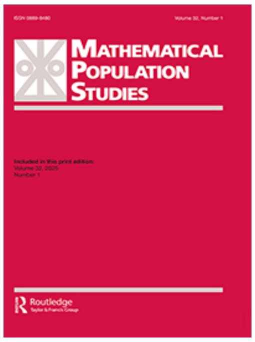

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Neutrosophic developments in Horvitz-Thompson type estimators

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ABSTRACT
In the realm of unequal probability sampling and Horvitz-Thompson type technique, addressing the challenge of estimating the mean in the presence of ambiguous data is one of the most important topics that has not yet been thoroughly discussed. The presence of ambiguous data not only complicates the estimation process but also affects its sensitivity, particularly when dealing with variables of varying significance. Leveraging auxiliary information becomes crucial to mitigate these challenges. This study presents a pioneering method that combines neutrosophic statistics, Horvitz-Thompson type estimators employing unequal probability sampling for mean estimation, and the integration of auxiliary information. Neutrosophic set theory, extending classical set theory to manage indeterminacy, offers a framework for addressing inherent uncertainties within the data. By merging neutrosophic statistics with the Horvitz-Thompson technique, the study develops innovative mean estimators adept at effectively managing ambiguous data within unequal probability sampling. To assess the effectiveness of this approach, the study conducts a numerical analysis utilizing real-world data. Findings from this analysis showcase the superior efficiency of the proposed methodology.

KEYWORDS
Horvitz-Thompson technique; inclusion probabilities; mean estimation; neutrosophic statistics

1. Introduction

Traditional statistical methods involve the utilization of precise numerical data, and numerous scholars have devised mean estimators, supplemented with additional data. In a specific study, when a robust correlation exists b/w the research and auxiliary variable, employing a ratio, i.e., mean per unit estimation technique rather than solely relying on the research variable can significantly mitigate sampling errors. As a result of this error reduction, there is a decrease in the necessary sample size while preserving accuracy, as emphasized by (Cochran, 1940). As a result, the mean per unit method has undergone extensive examination, resulting in the development of various types and applications over time. Researchers have utilized characteristics supplemented with additional data, investigating different transformations

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of these variables. Recent research indicates that the performance of such estimators can be enhanced by leveraging diverse forms of auxiliary information. Some scholars have advocated for the utilization of exponential-type ratio estimators, with efforts made to refine their efficacy. Previous research has delved into estimating the parameters using exponential and ratio approaches in cases of non-response. These advancements were discussed by Robson (1957) in his research.

In classical statistics, data are precise and without uncertainty in measurement. However, when dealing with data that is not precise, new methods are required. Fuzzy logic methods are advancing quickly and are widely utilized in decision-making environments. Complex fuzzy sets represent a further development in the field, with complex neutrosophic sets being their generalized form. In a recent study, Smarandache (2014) introduced a comprehensive flow chart that covers fuzzy sets and their generalizations. The chart outlines various functions associated with interval-valued neutrosophic sets.

In decision-making scenarios characterized by uncertainty, the neutrosophic set presents a more advantageous alternative compared to the fuzzy set. Neutrosophic sets are available in various types, such as trapezoidal bipolar neutrosophic numbers, which have been classified for decision-making problems in one study. A recent research presented a novel approach using generalized spherical fuzzy numbers for multi-criteria group decision-making (MCGDM) and outlined an analytical framework with associated techniques. Similarly, another study explored arithmetic and geometric operations with pentagonal neutrosophic numbers, which can be utilized in MCGDM for mobile communication. The growing interest in neutrosophic numbers is also apparent in recent research, where a team of scientists proposed an MCGDM plan utilizing the cylindrical neutrosophic domain.

Neutrosophic statistics come into play when data exhibit a level of uncertainty. This statistical approach surpasses traditional methods, proving valuable in scenarios where the data or sample displays neutrosophy. In particular, when observations in the population or sample are vague, uncertain, or indefinite, neutrosophic statistics, as elucidated by Smarandache (2014), offer significant utility. Tahir et al. (2021) introduced neutrosophic ratio estimators for SRS scheme, while Vishwakarma and Singh (2022) expanded upon this in the context of ranked set sampling. Yadav and Smarandache (2023), along with Kumar et al. (2022), outlined generalized families of neutrosophic ratio and exponential estimators. Yadav and Prasad (2023) extend the work under two-stage sampling. Ullah et al. (2023) first time introduced Intuitionistic fuzzy function in mean estimation as a cost function under neutrosophic frame work. Alomair and Shahzad (2023) constructed neutrosophic Hartley-Ross type estimators. They also introduced some basic randomized response models under neutrosophic frame work. Hence, they also presented their proposed neutrosophic estimators for the sensitive issue using neutrosophic randomized response models. Recently, Anwar et al. (2024) performed a novel study about neutrosophic predictive mean estimation where non-sampled part of the population is estimated through non-parametric kernel regression function. They also provided neutrosophic version of non-parametric kernel regression function. It is worthy to note that all the mentioned literature belongs to equal probability sampling scheme.

Nevertheless, there has been no exploration in this domain within the framework of unequal probability sampling design employing neutrosophic statistics. Consequently, we endeavor to advance this research by introducing ratio and exponential Horvitz Thompson type mean estimators under unequal probability sampling and derive expressions for their mean square error and bias.

The article is structured into multiple sections. Section 2 explores the concept of basic neutrosophic Horvitz-Thompson type mean estimators and adapted estimators within the framework of unequal PPS sampling for mean estimation. Section 3 presents our proposal for neutrosophic Horvitz-Thompson type mean estimators. Sections 4 and 5 are devoted to illustrating practical applications and analyzing the results of our proposed methodologies. Lastly, Section 6 summarizes the conclusions drawn in the article.

2. Unequal probability sampling and adapted neutrosophic Horvitz Thompson type estimators

In the preceding section, we delved into literature concerning simple random sampling (SRS), where each unit in the population holds an equal likelihood of being chosen for the sample. However, when there exists notable disparity in unit sizes, SRS may prove inadequate as it disregards the potential significance of unit size. In such scenarios, opting for unequal probability sampling could result in more effective estimators compared to equal probability sampling. This method, termed varying probability sampling or probability proportional to size (PPS) sampling, entails selecting units with probabilities proportionate to a specified measure of size. Typically, this size measure is denoted by an auxiliary variable, represented as X, closely linked to the study variable, denoted as Y. For instance, when estimating the total count of unemployed youth within a district, the quantity of households in a village could serve as a measure of size if villages are selected as sampling units. Likewise, in estimating the total number of tube wells within a specific district, factors such as the prior-period count of tube wells in a village or the net irrigated area for the village could be regarded as indicators of size. Ahmad and Shabbir (2018), Al-Marzouki et al. (2021) and, Ayinzoya and Jakperik (2023) utilized PPS sampling in the presence of extreme values and constructed mean estimators. Sinha and Khanna (2022) extended the work in presence of measurement error.

Horvitz and Thompson (1952) proposed an estimator for the total population that is applicable to various types of samples taken without replacement. However, this estimator necessitates computing inclusion probabilities for the sampled units. It proves to be effective for sampling methods where the inclusion probabilities for population units are directly proportional to their sizes. Such sampling methods, which yield these proportional inclusion probabilities, are recognized as IPPS (Inclusion Probability Proportional to Size) designs. Bacanli and Kadilar (2008) developed ratio-type estimators, and Bacanli (2015) developed regression-based mean estimators using the Horvitz and Thompson approach. Walia (2018) extended this work using exponential estimators. Khan, Gupta, and Farhat (2020) corrected the MSE expression for Bacanli and Kadilar (2008) estimators and developed some new estimators. However, these estimators are not available within the framework of unequal probability sampling designs employing neutrosophic statistics. To fill this gap in literature, we develop nutrosophic Horvitz Thompson type estimators.

Suppose a neutrosophic population Z_{z_a,z_b} of size N with study variable $Y_{z_a,z_b} \in [Y_{z_a}, Y_{z_b}]$ and auxiliary variable $X_{z_a,z_b} \in [X_{z_a}, X_{z_b}]$. Further, $\bar{Y}_{z_a,z_b} \in [\bar{Y}_{z_a}, \bar{Y}_{z_b}]$ and $\bar{X}_{z_a,z_b} \in [\bar{X}_{z_a}, \bar{X}_{z_b}]$ be the neutrosophic population means of study and auxiliary variables. Let $\bar{y}_{ht_{z_a,z_b}} \in \left[\bar{y}_{ht_{z_a}}, \bar{y}_{ht_{z_b}}\right]$ and $\bar{x}_{ht_{z_a,z_b}} \in \left[\bar{x}_{ht_{z_a}}, \bar{x}_{ht_{z_b}}\right]$ are representing the Horvitz Thompson type mean estimators, i.e.

$$\bar{y}_{ht_{z_a,z_b}} = \frac{1}{N}\sum_{i=1}^{n}\frac{Y_{iz_a,z_b}}{\Pi_{iz_a,z_b}} \tag{1}$$

$$\bar{x}_{ht_{z_a,z_b}} = \frac{1}{N}\sum_{i=1}^{n}\frac{X_{iz_a,z_b}}{\Pi_{iz_a,z_b}} \tag{2}$$

where Π_{iz_a,z_b} is representing neutrosophic inclusion probabilities. Also

$$\mathrm{Var}\left(\bar{y}_{ht_{z_a,z_b}}\right) = \frac{1}{2N^2}\left[\sum_{i=1}^{N}\sum_{j=i}^{N}\left(\Pi_{iz_a,z_b}\Pi_{jz_a,z_b} - \Pi_{ijz_a,z_b}\right)A_{z_a,z_b}\right]\text{where } i \neq j \tag{3}$$

and $A_{z_a,z_b} = \left(\frac{Y_{iz_a,z_b}}{\Pi_{iz_a,z_b}} - \frac{Y_{jz_a,z_b}}{\Pi_{jz_a,z_b}}\right)^2$.

$$Var\left(\bar{x}_{ht_{z_a,z_b}}\right) = \frac{1}{2N^2}\left[\sum_{i=1}^{N}\sum_{j=i}^{N}\left(\Pi_{iz_a,z_b}\Pi_{jz_a,z_b} - \Pi_{ijz_a,z_b}\right)B_{z_a,z_b}\right]\text{where } i \neq j \tag{4}$$

and $B_{z_a,z_b} = \left(\frac{X_{iz_a,z_b}}{\Pi_{iz_a,z_b}} - \frac{X_{jz_a,z_b}}{\Pi_{jz_a,z_b}}\right)^2$.

$$Cov\left(\bar{y}_{htz_{a,z_b}}, \bar{x}_{htz_{a,z_b}}\right) = \frac{1}{2N^2}\left[\sum_{i=1}^{N}\sum_{j=i}^{N}\left(\Pi_{iz_{a,z_b}}\Pi_{jz_{a,z_b}} - \Pi_{ijz_{a,z_b}}\right)A_{z_a,z_b}B_{z_a,z_b}\right] \quad (5)$$

where $i \neq j$ and $Var\left(\bar{y}_{htz_{a,z_b}}\right) \in \left[Var\left(\bar{y}_{htz_a}\right), Var\left(\bar{y}_{htz_b}\right)\right]$, $Var\left(\bar{x}_{htz_{a,z_b}}\right) \in \left[Var\left(\bar{x}_{htz_a}\right), Var\left(\bar{x}_{htz_b}\right)\right]$, $\Pi_{iz_{a,z_b}} \in [\Pi_{iz_a}, \Pi_{iz_b}]$, $\Pi_{jz_{a,z_b}} \in \left[\Pi_{jz_a}, \Pi_{jz_b}\right]$, $Cov\left(\bar{y}_{htz_{a,z_b}}, \bar{x}_{htz_{a,z_b}}\right) \in \left[Cov\left(\bar{y}_{htz_a}, \bar{x}_{htz_a}\right), Cov\left(\bar{y}_{htz_b}, \bar{x}_{htz_b}\right)\right]$.

Khoshnevisan et al. (2007) defined a class of mean estimators using an equal probability sampling scheme. Then, following the work of Khoshnevisan et al. (2007), we introduced a class of adapted neutrosophic Horvitz Thompson type estimators using unequal probability sampling scheme, as given by

$$\Gamma_{a(i)_{z_a,z_b}} = \bar{y}_{ht_{z_a,z_b}}\left[\frac{a_{z_a,z_b}\bar{X}_{z_a,z_b} + b_{z_a,z_b}}{\omega_{z_a,z_b}\left(a_{z_a,z_b}\bar{x}_{ht_{z_a,z_b}} + b_{z_a,z_b}\right) + \left(1-\omega_{z_a,z_b}\right)\left(a_{z_a,z_b}\bar{X}_{z_a,z_b} + b_{z_a,z_b}\right)}\right]^{g_{z_a,z_b}} \quad (6)$$

In the context of Eq. (6), ω_{z_a,z_b} represents a constant that is appropriate for the situation. Further, a_{z_a,z_b} and b_{z_a,z_b} representing binary numbers or functions of known parameters associated with the auxiliary variable, X, such as $\psi_{0_{z_a,z_b}} = S_{x_{z_a,z_b}}$, $\psi_{1_{z_a,z_b}} = C_{x_{z_a,z_b}}$, $\psi_{2z_{a},z_b} = \beta_1\left(x_{z_a,z_b}\right)$, $\psi_{3z_a,z_b} = \beta_2\left(x_{z_a,z_b}\right)$, and $\psi_{4z_a,z_b} = \rho_{z_a,z_b}$. It is worth noting that Eq. (6) yields nine ratio estimators for $i = 1, \ldots, 9$ with various parameter values, as detailed in Table 1. Notably, $\Gamma_{a(1)_{z_a,z_b}}$ in Table 1 corresponds to the classical neutrosophic ratio estimator.

If $e_{\bar{y}_{htz_a,z_b}} = \bar{y}_{htz_a,z_b} - \bar{y}_{z_a,z_b}$, $e_{\bar{x}_{htz_a,z_b}} = \bar{x}_{htz_a,z_b} - \bar{X}_{z_a,z_b}$, $e_{htuz_a,z_b} = \dfrac{e_{\bar{y}_{htz_a,z_b}}}{\bar{Y}_{z_a,z_b}}$, $e_{htvz_a,z_b} = \dfrac{e_{\bar{x}_{htz_a,z_b}}}{\bar{X}_{z_a,z_b}}$, and $E\left(e_{\bar{y}_{htz_a,z_b}}\right) = E\left(e_{\bar{x}_{htz_a,z_b}}\right) = 0$, $E\left(e^2_{htu_{z_a,z_b}}\right) = \dfrac{Var\left(\bar{y}_{htz_a,z_b}\right)}{\bar{Y}^2_{z_a,z_b}} = J_{uz_a,z_b}$, $E\left(e^2_{htv_{z_a,z_b}}\right) = \dfrac{Var\left(\bar{x}_{htz_a,z_b}\right)}{\bar{X}^2_{z_a,z_b}} = J_{vz_a,z_b}$, $E\left(e_{htuz_a,z_b}e_{htvz_a,z_b}\right) = \dfrac{Cov\left(\bar{x}_{htz_a,z_b}, \bar{y}_{htz_a,z_b}\right)}{\bar{X}_{z_a,z_b}\bar{Y}_{z_a,z_b}} J_{uvz_a,z_b}$

Let as express $\Gamma_{a(i)_{z_a,z_b}}$ in terms of $e_{htiz_a,z_b}(i = 0, 1)$. So, Eq. (6) can be written as

$$\Gamma_{a(i)_{z_a,z_b}} = \bar{Y}_{z_a,z_b}\left(1 + e_{htuz_a,z_b}\right)\left(1 + \omega_{z_a,z_b}v_{z_a,z_b}e_{htvz_a,z_b}\right)^{-g_{z_a,z_b}} \quad (7)$$

Table 1. Members of adapted class for $g_{z_a,z_b} = [1,1]$.

	Adapted Estimators	Weights	Binary values or X variable characteristics	
i	$\Gamma_{a(i)_{z_a,z_b}}$	ω_{z_a,z_b}	a_{z_a,z_b}	b_{z_a,z_b}
1	$\Gamma_{a(1)_{z_a,z_b}}$	$[1,1]$	$[1,1]$	$[0,0]$
2	$\Gamma_{a(2)_{z_a,z_b}}$	$[1,1]$	$[1,1]$	$\psi_{1_{z_a,z_b}}$
3	$\Gamma_{a(3)_{z_a,z_b}}$	$[1,1]$	$\psi_{3_{z_a,z_b}}$	$\psi_{1_{z_a,z_b}}$
4	$\Gamma_{a(4)_{z_a,z_b}}$	$[1,1]$	$\psi_{1_{z_a,z_b}}$	$\psi_{3_{z_a,z_b}}$
5	$\Gamma_{a(5)_{z_a,z_b}}$	$[1,1]$	$[1,1]$	$\psi_{0_{z_a,z_b}}$
6	$\Gamma_{a(6)_{z_a,z_b}}$	$[1,1]$	$\psi_{2_{z_a,z_b}}$	$\psi_{0_{z_a,z_b}}$
7	$\Gamma_{a(7)_{z_a,z_b}}$	$[1,1]$	$\psi_{3_{z_a,z_b}}$	$\psi_{0_{z_a,z_b}}$
8	$\Gamma_{a(8)_{z_a,z_b}}$	$[1,1]$	$[1,1]$	$\psi_{4_{z_a,z_b}}$
9	$\Gamma_{a(9)_{z_a,z_b}}$	$[1,1]$	$[1,1]$	$\psi_{3_{z_a,z_b}}$

where $v_{z_a,z_b} = \frac{a_{z_a,z_b}\bar{X}_{z_a,z_b}}{a_{z_a,z_b}\bar{X}_{z_a,z_b}+b_{z_a,z_b}}$. Assuming that $\left|\omega_{z_a,z_b} v_{z_a,z_b} e_{htv z_a,z_b}\right| < 1$, allowing $\left(1+\omega_{z_a,z_b} v_{z_a,z_b} e_{htv z_a,z_b}\right)^{-g_{z_a,z_b}}$ to be expandable. By expanding Eq. (7) to the first order of approximation, we derive

$$\Gamma_{a(i)_{z_a,z_b}} - \bar{Y}_{z_a,z_b} = \bar{Y}_{z_a,z_b}\left[-V_{z_a,z_b} + \frac{1}{2}g_{z_a,z_b}\left(g_{z_a,z_b}+1\right)\omega^2_{z_a,z_b} v^2_{z_a,z_b} e^2_{htv_{z_a,z_b}} + W_{z_a,z_b}\right] \tag{8}$$

where $V_{z_a,z_b} = g_{z_a,z_b}\omega_{z_a,z_b} v_{z_a,z_b} e_{htv z_a,z_b}$ and $W_{z_a,z_b} = e_{htu z_a,z_b} - g_{z_a,z_b}$ $\omega_{z_a,z_b} v_{z_a,z_b} e_{htu z_a,z_b} e_{htv z_a,z_b}$. By computing the expectation of Eq. (8), we determine the bias of the estimator $\Gamma_{a(i)_{z_a,z_b}}$ as

$$B\left(\Gamma_{a(i)_{z_a,z_b}}\right) = \bar{Y}_{z_a,z_b}\left[\frac{1}{2}g_{z_a,z_b}\left(g_{z_a,z_b}+1\right)\omega^2_{z_a,z_b} v^2_{z_a,z_b} J_{z_a,z_b} - g_{z_a,z_b}\omega_{z_a,z_b} v_{z_a,z_b} J_{uv z_a,z_b}\right] \tag{9}$$

After squaring both sides of Eq. (8) and subsequently computing the expectation, we ascertain the MSE of the estimator $\Gamma_{a(i)_{z_a,z_b}}$ to the first-order approximation as

$$MSE\left(\Gamma_{a(i)_{z_a,z_b}}\right) = \bar{Y}^2_{z_a,z_b}\left[g^2_{z_a,z_b}\omega^2_{z_a,z_b} v^2_{z_a,z_b} J_{v z_a,z_b} + J_{u z_a,z_b} - 2g_{z_a,z_b}\omega_{z_a,z_b} v_{z_a,z_b} J_{uv z_a,z_b}\right] \tag{10}$$

3. Proposed neutrosophic Horvitz Thompson type estimators

Past investigations in survey sampling have predominantly concentrated on data characterized by precision, certainty, and unambiguity. Nonetheless, these approaches might yield a solitary, unequivocal outcome that could be flawed, exaggerated, or underestimated, presenting a constraint in specific scenarios. Conversely, there exist circumstances where data exhibit a neutrosophic character, rendering classical statistical techniques inadequate. Data that embodies a neutrosophic nature often comprises uncertain and ambiguous observations, along with arguments lacking clarity and intervals with imprecise values. As a result, experimental or population data often manifest as interval-valued neutrosophic numbers (INN), indicating that the observation falls within the specified interval boundaries.

In practical terms, indeterminate data outweighs determinate data, emphasizing the necessity for continued advancement in neutrosophic statistical methodologies to proficiently analyze such data.

Therefore, the aim of this study is to investigate effective alternative estimators for survey practitioners utilizing the Horvitz-Thompson approach under neutrosophic unequal PPS sampling. These estimators offer an improved option compared to the adapted estimators outlined in the preceding section. The suggested class of neutrosophic exponential-type estimators is presented below:

$$\Gamma_{p(i)_{z_a,z_b}} = \bar{y}_{ht_{z_a,z_b}} exp\left[\frac{a_{z_a,z_b}\left(\bar{X}_{z_a,z_b} - \bar{x}_{ht_{z_a,z_b}}\right)}{a_{z_a,z_b}\left(\bar{X}_{z_a,z_b} - \bar{x}_{ht_{z_a,z_b}}\right) + 2b_{z_a,z_b}}\right] \text{for } i = 1, \ldots, 9 \quad (11)$$

Let we express $\Gamma_{p(i)_{z_a,z_b}}$in terms of $e_{ht_i z_a,z_b}(i = 0, 1)$. So, Eq. (11) can be written as

$$\Gamma_{p(i)_{z_a,z_b}} - \bar{Y}_{z_a,z_b} = \bar{Y}_{z_a,z_b}\left(1 + e_{ht_u z_a,z_b}\right)\left(1 - \frac{1}{2}v_{z_a,z_b}e_{ht_v z_a,z_b} + \frac{3}{8}v^2_{z_a,z_b}e^2_{htv_{z_a,z_b}}\right) - \bar{Y}_{z_a,z_b} \quad (12)$$

By expanding Eq. (12) to the first order of approximation, the bias of $\Gamma_{p(i)_{z_a,z_b}}$ is

$$B\left(\Gamma_{p(i)_{z_a,z_b}}\right) = \bar{Y}_{z_a,z_b}\left(\frac{3}{8}v^2_{z_a,z_b}J_{vz_a,z_b} - \frac{1}{2}v_{z_a,z_b}J_{uvz_a,z_b}\right)$$

After squaring both sides of Eq. (12) and subsequently computing the expectation, we ascertain the MSE of the estimator $\Gamma_{p(i)_{z_a,z_b}}$ to the first order approximation as

$$MSE\left(\Gamma_{p(i)_{z_a,z_b}}\right) = \bar{Y}^2_{z_a,z_b}\left[J_{uz_a,z_b} + \frac{1}{4}v^2_{z_a,z_b}J_{vz_a,z_b} - v_{z_a,z_b}J_{uvz_a,z_b}\right] \quad (13)$$

All the members of proposed class are provided in Table 2.

Table 2. Members of proposed class.

i	Adapted Estimators $\Gamma_{p(i)_{z_a,z_b}}$	Weights ω_{z_a,z_b}	Binary values or X variable characteristics a_{z_a,z_b}	b_{z_a,z_b}
1	$\Gamma_{p(1)_{z_a,z_b}}$	$[1,1]$	$[1,1]$	$[0,0]$
2	$\Gamma_{p(2)_{z_a,z_b}}$	$[1,1]$	$[1,1]$	$\psi_{1_{z_a,z_b}}$
3	$\Gamma_{p(3)_{z_a,z_b}}$	$[1,1]$	$\psi_{3_{z_a,z_b}}$	$\psi_{1_{z_a,z_b}}$
4	$\Gamma_{p(4)_{z_a,z_b}}$	$[1,1]$	$\psi_{1_{z_a,z_b}}$	$\psi_{3_{z_a,z_b}}$
5	$\Gamma_{p(5)_{z_a,z_b}}$	$[1,1]$	$[1,1]$	$\psi_{0_{z_a,z_b}}$
6	$\Gamma_{p(6)_{z_a,z_b}}$	$[1,1]$	$\psi_{2_{z_a,z_b}}$	$\psi_{0_{z_a,z_b}}$
7	$\Gamma_{p(7)_{z_a,z_b}}$	$[1,1]$	$\psi_{3_{z_a,z_b}}$	$\psi_{0_{z_a,z_b}}$
8	$\Gamma_{p(8)_{z_a,z_b}}$	$[1,1]$	$[1,1]$	$\psi_{4_{z_a,z_b}}$
9	$\Gamma_{p(9)_{z_a,z_b}}$	$[1,1]$	$[1,1]$	$\psi_{3_{z_a,z_b}}$

4. Numerical illustration

The utilization of weather information, particularly dew point, arises as a pivotal and innovative strategy within our endeavors, dedicated to enhancing the average of sample survey neutrosophic data. The dew point data of Pakistan—Karachi is considered for the current article.

The dew point measured in degrees Fahrenheit (°F) and depicts the temperature at which the air holding maximum amount of moisture reaches saturation point and dew forms if the temperature decreases more than this point. It is also an important variable in matters of humidity and climate. Dew point measurements are precedent to varying and uncertain factors such as changes in the environment, pressure alteration, temperature incongruity in certain territories and measurement inaccuracy. These issues are relevant to neutrosophic, because neutrosophic approach is designed to work with indeterminacy in data. In this analysis, the entire year's data of Karachi city in 2021 is taken as the auxiliary variable (X), and the data of year 2022 is taken as the study variable (Y). The neutrosophic approach is ideal here due to the fact that neutrosophic sets actively incorporate the inherent uncertainty, variability and imprecision characteristic of dew point measurements, hence providing a more reliable representation of the data collected and its estimation of parameters such as average.

The mean squared error (MSE) plays a critical role in evaluating the efficiency of estimators. Notably, our suggested neutrosophic estimators exhibit the lowest MSE value among the studied methods, further underscoring their superiority in handling complex data scenarios. The MSE results for neutrosophic weather data are presented in Tables 3, 4 , and 5. The PRE results are provided in Table 6 for sample sizes of $n = 3, 4, 5$.

For unequal probability sampling designs, the selection procedure outlined by Midzuno (1952) is employed. The probability of including the i^{th} unit is expressed as:

Table 3. MSE of Horvitz Thompson type estimators when $n = 3$.

	Populations					
Estimators	(1) Jan-Feb	(2) Mar-Apr	(3) May-Jun	(4) Jul-Aug	(5) Sep-Oct	(6) Nov-Dec
Adapted						
$\Gamma_{a(1)_{z_a z_b}}$	[0.2640, 0.0337]	[0.0549, 0.0215]	[0.0048, 0.0038]	[0.5423, 0.0024]	[0.0362, 0.0372]	[0.0920, 0.0519]
$\Gamma_{a(2)_{z_a z_b}}$	[0.2589, 0.0336]	[0.0545, 0.0215]	[0.0048, 0.0038]	[0.5423, 0.0024]	[0.0358, 0.0372]	[0.0904, 0.0517]
$\Gamma_{a(3)_{z_a z_b}}$	[0.2621, 0.0337]	[0.0548, 0.0215]	[0.0048, 0.0038]	[0.5423, 0.0024]	[0.0361, 0.0372]	[0.0915, 0.0518]
$\Gamma_{a(4)_{z_a z_b}}$	[0.2259, 0.0262]	[0.0469, 0.0110]	[0.0031, 0.0022]	[0.5128, 0.0023]	[0.0234, 0.0253]	[0.0707, 0.0445]
$\Gamma_{a(5)_{z_a z_b}}$	[0.1768, 0.0272]	[0.0413, 0.0194]	[0.0043, 0.0037]	[0.5380, 0.0024]	[0.0205, 0.0344]	[0.0618, 0.0443]
$\Gamma_{a(6)_{z_a z_b}}$	[0.0817, 0.0173]	[0.0397, 0.0203]	[0.0045, 0.0038]	[0.5422, 0.0023]	[0.0230, 0.0355]	[0.0412, 0.0160]
$\Gamma_{a(7)_{z_a z_b}}$	[0.2197, 0.0310]	[0.0494, 0.0210]	[0.0046, 0.0038]	[0.5422, 0.0024]	[0.0301, 0.0364]	[0.0798, 0.0478]
$\Gamma_{a(8)_{z_a z_b}}$	[0.2624, 0.0334]	[0.0542, 0.0213]	[0.0048, 0.0038]	[0.5424, 0.0024]	[0.0354, 0.0370]	[0.0917, 0.0518]
$\Gamma_{a(9)_{z_a z_b}}$	[0.2411, 0.0317]	[0.0515, 0.0198]	[0.0044, 0.0032]	[0.5278, 0.0024]	[0.0314, 0.0354]	[0.0810, 0.0501]
Proposed						
$\Gamma_{p(1)_{z_a z_b}}$	[0.1475, 0.0169]	[0.0314, 0.0099]	[0.0027, 0.0028]	[0.5245, 0.0023]	[0.0122, 0.0221]	[0.0460, 0.0309]
$\Gamma_{p(2)_{z_a z_b}}$	[0.1457, 0.0168]	[0.0313, 0.0099]	[0.0027, 0.0028]	[0.5245, 0.0023]	[0.0121, 0.0221]	[0.0455, 0.0309]
$\Gamma_{p(3)_{z_a z_b}}$	[0.1469, 0.0168]	[0.0314, 0.0099]	[0.0027, 0.0028]	[0.5245, 0.0023]	[0.0121, 0.0221]	[0.0459, 0.0309]
$\Gamma_{p(4)_{z_a z_b}}$	[0.1341, 0.0141]	[0.0284, 0.0064]	[0.0022, 0.0021]	[0.5099, 0.0023]	[0.0119, 0.0179]	[0.0388, 0.0281]
$\Gamma_{p(5)_{z_a z_b}}$	[0.1162, 0.0145]	[0.0262, 0.0093]	[0.0025, 0.0028]	[0.5224, 0.0023]	[0.0121, 0.0212]	[0.0356, 0.0280]
$\Gamma_{p(6)_{z_a z_b}}$	[0.0772, 0.0108]	[0.0256, 0.0095]	[0.0026, 0.0028]	[0.5245, 0.0023]	[0.0119, 0.0215]	[0.0281, 0.0157]
$\Gamma_{p(7)_{z_a z_b}}$	[0.1319, 0.0159]	[0.0294, 0.0098]	[0.0026, 0.0028]	[0.5245, 0.0023]	[0.0119, 0.0219]	[0.0419, 0.0294]
$\Gamma_{p(8)_{z_a z_b}}$	[0.1470, 0.0168]	[0.0312, 0.0099]	[0.0027, 0.0028]	[0.5246, 0.0023]	[0.0121, 0.0221]	[0.0459, 0.0309]
$\Gamma_{p(9)_{z_a z_b}}$	[0.1395, 0.0162]	[0.0302, 0.0094]	[0.0026, 0.0026]	[0.5174, 0.0023]	[0.0119, 0.0215]	[0.0423, 0.0303]

$$\Pi_{iz_a,z_b} = \frac{(N-n)\mathrm{P}_{iz_a,z_b} + (n-1)}{N-1}$$

and for the joint probability of including the i^{th}, j^{th} unit is

$$\Pi_{ijz_a,z_b} = \frac{n-1}{N-1}\left[\frac{N-n}{N-2}\left(\mathrm{P}_{iz_a,z_b} + \mathrm{P}_{jz_a,z_b}\right) + \frac{n-2}{N-2}\right]$$

where

$$\mathrm{P}_{iz_a,z_b} = \frac{X_{z_a,z_b}}{\sum_{i=1}^{N} X_{z_a,z_b}}$$

It is worthy to note that PRE of proposed versus adapted estimators calculated as given below:

Table 4. MSE of Horvitz Thompson type estimators when $n = 4$.

	Populations					
Estimators	(1) Jan-Feb	(2) Mar-Apr	(3) May-Jun	(4) Jul-Aug	(5) Sep-Oct	(6) Nov-Dec
Adapted						
$\Gamma_{a(1)_{z_a,z_b}}$	[0.3045, 0.1530]	[0.0696, 0.0295]	[0.0089, 0.0043]	[0.7642, 0.0047]	[0.0374, 0.0357]	[0.0970, 0.1326]
$\Gamma_{a(2)_{z_a,z_b}}$	[0.2978, 0.1521]	[0.0688, 0.0295]	[0.0089, 0.0043]	[0.7642, 0.0047]	[0.0369, 0.0357]	[0.0951, 0.1320]
$\Gamma_{a(3)_{z_a,z_b}}$	[0.3020, 0.1527]	[0.0693, 0.0295]	[0.0089, 0.0043]	[0.7642, 0.0047]	[0.0373, 0.0357]	[0.0964, 0.1323]
$\Gamma_{a(4)_{z_a,z_b}}$	[0.2552, 0.1130]	[0.0552, 0.0169]	[0.0053, 0.0026]	[0.7420, 0.0032]	[0.0228, 0.0249]	[0.0723, 0.1084]
$\Gamma_{a(5)_{z_a,z_b}}$	[0.1925, 0.1186]	[0.0465, 0.0270]	[0.0078, 0.0041]	[0.7610, 0.0046]	[0.0198, 0.0331]	[0.0622, 0.1076]
$\Gamma_{a(6)_{z_a,z_b}}$	[0.0768, 0.0696]	[0.0442, 0.0281]	[0.0084, 0.0042]	[0.7642, 0.0037]	[0.0224, 0.0341]	[0.0393, 0.0268]
$\Gamma_{a(7)_{z_a,z_b}}$	[0.2472, 0.1385]	[0.0596, 0.0289]	[0.0087, 0.0042]	[0.7642, 0.0046]	[0.0303, 0.0350]	[0.0828, 0.1191]
$\Gamma_{a(8)_{z_a,z_b}}$	[0.3025, 0.1513]	[0.0683, 0.0293]	[0.0089, 0.0042]	[0.7643, 0.0047]	[0.0364, 0.0355]	[0.0966, 0.1325]
$\Gamma_{a(9)_{z_a,z_b}}$	[0.2748, 0.1424]	[0.0633, 0.0275]	[0.0082, 0.0036]	[0.7532, 0.0046]	[0.0318, 0.0340]	[0.0842, 0.1268]
Proposed						
$\Gamma_{p(1)_{z_a,z_b}}$	[0.1557, 0.0676]	[0.0345, 0.0157]	[0.0044, 0.0032]	[0.7508, 0.0030]	[0.0138, 0.0223]	[0.0446, 0.0663]
$\Gamma_{p(2)_{z_a,z_b}}$	[0.1535, 0.0673]	[0.0344, 0.0157]	[0.0044, 0.0032]	[0.7508, 0.0030]	[0.0138, 0.0223]	[0.0440, 0.0661]
$\Gamma_{p(3)_{z_a,z_b}}$	[0.1549, 0.0675]	[0.0345, 0.0157]	[0.0044, 0.0032]	[0.7508, 0.0030]	[0.0138, 0.0223]	[0.0444, 0.0662]
$\Gamma_{p(4)_{z_a,z_b}}$	[0.1392, 0.0555]	[0.0320, 0.0119]	[0.0033, 0.0025]	[0.7399, 0.0026]	[0.0153, 0.0191]	[0.0367, 0.0580]
$\Gamma_{p(5)_{z_a,z_b}}$	[0.1173, 0.0572]	[0.0306, 0.0150]	[0.0041, 0.0031]	[0.7492, 0.0029]	[0.0161, 0.0215]	[0.0333, 0.0577]
$\Gamma_{p(6)_{z_a,z_b}}$	[0.0717, 0.0417]	[0.0303, 0.0153]	[0.0043, 0.0032]	[0.7507, 0.0027]	[0.0154, 0.0218]	[0.0253, 0.0262]
$\Gamma_{p(7)_{z_a,z_b}}$	[0.1364, 0.0632]	[0.0327, 0.0155]	[0.0043, 0.0032]	[0.7507, 0.0029]	[0.0142, 0.0221]	[0.0401, 0.0617]
$\Gamma_{p(8)_{z_a,z_b}}$	[0.1551, 0.0671]	[0.0343, 0.0156]	[0.0044, 0.0032]	[0.7508, 0.0030]	[0.0138, 0.0223]	[0.0445, 0.0662]
$\Gamma_{p(9)_{z_a,z_b}}$	[0.1458, 0.0644]	[0.0334, 0.0151]	[0.0042, 0.0029]	[0.7454, 0.0029]	[0.0141, 0.0218]	[0.0405, 0.0643]

$$\text{PRE} = \frac{\Gamma_{a(i)_{z_a,z_b}}}{\Gamma_{p(i)_{z_a,z_b}}} \times 100$$

5. Results discussion

We explore the following points from numerical investigation:

- Tables 3, 4 , and 5 are showing the neutrosophic MSE results of dew pints data with $n = 3, 4, 5$, respectively. Further, clear hierarchy of decrements in MSE can be seen in the Tables 3, 4 , and 5 by increasing sample size. Same pattern can be seen in PRE Table 6.
- The results in Tables 3, 4 , 5 , and 6 also showing that proposed estimators are superior as compared to adapted ones.

Table 5. MSE of Horvitz Thompson type estimators when $n = 5$.

	Populations					
Estimators	(1) Jan-Feb	(2) Mar-Apr	(3) May-Jun	(4) Jul-Aug	(5) Sep-Oct	(6) Nov-Dec
Adapted						
$\Gamma_{a(1)_{z_a z_b}}$	[0.3445, 0.1741]	[0.0768, 0.0293]	[0.0092, 0.0048]	[0.9047, 0.0071]	[0.0492, 0.0199]	[0.0960, 0.1467]
$\Gamma_{a(2)_{z_a z_b}}$	[0.3368, 0.1731]	[0.0761, 0.0292]	[0.0092, 0.0048]	[0.9046, 0.0071]	[0.0486, 0.0199]	[0.0941, 0.1459]
$\Gamma_{a(3)_{z_a z_b}}$	[0.3416, 0.1738]	[0.0766, 0.0293]	[0.0092, 0.0048]	[0.9047, 0.0071]	[0.0490, 0.0199]	[0.0954, 0.1463]
$\Gamma_{a(4)_{z_a z_b}}$	[0.2878, 0.1263]	[0.0636, 0.0160]	[0.0053, 0.0010]	[0.8750, 0.0047]	[0.0328, 0.0091]	[0.0705, 0.1150]
$\Gamma_{a(5)_{z_a z_b}}$	[0.2159, 0.1329]	[0.0562, 0.0266]	[0.0080, 0.0045]	[0.9003, 0.0069]	[0.0294, 0.0172]	[0.0601, 0.1139]
$\Gamma_{a(6)_{z_a z_b}}$	[0.0854, 0.0757]	[0.0544, 0.0277]	[0.0086, 0.0048]	[0.9046, 0.0054]	[0.0323, 0.0182]	[0.0369, 0.0223]
$\Gamma_{a(7)_{z_a z_b}}$	[0.2786, 0.1567]	[0.0675, 0.0286]	[0.0089, 0.0048]	[0.9046, 0.0070]	[0.0412, 0.0192]	[0.0813, 0.1288]
$\Gamma_{a(8)_{z_a z_b}}$	[0.3422, 0.1722]	[0.0757, 0.0290]	[0.0092, 0.0048]	[0.9047, 0.0071]	[0.0481, 0.0197]	[0.0956, 0.1466]
$\Gamma_{a(9)_{z_a z_b}}$	[0.3103, 0.1614]	[0.0709, 0.0271]	[0.0084, 0.0032]	[0.8900, 0.0069]	[0.0429, 0.0181]	[0.0828, 0.1390]
Proposed						
$\Gamma_{p(1)_{z_a z_b}}$	[0.1740, 0.0734]	[0.0479, 0.0148]	[0.0043, 0.0022]	[0.8867, 0.0043]	[0.0225, 0.0068]	[0.0422, 0.0627]
$\Gamma_{p(2)_{z_a z_b}}$	[0.1715, 0.0732]	[0.0479, 0.0148]	[0.0043, 0.0022]	[0.8867, 0.0043]	[0.0226, 0.0067]	[0.0416, 0.0624]
$\Gamma_{p(3)_{z_a z_b}}$	[0.1731, 0.0733]	[0.0479, 0.0148]	[0.0043, 0.0022]	[0.8867, 0.0043]	[0.0225, 0.0067]	[0.0420, 0.0626]
$\Gamma_{p(4)_{z_a z_b}}$	[0.1552, 0.0599]	[0.0470, 0.0110]	[0.0032, 0.0009]	[0.8721, 0.0036]	[0.0243, 0.0042]	[0.0343, 0.0531]
$\Gamma_{p(5)_{z_a z_b}}$	[0.1305, 0.0618]	[0.0469, 0.0140]	[0.0040, 0.0021]	[0.8846, 0.0042]	[0.0252, 0.0061]	[0.0309, 0.0528]
$\Gamma_{p(6)_{z_a z_b}}$	[0.0798, 0.0451]	[0.0470, 0.0144]	[0.0042, 0.0022]	[0.8866, 0.0038]	[0.0244, 0.0063]	[0.0232, 0.0218]
$\Gamma_{p(7)_{z_a z_b}}$	[0.1521, 0.0685]	[0.0472, 0.0146]	[0.0042, 0.0022]	[0.8866, 0.0042]	[0.0230, 0.0066]	[0.0376, 0.0573]
$\Gamma_{p(8)_{z_a z_b}}$	[0.1733, 0.0729]	[0.0478, 0.0147]	[0.0043, 0.0022]	[0.8867, 0.0043]	[0.0226, 0.0067]	[0.0420, 0.0627]
$\Gamma_{p(9)_{z_a z_b}}$	[0.1628, 0.0699]	[0.0474, 0.0142]	[0.0041, 0.0017]	[0.8795, 0.0042]	[0.0229, 0.0063]	[0.0381, 0.0604]

- These finding underscore the effectiveness of our innovative approach in enhancing estimation accuracy, particularly in scenarios involving ambiguous observations. These results affirm the capability of neutrosophic estimators to offer dependable statistical insights and more accurate estimations of population means.

6. Conclusion

In this study, an innovative approach resolves the challenge of estimating averages when dealing with ambiguous observations, a task not achievable using conventional estimation techniques under PPS scheme. The study is significant in the sense that it cannot remove the effect of ambiguous observations but instead retain them under PPS scheme. Therefore, a novel method is

Table 6. PRE of proposed estimators with sample sizes $n = 3, 4, 5$.

		Populations					
n	Estimators	(1) Jan-Feb	(2) Mar-Apr	(3) May-Jun	(4) Jul-Aug	(5) Sep-Oct	(6) Nov-Dec
3	$\Gamma_{p(1)_{za,z_b}}$	[178.95, 200.01]	[174.62, 216.62]	[179.20, 135.83]	[103.39, 103.72]	[297.97, 168.06]	[199.87, 167.55]
	$\Gamma_{p(2)_{za,z_b}}$	[177.62, 199.74]	[174.15, 216.52]	[179.04, 135.80]	[103.38, 103.72]	[294.98, 167.98]	[198.69, 167.36]
	$\Gamma_{p(3)_{za,z_b}}$	[178.45, 199.92]	[174.47, 216.60]	[179.17, 135.83]	[103.39, 103.72]	[297.14, 168.04]	[199.52, 167.46]
	$\Gamma_{p(4)_{za,z_b}}$	[168.45, 185.00]	[165.14, 171.87]	[143.38, 103.50]	[100.56, 101.07]	[196.42, 141.31]	[182.31, 158.59]
	$\Gamma_{p(5)_{za,z_b}}$	[152.18, 187.41]	[157.61, 210.02]	[169.56, 133.88]	[102.99, 103.43]	[169.92, 162.51]	[173.34, 158.25]
	$\Gamma_{p(6)_{za,z_b}}$	[105.86, 160.46]	[155.17, 212.98]	[174.76, 135.73]	[103.38, 101.84]	[193.06, 164.66]	[146.87, 101.97]
	$\Gamma_{p(7)_{za,z_b}}$	[166.57, 195.10]	[168.32, 215.11]	[176.92, 135.77]	[103.38, 103.57]	[253.08, 166.58]	[190.39, 162.74]
	$\Gamma_{p(8)_{za,z_b}}$	[178.55, 199.49]	[173.89, 216.06]	[179.08, 135.77]	[103.40, 103.72]	[292.06, 167.64]	[199.62, 167.54]
	$\Gamma_{p(9)_{za,z_b}}$	[172.82, 196.47]	[170.79, 211.29]	[173.17, 124.69]	[102.01, 103.52]	[263.03, 164.56]	[191.41, 165.55]
4	$\Gamma_{p(1)_{za,z_b}}$	[195.52, 226.36]	[201.43, 188.06]	[202.78, 133.93]	[101.79, 158.73]	[272.32, 159.89]	[217.43, 200.10]
	$\Gamma_{p(2)_{za,z_b}}$	[194.00, 225.95]	[199.94, 187.95]	[202.61, 133.90]	[101.79, 158.69]	[268.18, 159.79]	[216.11, 199.82]
	$\Gamma_{p(3)_{za,z_b}}$	[194.95, 226.21]	[200.96, 188.04]	[202.74, 133.93]	[101.79, 158.71]	[271.17, 159.87]	[217.04, 199.96]
	$\Gamma_{p(4)_{za,z_b}}$	[183.39, 203.76]	[172.58, 142.51]	[162.15, 102.99]	[100.29, 125.56]	[148.95, 130.66]	[197.40, 187.07]
	$\Gamma_{p(5)_{za,z_b}}$	[164.10, 207.41]	[152.07, 180.53]	[192.42, 131.96]	[101.57, 155.66]	[122.83, 153.44]	[186.94, 186.56]
	$\Gamma_{p(6)_{za,z_b}}$	[107.08, 166.95]	[146.03, 183.87]	[198.05, 133.83]	[101.79, 136.62]	[145.50, 155.92]	[155.39, 102.62]
	$\Gamma_{p(7)_{za,z_b}}$	[181.19, 219.02]	[181.93, 186.31]	[200.36, 133.86]	[101.79, 157.10]	[213.35, 158.16]	[206.69, 193.15]
	$\Gamma_{p(8)_{za,z_b}}$	[195.06, 225.58]	[199.12, 187.41]	[202.65, 133.86]	[101.80, 158.68]	[264.16, 159.39]	[217.15, 200.09]
	$\Gamma_{p(9)_{za,z_b}}$	[188.47, 221.07]	[189.43, 181.97]	[196.34, 122.82]	[101.05, 156.58]	[225.81, 155.81]	[207.85, 197.22]
5	$\Gamma_{p(1)_{za,z_b}}$	[197.96, 237.10]	[160.34, 198.09]	[211.98, 215.20]	[102.03, 165.87]	[218.36, 295.11]	[227.71, 234.04]
	$\Gamma_{p(2)_{za,z_b}}$	[196.39, 236.62]	[158.97, 197.96]	[211.80, 215.14]	[102.02, 165.82]	[215.59, 294.92]	[226.27, 233.67]
	$\Gamma_{p(3)_{za,z_b}}$	[197.37, 236.93]	[159.91, 198.06]	[211.94, 215.20]	[102.03, 165.84]	[217.59, 295.06]	[227.28, 233.86]
	$\Gamma_{p(4)_{za,z_b}}$	[185.40, 210.80]	[135.38, 144.95]	[166.94, 112.47]	[100.33, 130.32]	[135.02, 215.17]	[205.81, 216.40]
	$\Gamma_{p(5)_{za,z_b}}$	[165.39, 215.07]	[119.87, 189.34]	[200.59, 210.54]	[101.78, 162.66]	[116.74, 281.39]	[194.24, 215.71]
	$\Gamma_{p(6)_{za,z_b}}$	[106.98, 167.89]	[115.68, 193.23]	[206.79, 214.97]	[102.02, 142.43]	[132.63, 286.87]	[159.07, 102.35]
	$\Gamma_{p(7)_{za,z_b}}$	[183.12, 228.60]	[143.07, 196.06]	[209.34, 215.05]	[102.02, 164.16]	[178.83, 291.60]	[216.01, 224.72]
	$\Gamma_{p(8)_{za,z_b}}$	[197.48, 236.20]	[158.22, 197.34]	[211.84, 215.05]	[102.03, 165.81]	[212.90, 294.11]	[227.41, 234.02]
	$\Gamma_{p(9)_{za,z_b}}$	[190.66, 230.97]	[149.53, 191.01]	[204.92, 186.00]	[101.19, 163.62]	[187.20, 286.64]	[217.29, 230.21]

defined for ambiguous observations to estimate the mean under PPS sampling conditions, first time. The evaluation of the proposed estimators is conducted through computer simulations utilizing real-world data. The results demonstrate superiority of $\Gamma_{p(i)_{za,z_b}}$ over $\Gamma_{a(i)_{za,z_b}}$ estimators, even when outlier appears in neutrosophic scenarios. In future studies, the work can be extended in light of Shahzad et al. (2021, 2022, 2023).

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