N-Valued Interval Neutrosophic Sets and Their Application in Medical Diagnosis

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Abstract

In this paper a new concept is called n-valued interval neutrosophic sets is given. The basic operations are introduced on n-valued interval neutrosophic sets such as; union, intersection, addition, multiplication, scalar multiplication, scalar division, truth-favorite and false-favorite. Then, some distances between n-valued interval neutrosophic sets (NVINS) are proposed. Also, we propose an efficient approach for group multi-criteria decision making based on n-valued interval neutrosophic sets. An application of n-valued interval neutrosophic sets in medical diagnosis problem is given.

Keywords

Neutrosophic sets, n-valued neutrosophic set, interval neutrosophic sets, n-valued interval neutrosophic sets.

1 Introduction

In 1999, Smarandache [37] proposed the concept of neutrosophic set (NS for short) by adding an independent indeterminacy-membership function which is a generalization of classic set, fuzzy set [45], intuitionistic fuzzy set [3] and so on. In NS, the indeterminacy is quantified explicitly and truth-membership (T), indeterminacy (I) membership, and false-membership (F) are completely independent and from scientific or engineering point of view, the NS operators need to be specified. Therefore, Wang et al [39] defined a single valued neutrosophic set (SVNS) and then provided the set theoretic operations and various properties of single valued neutrosophic sets and Wang et al. [40] proposed the set theoretic operations on an instance of neutrosophic set is called interval valued neutrosophic set (IVNS) which is more flexible and practical than NS. The works on single valued neutrosophic set (SVNS) and interval valued neutrosophic sets (IVNS) and their hybrid structure in theories and application have been progressing rapidly (e.g., [1,2,4-19,21,22,24-26,28-30,36,41,43]). Also, neutrosophic sets extended neutrosophic models in [13,16] both theory and application by using [27,31].

The concept of intuitionistic fuzzy multiset and some propositions with applications is originaly presented by Rajarajeswari and Uma [32-35]. After Rajarajeswari and Uma, Smarandache [38] presented n-Valued neutrosophic sets with applications. Recently, Chatterjee et al. [20], Deli et al. [18, 23], Ye et al. [42] and Ye and Ye [44] initiated definition of neutrosophic multisets with some operations. Also, the authors gave some distance and similarity measures on neutrosophic multisets. In this paper, our objective is to generalize the concept of n-valued neutrosophic sets (or neutrosophic multi sets; or neutrosophic refined sets) to the case of n-valued interval neutrosophic sets.

The paper is structured as follows; in Section 2, we first recall the necessary background on neutrosophic sets, single valued neutrosophic sets, interval valued neutrosophic sets and n-valued neutrosophic sets (or neutrosophic multi sets). Section 3 presents the concept of n-valued interval neutrosophic sets and derive their respective properties with examples. Section 4 presents the distance between two n-valued interval neutrosophic sets. Section 5 presents an application of this concept in solving a decision making problem. Section 6 concludes the paper.

2 Preliminaries

This section gives a brief overview of concepts of neutrosophic set theory [37], n-valued neutrosophic set theory [42,44] and interval valued neutrosophic set theory [40]. More detailed explanations related to this subsection may be found in [18,20,23,37,40,42,44].

Definition 2.1. [37,39] Let X be an universe of discourse, with a generic element in X denoted by x, then a neutrosophic (NS) set A is an object having the form

$$A = \{< x: T_A(x), I_A(x), F_A(x)>, x \in X\}$$

where the functions T, I, F : X→ $]^-0, 1^+[$ define respectively the degree of membership (or Truth) , the degree of indeterminacy, and the degree of non-membership (or Falsehood) of the element x ∈ X to the set A with the condition.

$$^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^-0, 1^+[$. So instead of $]-0, 1^+[$ we need to take the interval [0, 1] for technical applications, because $]^-0, 1^+[$will be difficult to apply in the real applications such as in scientific and engineering problems.

For two NS, A_{NS} ={ <x , $T_A(x)$, $I_A(x)$, $F_A(x)$> | x ∈ X }

andB_{NS} ={ <x , $T_B(x)$, $I_B(x)$, $F_B(x)$> | x ∈ X } the two relations are defined as follows:

(1) $A_{NS} \subseteq B_{NS}$ if and only if $T_A(x) \leq T_B(x)$, $I_A(x) \geq I_B(x)$, $F_A(x) \geq F_B(x)$

(2) $A_{NS} = B_{NS}$ if and only if , $T_A(x) = T_B(x)$, $I_A(x) = I_B(x)$, $F_A(x) = F_B(x)$

Definition 2.2. [40] Let X be a space of points (objects) with generic elements in X denoted by x. An interval valued neutrosophic set (for short IVNS) A in X is characterized by truth-membership function $T_A(x)$, indeteminacy-membership function $I_A(x)$ and falsity-membership function $F_A(x)$. For each point x in X, we have that $T_A(x), I_A(x), F_A(x) \subseteq [0, 1]$.

For two IVNS

$$A_{\text{IVNS}}=\{<x, [\inf T_A^1(x), \sup T_A^1(x)], [\inf I_A^1(x), \sup I_A^1(x)], [\inf F_A^1(x), \sup F_A^1(x)] >: x \in X\}$$

and

$$B_{\text{IVNS}}= \{<x, \inf T_B^1(x), \sup T_B^1(x)], [\inf I_B^1(x), \sup I_B^1(x)], [\inf F_B^1(x), \sup F_B^1(x)]>: x \in X\}$$

Then,

1. $A_{\text{IVNS}} \subseteq B_{\text{IVNS}}$ if and only if

$\inf T_A^1(x) \leq \inf T_B^1(x)$, $\sup T_A^1(x) \leq \sup T_B^1(x)$,
$\inf I_A^1(x) \geq \inf I_B^1(x)$, $\sup I_A^1(x) \geq \sup I_B^1(x)$,
$\inf F_A^1(x) \geq \inf F_B^1(x)$, $\sup F_A^1(x) \geq \sup F_B^1(x)$,

for all x ∈ X.

2. $A_{\text{IVNS}} = B_{\text{IVNS}}$ if and only if ,
$\inf T_A^1(x) = \inf T_B^1(x)$, $\sup T_A^1(x) = \sup T_B^1(x)$,
$\inf I_A^1(x) = \inf I_B^1(x)$, $\sup I_A^1(x) = \sup I_B^1(x)$,
$\inf F_A^1(x) = \inf F_B^1(x)$, $\sup F_A^1(x) = \sup F_B^1(x)$,

for any x ∈ X.

3. $A_{\text{IVNS}}{}^c = \{x, [\inf F_A^1(x), \sup F_A^1(x)], [1 - \ sup\ I_A^1(x), 1 - inf\ I_A^1(x)],$
$[inf\ T_A^1(x), sup\ T_A^1(x)]: x \in X\}$

4. $A_{\text{IVNS}} \cap B_{\text{IVNS}}$={<x, $[\inf T_A^1(x) \wedge \inf T_B^1(x), \sup T_A^1(x) \wedge \sup T_B^1(x)]$,
$[\inf I_A^1(x) \vee \inf I_B^1(x), \sup I_A^1(x) \vee \sup I_B^1(x)]$,
$[\inf F_A^1(x) \vee \inf F_B^1(x), \sup F_A^1(x) \vee \sup F_B^1(x)]$>: x ∈ X}

5. $A_{\text{IVNS}} \cup B_{\text{IVNS}}$={<x, $[\inf T_A^1(x) \vee \inf T_B^1(x), \sup T_A^1(x) \vee \sup T_B^1(x)]$,
$[\inf I_A^1(x) \wedge \inf I_B^1(x), \sup I_A^1(x) \wedge \sup I_B^1(x)]$,
$[\inf F_A^1(x) \wedge \inf F_B^1(x), \sup F_A^1(x) \wedge \sup F_B^1(x)]$>: x ∈ X}

6. $A_{\text{IVNS}} \backslash$
B_{IVNS}={<x,[$\min\{\inf T_A^1(x), \inf F_B^1(x)\}, \min\{\sup T_A^1(x), \sup F_B^1(x)\}$],
[max(inf $I_A^1(x)$,1-sup $I_B^1(x)$), max(sup $I_A^1(x)$,1-inf $I_B^1(x)$)],
[max(inf$F_A^1(x)$,inf $T_B^1(x)$), max(sup $F_A^1(x)$, sup $T_B^1(x)$)]>: x ∈ X}

7. $A_{\text{IVNS}}+ B_{\text{IVNS}}$ ={<x,$[\min(\inf T_A^1(x) + \inf T_B^1(x), 1), \min(\sup T_A^1(x) + \sup T_B^1(x), 1)]$,
$[\min(\inf I_A^1(x) + \inf I_B^1(x), 1), \min(\sup I_A^1(x) + \sup I_B^1(x), 1)]$,
$[\min(\inf F_A^1(x) + \inf F_B^1(x), 1), \min(\sup F_A^1(x) + \sup F_B^1(x), 1)] >$
: x ∈ X},

8. A_{IVNS}.a={<x,$[\min(\inf T_A^1(x).\text{a}, 1), \min(\sup T_A^1(x).\text{a}, 1)]$,
$[\min(\inf I_A^1(x).\text{a}, 1), \min(\sup I_A^1(x).\text{a}, 1)]$,
$[\min(\inf F_A^1(x).\text{a}, 1), \min(\sup F_A^1(x).\text{a}, 1)]$,
$[\min(\inf I_A^1(x).\text{a}, 1), \min(\sup I_A^1(x).\text{a}, 1)]$ >: x ∈ X},

9. A_{IVNS}/a={<x,$[\min(\inf T_A^1(x)/\text{a}, 1), \min(\sup T_A^1(x)/\text{a}, 1)]$,
$[\min(\inf I_A^1(x)/\text{a}, 1), \min(\sup I_A^1(x)/\text{a}, 1)]$,
$[\min(\inf F_A^1(x)/\text{a}, 1), \min(\sup F_A^1(x).\text{a}, 1)]$ > : x ∈ X},

10. Δ A_{IVNS} ={<x,$[\min(\inf T_A^1(x) + \inf I_A^1(x), 1), \min(\sup T_A^1(x) + \sup I_B^1(x), 1)]$, [0,0],
$[\inf F_A^1(x), \sup F_A^1(x)]$ >: x ∈ X},

11. ∇ A_{IVNS} ={<x,$[\inf T_A^1(x), \sup T_A^1(x)]$, [0,0],

$[\min(\inf F_A^1(x) + \inf I_A^1(x), 1), \min(\sup F_A^1(x) + \sup I_B^1(x), 1)] >: x \in X\}$

Definition 2.3. [20,42] Let E be a universe. A n-valued neutrosophic sets on E can be defined as follows:

$$A = \{<x, (T_A^1(x), T_A^2(x), ..., T_A^p(x)), (I_A^1(x), I_A^2(x), ..., I_A^p(x)), (F_A^1(x), F_A^2(x), ..., F_A^p(x))>: x \in X\}$$

where

$T_A^1(x), T_A^2(x), ..., T_A^p(x), I_A^1(x), I_A^2(x), ..., I_A^p(x), F_A^1(x), F_A^2(x), ..., F_A^p(x): E \to [0, 1]$ such that

$$0 \leq T_A^i(x) + I_A^i(x) + F_A^i(x) \leq 3 \text{ for } i=1, 2, ..., p \text{ for any } x \in X,$$

Here, $(T_A^1(x), T_A^2(x), ..., T_A^p(x))$, $(I_A^1(x), I_A^2(x), ..., I_A^p(x))$ and $(F_A^1(x), F_A^2(x), ..., F_A^p(x))$

is the truth-membership sequence, indeterminacy-membership sequence and falsity-membership sequence of the element x, respectively. Also, P is called the dimension of n-valued neutrosophic sets (NVNS) A.

3 N-Valued Interval Neutrosophic Sets

Following the n-valued neutrosophic sets (multiset or refined set) and interval neutrosophic sets defined in [20,38,42,44] and Wang et al. in [40], respectively. In this section, we extend these sets to n-valued interval valued neutrosophic sets.

Definition 3.1. Let X be a universe, a n-valued interval neutrosophic sets (NVINS) on X can be defined as follows:

$$A = \{x, \begin{pmatrix} [\inf T_A^1(x), \sup T_A^1(x)], [\inf T_A^2(x), \sup T_A^2(x)], ..., \\ [\inf T_A^p(x), \sup T_A^p(x)] \end{pmatrix},$$

$$\begin{pmatrix} [\inf I_A^1(x), \sup I_A^1(x)], [\inf I_A^2(x), \sup I_A^2(x)], ..., \\ [\inf I_A^p(x), \sup I_A^q(x)] \end{pmatrix},$$

$$[\inf F_A^1(x), \sup F_A^1(x)], ([inf\ F_A^2(x), sup\ F_A^2(x)], ..., ([inf\ F_A^p(x), sup\ F_A^r(x)]): x \in X\}$$

where

$\inf T_A^1(x), \inf T_A^2(x), ..., \inf T_A^p(x), \inf I_A^1(x), \inf I_A^2(x), ..., \inf I_A^p(x), \inf F_A^1(x), \inf F_A^2(x), ..., \inf F_A^q(x), \sup T_A^1(x), \sup T_A^2(x), ..., \sup T_A^p(x),$

$\sup\ I_A^1(x)$, $\sup\ I_A^2(x)$,…,$\sup\ I_A^q(x)$,
$\sup F_A^1(x)$, $\sup F_A^2(x)$,…,$\sup\ F_A^r(x)\in[0, 1]$

such that $0\leq \sup T_A^i(x) + \sup I_A^i(x) + \sup F_A^i(x) \leq 3$, $\forall$ i=1, 2,…, p.

In our study, we focus only on the case where p=q=r is the interval truth-membership sequence, interval indeterminacy-membership sequence and interval falsity-membership sequence of the element x, respectively. Also, p is called the dimension of n-valued interval (NVINS) A. Obviously, when the upper and lower ends of the interval values of $T_A^i(x)$, $I_A^i(x)$, $F_A^i(x)$ in a NVINS are equal, the NVINS reduces to n-valued neutrosophic set (or neutrosophic multiset proposed in [17,20]).

The set of all n-valued interval neutrosophic set on X is denoted by NVINS(X).

Example 3.2. Let X={x_1, x_2} be the universe and A is an n- valued interval neutrosophic sets

A= { <x_1,{[.1, .2], [.2, .3]},{[.3, .4],[.1, .5]},{[.3, .4], [.2, .5]}>,
<x_2,{[.3, .4], [.2,.4]},{[.3, .5],[.2, .4]},{[.1, .2], [.3, .4]}>}

Definition 3.3. The complement of A is denoted by A^c and is defined by

$$A^c=\{x, \big([\inf F_A^1(x), \sup F_A^1(x)] , ([\inf F_A^2(x), \sup F_A^2(x)], \dots, \\ ([inf\ F_A^p(x), sup\ F_A^p(x)]), \\ \begin{pmatrix} [1-\ sup\ I_A^1(x) , 1-inf\ I_A^1(x)], [1-sup\ I_A^2(x) , 1-inf\ I_A^2(x)], \dots, \\ [1-\ sup\ I_A^p(x)\ ,\ 1-inf\ I_A^p(x)] \end{pmatrix}, \\ ([inf\ T_A^1(x) , sup\ T_A^1(x)], [inf\ T_A^2(x) , sup\ T_A^2(x)], \dots, \\ [inf\ T_A^p(x) , sup\ T_A^p(x)]\ ): x\in X\}.$$

Example 3.4. Let us consider the Example 3.5. Then we have,

A^c= { <x_1,{[.3,.4],[.2,.5]},{[.6,.7],[.5,.9]},{[.1,.2],[.2,.3]}>,

<x_2,{[.1,.2],[.3,.4]},{[.5,.7],[.6,.8]},{[.3,.4],[.2,.4]}>}

Definition 3.5. For $\forall$ i=1, 2,…,p if $\inf T_A^i(x) = \sup T_A^i(x) = 0$ and $\inf I_A^i(x) = \sup I_A^i(x) = \inf F_A^i(x) = \sup F_A^i(x) = 1$, then A is called null n-valued interval neutrosophic set denoted by Φ, for all x ∈ X.

Example 3.6. Let X={x_1, x_2} be the universe and A is an n-valued interval neutrosophic sets

Φ= { <x_1,{[0, 0], [0, 0]},{[1, 1], [1, 1] },{[1, 1], [1, 1]}>,
<x_2,{[0, 0], [0, 0]},{[1, 1], [1, 1]},{[1, 1], [1, 1]}>}.

Definition 3.7. For ∀ i=1,2,…,p if $\inf T_A^i(x)$ = $\sup T_A^i(x)$ =1 and $\inf I_A^i(x)$=$\sup I_A^i(x)$=$\inf F_A^i(x)$ =$\sup F_A^i(x)$=0, then A is called universal n-valued interval neutrosophic set denoted by E, for all x ∈ X.

Example 3.8. Let X={x_1, x_2} be the universe and A is an n-valued interval neutrosophic sets

E= { <x_1,{ [1, 1], [1, 1] },{[0, 0], [0, 0] },{[0, 0], [0, 0]}>,
<x_2{ [1, 1], [1, 1] },{[0, 0], [0, 0] },{[0, 0], [0, 0]}>}.

Definition 3.9. A n-valued interval neutrosophic set A is contained in the other n-valued interval neutrosophic set B, denoted by A⊆B, if and only if

$\inf T_A^1(x) \leq \inf T_B^1(x)$, $\inf T_A^2(x) \leq \inf T_B^2(x)$,..., $\inf T_A^p(x) \leq \inf T_B^p(x)$,
$\sup T_A^1(x) \leq \sup T_B^1(x)$, $\sup T_A^2(x) \leq \sup T_B^2(x)$,..., $\sup T_A^p(x) \leq \sup T_B^p(x)$,
$\inf I_A^1(x) \geq \inf I_B^1(x)$, $\inf I_A^2(x) \geq \inf I_B^2(x)$,..., $\inf I_A^p(x) \geq \inf I_B^p(x)$,
$\sup I_A^1(x) \geq \sup I_B^1(x)$, $\sup I_A^2(x) \geq \sup I_B^2(x)$,..., $\sup I_A^p(x) \geq \sup I_B^p(x)$,
$\inf F_A^1(x) \geq \inf F_B^1(x)$, $\inf F_A^2(x) \geq \inf F_B^2(x)$,..., $\inf F_A^p(x) \geq \inf F_B^p(x)$,
$\sup F_A^1(x) \geq \sup F_B^1(x)$, $\sup F_A^2(x) \geq \sup F_B^2(x)$,..., $\sup F_A^p(x) \geq \sup F_B^p(x)$

for all x ∈ X.

Example 3.10. Let X={x_1, x_2} be the universe and A and B are two n-valued interval neutrosophic sets

A= { <x_1,{[.1, .2], [.2, .3]},{[.4, .5], [.6, .7]},{[.5, .6],[.7, .8]}>,
<x_2,{[.1, .4],[.1, .3]},{[.6, .8], [.4, .6]},{[.5, .6], [.6, .7]}>}

and

B= { <x_1,{[.5, .7], [.4, .5]},{[.3,.4],[.1,.5]},{[.3,.4],[.2,.5]}>,
<x_2,{[.2, .5],[.3, .6]},{[.3, .5],[.2,.4]},{[.1, .2], [.3, .4]}>}

Then, we have A ⊆ B.

Definition 3.11. Let A and B be two n-valued interval neutrosophic sets. Then, A and B are equal, denoted by A= B if and only if A ⊆ B and B⊆ A.

Proposition 3.12. Let A, B, C ∈NVINS(X).Then,

1. ∅⊆ A
2. A ⊆ A

3. A ⊆ E
4. A ⊆ B and B ⊆ C → A ⊆ C
5. K=L and L=M ↔ $K = M$
6. K⊆L and L⊆K ↔ $K = L$.

Definition 3.13. Let A and B be two n-valued interval neutrosophic sets. Then, intersection of A and B, denoted by A∩B, is defined by

$$A \cap B=\{<x,\big([\inf T_A^1(x) \wedge \inf T_B^1(x), \sup T_A^1(x) \wedge \sup T_B^1(x)], [\inf T_A^2(x) \wedge \inf T_B^2(x), \sup T_A^2(x) \wedge \sup T_B^2(x)], \dots, [\inf T_A^p(x) \wedge \inf T_B^p(x), \sup T_A^P(x) \wedge \sup T_B^P(x)]\big),\big([\inf I_A^1(x) \vee \inf I_B^1(x), \sup I_A^1(x) \vee \sup I_B^1(x)], [\inf I_A^2(x) \vee \inf I_B^2(x), \sup I_A^2(x) \vee \sup I_B^2(x)], \dots, [\inf I_A^p(x) \vee \inf I_B^p(x), \sup I_A^P(x) \vee \sup I_B^P(x)]\big), \big([\inf F_A^1(x) \vee \inf F_B^1(x), \sup F_A^1(x) \vee \sup F_B^1(x)], [\inf F_A^2(x) \vee \inf F_B^2(x), \sup F_A^2(x) \vee \sup F_B^2(x)], \dots, [\inf F_A^p(x) \vee \inf F_B^p(x), \sup F_A^P(x) \vee \sup F_B^P(x)]\big)>: x \in X\}$$

Example 3.14. Let U={x_1, x_2} be the universe and A and B are two n-valued interval neutrosophic sets

A= { <x_1,{[.1, .2], [.2, .3]},{[.4, .5], [.6, .7]},{[.5, .6], [.7, .8]}>,
<x_2,{[.1, .4], [.1,.3]},{[.6, .8], [.4, .6]},{[.3, .4], [.2, .7]}>}

and

B= { <x_1,{[.3, .7], [.3, .5]},{[.2, .4], [.3, .5]},{[.3, .6], [.2, .7]}>,
<x_2,{[.3, .5], [.4,.6]},{[.3, .5], [.4, .5]},{[.3, .4], [.1, .2]}>}

Then,

A∩B={<x_1,{[.1,.2],[.2,.3]},{[.4,.5],[.6,.7]},{[.5,.6],[.7,.8]}>,
<x_2,{[.1,.4],[.1,.3]},{[.6,.8],[.4,.6]},{[.3,.4],[.2,7]}>}

Proposition 3.15. Let A, B, C∈NVINS(X). Then,

1. A∩A=A
2. A∩ $\emptyset = \emptyset$.
3. A∩E=A
4. A∩B= B∩A
5. $(A \cap B) \cap C = A \cap (B \cap C)$.

Proof: The proof is straightforward.

Definition 3.16. Let A and B be two n-valued interval neutrosophic sets. Then, union of A and B, denoted by A∪B, is defined by

A∪B = {x,<$\big([\inf T_A^1(x) \vee \inf T_B^1(x), \sup T_A^1(x) \vee \sup T_B^1(x)], [\inf T_A^2(x) \vee \inf T_B^2(x), \sup T_A^2(x) \vee \sup T_B^2(x)], \ldots, [\inf T_A^p(x) \vee \inf T_B^p(x), \sup T_A^P(x) \vee \sup T_B^P(x)]\big)$,$\big([\inf I_A^1(x) \wedge \inf I_B^1(x), \sup I_A^1(x) \wedge \sup I_B^1(x)], [\inf I_A^2(x) \wedge \inf I_B^2(x), \sup I_A^2(x) \wedge \sup I_B^2(x)], \ldots, [\inf I_A^p(x) \wedge \inf I_B^p(x), \sup I_A^P(x) \wedge \sup I_B^P(x)]\big)$, $\big([\inf F_A^1(x) \wedge \inf F_B^1(x), \sup F_A^1(x) \wedge \sup F_B^1(x)], [\inf F_A^2(x) \wedge \inf F_B^2(x), \sup F_A^2(x) \wedge \sup F_B^2(x)], \ldots, [\inf F_A^p(x) \wedge \inf F_B^p(x), \sup F_A^P(x) \wedge \sup F_B^P(x)]\big)$>: x ∈ X}

Proposition 3.17. Let A, B, C ∈ NVINS(X).Then,

1. A∪A=A.
2. A∪ ∅ = A.
3. A∪E=E.
4. A∪B= B∪A.
5. $(A \cup B) \cup C = A \cup (B \cup C)$.

Proof: The proof is straightforward.

Definition 3.18. Let A and B be two n-valued interval neutrosophic sets. Then, difference of A and B, denoted by A \B, is defined by

A\B={x,([min{$\inf T_A^1(x), \inf F_B^1(x)$} , min{$\sup T_A^1(x), \sup F_B^1(x)$}], [min{$\inf T_A^2(x), \inf F_B^2(x)$} , min{$\sup T_A^2(x), \sup F_B^2(x)$}],..., [min{$\inf T_A^p(x), \inf F_B^p(x)$} , min{$\sup T_A^p(x), \sup F_B^p(x)$}]),([max(inf $I_A^1(x)$,1-sup $I_B^1(x)$),max(sup$I_A^1(x)$,1-inf$I_B^1(x)$)],[max(inf$I_A^2(x)$,1-sup$I_B^2(x)$),max(sup$I_A^2(x)$,1-inf $I_B^2(x)$)],...,[max(inf$I_A^p(x)$,1-sup$I_B^p(x)$),max(sup$I_A^p(x)$,1-inf$I_B^p(x)$)]),([max(inf $F_A^1(x)$,inf$T_B^1(x)$), max(sup $F_A^1(x)$, sup$T_B^1(x)$)], [max(inf $F_A^2(x)$,inf$T_B^2(x)$), max(sup$F_A^2(x)$,Sup$T_B^2(x)$)],..., [max(inf $F_A^p(x)$,inf$T_B^p(x)$), max(sup $F_A^p(x)$, sup$T_B^p(x)$)]): x ∈ X}

Example 3.19. Let X= {x_1, x_2} be the universe and A and B are two n-valued interval neutrosophic sets

A= { <x_1,{[.1, .2], [.2, .3]},{[.4, .5], [.6, .7]},{[.5, .6], [.7, .8]}>,
<x_2,{[.1, .4], [.1, .3]},{[.6,.8], [.4, .6]},{[.3, .4], [.2, .7]}>}

and

B= { <x_1,{[.3, .7], [.3, .5]},{[.2, .4], [.3, .5]},{[.3, .6], [.2, .7]}>,
<x_2,{[.3, .5], [.4, .6]},{[.3,.5], [.4, .5]},{[.3, .4], [.1, .2]}>}

Then,

A \ B={ < x_1 ,{[.1, .2], [.2, .3]},{[.6 ,.8], [.6, .7]},{[.5, .7], [.7, .8]}>,
< x_2 ,{[.1, .4], [.1, .2]},{[.6,.8], [.5, .6]},{[.3, .5], [.4, 7]}>}

Definition 3.20. Let A and B be two n-valued interval neutrosophic sets. Then, addition of A and B, denoted by A $\tilde{\mp}$B, is defined by

$$A\tilde{\mp} = \{<x,([\min(\inf T_A^1(x) + \inf T_B^1(x),1), \min(\sup T_A^1(x) + \sup T_B^1(x),1)], [\min(\inf T_A^2(x) + \inf T_B^2(x),1), \min(\sup T_A^2(x) + \sup T_B^2(x),1)], ..., [\min(\inf T_A^p(x) + \inf F_B^p(x),1), \min(\sup T_A^p(x) + \sup T_B^p(x),1)]) ([\min(\inf I_A^1(x) + \inf I_B^1(x),1), \min(\sup I_A^1(x) + \sup I_B^1(x),1)], [\min(\inf I_A^2(x) + \inf I_B^2(x),1), \min(\sup I_A^2(x) + \sup I_B^2(x),1)], ..., [\min(\inf T_A^p(x) + \inf T_B^p(x),1), \min(\sup T_A^p(x) + \sup T_B^p(x),1)]), ([\min(\inf F_A^1(x) + \inf F_B^1(x),1), \min(\sup F_A^1(x) + \sup F_B^1(x),1)], [\min(\inf F_A^2(x) + \inf F_B^2(x),1), \min(\sup F_A^2(x) + \sup F_B^2(x),1)], ..., [\min(\inf F_A^p(x) + \inf F_B^p(x),1), \min(\sup F_A^p(x) + \sup F_B^p(x),1)] >: x \in X\}.$$

Example 3.21. Let X={x_1, x_2} be the universe and A and B are two n-valued interval neutrosophic sets

A= { < x_1 ,{[.1, .2], [.2, .3]},{[.4, .5], [.6, .7]},{[.5, .6], [.7, .8]}>,
< x_2 ,{[.1, .4], [.1, .3]},{[.6, .8], [.4, .6]},{[.3, .4], [.2, .7]}>}

and

B= { < x_1 ,{[.3, .7], [.3, .5]},{[.2, .4], [.3, .5]},{[.3, .6], [.2, .7]}>,
< x_2 ,{[.3, .5], [.4, .6]},{[.3, .5], [.4, .5]},{[.3, .4], [.1, .2]}>}

then,

A$\tilde{\mp}$B={< x_1 ,{[.4,.9],[.5,.8]},{[.6,.9],[.9,1]},{[.8,1],[.9,1]}>,
< x_2 ,{[.4,.9],[.5,.9]},{[.9,1],[.8,1]},{[.6,.8],[.3,9]}>}.

Proposition 3.22. Let A, B, C ∈ NVINS(X).Then,

1. A$\tilde{\mp}$B=B$\tilde{\mp}$A.
2.(A$\tilde{\mp}$B)$\tilde{\mp}$C = A$\tilde{\mp}$(B$\tilde{\mp}$C).

Proof: The proof is straightforward.

Definition 3.23. Let A and B be two n-valued interval neutrosophic sets. Then, scalar multiplication of A, denoted by A $\tilde{.}$a, is defined by

$$A\tilde{.}a = \{x, ([\min(\inf T_A^1(x).a,1), \min(\sup T_A^1(x).a,1)], \begin{bmatrix}\min(\inf T_A^2(x).a,1), \\ \min(\sup T_A^2(x).a,1)\end{bmatrix}$$

$,\ldots,\big[\min(\inf T_A^p(x).\mathrm{a},1), \min(\sup T_A^p(x).\mathrm{a},1)\big]),$
$([\min(\inf I_A^1(x).\mathrm{a},1), \min(\sup I_A^1(x).\mathrm{a},1)],$
$[\min(\inf I_A^2(x).\mathrm{a},1), \min(\sup I_A^2(x).\mathrm{a},1)],\ldots,$
$\big[\min(\inf I_A^p(x).\mathrm{a},1), \min(\sup I_A^p(x).\mathrm{a},1)\big]),$
$([\min(\inf F_A^1(x).\mathrm{a},1), \min(\sup F_A^1(x).\mathrm{a},1)],$
$[\min(\inf I_A^1(x).\mathrm{a},1), \min(\sup I_A^1(x).\mathrm{a},1)],\ldots,$
$\big[\min(\inf I_A^p(x).\mathrm{a},1), \min(\sup I_A^p(x).\mathrm{a},1)\big]) \colon x \in X\}.$

Example 3.24. Let X={x_1, x_2} be the universe and A and B are two n-valued interval neutrosophic sets

A= { <x_1,{[.1, .2], [.2, .3]},{[.4, .5], [.6, .7]},{[.5, .6], [.7, .8]}>,
<x_2,{[.1, .4], [.1, .3]},{[.6,.8], [.4, .6]},{[.3, .4], [.2, .7]}>}

and

B= { <x_1,{[.3, .7], [.3, .5]},{[.2, .4], [.3, .5]},{[.3, .6], [.2, .7]}>,
<x_2,{[.3, .5], [.4, .6]},{[.3,.5], [.4, .5]},{[.3, .4], [.1, .2]}>},

then,

A$\tilde{.}$2 = { <x_1,{[.2, .4] ,[.4 ,.6]},{[.8, 1],[1, 1]},{[1, 1],[1, 1]}>,
<x_2,{[.2,.8],[.2, .6]},{[1, 1],[.8, 1]},{[.6, .8],[.4, 1]}>}

Proposition 3.25. Let A, B, C ∈ NVINS(X). Then,

1. A$\tilde{.}$B=B$\tilde{.}$A
2. (A$\tilde{\tilde{.}}$B)$\tilde{.}C$ = A$\tilde{.}$(B$\tilde{.}C$)

Proof: The proof is straightforward.

Definition 3.26. Let A and B be two n-valued interval neutrosophic sets. Then, scalar division of A, denoted by A $\tilde{/}$ a, is defined by

A$\tilde{/}$ a={x,$[\min(\inf T_A^1(x)/\mathrm{a},1), \min(\sup T_A^1(x)/\mathrm{a},1)],$
$[\min(\inf T_A^2(x)/\mathrm{a},1), \min(\sup T_A^2(x)/\mathrm{a},1)]$
$,\ldots,\big[\min(\inf T_A^p(x)/\mathrm{a},1), \min(\sup T_A^p(x)/\mathrm{a},1)\big]),([\min(\inf I_A^1(x)/$
$\mathrm{a},1), \min(\sup I_A^1(x)/\mathrm{a},1)], [\min(\inf I_A^2(x)/\mathrm{a},1), \min(\sup I_A^2(x)/$
$\mathrm{a},1)],\ldots,\big[\min(\inf I_A^p(x)/\mathrm{a},1), \min(\sup I_A^p(x)/\mathrm{a},1)\big]),$
$\big[[\min(\inf F_A^1(x)/\mathrm{a},1), \min(\sup F_A^1(x).\mathrm{a},1)]\big], [\min(\inf F_A^2(x)/$
$\mathrm{a},1), \min(\sup F_A^2(x)/\mathrm{a},1)], \ldots,\big[\min(\inf I_A^p(x)/$
$\mathrm{a},1), \min(\sup I_A^p(x)/\mathrm{a},1)\big]) \colon x \in X\}.$

Example 3.27. Let X={x_1, x_2} be the universe and A and B are two n-valued interval neutrosophic sets

A= { <x$_1$,{[.1, .2], [.2, .3]},{[.4, .5], [.6, .7]},{[.5, .6], [.7, .8]}>,
<x$_2$,{[.1, .4], [.1, .3]},{[.6, .8], [.4, .6]},{[.3, .4], [.2, .7]}>}

and

B= { <x$_1$,{[.3, .7], [.3, .5]},{[.2, .4], [.3, .5]},{[.3, .6], [.2, .7]}>,
<x$_2$,{[.3, .5], [.4, .6]},{[.3, .5], [.4, .5]},{[.3, .4], [.1, .2]}>},

then,

A/˜2= { <x$_1$,{[.05, .1], [.1, .15]},{[.2, .25], [.3, .35]},{[.25, .3],[.35, .4]}>,
<x$_2$,{[.05,.2], [.05, .15]},{[.3, .4],[.2, .3]},{[.15, .2], [.1, .35]}>}

Definition 3.28. Let A and B be two n-valued interval neutrosophic sets. Then, truth-Favorite of A, denoted by $\tilde{\Delta}$A, is defined by

$$\tilde{\Delta}A=\{x,([\min(\inf T_A^1(x)+\inf I_A^1(x),1),\min(\sup T_A^1(x)+\sup I_B^1(x),1)],[\min(\inf T_A^2(x)+\inf I_A^2(x),1),\min(\sup T_A^2(x)+\sup I_A^2(x),1)],...,[\min(\inf T_A^p(x)+\inf I_A^p(x),1),\min(\sup T_A^p(x)+\sup I_A^p(x),1)]),([0,0],[0,0],...,[0,0]),([\inf F_A^1(x),\sup F_A^1(x)],[\inf F_A^2(x),\sup F_A^2(x)],...,[\inf F_A^p(x),\sup F_A^p(x)]):x\in X\}$$

Example 3.29. Let X={x$_1$, x$_2$} be the universe and A and B are two n-valued interval neutrosophic sets

A= { <x$_1$,{[.1, .2], [.2, .3]},{[.4, .5], [.6, .7]},{[.5, .6], [.7, .8]}>,
<x$_2$,{[.1, .4], [.1, .3]},{[.6, .8], [.4, .6]},{[.3, .4], [.2, .7]}>}

and

B= { <x$_1$,{[.3, .7], [.3, .5]},{[.2, .4], [.3, .5]},{[.3, .6], [.2, .7]}>,
<x$_2$,{[.3, .5], [.4, .6]},{[.3,.5], [.4, .5]},{[.3, .4], [.1, .2]}>}

Then,

$\tilde{\Delta}$A= { <x$_1$,{[.5, .7],[.8, 1]},{[0, 0],[0, 0]},{[. 5, .6], [.7, .8]}>, <x$_2$,{[.7, 1], [. 5, .9]},{[0, 0],[0, 0]},{[. 3, .4],[.2, .7]}>}

Proposition 3.30. Let A, B, C ∈ NVINS(X).Then,

1. $\tilde{\Delta}\tilde{\Delta}A=\tilde{\nabla}A$.
2. $\tilde{\Delta}(A\cup B)\subseteq\tilde{\Delta}A\cup\tilde{\Delta}B$.
3. $\tilde{\Delta}(A\cap B)\subseteq\tilde{\Delta}A\cap\tilde{\Delta}B$
4. $\tilde{\Delta}(A\tilde{\mp}B)\subseteq\tilde{\Delta}A\tilde{\mp}\tilde{\Delta}B$.

Proof: The proof is straightforward.

Definition 3.31. Let A and B be two n-valued interval neutrosophic sets. Then, false-Favorite of A, denoted by $\tilde{\nabla}$A, is defined by

$\widetilde{\nabla}$A=
{x([$\inf T_A^1(x)$, $\sup T_A^1(x)$], [$\inf T_A^2(x)$, $\sup T_A^2(x)$],...,
[$\inf T_A^p(x)$, $\sup T_A^p(x)$]),([0,0], [0,0],...,[0,0]), [min($\inf F_A^1(x)+$
$\inf I_A^1(x), 1)$, min($\sup F_A^1(x) +$ $\sup I_B^1(x), 1)$], [min($\inf F_A^2(x) +$
$\inf I_A^2(x), 1)$, min($\sup F_A^2(x) +$ $\sup I_A^2(x), 1)$],...,[min($\inf F_A^p(x) +$
$\inf I_A^p(x), 1)$, min($\sup F_A^p(x) +$ $\sup I_A^p(x), 1)$]): x ∈ X }

Example 3.32. Let X={x_1, x_2} be the universe and A and B are two n-valued interval neutrosophic sets

A= { <x_1,{[.1, .2], [.2, .3]},{[.4, .5], [.6, .7]},{[.5, .6], [.7, .8]}>,
<x_2,{[.1, .4], [.1, .3]},{[.6,.8], [.4, .6]},{[.3, .4], [.2, .7]}>}

and

B= { <x_1,{[.3, .7], [.3, .5]},{[.2, .4], [.3, .5]},{[.3, .6], [.2, .7]}>,
<x_2,{[.3, .5], [.4, .6]},{[.3,.5], [.4, .5]},{[.3, .4], [.1, .2]}>}

Then,

$\widetilde{\nabla}$A = { <x_1,{[.1, .2],[.2, .3]},{[0, 0],[0, 0]},{[. 9, 1], [1, 1]}>,
<x_2,{[.1, .4], [. 1, .3]},{[0, 0], [0, 0]},{[. 9, 1], [.6, 1]}>}

Proposition 3.33. Let A, B, C ∈ NVINS(X). Then,

1. $\widetilde{\nabla}\widetilde{\nabla}$A = $\widetilde{\nabla}A$.
2. $\widetilde{\nabla}(A \cup B) \subseteq \widetilde{\nabla}A \cup \widetilde{\nabla}B$.
3. $\widetilde{\nabla}(A \cap B) \subseteq \widetilde{\nabla}A \cap \widetilde{\nabla}B$.
4. $\widetilde{\nabla}(A \widetilde{\mp} B) \subseteq \widetilde{\nabla}A \widetilde{\mp} \widetilde{\nabla}B$.

Proof: The proof is straightforward.

Here ∨, ∧, +, . , /, $\tilde{.}$, $\tilde{/}$ denotes maximum, minimum, addition, multiplication, scalar multiplication, scalar division of real numbers respectively.

Definition 3.34. Let E is a real Euclidean space E^n. Then, a NVINS A is convex if and only if

$\inf T_A^i(x)(\lambda x_1 +(1-\lambda)\, x_2) \geq \min(\inf T_A^i(x_1), \inf T_A^i(x_2))$,
$\sup T_A^i(x)(\lambda x_1 +(1-\lambda)\, x_2) \geq \min(\sup T_A^i(x_1), \sup T_A^i(x_2))$,
$\inf I_A^i(x)(\lambda x_1 +(1-\lambda)\, x_2) \leq \min(\inf I_A^i(x_1), \inf I_A^i(x_2))$,
$\sup I_A^i(x)(\lambda x_1 +(1-\lambda)\, x_2) \leq \min(\sup i_A^i(x_1), \sup I_A^i(x_2))$,
$\inf F_A^i(x)(\lambda x_1 +(1-\lambda)\, x_2) \leq \min(\inf F_A^i(x_1), \inf F_A^i(x_2))$,
$\sup F_A^i(x)(\lambda x_1 +(1-\lambda)\, x_2) \leq \min(\sup F_A^i(x_1), \sup F_A^i(x_2))$,

for allx_1, x_2 ∈ E and all λ ∈ [0 ,1] and i= 1, 2,..., p.

Theorem 3.35. If A and B are convex, so is their intersection.

Proof: Let C= A ∩ B

InfT_C^j $(\lambda x_1$ +(1- $\lambda)x_2)$ ≥ min (Inf T_A^j $(\lambda x_1$ +(1- $\lambda)x_2)$, Inf T_B^j $(\lambda x_1$ +(1- $\lambda)x_2)$) ,supT_C^j $(\lambda x_1$ +(1- $\lambda)x_2)$ ≥ min (sup T_A^j $(\lambda x_1$ +(1- $\lambda)x_2)$, sup T_B^j $(\lambda x_1$ +(1- $\lambda)x_2)$) , , Inf I_C^j $(\lambda x_1$ +(1- $\lambda)x_2)$≤ max (Inf I_A^j $(\lambda x_1$ +(1- $\lambda)x_2)$, Inf I_B^j $(\lambda x_1$ +(1- $\lambda)x_2)$), supI_C^j $(\lambda x_1$ +(1- $\lambda)x_2)$ ≤ max (sup I_A^j $(\lambda x_1$ +(1- $\lambda)x_2)$, sup i_B^j $(\lambda x_1$ +(1- $\lambda)x_2)$) , Inf F_C^j $(\lambda x_1$ +(1- $\lambda)x_2)$≤ max (Inf F_A^j $(\lambda x_1$ +(1- $\lambda)x_2)$, Inf F_B^j $(\lambda x_1$ +(1- $\lambda)x_2)$), supF_C^j $(\lambda x_1$ +(1- $\lambda)x_2)$ ≤ max (inf F_A^j $(\lambda x_1$ +(1- $\lambda)x_2)$,inf F_B^j $(\lambda x_1$ +(1- $\lambda)x_2)$) since A and are convex: Inf T_A^j $(\lambda x_1$ +(1- $\lambda)x_2)$ ≥ min (Inf T_A^j (x_1)+ Inf T_A^j (x_2)), sup T_A^j $(\lambda x_1$ +(1- $\lambda)x_2)$ ≥ min (sup T_A^j (x_1)+ sup T_A^j (x_2)) , Inf I_A^j $(\lambda x_1$ +(1- $\lambda)x_2)$ ≤ max (Inf I_A^j (x_1)+ Inf I_A^j (x_2)) , sup I_A^j $(\lambda x_1$ +(1- $\lambda)x_2)$≤max (sup I_A^j (x_1)+ sup I_A^j (x_2)) ,inf F_A^j $(\lambda x_1$ +(1- $\lambda)x_2)$≤max (inf F (x_1)+ inf F_A^j (x_2)) , sup F_A^j $(\lambda x_1$ +(1- $\lambda)x_2)$≤max (sup F_A^j (x_1)+ sup F_A^j (x_2)) , inf T_B^j $(\lambda x_1$ +(1- $\lambda)x_2)$≥max (inf T_B^j (x_1)+ inf T_B^j (x_2)) , sup T_B^j $(\lambda x_1$ +(1- $\lambda)x_2)$≥min (sup T_B^j (x_1)+ sup T_A^j (x_2)) , inf I_B^j $(\lambda x_1$ +(1- $\lambda)x_2)$≤max (inf I_B^j (x_1)+ inf I_A^j (x_2)) , sup I_B^j $(\lambda x_1$ +(1- $\lambda)x_2$ ≤max (sup I_B^j (x_1)+ sup I_A^j (x_2)) , inf F_B^j $(\lambda x_1$ +(1- $\lambda)x_2)$≤max (inf F_B^j (x_1)+ inf F_A^j (x_2)) , sup F_B^j $(\lambda x_1$+(1- $\lambda)x_2)$≤max (sup F_B^j (x_1)+ sup F_A^j (x_2)).

Hence,

infT_C^j $(\lambda x_1$ +(1- $\lambda)x_2)$≥min(min (inf T_A^j (x_1)+ inf T_A^j (x_2)) , min (inf T_B^j (x_1)+ inf T_B^j (x_2)))= min (inf T_A^j (x_1)+ inf T_B^j (x_1)), min (inf T_A^j (x_2)+ inf T_B^j (x_2))= min (inf T_C^j (x_1)+ inf T_C^j (x_2)),
sup T_C^j $(\lambda x_1$ +(1- $\lambda)x_2)$≥min(min (supT_A^j (x_1)+ sup T_A^j (x_2)) , min (sup T_B^j (x_1)+ sup T_B^j (x_2)))= min (supT_A^j (x_1)+sup T_B^j (x_1)), min (sup T_A^j (x_2)+ sup T_B^j (x_2))= min (sup T_C^j (x_1)+ sup T_C^j (x_2)),
infI_C^j $(\lambda x_1$ +(1- $\lambda)x_2)$≤max(max (inf I_A^j (x_1)+ inf I_A^j (x_2)) , max(inf I_B^j (x_1)+ inf I_B^j (x_2)))= max(infI_A^j (x_1)+inf I_B^j (x_1)),max (inf I_A^j (x_2)+ inf I_B^j (x_2))= max (inf I_C^j (x_1)+ inf I_C^j (x_2)).

Definition 3.36. An n-valued interval neutrosophic set is strongly convex if for any two points x_1 and $x_2$and any λ in the open interval (0.1).

inf $T_A^i(x)(\lambda x_1$ +(1- λ) $x_2)$> min(inf $T_A^i(x_1)$, inf $T_A^i(x_2)$),

$\sup T_A^i(x)(\lambda x_1 + (1-\lambda)\, x_2) > \min(\sup T_A^i(x_1), \sup T_A^i(x_2))$,
$\inf I_A^i(x)(\lambda x_1 + (1-\lambda)\, x_2) < \min(\inf I_A^i(x_1), \inf I_A^i(x_2))$,
$\sup I_A^i(x)(\lambda x_1 + (1-\lambda)\, x_2) < \min(\sup i_A^i(x_1), \sup I_A^i(x_2))$,
$\inf F_A^i(x)(\lambda x_1 + (1-\lambda)\, x_2) < \min(\inf F_A^i(x_1), \inf F_A^i(x_2))$,
$\sup F_A^i(x)(\lambda x_1 + (1-\lambda)\, x_2) < \min(\sup F_A^i(x_1), \sup F_A^i(x_2))$,
for all x_1, x_2 in X and all λ in [0 ,1] and i= 1, 2,...,p.

Theorem 3.37. If A and B are strongly convex, so is their intersection.

Proof: The proof is similar to Theorem 3.25

4 Distances between n-valued interval neutrosophic sets

In this section, we present the definitions of the Hamming, Euclidean distances between n-valued interval neutrosophic sets, generalized weighted distance and the similarity measures between n-valued interval neutrosophic sets based on the distances, which can be used in real scientific and engineering applications.

On the basis of the Hamming distance and Euclidean distance between two interval neutrosophic set defined by Ye in [43], we give the following Hamming distance and Euclidean distance between NVINSs as follows:

Definition 4.1 Let A and B two n-valued interval neutrosophic sets, Then, the Hamming distance is defined by:

1- $d_{HD} = \frac{1}{p}\sum_{j=1}^{p}\frac{1}{6}\sum_{i=1}^{n}\left[\left|\mathrm{infT}_A^j(\mathrm{x_i}) - \mathrm{infT}_B^j(\mathrm{x_i})\right| + \left|\mathrm{supT}_A^j(\mathrm{x_i}) - \mathrm{supT}_B^j(\mathrm{x_i})\right| + \left|\mathrm{infI}_A^j(\mathrm{x_i}) - \mathrm{infI}_B^j(\mathrm{x_i})\right| + \left|\mathrm{supI}_A^j(\mathrm{x_i}) - \mathrm{supI}_B^j(\mathrm{x_i})\right| + \left|inf\,\mathrm{F}_A^j(\mathrm{x_i}) - \mathrm{infF}_B^j(\mathrm{x})\right| + \left|sup\,\mathrm{F}_A^j(\mathrm{x_i}) - \mathrm{supF}_B^j(\mathrm{x})\right|\right]$

The normalized Hamming distance is defined by:

2- $d_{NHD} = \frac{1}{p}\sum_{j=1}^{p}\frac{1}{6\,n}\sum_{i=1}^{n}\left[\left|\mathrm{infT}_A^j(\mathrm{x_i}) - \mathrm{infT}_B^j(\mathrm{x_i})\right| + \left|\mathrm{supT}_A^j(\mathrm{x_i}) - \mathrm{supT}_B^j(\mathrm{x_i})\right| + \left|\mathrm{infI}_A^j(\mathrm{x_i}) - \mathrm{infI}_B^j(\mathrm{x_i})\right| + \left|\mathrm{supI}_A^j(\mathrm{x_i}) - \mathrm{supI}_B^j(\mathrm{x_i})\right| + \left|inf\,\mathrm{F}_A^j(\mathrm{x_i}) - \mathrm{infF}_B^j(\mathrm{x})\right| + \left|sup\,\mathrm{F}_A^j(\mathrm{x_i}) - \mathrm{supF}_B^j(\mathrm{x})\right|\right]$

However, the difference of importance is considered in the elements in the universe. Therefore, we need to consider the weights of the elements $\mathrm{x_i}$ (i=1, 2,..., n) into account. In the following, we defined the weighted Hamming distance with w={$\mathrm{w_1}$,$\mathrm{w_2}$,...,$\mathrm{w_n}$}

3- eighted normalized Hamming distance is defined by:

$$d_{wHD}=\frac{1}{p}\sum_{j=1}^{p}\frac{1}{6\,n}\sum_{i=1}^{n}w_i\Big[\left|\mathrm{infT}_A^j(x_i)-\mathrm{infT}_B^j(x_i)\right|+\left|\mathrm{supT}_A^j(x_i)-\mathrm{supT}_B^j(x_i)\right|+\left|\mathrm{infI}_A^j(x_i)-\mathrm{infI}_B^j(x_i)\right|+\left|\mathrm{supI}_A^j(x_i)-\mathrm{supI}_B^j(x_i)\right|+\left|inf\mathrm{F}_A^j(x_i)-\mathrm{infF}_B^j(x)\right|+\left|sup\mathrm{F}_A^j(x_i)-\mathrm{supF}_B^j(x)\right|\Big]$$

If $w_i=(\frac{1}{n},\frac{1}{n},\ldots,\frac{1}{n})$, then (3) reduces to the Normalized Hamming distance.

Example 4.2. Let X={x_1, x_2} be the universe and A and B are two n- valued interval neutrosophic sets

A= { <x_1,{[.1, .2], [.2, .3]},{[.4, .5], [.6, .7]},{[.5, .6], [.7, .8]}>, <x_2,{[.1, .4], [.1, .3]},{[.6,.8], [.4, .6]},{[.3, .4], [.2, .7]}>}

and

B= { <x_1,{[.3, .7], [.3, .5]},{[.2, .4], [.3, .5]},{[.3, .6], [.2, .7]}>, <x_2,{[.3, .5], [.4, .6]},{[.3,.5], [.4, .5]},{[.3, .4], [.1, .2]}>}

Then, we have d_{HD}= 0.4.

Definition 4.3. Let A, B two n-valued interval neutrosophic sets. Thus,

1. The Euclidean distance d_{ED} is defined by:

$$d_{ED}=\left\{\frac{1}{p}\sum_{j=1}^{p}\frac{1}{6}\sum_{i=1}^{n}\left[\left(\mathrm{infT}_A^j(x_i)-\mathrm{infT}_B^j(x_i)\right)^2+\left(\mathrm{supT}_A^j(x_i)-\mathrm{supT}_B^j(x_i)\right)^2+\left(\mathrm{infI}_A^j(x_i)-\mathrm{infI}_B^j(x_i)\right)^2+\left(\mathrm{supI}_A^j(x_i)-\mathrm{supI}_B^j(x_i)\right)^2+\left(inf\mathrm{F}_A^j(x_i)-\mathrm{infF}_B^j(x)\right)^2+\left(sup\mathrm{F}_A^j(x_i)-\mathrm{supF}_B^j(x)\right)^2\right]\right\}^{\frac{1}{2}}$$

2. The normalized Euclidean distance d_{NED} is defined by:

$$d_{NED}=\left\{\frac{1}{p}\sum_{j=1}^{p}\frac{1}{6\,n}\sum_{i=1}^{n}\left[\left(\mathrm{infT}_A^j(x_i)-\mathrm{infT}_B^j(x_i)\right)^2+\left(\mathrm{supT}_A^j(x_i)-\mathrm{supT}_B^j(x_i)\right)^2+\left(\mathrm{infI}_A^j(x_i)-\mathrm{infI}_B^j(x_i)\right)^2+\left(\mathrm{supI}_A^j(x_i)-\mathrm{supI}_B^j(x_i)\right)^2+\left(inf\mathrm{F}_A^j(x_i)-\mathrm{infF}_B^j(x_i)\right)^2+\left(sup\mathrm{F}_A^j(x_i)-\mathrm{supF}_B^j(x)\right)^2\right]\right\}^{\frac{1}{2}}$$

However, the difference of importance is considered in the elements in the universe. Therefore, we need to consider the weights of the elements x_i (i=1, 2,..., n) into account. In the following, we defined the weighted Euclidean distance with w={w_1 ,w_2,...,w_n}

3. The weighted Euclidean distance d_{WED} is defined by:

$$d_{WED}=\left\{\frac{1}{p}\sum_{j=1}^{p}\frac{1}{6\,n}\sum_{i=1}^{n}w_i\left[\left(\inf T_A^j(x_i)-\inf T_B^j(x_i)\right)^2+\left(\sup T_A^j(x_i)-\sup T_B^j(x_i)\right)^2+\left(\inf I_A^j(x_i)-\inf I_B^j(x_i)\right)^2+\left(\sup I_A^j(x_i)-\sup I_B^j(x_i)\right)^2+\left(\inf F_A^j(x_i)-\inf F_B^j(x)\right)^2+\left(\sup F_A^j(x_i)-\sup F_B^j(x)\right)^2\right]\right\}^{\frac{1}{2}}$$

If $w_i=(\frac{1}{n},\frac{1}{n},\ldots,\frac{1}{n})$, then (3) reduces to the Normalized Euclidean distance.

Example 4.4. Let X={x_1, x_2} be the universe and A and B are two n- valued interval neutrosophic sets

A= { <x_1,{[.1, .2], [.2, .3]},{[.4, .5], [.6, .7]},{[.5, .6], [.7, .8]}>,
<x_2,{[.1, .4], [.1, .3]},{[.6,.8], [.4, .6]},{[.3, .4], [.2, .7]}>}

and

B= { <x_1,{[.3, .7], [.3, .5]},{[.2, .4], [.3, .5]},{[.3, .6], [.2, .7]}>,
<x_2,{[.3, .5], [.4, .6]},{[.3,.5], [.4, .5]},{[.3, .4], [.1, .2]}>},

then, we have d_{ED}= 0.125.

Definition 4.5. Let A, B two n-valued interval neutrosophic sets. Then based on Broumi et al.[11] we proposed a generalized interval valued neutrosophic weighted distance measure between A and B as follows:

$$d_\lambda(A\,,B)=\left\{\frac{1}{p}\sum_{j=1}^{p}\frac{1}{6\,n}\sum_{i=1}^{n}w_i\left[\left|\inf T_A^j(x_i)-\inf T_B^j(x_i)\right|^\lambda+\left|\sup T_A^j(x_i)-\sup T_B^j(x_i)\right|^\lambda+\left|\inf I_A^j(x_i)-\inf I_B^j(x_i)\right|^\lambda+\left|\sup I_A^j(x_i)-\sup I_B^j(x_i)\right|^\lambda+\left|\inf F_A^j(x_i)-\inf F_B^j(x)\right|^\lambda+\left|\sup F_A^j(x_i)-\sup F_B^j(x)\right|^\lambda\right]\right\}^{\frac{1}{\lambda}}$$

If λ=1 and $w_i=(\frac{1}{n},\frac{1}{n},\ldots,\frac{1}{n})$, then the above equation reduces to the normalized Hamming distance.

If λ=2 and $w_i=(\frac{1}{n},\frac{1}{n},\ldots,\frac{1}{n})$,then the above equation reduces to the normalized Euclidean distance.

Theorem 4.6. The defined distance $d_K(A,B)$ between NVINSs A and B satisfies the following properties (1-4), for (k=HD, NHD, ED, NED);

1. $d_k(A,B)\geq 0$,
2. $d_k(A,B)=0$ if and only if A = B; for all A,B ∈ NVINSs,

3. $d_k(A, B) = d_k(B, A)$,
4. If $A \subseteq B \subseteq C$, for A, B, C ∈ NVINSs, then $d_k(A, C) \geq d_k(A, B)$ and $d_k(A, C) \geq d_k(B, C)$.

Proof: it is easy to see that $d_k(A, B)$ satisfies the properties (D1)-(D3). Therefore, we only prove (D4). Let $A \subseteq B \subseteq C$, then,

$$\inf T_A^j(x_i) \leq \inf T_B^j(x_i) \leq \inf T_C^j(x_i), \text{supf } T_A^j(x_i) \leq \sup T_B^j(x_i) \leq \sup T_C^j(x_i), \inf I_A^j(x_i) \geq \inf I_B^j(x_i) \geq \inf I_C^j(x_i), \text{supf } I_A^j(x_i) \geq \sup I_B^j(x_i) \geq \sup I_C^j(x_i),$$

and

$$\inf F_A^j(x_i) \geq \inf F_B^j(x_i) \geq \inf F_C^j(x_i), \text{supf } F_A^j(x_i) \geq \sup F_B^j(x_i) \geq \sup F_C^j(x_i),$$

for k= (HD, NHD, ED, NED), we have

$$\left|\inf T_A^j(x_i) - \inf T_B^j(x_i)\right|^k \leq \left|\inf T_A^j(x_i) - \inf T_C^j(x_i)\right|^k, \left|\sup T_A^j(x_i) - \sup T_B^j(x_i)\right|^k \leq \left|\sup T_A^j(x_i) - \sup T_C^j(x_i)\right|^k,$$
$$\left|\inf T_B^j(x_i) - \inf T_C^j(x_i)\right|^k \leq \left|\inf T_A^j(x_i) - \inf T_C^j(x_i)\right|^k, \left|\sup T_B^j(x_i) - \sup T_B^j(x_i)\right|^k \leq \left|\sup T_A^j(x_i) - \sup T_C^j(x_i)\right|^k,$$
$$\left|\inf I_A^j(x_i) - \inf I_B^j(x_i)\right|^k \leq \left|\inf I_A^j(x_i) - \inf I_C^j(x_i)\right|^k, \left|\sup I_A^j(x_i) - \sup I_B^j(x_i)\right|^k \leq \left|\sup I_A^j(x_i) - \sup I_C^j(x_i)\right|^k,$$
$$\left|\inf I_B^j(x_i) - \inf I_C^j(x_i)\right|^k \leq \left|\inf I_A^j(x_i) - \inf I_C^j(x_i)\right|^k, \left|\sup I_B^j(x_i) - \sup I_B^j(x_i)\right|^k \leq \left|\sup I_A^j(x_i) - \sup I_C^j(x_i)\right|^k$$
$$\left|\inf I_A^j(x_i) - \inf I_B^j(x_i)\right|^k \leq \left|\inf I_A^j(x_i) - \inf I_C^j(x_i)\right|^k, \left|\sup I_A^j(x_i) - \sup I_B^j(x_i)\right|^k \leq \left|\sup I_A^j(x_i) - \sup I_C^j(x_i)\right|^k,$$
$$\left|\inf I_B^j(x_i) - \inf I_C^j(x_i)\right|^k \leq \left|\inf I_A^j(x_i) - \inf I_C^j(x_i)\right|^k, \left|\sup I_B^j(x_i) - \sup I_B^j(x_i)\right|^k \leq \left|\sup I_A^j(x_i) - \sup I_C^j(x_i)\right|^k,$$
$$\left|\inf F_A^j(x_i) - \inf F_B^j(x_i)\right|^k \leq \left|\inf F_A^j(x_i) - \inf F_C^j(x_i)\right|^k, \left|\sup F_A^j(x_i) - \sup F_B^j(x_i)\right|^k \leq \left|\sup F_A^j(x_i) - \sup F_C^j(x_i)\right|^k,$$
$$\left|\inf F_B^j(x_i) - \inf F_C^j(x_i)\right|^k \leq \left|\inf F_A^j(x_i) - \inf F_C^j(x_i)\right|^k,$$
$$\left|\sup F_B^j(x_i) - \sup F_B^j(x_i)\right|^k \leq \left|\sup F_A^j(x_i) - \sup F_C^j(x_i)\right|^k$$

Hence

$$\left|\inf T_A^j(x_i) - \inf T_B^j(x_i)\right|^k + \left|\sup T_A^j(x_i) - \sup T_B^j(x_i)\right|^k + \left|\inf I_A^j(x_i) - \inf I_B^j(x_i)\right|^k + \left|\sup I_A^j(x_i) - \sup I_B^j(x_i)\right|^k + \left|\inf F_A^j(x_i) - \inf F_B^j(x_i)\right|^k + \left|\sup F_A^j(x_i) - \sup F_B^j(x_i)\right|^k \leq \left|\inf T_A^j(x_i) - \inf T_C^j(x_i)\right|^k + \left|\sup T_A^j(x_i) - \sup T_C^j(x_i)\right|^k + \left|\inf I_A^j(x_i) - \inf I_C^j(x_i)\right|^k + \left|\sup I_A^j(x_i) - \sup I_C^j(x_i)\right|^k + \left|\inf F_A^j(x_i) - \inf F_C^j(x_i)\right|^k + \left|\sup F_A^j(x_i) - \sup F_C^j(x_i)\right|^k$$

$$\begin{aligned}
\frac{1}{p}\sum_{j=1}^{p}\frac{1}{6}\sum_{i=1}^{n}&\left|\inf T_A^j(x_i) - \inf T_B^j(x_i)\right|^k + \left|\sup T_A^j(x_i) - \sup T_B^j(x_i)\right|^k \\
&+ \left|\inf I_A^j(x_i) - \inf I_B^j(x_i)\right|^k \\
&+ \left|\sup I_A^j(x_i) - \sup I_B^j(x_i)\right|^k \\
&+ \left|\inf F_A^j(x_i) - \inf F_B^j(x_i)\right|^k \\
&+ \left|\sup F_A^j(x_i) - \sup F_B^j(x_i)\right|^k \\
&\leq \frac{1}{p}\sum_{j=1}^{p}\frac{1}{6}\sum_{i=1}^{n}\left|\inf T_A^j(x_i) - \inf T_C^j(x_i)\right|^k \\
&+ \left|\sup T_A^j(x_i) - \sup T_C^j(x_i)\right|^k \\
&+ \left|\inf I_A^j(x_i) - \inf I_C^j(x_i)\right|^k \\
&+ \left|\sup I_A^j(x_i) - \sup I_C^j(x_i)\right|^k \\
&+ \left|\inf F_A^j(x_i) - \inf F_C^j(x_i)\right|^k \\
&+ \left|\sup F_A^j(x_i) - \sup F_C^j(x_i)\right|^k
\end{aligned}$$

Then $d_k\,(A, B) \leq d_k(A, C)$

$$\left|\inf T_B^j(x_i) - \inf T_C^j(x_i)\right|^k + \left|\sup T_B^j(x_i) - \sup T_C^j(x_i)\right|^k + \left|\inf I_B^j(x_i) - \inf I_C^j(x_i)\right|^k + \left|\sup I_B^j(x_i) - \sup I_C^j(x_i)\right|^k + \left|\inf F_B^j(x_i) - \inf F_C^j(x_i)\right|^k + \left|\sup F_B^j(x_i) - \sup F_C^j(x_i)\right|^k \leq \left|\inf T_A^j(x_i) - \inf T_C^j(x_i)\right|^k + \left|\sup T_A^j(x_i) - \sup T_C^j(x_i)\right|^k + \left|\inf I_A^j(x_i) - \inf I_C^j(x_i)\right|^k + \left|\sup I_A^j(x_i) - \sup I_C^j(x_i)\right|^k + \left|\inf F_A^j(x_i) - \inf F_C^j(x_i)\right|^k + \left|\sup F_A^j(x_i) - \sup F_C^j(x_i)\right|^k$$

$$\frac{1}{p}\sum_{j=1}^{p}\frac{1}{6}\sum_{i=1}^{n}\left|\inf T_B^j(x_i)-\inf T_C^j(x_i)\right|^k+\left|\sup T_B^j(x_i)-\sup T_C^j(x_i)\right|^k$$
$$+\left|\inf I_B^j(x_i)-\inf I_C^j(x_i)\right|^k+\left|\sup I_B^j(x_i)-\sup I_C^j(x_i)\right|^k$$
$$+\left|\inf F_B^j(x_i)-\inf F_C^j(x_i)\right|^k+\left|\sup F_B^j(x_i)-\sup F_C^j(x_i)\right|^k$$
$$\leq\frac{1}{p}\sum_{j=1}^{p}\frac{1}{6}\sum_{i=1}^{n}\left|\inf T_A^j(x_i)-\inf T_C^j(x_i)\right|^k$$
$$+\left|\sup T_A^j(x_i)-\sup T_C^j(x_i)\right|^k+\left|\inf I_A^j(x_i)-\inf I_C^j(x_i)\right|^k$$
$$+\left|\sup I_A^j(x_i)-\sup I_C^j(x_i)\right|^k+\left|\inf F_A^j(x_i)-\inf F_C^j(x_i)\right|^k$$
$$+\left|\sup F_A^j(x_i)-\sup F_C^j(x_i)\right|^k$$

Then d_k (B, C) $\leq d_k$(A, C).

Combining the above inequalities with the above defined distance formulas (1)-(4), we can obtain that d_k (A, B) $\leq d_k$ (A, C) and d_k (B, C) $\leq d_k$ (A, C) for k= (HD, NHD, ED, NED).

Thus the property (D4) is obtained.

It is well known that similarity measure can be generated from distance measure. Therefore we may use the proposed distance measure to define similarity measures.

Based on the relationship of similarity measure and distance we can define some similarity measures between NVINSs A and B as follows:

Definition 4.7. The similarity measure based on s_{NVINS}(A, B)= 1-d_k(A, B), s_{NVINS}(A,B) is said to be the similarity measure between A and B, where A, B ∈ NVINS.

Theorem 4.8. The defined similarity measure s_{NVINS}(A, B) between NVINSs A and B satisfies the following properties (1-4),

1. s_{NVINS}(A, B) = s_{NVINS}(B, A).
2. s_{NVINS}(A, B) = (1, 0, 0)=$\underline{1}$ .if A=B for all A, B ∈ NVINSs.
3. s_{NVINS}(A, B) ∈[0,1]
4. If A⊆B⊆C for all A, B, C ∈ NVNSs then s_{NVINS}(A, B)≥ s_{NVINS}(A, C) and s_{NVINS}(B, C) ≥ s_{NVINS}(A, C).

From now on, we use

$$A=\{x,\begin{pmatrix}[\inf T_A^1(x),\sup T_A^1(x)],[\inf I_A^1(x),\sup I_A^1(x)],\\ [inf\ F_A^1(x), sup\ F_A^1(x)]\end{pmatrix},\ldots,$$

$$\begin{pmatrix}[\inf T_A^P(x)\,,\sup T_A^P(x)],[\inf I_A^P(x)\,,\sup I_A^P(x)],\\ [inf\ F_A^P(x), sup\ F_A^P(x)]),\end{pmatrix}): \mathrm{x} \in \mathrm{E}\}$$

instead of

$$\mathrm{A}=\{x,\begin{pmatrix}[\inf T_A^1(x)\,,\sup T_A^1(x)],[\inf T_A^2(x)\,,\sup T_A^2(x)],\dots,\\ [\inf T_A^p(x)\,,\sup T_A^p(x)]\end{pmatrix},$$

$$\begin{pmatrix}[\inf I_A^1(x)\,,\sup I_A^1(x)],[\inf I_A^2(x)\,,\sup I_A^2(x)],\dots.,\\ [\inf I_A^p(x)\,,\sup I_A^q(x)]\end{pmatrix},$$

$$(\ [\inf F_A^1(x), \sup F_A^1(x)]\ ,([\inf F_A^2(x), \sup F_A^2(x)],\dots.,$$

$$([inf\ F_A^p(x), sup\ F_A^r(x)]): \mathrm{x} \in \mathrm{E}\}.$$

5 Medical Diagnosis using NVINS

In what follows, let us consider an illustrative example adopted from Rajarajeswari and Uma [32] with minor changes and typically considered in [17,20,37]. Obviously, the application is an extension of intuitionistic fuzzy multi sets [17,20,32,33,34].

"As Medical diagnosis contains lots of uncertainties and increased volume of information available to physicians from new medical technologies, the process of classifying different set of symptoms under a single name of disease becomes difficult. In some practical situations, there is the possibility of each element having different truth membership, indeterminate and false membership functions. The proposed similarity measure among the patients Vs symptoms and symptoms Vs diseases gives the proper medical diagnosis. The unique feature of this proposed method is that it considers multi truth membership, indeterminate and false membership. By taking one time inspection, there may be error in diagnosis. Hence, this multi time inspection, by taking the samples of the same patient at different times gives best diagnosis" [32].

Now, an example of a medical diagnosis will be presented.

Example 5.1. Let P={P_1,P_2,P_3} be a set of patients, D={Viral Fever, Tuberculosis, Typhoid, Throat disease} be a set of diseases and S={Temperature, cough, throat pain, headache, body pain} be a set of symptoms. Our solution is to examine the patient at different time intervals (three times a day), which in turn give arise to different truth membership, indeterminate and false membership function for each patient.

Table I: Q (the relation Between Patient and Symptoms)

Q	Temperature	Cough	Throat pain	Headache	Body Pain
P_1	[.3,.4],[.4,.5],[.3,.7] [0,.3],[.1,.3],[0,.5] [0,.6],[.4,.5],[.3,.4]	[.1,.2],[.3,.6],[.6,.8] [0,.5],[.4,.7],[.4,.5] [.2,.3],[0,.5],[.4,.6]	[0,.5],[.2,.6],[0,.4] [.3,.4],[.2,.3,[.3,.4] [0,.7],[.3,.7],[.3,.5]	[.2,.3],[.3,.5],[0,.7] [.4,.5],[.4,.7],[.3,.6] [.2,.6],[0,.6],[.3,.4]	[0,.4],[.6,.7],[.2,.5] [.2,.4],[.4,.5],[.1,.2] [.1,.3],[.1,.3],[.2,.3]
P2	[.2,.3],[.4,.5],[.1,.2] [.4,.5],[.2,.5],[0,.3] [.6,.7,[.4,.5],[.4,.5]	[.5,.7],[0,.4],[.7,.8] [.6,.7],[0,.5],[.4,.5] [.4,.6],[.2,.7],[0,.3]	[.5,.6],[0,.6],[.2,.3] [.4,.7],[.4,.6],[.3,.4] [.1,.3],[.2,.3],[.5,.7]	[.2,.5],[.5,.6],[.1,.5] [.2,.3],[.2,.5],[.5,.6] [.1,.3],[.3,.4],[.4,.5]	[.2,.4],[.4,.6],[.1,.4] [0,.5],[.2,.4],[.5,.6] [.5,.7],[0,.7],[.2,.4]
P3	[.1,.3],[0,.5],[.4,.6] [.1,.2],[.3,.4],[.2,.5] [.2,.4],[.4,.5],[.3,.7]	[.2,.3],[.0,.7],[.1,.4] [.5,.6],[0,.3],[.3,.5] [.3,.5],[.2,.5],[.4,.6]	[.2,.4],[.3,.6],[0,.6] [.4,.5],[0,.3],[.3,.4] [.5,.7],[.4,.6],[.3,.7]	[.2,.3],[.5,.6],[.4,.5] [.2,.4],[0,.4],[.2,.7] [.4,.5],[.2,.3],[.3,.5]	[0,.6],[.4,.7],[.2,.3] [.2,.3],[.2,.3],[.1,.2] [.0,.6],[.2,.4],[.4,.6]

Let the samples be taken at three different timings in a day (in 08:00,16:00,24:00).

Table II: R (the relation among Symptoms and Diseases)

R	Viral Fever	Tuberculosis	Typhoid	Throat diseas
Temerature	[.2,.4],[.3,.5],[.3,.7]	[.1,.4],[.2,.6],[.6,.7]	[0,.3],[.4,.6],[0,.2]	[.3,.4],[.2,.5],[0,.6]
Cough	[.2,.4],[.2,.3],[0,.5]	[.3,.4],[.2,.5],[.7,.8]	[.3,.4],[.2,.3],[.1,.2]	[.4,.5],[.1,.3],[0,.5]
Throat Pain	[0,.4],[.2,.4],[.2,.4]	[0,.2,[.3,.6],[.6,.7]	[.1,.2],[.4,.5],[.3,.4]	[.2,.4],[.2,.5],[.3,.7]
Headache	[.4,.7],[0,.3],[.3,.5]	[.1,.2],[0,.5],[0,.6]	[.3,.4],[.2,.3],[.2,.5]	[0,.3],[.3,.6],[.2,.5]
Body Pain	[.1,.4],[.2,.5],[.3,.4]	[.5,.7],[.4,.5],[.2.5]	[.2,.3],[.2,.4],[.2.3]	[0,.4],[.1,.2],[.1.3]

Table III: The Hamming distance between NVINS Q and R

Hamming Distance	Viral Fever	Tuberculosis	Typhoid	Throat diseas
P_1	**0.1511**	0.1911	0.1678	0.1689
P_2	0.1911	0.2089	0.1789	**0.1644**
P_3	**0.1433**	0.1967	0.1533	0.1456

Table IV: The similarity Measure between NVINS Q and R

Similarity measure	Viral Fever	Tuberculosis	Typhoid	Throat diseas
P_1	**0.8489**	0.8089	0.8322	0.8311
P_2	0.8089	0.7911	0.8211	**0.8356**
P_3	**0.8567**	0.8033	0.8467	0.8544

The highest similarity measure from the Table IV gives the proper medical diagnosis. Therefore, patient P_1 suffers from Viral Fever, P_2 suffers from Throat disease and P_3 suffers from Viral Fever.

6 Conclusion

In this paper, we give n-valued interval neutrosophic sets and desired operations such as; union, intersection, addition, multiplication, scalar multiplication, scalar division, truth-favorite and false-favorite. The concept of n-valued interval neutrosophic set is a generalization of interval valued neutrosophic set, single valued neutrosophic sets and single valued neutrosophic multi sets. Then, we introduce some distances between n-valued interval neutrosophic sets (NVINS) and propose an efficient approach for group multi-criteria decision making based on n-valued interval neutrosophic sets. The distances have natural applications in the field of pattern recognition, feature extraction, region extraction, image processing, coding theory etc.

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