, showcasing their broad applicability across various domains.

1.2 Locality in Neutrosophic Logic

In Neutrosophic Logic, indeterminacy often emerges from real-world factors. This paper investigates locality as a key determinant of indeterminacy, building on the framework proposed by F. Smarandache in [73].

Locality describes processes confined to a specific region, where an object is influenced by its immediate surroundings. It can be Total Locality (100 percent, all interactions are local) or Partial Locality (greater than 0 and less than 100 percent). Conversely, Nonlocality involves effects spanning space or time, with changes in one location influencing another. Like locality, nonlocality may be Total or Partial.

Indeterminacy arises when a system is neither fully local nor nonlocal, often due to hidden variables or environmental uncertainty. It too can range from Total to Partial, depending on the extent of ambiguity or mixed characteristics.

1.3 Contributions of This Paper

This paper makes several key contributions:

1. It introduces the novel concepts of Local-Neutrosophic Logic and Local-Neutrosophic Set, incorporating the notion of locality into the framework of Neutrosophic Logic.
2. It provides precise mathematical definitions for these concepts, laying a robust theoretical foundation.
3. It explores potential applications, demonstrating the practicality and relevance of these ideas in addressing uncertainty and contextual dependencies.

1.4 The Structure of the Paper

The structure of this paper is as follows.

2 Preliminaries

2.1 Basic Definition of Formal Language

To explore Upside-Down Logic, several key concepts are introduced below. For further details, readers are encouraged to consult the respective lecture notes and surveys on these topics (ex. [16, 29, 30, 33, 40]).

Definition 2.1 (Set). [33] A *set* is a collection of distinct and clearly defined objects, known as elements, such that any object can be identified as either a member of the set or not. If A is a set and x is an element of A, this membership is denoted by $x \in A$. Sets are commonly represented using curly brackets, for example, $A = \{x_1, x_2, \dots, x_n\}$.

Definition 2.2 (Formal Language). [29, 49] A *formal language* $\mathcal{L}$ is defined as a set of strings (or sequences) formed from a finite alphabet Σ, adhering to specific syntactic rules. Formally:

$$\mathcal{L} \subseteq \Sigma^*,$$

where Σ^* represents the set of all finite strings over the alphabet Σ. The strings in $\mathcal{L}$ are referred to as *well-formed formulas (WFFs)*.

A formal language $\mathcal{L}$ is typically characterized by:

- A set of *symbols* (or *alphabet*) Σ, which may include logical connectives (e.g., $\wedge$, $\vee$, $\neg$), quantifiers (e.g., $\forall$, $\exists$), variables, and parentheses.
- A set of *formation rules* specifying which strings in Σ^* qualify as well-formed.

Definition 2.3 (Logical System)**.** (cf. [37]) A *logical system* $\mathcal{M}$ is a mathematical structure used to formalize reasoning. It is defined as:

$$\mathcal{M} = (\mathcal{P}, \mathcal{V}, v),$$

where:

- $\mathcal{P}$ is the set of propositions (or statements) expressed in the formal language $\mathcal{L}$.
- $\mathcal{V}$ is the set of truth values, such as $\{\text{True}, \text{False}\}$ in classical logic.
- $v : \mathcal{P} \to \mathcal{V}$ is a *valuation function* (or interpretation function) that assigns a truth value to each proposition in $\mathcal{P}$.

Additionally, a logical system may include:

- A set of *axioms* $\mathcal{A} \subseteq \mathcal{P}$, propositions assumed to be true within the system.
- A set of *inference rules* $\mathcal{I}$, defining valid methods of deriving new truths from existing propositions.

2.2 Neutrosophic Logic

In this subsection, we explore the relationship between Neutrosophic Logic and Upside-Down Logic. First, we present the definition of Neutrosophic Logic below [21, 58]. Note that Neutrosophic Logic is known to generalize Fuzzy Logic (cf. [58]).

Definition 2.4 (Neutrosophic Logic)**.** [58] Neutrosophic Logic extends classical logic by assigning to each proposition a truth value comprising three components:

$$v(A) = (T, I, F),$$

where $T, I, F \in [0, 1]$ represent the degrees of truth, indeterminacy, and falsity, respectively.

Example 2.5 (Student Performance Evaluation)**.** Student performance evaluation assesses academic progress using metrics like grades, participation, and skills, identifying strengths and areas for improvement (cf. [7,36]).

In education, Neutrosophic Logic can assess student performance when data is uncertain or incomplete. For example, consider the proposition ”The student will perform well in the final exam,” represented as:

$$v(A) = (0.8, 0.1, 0.1),$$

where:

- $T = 0.8$: An 80% chance of good performance based on past grades and class participation.
- $I = 0.1$: A 10% level of indeterminacy due to unmeasured factors like stress or unforeseen circumstances.
- $F = 0.1$: A 10% chance of poor performance due to lack of preparation or external distractions.

This approach enables educators to provide personalized feedback and prepare targeted interventions to improve student outcomes(cf. [3, 47, 56]).

Example 2.6 (Medical Diagnosis)**.** In the field of medical diagnosis(cf. [5, 43]), Neutrosophic Logic is employed to handle uncertain and incomplete information. For instance, consider the proposition ”The patient has Disease X,” which can be represented as:

$$v(A) = (0.6, 0.2, 0.2),$$

where:

- $T = 0.6$: A 60% probability that the patient has the disease.
- $I = 0.2$: A 20% of the data is inconclusive due to uncertainty in test results or conflicting evidence.
- $F = 0.2$: A 20% probability that the patient does not have the disease.

By integrating test results, symptoms, and expert opinions, this approach enables healthcare professionals to make more informed diagnostic decisions (cf. [9, 11, 13, 55]).

Example 2.7 (Project Management)**.** Project management involves planning, organizing, and executing tasks to achieve specific goals within constraints like time, budget, and resources(cf. [15, 17, 18, 80]).

In project management, Neutrosophic Logic can aid in handling uncertainties and risks associated with project timelines and outcomes. Consider the proposition ”The project will be completed on time,” represented as:

$$v(A) = (0.6, 0.25, 0.15),$$

where:

- $T = 0.6$: A 60% chance that the project will be completed on schedule, based on current progress and resource availability.
- $I = 0.25$: A 25% level of indeterminacy due to uncertainties like unexpected delays, resource shortages, or scope changes.
- $F = 0.15$: A 15% chance that the project will not meet the deadline, based on known risks or past trends in similar projects.

By quantifying these components, project managers can better assess risks and devise strategies such as resource reallocation or timeline adjustments to mitigate potential delays. This enhances decision-making under uncertainty and improves project success rates(cf. [2, 6, 34, 45, 50]).

Example 2.8 (Decision-Making in Business)**.** Neutrosophic Logic also plays a critical role in business decision-making, particularly under conditions of uncertainty. For example, when evaluating whether to invest in a project, the proposition ”The project will yield profit” can be represented as:

$$v(A) = (0.7, 0.1, 0.2),$$

where:

- $T = 0.7$: A 70% chance that the project will be profitable.
- $I = 0.1$: A 10% level of uncertainty due to incomplete or ambiguous market data.
- $F = 0.2$: A 20% chance that the project will not be profitable.

This representation allows decision-makers to assess risks and rewards quantitatively, facilitating more effective strategy formulation (cf. [4, 52]).

2.3 Neutrosophic Set

In this subsection, we explain the concept of the Neutrosophic Set [58]. Intuitively, a Neutrosophic Set can be understood as the set-theoretic extension of Neutrosophic Logic. It is known as a generalization of several classical and modern set concepts, including the Crisp Set (Classic Set), Fuzzy Set [87,88,91–93], Intuitionistic Fuzzy Set [61, 76], Vague Set [10, 96], and Paraconsistent Set [83, 84]. The related definitions are provided below.

Definition 2.9 (Crisp Set)**.** [48] Let X be a universe set, and let $P(X)$ denote the power set of X, which represents all subsets of X. A *crisp set* $A \subseteq X$ is defined by a characteristic function $\chi_A : X \to \{0, 1\}$, where:

$$\chi_A(x) = \begin{cases} 1 & \text{if } x \in A, \\ 0 & \text{if } x \notin A. \end{cases}$$

This function χ_A assigns a value of 1 to elements within the set A and 0 to those outside it, creating a clear boundary. Crisp sets are thus bivalent and follow the principle of binary classification, where each element is either a member of the set or not.

Definition 2.10. [58, 60, 78] Let X be a given set. A Neutrosophic Set A on X is characterized by three membership functions:

$$T_A : X \to [0, 1], \quad I_A : X \to [0, 1], \quad F_A : X \to [0, 1],$$

where for each $x \in X$, the values $T_A(x)$, $I_A(x)$, and $F_A(x)$ represent the degree of truth, indeterminacy, and falsity, respectively. These values satisfy the following condition:

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3.$$

3 Mathematical Framework of Locality, Indeterminacy, and Nonlocality in Neutrosophic Logic

This section discusses the Mathematical Framework of Locality, Indeterminacy, and Nonlocality in Neutrosophic Logic. It redefines these concepts within the context of Neutrosophic Logic, providing basic considerations and illustrative examples.

3.1 Notations and Definitions

Below, we present the Notations and Definitions of Locality, Indeterminacy, and Nonlocality in Neutrosophic Logic.

Notation 3.1. *Let $\mathcal{P}$ denote the set of propositions, $\mathcal{C}$ the set of contexts, and $T : \mathcal{P} \times \mathcal{C} \to \{True, False, Indeterminate\}$ a truth valuation function. Each proposition $A \in \mathcal{P}$ is associated with a neutrosophic truth value $v(A) = (T, I, F)$, where:*

$$T, I, F \in [0, 1], \quad T + I + F \leq 1,$$

represent the degrees of truth (T), indeterminacy (I), and falsity (F), respectively.

Remark 3.2. The sum $T + I + F$ does not necessarily equal 1, allowing for partial states. This flexibility is a core feature of neutrosophic logic.

Definition 3.3 (Context)**.** [74] A *context* C is a set of parameters or conditions under which propositions are evaluated. This may include spatial, temporal, semantic, or interpretative settings.

Definition 3.4 (Locality)**.** (cf. [73]) A proposition A exhibits *locality* if its truth value depends solely on a single context $C \in \mathcal{C}$. Formally:

$$v(A) = T(A, C),$$

where C represents the immediate spatial, temporal, or conceptual domain affecting A.

Definition 3.5 (Indeterminacy)**.** (cf. [73]) A proposition A exhibits *indeterminacy* if its truth value includes a non-zero degree of I due to hidden variables, insufficient information, or ambiguity within the context C. Formally:

$$v(A) = (T, I, F), \quad \text{with } I > 0.$$

Definition 3.6 (Nonlocality)**.** (cf. [73]) A proposition A exhibits *nonlocality* if its truth value depends on multiple, spatially or conceptually separate contexts $C_1, C_2 \in C$. Formally:

$$v(A) = T(A, C_1, C_2), \quad \text{with } C_1 \cap C_2 = \emptyset.$$

Definition 3.7 (Multilocality)**.** (cf. [73]) A proposition A exhibits *multilocality* if its truth value depends on a set of interacting local contexts $\{C_i\}_{i=1}^n$. Formally:

$$v(A) = T(A, \{C_i\}_{i=1}^n),$$

where each C_i is confined to a specific local domain.

Definition 3.8 (Multiindeterminacy)**.** (cf. [73]) A proposition A exhibits *multiindeterminacy* if its truth value includes cumulative indeterminacy across multiple contexts:

$$v(A) = (T, I, F), \quad \text{where } I = \sum_{i=1}^{n} I_i > 0,$$

and I_i represents the degree of indeterminacy in each context C_i.

Definition 3.9 (Multinonlocality)**.** (cf. [73]) A proposition A exhibits *multinonlocality* if its truth value depends on interactions across multiple nonlocal contexts $\{C_i, C_j\}_{i \neq j}$:

$$v(A) = T(A, \{C_i, C_j\}_{i \neq j}),$$

where $C_i \cap C_j = \emptyset$.

3.2 Some Real-Life Examples of Locality, Indeterminacy, and Nonlocality

This subsection presents some real-life examples of locality, indeterminacy, and nonlocality.

Example 3.10 (Traffic Flow (Locality))**.** Let A: "The traffic density on road segment R is high."

Context C: The immediate local parameters affecting R, such as vehicle count, average speed, and weather conditions, determine $v(A)$:

$$v(A) = (0.8, 0.1, 0.1),$$

where $T = 0.8$ indicates high traffic density based on local observations. Locality is evident as A depends solely on C.

Example 3.11 (Quantum Entanglement (Nonlocality))**.** Quantum entanglement is a phenomenon where particles share linked states, such that changing one instantly affects the other, regardless of distance (cf. [31, 85]).

Let A: "The spin of particle P_1 is up."

Contexts C_1, C_2**:** Measurement of P_1 in C_1 instantly determines the spin of P_2 in C_2:

$$v(A) = (1, 0, 0), \quad \text{if } P_2 \text{ spin is down in } C_2.$$

Nonlocality is evident as $v(A)$ spans C_1 and C_2.

Example 3.12 (Stock Market Volatility (Indeterminacy))**.** Stock market volatility measures rapid price fluctuations in financial markets, influenced by economic events, investor behavior, and uncertainty (cf. [14, 41]).

Let B: "The stock price of company X will increase tomorrow."

Context C: Factors such as market trends, global events, and investor sentiment introduce indeterminacy. The truth value is:

$$v(B) = (0.5, 0.4, 0.1),$$

where $I = 0.4$ reflects uncertainty due to incomplete or ambiguous data. Indeterminacy arises from unpredictable market influences.

Example 3.13 (Climate Models (Multiindeterminacy))**.** Let B: "Global temperature will rise by 1°C in 50 years."

Contexts $\{C_i\}_{i=1}^3$**:** Varying predictions from three models yield:

$$v(B) = (0.6, 0.3, 0.1), \quad \text{where } I = 0.3 = \sum_{i=1}^{3} I_i.$$

Multiindeterminacy arises from differing model assumptions.

Example 3.14 (Global Communications (Multinonlocality))**.** Let E: "Information is transmitted successfully across the network."

Contexts $\{C_i, C_j\}_{i\neq j}$: The reliability of nodes in separate regions C_i and C_j determines:

$$v(E) = (0.9, 0.05, 0.05),$$

where multinonlocality reflects the interaction between contexts C_i and C_j across the global network.

3.3 Some Basic Theorem of Locality, Indeterminacy, and Nonlocality

In this subsection, we present Some Basic Theorems of Locality, Indeterminacy, and Nonlocality.

Theorem 3.15 (Consistency of Locality)**.** *If a proposition A is local, then the truth value $v(A)$ is unaffected by nonlocal contexts. Formally:*

$$T(A, C_1) = T(A, C_2) \quad \textit{for all } C_1, C_2 \neq C.$$

Proof. Locality assumes A is influenced only by C. For $C_1, C_2 \neq C$, the absence of influence implies:

$$v(A) = T(A, C),$$

and thus $T(A, C_1) = T(A, C_2)$ follows trivially. □

Theorem 3.16 (Additivity of Multiindeterminacy)**.** *For a proposition A exhibiting multiindeterminacy across n contexts $\{C_i\}_{i=1}^n$, the total indeterminacy satisfies:*

$$I = \sum_{i=1}^{n} I_i, \quad \textit{where } I_i > 0 \textit{ for each } C_i.$$

Proof. By definition, multiindeterminacy aggregates indeterminacy from individual contexts:

$$v(A) = (T, I, F), \quad I = \sum_{i=1}^{n} I_i.$$

The constraint $I_i > 0$ ensures that each context contributes to the total indeterminacy. □

4 Mathematical Framework of Partial Locality, Partial Non-Locality, and Partial Indeterminacy

In this section, we examine the Mathematical Framework of Partial Locality, Partial Non-Locality, and Partial Indeterminacy.

An overview of Partial Locality, Partial Non-Locality, and Partial Indeterminacy is provided below:

- *Partial Locality* refers to a situation where an object or system is partially influenced by its immediate surroundings, with limited external interactions or dependencies.
- *Partial Non-Locality* describes a condition where an object or system is partially influenced by distant factors or entities without direct physical contact.
- *Partial Indeterminacy* represents a system exhibiting unclear or mixed characteristics, influenced neither entirely locally nor entirely non-locally.

4.1 Definitions of Partial Locality, Partial Non-Locality, and Partial Indeterminacy

In this subsection, we consider about Definitions of Partial Locality, Partial Non-Locality, and Partial Indeterminacy.

Definition 4.1 (Partial Locality)**.** (cf. [73]) A proposition $A \in \mathcal{P}$ exhibits *partial locality* if its truth value is influenced by its immediate surroundings with a fractional dependency denoted by $\alpha \in [0, 1]$. Formally:

$$v(A) = \alpha \cdot T(A, C_{\text{local}}) + (1 - \alpha) \cdot \text{Residual Effects},$$

where C_{local} is the immediate local context and α represents the degree of locality.

Definition 4.2 (Partial Non-Locality)**.** (cf. [73]) A proposition $A \in \mathcal{P}$ exhibits *partial non-locality* if its truth value is influenced by distant or separate contexts with a fractional dependency $\beta \in [0, 1]$. Formally:

$$v(A) = \beta \cdot T(A, C_{\text{nonlocal}}) + (1 - \beta) \cdot \text{Residual Effects},$$

where C_{nonlocal} refers to nonlocal contexts influencing A, and β represents the degree of non-locality.

Definition 4.3 (Partial Indeterminacy)**.** (cf. [73]) A proposition $A \in \mathcal{P}$ exhibits *partial indeterminacy* if its truth value includes an indeterminate component $\gamma \in [0, 1]$ due to hidden variables or ambiguous influences. Formally:

$$v(A) = (T, \gamma \cdot I, F),$$

where I is the total indeterminacy, and γ represents the degree of partial indeterminacy.

Remark 4.4. Partial locality, non-locality, and indeterminacy may coexist for a single proposition, forming a composite influence model:

$$v(A) = \alpha \cdot T(A, C_{\text{local}}) + \beta \cdot T(A, C_{\text{nonlocal}}) + \gamma \cdot I,$$

where $\alpha, \beta, \gamma \in [0, 1]$ and $\alpha + \beta + \gamma \leq 1$.

4.2 Theorems and Proofs of Partial Locality, Partial Non-Locality, and Partial Indeterminacy

In this subsection, we present Some Basic Theorems of artial Locality, Partial Non-Locality, and Partial Indeterminacy.

Theorem 4.5 (Consistency of Partial Locality)**.** *If a proposition A exhibits partial locality with degree α, then the influence from nonlocal contexts diminishes proportionally. Formally:*

$$v(A) = \alpha \cdot T(A, C_{local}) + (1 - \alpha) \cdot T(A, C_{nonlocal}),$$

where $\alpha \to 1$ implies pure locality.

Proof. By definition, α scales the influence of C_{local}, and $1 - \alpha$ scales the complementary effect from C_{nonlocal}. As $\alpha \to 1$, the term $(1 - \alpha) \cdot T(A, C_{\text{nonlocal}})$ vanishes, yielding pure locality. □

Theorem 4.6 (Superposition of Influences)**.** *The total influence on a proposition A can be represented as a superposition of partial locality, partial non-locality, and partial indeterminacy:*

$$v(A) = \alpha \cdot T(A, C_{local}) + \beta \cdot T(A, C_{nonlocal}) + \gamma \cdot I.$$

Proof. The influence components are orthogonal by construction: C_{local} affects A directly, C_{nonlocal} introduces distant dependencies, and I incorporates ambiguity. Thus, the superposition holds. □

4.3 Examples of Partial Locality, Partial Non-Locality, and Partial Indeterminacy

In this subsection, we present Examples of Partial Locality, Partial Non-Locality, and Partial Indeterminacy.

Example 4.7 (Quantum Physics: Aharonov-Bohm Effect)**.** Quantum physics studies the behavior of particles at atomic and subatomic scales, governed by principles like wave-particle duality and superposition (cf. [8,27]).

Let A: "A charged particle is influenced by an electromagnetic potential."

Local Context C_{local}: The particle exists in a region with no magnetic field intensity.

Nonlocal Context C_{nonlocal}: The electromagnetic potential resides outside the particle's local region.

$$v(A) = \alpha \cdot T(A, C_{\text{local}}) + \beta \cdot T(A, C_{\text{nonlocal}}),$$

where $\beta > 0$ captures the nonlocal influence of the potential.

Example 4.8 (Ecology: Migratory Birds)**.** Let B: "Birds impact the nutrient cycle."

Local Context C_{local}: Birds forage and nest locally.

Nonlocal Context C_{nonlocal}: Migratory behavior spreads nutrients across ecosystems.

$$v(B) = \alpha \cdot T(B, C_{\text{local}}) + \beta \cdot T(B, C_{\text{nonlocal}}),$$

where $\beta > 0$ quantifies the nonlocal nutrient transfer.

5 New Definition of Local-Neutrosophic Logic and Set

In this section, we introduce a new concept called Local-Neutrosophic Logic and Set. This concept extends Neutrosophic Logic by incorporating the notion of locality. The definition is provided below.

Definition 5.1 (Local-Neutrosophic Logic)**.** Local-Neutrosophic Logic assigns to each proposition $A \in \mathcal{P}$ a truth value of the form:

$$v(A) = (T, I, L, F),$$

where:

- $T \in [0, 1]$: Degree of truth.
- $I \in [0, 1]$: Degree of indeterminacy.
- $L \in [0, 1]$: Degree of locality, representing the influence of immediate contextual or spatial factors.
- $F \in [0, 1]$: Degree of falsity.

These components satisfy the constraint:

$$T + I + L + F \leq 1.$$

Remark 5.2 (Transformation Rules)**.** Transformation Rules of Local-Neutrosophic Logic are following.

- **Locality-to-Truth Transformation:** When locality L provides strong supporting evidence for truth:

$$v(U_{LT}(A)) = (T + L, I, 0, F).$$

- **Locality-to-Falsity Transformation:** When locality L provides strong evidence against truth:

$$v(U_{LF}(A)) = (T, I, 0, F + L).$$

- **Locality-to-Indeterminacy Transformation:** When locality L introduces ambiguity or uncertainty:

$$v(U_{LI}(A)) = (T, I + L, 0, F).$$

- **Indeterminacy-to-Locality Transformation:** When indeterminacy I is clarified by locality L:

$$v(U_{IL}(A)) = (T, 0, I + L, F).$$

The definition of the Local-Neutrosophic Set, which extends Local-Neutrosophic Logic to sets, is as follows. It is anticipated that future research will explore the specific mathematical structures and applications of this concept.

Definition 5.3 (Local-Neutrosophic Set)**.** Let X be a given universe of discourse. A *Local-Neutrosophic Set* A on X is characterized by four membership functions:

$$T_A : X \to [0, 1], \quad I_A : X \to [0, 1], \quad L_A : X \to [0, 1], \quad F_A : X \to [0, 1],$$

where, for each $x \in X$:

- $T_A(x)$: The degree of truth of x in A.
- $I_A(x)$: The degree of indeterminacy of x in A.
- $L_A(x)$: The degree of locality of x in A, representing the influence of immediate spatial, contextual, or environmental factors.
- $F_A(x)$: The degree of falsity of x in A.

These membership values satisfy the following constraint:

$$0 \leq T_A(x) + I_A(x) + L_A(x) + F_A(x) \leq 4.$$

Remark 5.4 (Local-Neutrosophic Set)**.** Compared to other sets:

- A *Fuzzy Set* has a single membership function, $\mu_A(x)$, representing truth.
- An *Intuitionistic Fuzzy Set* adds a falsity component, $\nu_A(x)$, to truth.
- A *Neutrosophic Set* further includes indeterminacy, $I_A(x)$.
- The *Local-Neutrosophic Set* expands these by adding locality ($L_A(x)$) to model systems influenced by contextual factors.

5.1 Basic Theorem of Local-Neutrosophic Logic

We outline several basic theorems of Local-Neutrosophic Logic below.

Theorem 5.5 (Preservation of Total Degree)**.** *The total degree of the truth valuation remains invariant under transformations:*

$$T + I + L + F = T' + I' + L' + F'.$$

Proof. Each transformation redistributes the components among T, I, L, and F, preserving their total sum. □

Theorem 5.6 (Superposition of Influences)**.** *The truth value of a proposition A in Local-Neutrosophic Logic can be expressed as:*

$$v(A) = \alpha \cdot T + \beta \cdot I + \gamma \cdot L + \delta \cdot F,$$

where $\alpha, \beta, \gamma, \delta \in [0, 1]$ represent the relative weights of each component and $\alpha + \beta + \gamma + \delta = 1$.

Proof. The superposition follows directly from the normalized representation of truth valuation components. □

Theorem 5.7. *Local-Neutrosophic Logic extends Neutrosophic Logic by introducing an additional degree of locality L, which represents the influence of immediate contextual or spatial factors. Specifically, every proposition in Neutrosophic Logic can be represented as a special case of Local-Neutrosophic Logic where $L = 0$.*

Proof. The truth value in Neutrosophic Logic is defined as:

$$v_{\mathrm{NL}}(A) = (T, I, F),$$

where $T, I, F \in [0, 1]$ and satisfy the constraint:

$$T + I + F \leq 1.$$

In Local-Neutrosophic Logic, the truth value is extended to:

$$v_{\mathrm{LNL}}(A) = (T, I, L, F),$$

where $T, I, L, F \in [0, 1]$ and satisfy the constraint:

$$T + I + L + F \leq 1.$$

If we set $L = 0$ in Local-Neutrosophic Logic, the truth value simplifies to:

$$v_{\mathrm{LNL}}(A) = (T, I, 0, F) = (T, I, F),$$

which is identical to the truth value in Neutrosophic Logic. Therefore, every truth value in Neutrosophic Logic is a valid truth value in Local-Neutrosophic Logic.

The addition of L in Local-Neutrosophic Logic allows for the representation of an additional degree of influence from immediate contextual or spatial factors, which is not captured in Neutrosophic Logic. This makes Local-Neutrosophic Logic a generalized framework.

The constraint $T + I + F \leq 1$ in Neutrosophic Logic is preserved in Local-Neutrosophic Logic because setting $L = 0$ satisfies:

$$T + I + L + F \leq 1.$$

Local-Neutrosophic Logic reduces to Neutrosophic Logic when $L = 0$, but it also allows for additional flexibility when $L > 0$. Hence, Local-Neutrosophic Logic is a strict extension of Neutrosophic Logic. □

Theorem 5.8. *A Local-Neutrosophic Set is an extension of a Neutrosophic Set. Specifically, every Neutrosophic Set can be represented as a Local-Neutrosophic Set where the degree of locality $L_A(x) = 0$ for all $x \in X$.*

Proof. The membership functions of a Neutrosophic Set A on X are:

$$T_A : X \to [0, 1], \quad I_A : X \to [0, 1], \quad F_A : X \to [0, 1],$$

with the constraint:

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3, \quad \forall x \in X.$$

For a Local-Neutrosophic Set A on X, the membership functions are extended to include L_A:

$$T_A : X \to [0,1], \quad I_A : X \to [0,1], \quad L_A : X \to [0,1], \quad F_A : X \to [0,1],$$

with the constraint:

$$0 \le T_A(x) + I_A(x) + L_A(x) + F_A(x) \le 4, \quad \forall x \in X.$$

If $L_A(x) = 0$ for all $x \in X$, the membership functions of a Local-Neutrosophic Set reduce to:

$$T_A : X \to [0,1], \quad I_A : X \to [0,1], \quad F_A : X \to [0,1],$$

with the constraint:

$$0 \le T_A(x) + I_A(x) + F_A(x) \le 3, \quad \forall x \in X,$$

which is identical to the structure of a Neutrosophic Set.

The additional component $L_A(x)$ in a Local-Neutrosophic Set allows for the representation of an additional degree of locality, representing the influence of spatial, contextual, or environmental factors. This flexibility generalizes the concept of Neutrosophic Sets.

For any $x \in X$, the constraint $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$ in a Neutrosophic Set is preserved in a Local-Neutrosophic Set when $L_A(x) = 0$, satisfying:

$$0 \le T_A(x) + I_A(x) + L_A(x) + F_A(x) \le 4.$$

A Local-Neutrosophic Set reduces to a Neutrosophic Set when $L_A(x) = 0$ for all $x \in X$, but it also allows for additional flexibility when $L_A(x) > 0$. Hence, the Local-Neutrosophic Set is a strict extension of the Neutrosophic Set. □

Theorem 5.9. *Local-Neutrosophic Logic can represent locality, non-locality, and indeterminacy through the values T, I, L, F assigned to propositions in the form $v(A) = (T, I, L, F)$, satisfying $T + I + L + F \le 1$. Specifically:*

- *Locality is captured by L, representing the influence of immediate surroundings.*
- *Non-locality is represented by the truth or falsity components T and F, influenced by distant contexts.*
- *Indeterminacy is represented by I, capturing the uncertainty or ambiguity.*

Proof. Let $A \in \mathcal{P}$ be a proposition evaluated in Local-Neutrosophic Logic with $v(A) = (T, I, L, F)$.

From the definition of locality:

$$v(A) = (T, I, L, F), \quad L > 0.$$

Here, L explicitly represents the degree to which A is influenced by its immediate surroundings (local context). If $L = 0$, A has no local influence. Thus, locality is embedded within the L component.

Non-locality is captured when L is small ($L \to 0$) and the remaining truth (T) or falsity (F) values depend on distant contexts. Specifically:

$$v(A) = (\beta \cdot T(A, C_{\text{nonlocal}}), I, L, \beta \cdot F(A, C_{\text{nonlocal}})),$$

where C_{nonlocal} represents nonlocal contexts and β $(0 < \beta \le 1)$ denotes the degree of non-locality. Non-locality arises when T or F depends on contexts spatially or conceptually separated from the local context.

Indeterminacy is directly captured by the I component in $v(A)$. If $I > 0$, there exists a degree of uncertainty or ambiguity in the truth value of A. Indeterminacy arises from hidden variables, conflicting evidence, or incomplete information. Formally:

$$v(A) = (T, \gamma \cdot I, L, F), \quad \text{where } \gamma \in [0,1].$$

Here, γ controls the contribution of indeterminacy to the overall evaluation.

By definition, Local-Neutrosophic Logic satisfies:

$$T + I + L + F \leq 1.$$

This constraint ensures that locality, non-locality, and indeterminacy are mathematically consistent and their contributions to $v(A)$ are bounded.

Hence, Local-Neutrosophic Logic effectively represents locality, non-locality, and indeterminacy through the values T, I, L, F, satisfying the stated constraint. □

5.2 Examples of Local-Neutrosophic Logic in real-life scenarios

In this subsection, we explain examples of Local-Neutrosophic Logic in real-life scenarios.

Example 5.10 (Quantum Physics: Measurement Locality)**.** Quantum physics studies the behavior of particles at atomic and subatomic scales, governed by principles like wave-particle duality and superposition (cf. [8,27]).

Let A: "The spin of a particle is up."

Truth Components:

$$v(A) = (T, I, L, F) = (0.6, 0.2, 0.1, 0.1),$$

where:

- $T = 0.6$: Evidence strongly supports the particle's spin being up.
- $I = 0.2$: Uncertainty due to measurement limitations.
- $L = 0.1$: Local experimental context influences spin alignment.
- $F = 0.1$: Weak evidence against the proposition.

Example 5.11 (Ecology: Pollination Dynamics)**.** Pollination dynamics refers to the interactions between plants and pollinators, such as bees or birds, facilitating plant reproduction and ecosystem stability (cf. [28]).

Let B: "Pollinators improve crop yield in a region."

Truth Components:

$$v(B) = (T, I, L, F) = (0.7, 0.1, 0.15, 0.05),$$

where:

- $T = 0.7$: Direct observations confirm significant pollination effects.
- $I = 0.1$: Uncertainty due to unmeasured ecological variables.
- $L = 0.15$: Local interactions between pollinators and plants are notable.
- $F = 0.05$: Minimal evidence contradicts the proposition.

Example 5.12 (Medical Diagnosis(Locality-to-Truth Transformation))**.** Let A: "The patient has a specific viral infection."

Initial Truth Value:

$$v(A) = (T, I, L, F) = (0.5, 0.3, 0.2, 0.0),$$

where:

- $T = 0.5$: Initial test results partially support the diagnosis.

- $I = 0.3$: Indeterminate due to conflicting symptoms.
- $L = 0.2$: Locality reflects observations by a specialist.
- $F = 0.0$: No evidence against the diagnosis.

Transformation: Given that L provides strong supporting evidence (e.g., specialist confirmation), the transformation U_{LT} is applied:

$$v(U_{LT}(A)) = (T + L, I, 0, F) = (0.7, 0.3, 0.0, 0.0).$$

Interpretation: The patient's diagnosis is now more likely to be true, as locality strongly supports the proposition.

Example 5.13 (Weather Prediction(Locality-to-Falsity Transformation))**.** Let B: "It will rain tomorrow."

Initial Truth Value:

$$v(B) = (T, I, L, F) = (0.4, 0.3, 0.3, 0.0),$$

where:

- $T = 0.4$: Weak prediction models suggest rain.
- $I = 0.3$: Indeterminacy due to uncertainty in weather models.
- $L = 0.3$: Local observations (e.g., clear skies).
- $F = 0.0$: No significant evidence against rain.

Transformation: Since L strongly opposes T (e.g., clear skies observed), the transformation U_{LF} is applied:

$$v(U_{LF}(B)) = (T, I, 0, F + L) = (0.4, 0.3, 0.0, 0.3).$$

Interpretation: The proposition becomes less likely, as local observations indicate no rain.

Example 5.14 (Ecological Impact(Locality-to-Indeterminacy Transformation))**.** Let C: "Reintroducing wolves to a forest will improve biodiversity."

Initial Truth Value:

$$v(C) = (T, I, L, F) = (0.6, 0.2, 0.2, 0.0),$$

where:

- $T = 0.6$: Prior studies support this outcome.
- $I = 0.2$: Some uncertainty due to unknown ecological factors.
- $L = 0.2$: Local reports suggest potential unintended consequences.
- $F = 0.0$: No evidence contradicting the proposition.

Transformation: Local observations introduce ambiguity, applying U_{LI}:

$$v(U_{LI}(C)) = (T, I + L, 0, F) = (0.6, 0.4, 0.0, 0.0).$$

Interpretation: The uncertainty in the proposition increases due to conflicting local data.

Example 5.15 (Supply Chain Disruption (Indeterminacy-to-Locality Transformation))**.** Let D: "A factory shutdown will disrupt global supply chains."

Initial Truth Value:

$$v(D) = (T, I, L, F) = (0.5, 0.4, 0.1, 0.0),$$

where:

- $T = 0.5$: Preliminary analysis suggests a significant impact.
- $I = 0.4$: Indeterminacy due to lack of specific data.
- $L = 0.1$: Localized reports provide clarity on immediate effects.
- $F = 0.0$: No evidence against the proposition.

Transformation: Local evidence reduces indeterminacy, applying U_{IL}:

$$v(U_{IL}(D)) = (T, 0, I + L, F) = (0.5, 0.0, 0.5, 0.0).$$

Interpretation: The proposition's reliance on locality increases as specific data becomes available.

6 Future Tasks of this research

In this section, we consider future tasks of this research.

6.1 Some Extension of Local-Neutrosophic Logic (Open Question)

There is interest in exploring the possibility of extending the above logic using the following set concepts. Further research in this direction is anticipated.

Question 6.1. Can the logic be extended using the following sets? Additionally, what are the mathematical characteristics of these extensions, their relationships with other uncertain concepts, and their potential applications?

- Double-Valued Neutrosophic Sets [35, 38]
- Interval-Valued Neutrosophic Sets [86, 94, 95]
- Plithogenic Sets [25, 66, 67, 79]
- Soft Sets [44, 46]
- Hypersoft Sets [1, 19, 24, 53, 65]
- Neutrosophic Offset [57, 63, 64, 69, 70, 72]

6.2 Neutrosophic Dynamic Systems

A *Neutrosophic Dynamic System* (NDS) is a generalized framework for modeling systems characterized by uncertainty, incompleteness, or contradictions. The definition is provided below [71]. There is particular interest in exploring how the current Local-Neutrosophic Logic can be extended to Neutrosophic Dynamic Systems.

Definition 6.2 (Neutrosophic Dynamic Systems)**.** [71] Let $\mathcal{U}$ be the universe of discourse. A *Neutrosophic Dynamic System* is defined as:

$$\mathcal{D}_N = (\Omega, \mathcal{E}, \mathcal{R}),$$

where:

- $\Omega \subseteq \mathcal{U}$: The neutrosophic space (or state space), representing the elements of the system. It is defined as:

$$\Omega = \{x_i(T_i, I_i, F_i) \mid x_i \in \Omega,\ T_i, I_i, F_i \in [0,1],\ i \in \{1, 2, \ldots, n\}\},$$

  where:

  - T_i: The degree of membership (truth) of x_i in Ω.
  - I_i: The degree of indeterminacy (uncertainty) of x_i in Ω.
  - F_i: The degree of non-membership (falsity) of x_i in Ω.

- $\mathcal{E}$: The set of elements within Ω. Each element x_i is associated with time-varying neutrosophic degrees (T_i, I_i, F_i), which evolve over time.

- $\mathcal{R}$: The set of neutrosophic hyperrelationships representing interactions within the system. A neutrosophic hyperrelationship is defined as:

$$\mathcal{R}_{\text{HR}} : \Omega^k \times C(\Omega)^l \to \mathcal{P}([0,1]),$$

  where:

  - $\mathcal{R}_{\text{HR}}(x_{i_1}, x_{i_2}, \ldots, x_{i_k}, y_{j_1}, y_{j_2}, \ldots, y_{j_l}) = (T_{\mathcal{R}}, I_{\mathcal{R}}, F_{\mathcal{R}})$,
  - $T_{\mathcal{R}}, I_{\mathcal{R}}, F_{\mathcal{R}} \in [0,1]$: Degrees of truth, indeterminacy, and falsity for the hyperrelationship.
  - k: Number of interacting elements within Ω.
  - l: Number of interacting elements between Ω and $C(\Omega)$, the complement of Ω in $\mathcal{U}$.

Definition 6.3 (Open and Closed Neutrosophic Systems)**.** [71]

- A system is *closed* if $l = 0$, meaning all relationships are confined to Ω.
- A system is *open* if $l \geq 1$, allowing interactions between Ω and $C(\Omega)$.

Definition 6.4 (Time-Dependent Neutrosophic dynamic system)**.** [71] The neutrosophic dynamic system evolves over time, with changes occurring in its space, elements, and relationships:

$$\mathcal{D}_N(t) = (\Omega(t), \mathcal{E}(t), \mathcal{R}(t)).$$

- **Element Dynamics**: The degrees of membership, indeterminacy, and non-membership for each element vary over time:

$$T_i(t), \quad I_i(t), \quad F_i(t),$$

  subject to the constraint:

$$T_i(t) + I_i(t) + F_i(t) \leq 1, \quad \forall i,\ \forall t.$$

- **Relationship Dynamics**: The hyperrelationships evolve over time, represented as:

$$\mathcal{R}_{\text{HR}}(t) = (T_{\mathcal{R}}(t), I_{\mathcal{R}}(t), F_{\mathcal{R}}(t)).$$

- **Space Dynamics**: The neutrosophic space Ω may change due to:
  - Addition of new elements to Ω.
  - Removal of existing elements from Ω.
  - Changes in the neutrosophic degrees (T_i, I_i, F_i) of elements within Ω.

Example 6.5 (Ecosystem Dynamics)**.** A biological ecosystem is a community of living organisms interacting with each other and their environment, including air, water, and soil (cf. [54, 82]).

Consider a biological ecosystem modeled as a neutrosophic dynamic system:

- $\Omega = \{x_1, x_2, \ldots, x_n\}$: The set of species in the ecosystem, where each species x_i is characterized by its neutrosophic attributes (T_i, I_i, F_i).

- $T_i(t)$: Degree to which species x_i is adapted to the environment at time t. For example, $T_i(t)$ may increase if x_i develops traits that improve survival under current environmental conditions.
- $I_i(t)$: Degree of uncertainty in species x_i's role or impact in the ecosystem. This reflects incomplete knowledge about how x_i interacts with other species or adapts to environmental changes.
- $F_i(t)$: Degree to which species x_i is maladapted or detrimental to the ecosystem. For instance, $F_i(t)$ may increase if x_i contributes to ecosystem imbalance.

The hyperrelationships $\mathcal{R}_{\text{HR}}(x_{i_1}, x_{i_2}, \ldots, x_{i_k}) = (T_{\mathcal{R}}, I_{\mathcal{R}}, F_{\mathcal{R}})$ capture the dynamic interactions among species:

- $T_{\mathcal{R}}$: Degree to which the interaction benefits the ecosystem (e.g., symbiosis).
- $I_{\mathcal{R}}$: Degree of indeterminacy in the interaction (e.g., uncertain impact of competition).
- $F_{\mathcal{R}}$: Degree to which the interaction harms the ecosystem (e.g., predation imbalance).

As the ecosystem evolves, the following dynamics may occur:

- Species x_i may join or leave Ω due to migration, extinction, or introduction.
- The neutrosophic degrees $T_i(t), I_i(t), F_i(t)$ and hyperrelationship values $T_{\mathcal{R}}(t), I_{\mathcal{R}}(t), F_{\mathcal{R}}(t)$ change over time based on environmental conditions, resource availability, and species interactions.
- External influences, such as human intervention or climate change, may alter the system by introducing new relationships $\mathcal{R}_{\text{HR}}$ or modifying Ω.

This framework provides a dynamic, nuanced representation of ecosystem behavior, accommodating uncertainty and variability in species interactions.

Example 6.6 (Social Networks)**.** Social networks are structures of individuals or groups connected through relationships like communication, collaboration, or shared interests, often facilitated by technology(cf. [32, 39, 42]).

Consider a social network modeled as a neutrosophic dynamic system:

- $\Omega = \{x_1, x_2, \ldots, x_n\}$: The set of individuals in the network, where each individual x_i is characterized by neutrosophic attributes (T_i, I_i, F_i).
- $T_i(t)$: Degree to which individual x_i positively contributes to the network at time t. For instance, $T_i(t)$ may increase if x_i actively collaborates or shares valuable information.
- $I_i(t)$: Degree of neutrality or indifference of x_i in the network. This could reflect an individual's limited or ambiguous involvement in network activities.
- $F_i(t)$: Degree to which x_i detracts from the network, such as spreading misinformation or creating conflicts.

The hyperrelationships $\mathcal{R}_{\text{HR}}(x_{i_1}, x_{i_2}, \ldots, x_{i_k}) = (T_{\mathcal{R}}, I_{\mathcal{R}}, F_{\mathcal{R}})$ represent interactions within the network:

- $T_{\mathcal{R}}$: Degree to which the interaction enhances network cohesion (e.g., collaborative projects).
- $I_{\mathcal{R}}$: Degree of indeterminacy in the interaction (e.g., ambiguous communication).
- $F_{\mathcal{R}}$: Degree to which the interaction harms the network (e.g., disputes or competitive behavior).

The dynamic behavior of the network includes:

- Addition or removal of individuals (x_i) to/from Ω, reflecting network growth or attrition.
- Changes in the neutrosophic degrees $T_i(t), I_i(t), F_i(t)$ of individuals based on their evolving roles and contributions.
- Evolution of hyperrelationships $\mathcal{R}_{\text{HR}}(t)$ as collaboration patterns, social dynamics, or external influences (e.g., new policies or technological changes) reshape the network.

This model captures the complexity and variability of social interactions, accommodating the uncertainty and contradictions inherent in human networks.

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Data Availability

This paper does not involve any data analysis.

Ethical Approval

This article does not involve any research with human participants or animals.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Disclaimer

This study primarily focuses on theoretical aspects, and its application to practical scenarios has not yet been validated. Future research may involve empirical testing and refinement of the proposed methods. The authors have made every effort to ensure that all references cited in this paper are accurate and appropriately attributed. However, unintentional errors or omissions may occur. The authors bear no legal responsibility for inaccuracies in external sources, and readers are encouraged to verify the information provided in the references independently. Furthermore, the interpretations and opinions expressed in this paper are solely those of the authors and do not necessarily reflect the views of any affiliated institutions.

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