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Chapter 15
Neutrosophic TreeSoft Expert Set and ForestSoft Set

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Abstract

Concepts such as Fuzzy Sets [28,57], Neutrosophic Sets [42,44], and Plithogenic Sets [48] have been extensively studied to address uncertainty, finding diverse applications across various fields. The Soft Set provides a framework that associates each parameter with subsets of a universal set, enabling flexible approximations [31]. The TreeSoft Set extends the Soft Set by introducing hierarchical, tree-structured parameters, allowing for multi-level data representation [53].

In this paper, we revisit the concept of the Neutrosophic TreeSoft Set, which has been discussed in other studies [8, 34]. Additionally, we propose and examine the Neutrosophic TreeSoft Expert Set by incorporating the framework of the Neutrosophic Soft Expert Set. Furthermore, we revisit the ForestSoft Set, an extension of the TreeSoft Set, and explore related concepts, including the Neutrosophic ForestSoft Set.

Keywords: Neutrosophic Set, Soft Set, Treesoft Set, Neutrosophic Treesoft Set, ForestSoft Set

1 Preliminaries and Definitions

This section provides an introduction to the foundational concepts and definitions required for the discussions in this paper.

1.1 Neutrosophic Set

Neutrosophic Sets extend Fuzzy Sets by introducing the concept of indeterminacy, which accounts for situations that are neither entirely true nor entirely false [17–19, 21, 27, 43, 45–47, 54, 55].

Definition 1.1 (Neutrosophic Set)**.** [44, 45] Let X be a non-empty set. A *Neutrosophic Set (NS) A* on X is characterized by three membership functions:

$$T_A : X \to [0,1], \quad I_A : X \to [0,1], \quad F_A : X \to [0,1],$$

where for each $x \in X$, the values $T_A(x)$, $I_A(x)$, and $F_A(x)$ represent the degrees of truth, indeterminacy, and falsity, respectively. These values satisfy the following condition:

$$0 \le T_A(x) + I_A(x) + F_A(x) \le 3.$$

1.2 Soft Set and TreeSoft Set

A *Soft Set* (F, E) associates each parameter in a set E with a subset of a universal set U. This provides a flexible framework for approximating objects within U [24, 30, 31]. A *TreeSoft Set* is a mapping from subsets of a hierarchical, tree-like parameter structure Tree(A) to subsets of a universal set U. This structure supports multi-level attributes for more refined and detailed analyses [8, 14, 22, 32, 34, 36, 53]. Related concepts include the *Hypersoft Set* [20, 49] and the *SuperHypersoft Set* [15, 16, 50]. The definitions of Soft Set and TreeSoft Set are provided below.

Definition 1.2. [30] Let U be a universal set and E a set of parameters. A *soft set* over U is defined as an ordered pair (F, E), where F is a mapping from E to the power set $\mathcal{P}(U)$:

$$F : E \to \mathcal{P}(U).$$

For each parameter $e \in E$, $F(e) \subseteq U$ represents the set of e-approximate elements in U, with (F, E) forming a parameterized family of subsets of U.

Definition 1.3. [51] Let U be a universe of discourse, and let H be a non-empty subset of U, with $P(H)$ denoting the power set of H. Let $A = \{A_1, A_2, \ldots, A_n\}$ be a set of attributes (parameters, factors, etc.), for some integer $n \geq 1$, where each attribute A_i (for $1 \leq i \leq n$) is considered a first-level attribute.

Each first-level attribute A_i consists of sub-attributes, defined as:

$$A_i = \{A_{i,1}, A_{i,2}, \ldots\},$$

where the elements $A_{i,j}$ (for $j = 1, 2, \ldots$) are second-level sub-attributes of A_i. Each second-level sub-attribute $A_{i,j}$ may further contain sub-sub-attributes, defined as:

$$A_{i,j} = \{A_{i,j,1}, A_{i,j,2}, \ldots\},$$

and so on, allowing for as many levels of refinement as needed. Thus, we can define sub-attributes of an m-th level with indices $A_{i_1,i_2,\ldots,i_m}$, where each i_k (for $k = 1, \ldots, m$) denotes the position at each level.

This hierarchical structure forms a tree-like graph, which we denote as $\text{Tree}(A)$, with root A (level 0) and successive levels from 1 up to m, where m is the depth of the tree. The terminal nodes (nodes without descendants) are called *leaves* of the graph-tree.

A *TreeSoft Set F* is defined as a function:

$$F : P(\text{Tree}(A)) \to P(H),$$

where $\text{Tree}(A)$ represents the set of all nodes and leaves (from level 1 to level m) of the graph-tree, and $P(\text{Tree}(A))$ denotes its power set.

1.3 Neutrosophic Soft Set

The Neutrosophic Soft Set is a concept that combines the principles of Neutrosophic Sets and Soft Sets [2,5,6,9–11,25,33]. The definition is provided below.

Definition 1.4 (Neutrosophic Soft Set [26,29])**.** Let U be a universe and E a set of parameters. A *Neutrosophic Soft Set (NSS)* over U is defined as a pair (F, A), where $A \subseteq E$ and

$$F \;:\; A \;\longrightarrow\; P(U),$$

with $P(U)$ being the collection of *Neutrosophic Sets* on U. Hence for each parameter $e \in A$,

$$F(e) = \left(T_{F(e)}, I_{F(e)}, F_{F(e)}\right)$$

is a Neutrosophic Set on U, satisfying

$$0 \;\leq\; T_{F(e)}(x) \;+\; I_{F(e)}(x) \;+\; F_{F(e)}(x) \;\leq\; 3, \quad \forall x \in U.$$

1.4 Neutrosophic Soft Expert Set

The Neutrosophic Soft Expert Set [3,37–39,56] is an extension of the Neutrosophic Soft Set, incorporating the framework of the Soft Expert Set (cf. [1,4,7,23,35,41]). The formal definition is provided below.

Definition 1.5 (Neutrosophic Soft Expert Set (NSES))**.** (cf. [3,38,39,56]) Let U be a universe, E a set of parameters, X a set of experts (agents), and $O = \{1, 0\}$ a set of opinions, where 1 indicates *agreement* and 0 indicates *disagreement*. Define $Z = E \times X \times O$, and let $A \subseteq Z$.

A *Neutrosophic Soft Expert Set (NSES)* over U is a pair (F, A), where $A \subseteq Z$ and:

$$F : A \to P(U),$$

where $P(U)$ denotes the power set of Neutrosophic Sets on U. That is, for each parameter $e = (p, x, o) \in A$, $F(e)$ is a Neutrosophic Set $\left(T_{F(e)}, I_{F(e)}, F_{F(e)}\right)$ defined on U. The values of $T_{F(e)}(u)$, $I_{F(e)}(u)$, and $F_{F(e)}(u)$ satisfy:

$$0 \leq T_{F(e)}(u) + I_{F(e)}(u) + F_{F(e)}(u) \leq 3, \quad \forall u \in U.$$

2 Results in This Paper

The results derived in this paper are presented below.

2.1 Neutrosophic Treesoft Set (Revisit)

A Neutrosophic Treesoft Set maps hierarchical attribute subsets to neutrosophic sets, representing truth, indeterminacy, and falsity on a universe.

Definition 2.1 (Neutrosophic Treesoft Set)**.** Let $H \subseteq U$ be a non-empty subset of a universe U, and $\mathrm{Tree}(A)$ be a hierarchical structure of attributes as defined previously. A *Neutrosophic Treesoft Set* is a mapping

$$\mathcal{F} : P\big(\mathrm{Tree}(A)\big) \longrightarrow N(H),$$

where each value $\mathcal{F}(\Gamma)$ is a Neutrosophic Set on H. Namely, for each $\Gamma \subseteq \mathrm{Tree}(A)$,

$$\mathcal{F}(\Gamma) = \Big(T_{\mathcal{F}(\Gamma)},\ I_{\mathcal{F}(\Gamma)},\ F_{\mathcal{F}(\Gamma)}\Big),$$

with $T_{\mathcal{F}(\Gamma)}, I_{\mathcal{F}(\Gamma)}, F_{\mathcal{F}(\Gamma)} : H \to [0,1]$ satisfying

$$0 \ \le\ T_{\mathcal{F}(\Gamma)}(h) + I_{\mathcal{F}(\Gamma)}(h) + F_{\mathcal{F}(\Gamma)}(h) \ \le\ 3 \quad \forall\, h \in H.$$

Theorem 2.2 (Neutrosophic Soft Set as a Special Case of Neutrosophic Treesoft Set)**.** *Every Neutrosophic Soft Set can be naturally embedded into a Neutrosophic Treesoft Set.*

More precisely, let $\big(F, A\big)$ be a Neutrosophic Soft Set on universe U, where $F : A \to P(U)$ and each $F(e)$ is a Neutrosophic Set in U. Define a single-level tree *of attributes $Tree(A)$ whose nodes are exactly the distinct parameters in A (no further sub-attributes). Set $H := U$. Then we can construct a Neutrosophic Treesoft Set*

$$\mathcal{F} : P\big(Tree(A)\big) \longrightarrow N(H)$$

such that $\mathcal{F}(\{e\}) = F(e)$ for each $e \in A$. Thus $\big(F, A\big)$ appears as the restriction of $\mathcal{F}$ to singletons in $Tree(A)$.

Proof. Since A is the set of parameters used in $\big(F, A\big)$, we treat it as a *single-level* tree:

$$\mathrm{Tree}(A) \ =\ \{\, A_1, A_2, \ldots, A_n\},$$

where each $A_i \in A$. There are *no* additional sub-attributes, i.e., no deeper levels. Hence any $\Gamma \subseteq \mathrm{Tree}(A)$ is simply a subset $\Gamma \subseteq A$.

We wish to define $\mathcal{F} : P(\mathrm{Tree}(A)) \to N(H)$ so that:

$$\mathcal{F}(\{A_i\}) \ =\ F(A_i),$$

where $F(A_i)$ is already a Neutrosophic Set on U. Since $H = U$, we have $F(A_i) \in N(H)$.

A simple way is to let $\mathcal{F}(\Gamma)$ be the *pointwise union* (in the neutrosophic sense) of the Neutrosophic Sets $\{F(e) \mid e \in \Gamma\}$. Concretely, for each $h \in U$:

$$T_{\mathcal{F}(\Gamma)}(h) \ =\ \max_{e\in\Gamma}\big\{\, T_{F(e)}(h)\big\}, \quad I_{\mathcal{F}(\Gamma)}(h) \ =\ \min_{e\in\Gamma}\big\{\, I_{F(e)}(h)\big\}, \quad F_{\mathcal{F}(\Gamma)}(h) \ =\ \max_{e\in\Gamma}\big\{\, F_{F(e)}(h)\big\}.$$

(Or any other appropriate aggregator, e.g. t-norm/t-conorm pairs, depending on the application.)

Verification of Neutrosophic Condition. Because each $F(e)$ is a Neutrosophic Set, we have

$$0 \ \le\ T_{F(e)}(h) + I_{F(e)}(h) + F_{F(e)}(h) \ \le\ 3,$$

for all $e \in A$ and all $h \in U$. Taking pointwise maxima or minima of these values across $e \in \Gamma$ keeps us within the bounds $[0, 3]$. Thus

$$0 \ \le\ T_{\mathcal{F}(\Gamma)}(h) \ +\ I_{\mathcal{F}(\Gamma)}(h) \ +\ F_{\mathcal{F}(\Gamma)}(h) \ \le\ 3.$$

Hence $\mathcal{F}(\Gamma)$ is indeed a Neutrosophic Set on $H = U$.

If $\Gamma = \{e\} \subseteq A$, then by definition,

$$\mathcal{F}(\{e\}) \; = \; F(e).$$

Thus on singletons, $\mathcal{F}$ and F agree exactly. In other words, (F, A) is embedded into the Neutrosophic Treesoft structure $\mathcal{F}$.

Therefore, (F, A) emerges as a special (single-level) restriction of $\mathcal{F}$. This completes the proof. □

Theorem 2.3 (Restriction to TreeSoft Set)**.** *Let $\mathcal{F}$ be a Neutrosophic Treesoft Set as in Definition. For each $\Gamma \subseteq Tree(A)$, define*

$$G(\Gamma) \; = \; \{\, h \in H \;\big|\; T_{\mathcal{F}(\Gamma)}(h) \geq \alpha \;\; and \;\; I_{\mathcal{F}(\Gamma)}(h) \leq \beta \,\},$$

for some fixed thresholds $0 \leq \alpha, \beta \leq 1$. Then G is a (classical) TreeSoft Set in the sense of Definition.

Proof. Since $\mathcal{F}(\Gamma)$ is a Neutrosophic Set on H, we have numeric values $T_{\mathcal{F}(\Gamma)}(h)$ and $I_{\mathcal{F}(\Gamma)}(h)$. If we pick thresholds α and β, the set of all $h \in H$ satisfying $T_{\mathcal{F}(\Gamma)}(h) \geq \alpha$ and $I_{\mathcal{F}(\Gamma)}(h) \leq \beta$ is indeed a subset of H. This procedure, repeated for each $\Gamma \subseteq \mathrm{Tree}(A)$, defines a mapping

$$\Gamma \; \longmapsto \; G(\Gamma) \; \subseteq \; H.$$

But by Definition, a TreeSoft Set is any function from $P(\mathrm{Tree}(A))$ to $P(H)$. Hence G is precisely a classical TreeSoft Set, restricted by the chosen thresholds on the neutrosophic membership functions of $\mathcal{F}(\Gamma)$. □

Theorem 2.4 (Union and Intersection in a Neutrosophic Treesoft Set)**.** *Let $\mathcal{F}_1$ and $\mathcal{F}_2$ be two Neutrosophic Treesoft Sets, both mapping*

$$\mathcal{F}_1, \; \mathcal{F}_2 : P(Tree(A)) \; \longrightarrow \; N(H).$$

Define new mappings $\mathcal{F}^{\cup}$ and $\mathcal{F}^{\cap}$ by

$$\mathcal{F}^{\cup}(\Gamma) \; = \; \Big(T_{\mathcal{F}_1(\Gamma)} \vee T_{\mathcal{F}_2(\Gamma)}, \;\; I_{\mathcal{F}_1(\Gamma)} \wedge I_{\mathcal{F}_2(\Gamma)}, \;\; F_{\mathcal{F}_1(\Gamma)} \vee F_{\mathcal{F}_2(\Gamma)}\Big),$$

$$\mathcal{F}^{\cap}(\Gamma) \; = \; \Big(T_{\mathcal{F}_1(\Gamma)} \wedge T_{\mathcal{F}_2(\Gamma)}, \;\; I_{\mathcal{F}_1(\Gamma)} \vee I_{\mathcal{F}_2(\Gamma)}, \;\; F_{\mathcal{F}_1(\Gamma)} \wedge F_{\mathcal{F}_2(\Gamma)}\Big),$$

where $\vee$ and $\wedge$ are pointwise max *and* min *operators, respectively (or any suitable t-conorm/t-norm pair in $[0, 1]$). Then $\mathcal{F}^{\cup}$ and $\mathcal{F}^{\cap}$ are also Neutrosophic Treesoft Sets on H.*

Proof. For every $\Gamma \subseteq \mathrm{Tree}(A)$ and each $h \in H$, we define

$$T_{\mathcal{F}^{\cup}(\Gamma)}(h) := \max\{T_{\mathcal{F}_1(\Gamma)}(h), \; T_{\mathcal{F}_2(\Gamma)}(h)\}.$$

Similarly for $I_{\mathcal{F}^{\cup}(\Gamma)}(h)$ using min or max, depending on the intended aggregator, and for $F_{\mathcal{F}^{\cup}(\Gamma)}(h)$. Since each of $T_{\mathcal{F}_i(\Gamma)}, I_{\mathcal{F}_i(\Gamma)}, F_{\mathcal{F}_i(\Gamma)}$ lies in $[0, 1]$, their max and min also lie in $[0, 1]$. Thus $\big(T_{\mathcal{F}^{\cup}(\Gamma)}, I_{\mathcal{F}^{\cup}(\Gamma)}, F_{\mathcal{F}^{\cup}(\Gamma)}\big)$ is a well-defined triple of functions $H \to [0, 1]$.

We must show

$$0 \;\; \leq \;\; T_{\mathcal{F}^{\cup}(\Gamma)}(h) \; + \; I_{\mathcal{F}^{\cup}(\Gamma)}(h) \; + \; F_{\mathcal{F}^{\cup}(\Gamma)}(h) \;\; \leq \;\; 3,$$

and similarly for $\mathcal{F}^{\cap}$. Since

$$T_{\mathcal{F}_i(\Gamma)}(h) + I_{\mathcal{F}_i(\Gamma)}(h) + F_{\mathcal{F}_i(\Gamma)}(h) \;\; \leq \;\; 3$$

(for $i = 1, 2$), the pointwise max or min among the corresponding membership values also cannot exceed 3 in sum. Indeed, for any real numbers $a_1 + b_1 + c_1 \leq 3$ and $a_2 + b_2 + c_2 \leq 3$, taking $\max(a_1, a_2) + \max(b_1, b_2) + \max(c_1, c_2)$ or $\min(a_1, a_2) + \min(b_1, b_2) + \min(c_1, c_2)$ is at most 3. Clearly, the sum is also non-negative.

Hence for each Γ, $\mathcal{F}^{\cup}(\Gamma)$ and $\mathcal{F}^{\cap}(\Gamma)$ satisfy the neutrosophic condition on $[0, 1]^3$. This shows that $\mathcal{F}^{\cup}$ and $\mathcal{F}^{\cap}$ are indeed functions from $P(\mathrm{Tree}(A))$ into $N(H)$. Therefore, they qualify as Neutrosophic Treesoft Sets. □

2.2 Neutrosophic TreeSoft Expert Set

The Neutrosophic TreeSoft Expert Set is an extension of the TreeSoft Set, incorporating the framework of the Neutrosophic Soft Expert Set. A related concept, the TreeSoft Expert Set, is also well-known [13].

Definition 2.5 (Neutrosophic TreeSoft Expert Set (NTSES))**.** Let:

- $H \subseteq U$ be a non-empty subset of a universe U.
- $\mathrm{Tree}(A)$ be a hierarchical attribute structure with root A and possibly multiple levels of sub-attributes.
- X be a set of experts.
- $O = \{1, 0\}$ a set of opinions, where 1 indicates *agreement* and 0 indicates *disagreement*.

Define

$$Z = P\big(\mathrm{Tree}(A)\big) \times X \times O.$$

Let $S \subseteq Z$. A *Neutrosophic TreeSoft Expert Set (NTSES)* on H is the pair $(\mathcal{F}, S)$ where $\mathcal{F}$ is a mapping

$$\mathcal{F} : S \longrightarrow \mathcal{P}_{\mathrm{NS}}(H),$$

with $\mathcal{P}_{\mathrm{NS}}(H)$ denoting the collection of Neutrosophic Sets on H. Concretely, for each triple $(\Gamma, x, o) \in S$, where $\Gamma \subseteq \mathrm{Tree}(A)$, $x \in X$, and $o \in O$,

$$\mathcal{F}(\Gamma, x, o) = \Big(T_{\Gamma,x,o},\ I_{\Gamma,x,o},\ F_{\Gamma,x,o}\Big),$$

where

$$T_{\Gamma,x,o},\ I_{\Gamma,x,o},\ F_{\Gamma,x,o} : H \longrightarrow [0,1]$$

satisfy

$$0 \leq T_{\Gamma,x,o}(h) + I_{\Gamma,x,o}(h) + F_{\Gamma,x,o}(h) \leq 3, \quad \forall\, h \in H.$$

Remark 2.6. In words, for each *subset of the attribute tree* Γ, each *expert* x, and each *opinion* $o \in \{1, 0\}$, the NTSES assigns a *Neutrosophic* evaluation (T, I, F) on the domain H. This merges three main components:

1. The *hierarchical attribute structure* (TreeSoft notion),
2. The *expert-based* positive/negative opinion (Soft Expert notion),
3. The *Neutrosophic membership functions* for each element in H.

Theorem 2.7 (Reduction to Neutrosophic Soft Expert Set)**.** *Let* $(\mathcal{F}, S)$ *be a Neutrosophic TreeSoft Expert Set as in Definition 2.5. Suppose:*

- *The tree* $\mathrm{Tree}(A)$ *is* single-level *(i.e., it is isomorphic to a simple parameter set* E *with no deeper sub-attributes).*
- *We identify each node in* $\Gamma \subseteq \mathrm{Tree}(A)$ *with a parameter* $p \in E$.

Then, by restricting Γ *to singletons and letting* $S \subseteq E \times X \times O$*, the NTSES* $(\mathcal{F}, S)$ *becomes a standard Neutrosophic Soft Expert Set* (F, A).

Proof. If $\mathrm{Tree}(A)$ has only one level (no sub-attributes), then each $\Gamma \subseteq \mathrm{Tree}(A)$ is simply a subset of a finite set E. In the *Soft Expert* scenario, we typically select $\Gamma = \{p\} \subseteq E$.

Consider the restriction

$$S' = \big\{(\{p\}, x, o) \mid (\{p\}, x, o) \in S\big\}.$$

In other words, only the singletons $\{p\} \subseteq E$. On such triples, define

$$F(p, x, o) = \mathcal{F}(\{p\}, x, o).$$

Since $\mathcal{F}(\{p\}, x, o)$ is a Neutrosophic Set on $H \subseteq U$, we get exactly the form required by a Neutrosophic Soft Expert Set.

Hence the mapping $F : A \to \mathcal{P}_{\mathrm{NS}}(U)$ recovers the definition of an NSES, with $A = S' \subseteq E \times X \times O$. This completes the reduction proof. □

Theorem 2.8 (Reduction to TreeSoft Set). *Let $(\mathcal{F}, S)$ be a Neutrosophic TreeSoft Expert Set on H. Suppose we drop both the* expert *dimension X and the* opinion *set O by fixing a trivial single-expert set $\{x_0\}$ and a single-opinion set $\{1\}$. Then $(\mathcal{F}, S)$ reduces to a classical TreeSoft Set*

$$\widetilde{F} : P(\mathrm{Tree}(A)) \longrightarrow P(H),$$

by selecting, for each $\Gamma \subseteq \mathrm{Tree}(A)$, a crisp subset $\widetilde{F}(\Gamma) \subseteq H$ from the corresponding neutrosophic membership.

Proof. Let $X = \{x_0\}$ and $O = \{1\}$. Then

$$Z = P(\mathrm{Tree}(A)) \times X \times O = P(\mathrm{Tree}(A)) \times \{x_0\} \times \{1\}.$$

Any subset $S \subseteq Z$ effectively identifies a collection of $\Gamma_i \subseteq \mathrm{Tree}(A)$.

Since $\mathcal{F}(\Gamma, x_0, 1)$ is a Neutrosophic Set $(T_\Gamma, I_\Gamma, F_\Gamma)$ on H, one can define

$$\widetilde{F}(\Gamma) = \{h \in H \mid T_\Gamma(h) \geq \alpha\},$$

or any other threshold-based selection from $\{T_\Gamma, I_\Gamma, F_\Gamma\}$ (e.g. "include h if the truth-degree is sufficiently large and the false-degree is sufficiently small"). This yields a crisp subset $\widetilde{F}(\Gamma) \subseteq H$.

This mapping $\Gamma \mapsto \widetilde{F}(\Gamma)$ is precisely a function from $P(\mathrm{Tree}(A))$ into $P(H)$. By Definition, it constitutes a TreeSoft Set. Thus the NTSES collapses to a classic TreeSoft Set once the expert and opinion dimensions are trivialized and the neutrosophic membership is interpreted in a crisp manner. □

3 Additional Results of This Paper

As additional results of this paper, we explore the concept of the ForestSoft Set and its extended variants [12, 40, 52].

3.1 ForestSoft Set (Revisit)

A *ForestSoft Set* is formed by taking a collection of TreeSoft Sets and "gluing" (uniting) them together so as to obtain a single function whose domain is the union of all tree-nodes' power sets and whose values in $P(H)$ combine the images given by the individual TreeSoft Sets.

Definition 3.1 (ForestSoft Set). [52] Let U be a universe of discourse, $H \subseteq U$ be a non-empty subset, and $P(H)$ be the power set of H. Suppose we have a finite (or countable) collection of TreeSoft Sets

$$\{F_t : P(\mathrm{Tree}(A^{(t)})) \to P(H)\}_{t \in T},$$

where each F_t is a TreeSoft Set corresponding to a tree $\mathrm{Tree}(A^{(t)})$ of attributes $A^{(t)}$.

We construct a *forest* by taking the (disjoint) union of all these trees:

$$\mathrm{Forest}(\{A^{(t)}\}_{t \in T}) = \bigsqcup_{t \in T} \mathrm{Tree}(A^{(t)}).$$

A *ForestSoft Set*, denoted by

$$\mathbf{F} : P(\mathrm{Forest}(\{A^{(t)}\})) \longrightarrow P(H),$$

is defined as the *union* of all TreeSoft Set mappings F_t. Concretely, for any element $X \in P(\mathrm{Forest}(\{A^{(t)}\}))$, we set

$$\mathbf{F}(X) = \bigcup_{\substack{t \in T \\ X \cap \mathrm{Tree}(A^{(t)}) \neq \emptyset}} F_t(X \cap \mathrm{Tree}(A^{(t)})),$$

where we only apply F_t to that portion of X belonging to the tree $\mathrm{Tree}(A^{(t)})$.

3.2 Neutrosophic ForestSoft Set

A Neutrosophic ForestSoft Set maps hierarchical multi-tree structures to neutrosophic sets, enabling multi-level uncertainty representation across multiple attribute domains.

Definition 3.2 (Neutrosophic Forestsoft Set (NFS))**.** Let $H \subseteq U$ be a non-empty subset of a universe U. For each $t \in T$, suppose we have a *Neutrosophic Treesoft Set*:

$$\mathcal{F}_t \;:\; P\big(\mathrm{Tree}(A^{(t)})\big) \;\longrightarrow\; N(H).$$

The *forest* of attribute trees is

$$\mathrm{Forest}\big(\{A^{(t)}\}_{t\in T}\big) \;=\; \bigsqcup_{t\in T} \mathrm{Tree}\big(A^{(t)}\big).$$

Then a *Neutrosophic Forestsoft Set* **F** is a function

$$\mathbf{F} : P\Big(\mathrm{Forest}\big(\{A^{(t)}\}\big)\Big) \;\longrightarrow\; N(H),$$

defined by "combining" the outputs of $\mathcal{F}_t$. Concretely, for each

$$X \;\in\; P\Big(\mathrm{Forest}\big(\{A^{(t)}\}_{t\in T}\big)\Big),$$

we decompose X into its parts

$$X_t \;:=\; X \cap \mathrm{Tree}(A^{(t)}),$$

and define for each $h \in H$,

$$\begin{aligned}
T_{\mathbf{F}(X)}(h) &= \max_{t\in T:\, X_t\neq\varnothing} \Big\{T_{\mathcal{F}_t(X_t)}(h)\Big\},\\
I_{\mathbf{F}(X)}(h) &= \min_{t\in T:\, X_t\neq\varnothing} \Big\{I_{\mathcal{F}_t(X_t)}(h)\Big\},\\
F_{\mathbf{F}(X)}(h) &= \max_{t\in T:\, X_t\neq\varnothing} \Big\{F_{\mathcal{F}_t(X_t)}(h)\Big\}.
\end{aligned}$$

(One may also choose alternative aggregators, e.g. t-norm / t-conorm, as desired.) Thus,

$$\mathbf{F}(X) \;=\; \Big(T_{\mathbf{F}(X)},\; I_{\mathbf{F}(X)},\; F_{\mathbf{F}(X)}\Big)$$

is a Neutrosophic Set on H.

Remark 3.3. If $X \cap \mathrm{Tree}(A^{(t)}) = \varnothing$ for some t, that tree does not contribute to the aggregator. One could also define a "universal aggregator" over all $t \in T$, ignoring whether X_t is empty; practical usage may vary. The definitions above ensure that each portion $X_t \subseteq \mathrm{Tree}(A^{(t)})$ is mapped by $\mathcal{F}_t$, and then the results are *combined* in a neutrosophic manner.

Theorem 3.4 (Well-definedness of Neutrosophic Forestsoft Set)**.** *With notation as in Definition 3.2, let* **F** *be constructed from* $\{\mathcal{F}_t\}_{t\in T}$*. Then for every* $X \subseteq \mathrm{Forest}(\{A^{(t)}\})$*, the triple* $\mathbf{F}(X) = (T_{\mathbf{F}(X)}, I_{\mathbf{F}(X)}, F_{\mathbf{F}(X)})$ *is a valid Neutrosophic Set on* H*.*

Proof. Fix $X \subseteq$ Forest. For each t, write $\mathcal{F}_t(X_t) = (T_{t,X_t}, I_{t,X_t}, F_{t,X_t})$, where

$$0 \;\leq\; T_{t,X_t}(h) + I_{t,X_t}(h) + F_{t,X_t}(h) \;\leq\; 3 \quad \text{for all } h \in H.$$

Then

$$T_{\mathbf{F}(X)}(h) \;=\; \max\big\{\,T_{t,X_t}(h)\big\}_{t\in T^*},$$

where $T^* = \{t \in T \mid X_t \neq \varnothing\}$. Clearly, $\max\{\dots\} \in [0,1]$. Analogous statements hold for $I_{\mathbf{F}(X)}(h)$ (using min) and $F_{\mathbf{F}(X)}(h)$ (using max).

Sum check: For each h, let

$$a_t = T_{t,X_t}(h), \quad b_t = I_{t,X_t}(h), \quad c_t = F_{t,X_t}(h).$$

Since $a_t + b_t + c_t \leq 3$ for every t, we must show

$$T_{\mathbf{F}(X)}(h) \ + \ I_{\mathbf{F}(X)}(h) \ + \ F_{\mathbf{F}(X)}(h) \ \ \leq \ 3.$$

But

$$T_{\mathbf{F}(X)}(h) = \max_{t\in T^*} a_t, \quad I_{\mathbf{F}(X)}(h) = \min_{t\in T^*} b_t, \quad F_{\mathbf{F}(X)}(h) = \max_{t\in T^*} c_t.$$

In general, for real numbers $\{a_t, b_t, c_t\} \subseteq [0,1]$ with each $a_t + b_t + c_t \leq 3$, the combination $\max(a_t) + \min(b_t) + \max(c_t) \leq 3$. Indeed:

$$\max(a_t) + \max(c_t) \ \ \leq \ \ \max(a_t + c_t) \ \ \leq \ \ \max(a_t + b_t + c_t) \ \ \leq \ 3,$$

and adding $\min(b_t) \leq \max(b_t)$ maintains a sum ≤ 3. Hence

$$0 \ \ \leq \ \ T_{\mathbf{F}(X)}(h) + I_{\mathbf{F}(X)}(h) + F_{\mathbf{F}(X)}(h) \ \ \leq \ 3.$$

Thus $\mathbf{F}(X)$ is indeed a Neutrosophic Set on H. □

Theorem 3.5 (Generalization of Neutrosophic Treesoft Set)**.** *A Neutrosophic Forestsoft Set generalizes the Neutrosophic Treesoft Set. Concretely, if* $\left|\{A^{(t)}\}_{t\in T}\right| = 1$*, i.e. there is only* one *tree in the forest, then the Neutrosophic Forestsoft Set reduces to a Neutrosophic Treesoft Set.*

Proof. Take $T = \{t_0\}$. Then we have only one Neutrosophic Treesoft Set $\mathcal{F}_{t_0} : P(\mathrm{Tree}(A^{(t_0)})) \to N(H)$. The forest is

$$\mathrm{Forest}\big(\{A^{(t_0)}\}\big) \ = \ \mathrm{Tree}\big(A^{(t_0)}\big).$$

For $X \subseteq \mathrm{Tree}(A^{(t_0)})$, define

$$X_{t_0} = X \cap \mathrm{Tree}(A^{(t_0)}),$$

but $X_{t_0} = X$ since there is only one tree. The aggregator in Definition 3.2 simply picks

$$T_{\mathbf{F}(X)}(h) = T_{\mathcal{F}_{t_0}(X_{t_0})}(h), \quad I_{\mathbf{F}(X)}(h) = I_{\mathcal{F}_{t_0}(X_{t_0})}(h), \quad F_{\mathbf{F}(X)}(h) = F_{\mathcal{F}_{t_0}(X_{t_0})}(h).$$

Hence $\mathbf{F}(X) = \mathcal{F}_{t_0}(X)$. So $\mathbf{F}$ is exactly the same mapping as $\mathcal{F}_{t_0}$. Consequently, the Neutrosophic Forestsoft Set and the Neutrosophic Treesoft Set coincide when the "forest" has only one tree. □

Theorem 3.6 (Union and Intersection in a Neutrosophic Forestsoft Set)**.** *Let* $\mathbf{F}_1$ *and* $\mathbf{F}_2$ *be two Neutrosophic Forestsoft Sets, both mapping*

$$\mathbf{F}_1, \ \mathbf{F}_2 : P\Big(\mathrm{Forest}(\{A^{(t)}\}_{t\in T})\Big) \ \longrightarrow \ N(H).$$

Define new mappings $\mathbf{F}^{\cup}$ *and* $\mathbf{F}^{\cap}$ *by*

$$\mathbf{F}^{\cup}(X) \ = \ \Big(T_{\mathbf{F}_1(X)} \ \vee \ T_{\mathbf{F}_2(X)}, \ \ I_{\mathbf{F}_1(X)} \ \wedge \ I_{\mathbf{F}_2(X)}, \ \ F_{\mathbf{F}_1(X)} \ \vee \ F_{\mathbf{F}_2(X)}\Big),$$

$$\mathbf{F}^{\cap}(X) \ = \ \Big(T_{\mathbf{F}_1(X)} \ \wedge \ T_{\mathbf{F}_2(X)}, \ \ I_{\mathbf{F}_1(X)} \ \vee \ I_{\mathbf{F}_2(X)}, \ \ F_{\mathbf{F}_1(X)} \ \wedge \ F_{\mathbf{F}_2(X)}\Big),$$

where $\vee$ *and* $\wedge$ *are pointwise* max *and* min *operators in* $[0,1]$*. Then* $\mathbf{F}^{\cup}$ *and* $\mathbf{F}^{\cap}$ *are also Neutrosophic Forestsoft Sets on* H*.*

Proof. For each $X \subseteq \mathrm{Forest}(\{A^{(t)}\})$, we have $\mathbf{F}_1(X), \mathbf{F}_2(X) \in N(H)$. So

$$T_{\mathbf{F}_1(X)}, \ I_{\mathbf{F}_1(X)}, \ F_{\mathbf{F}_1(X)} \quad \text{and} \quad T_{\mathbf{F}_2(X)}, \ I_{\mathbf{F}_2(X)}, \ F_{\mathbf{F}_2(X)}$$

all lie in $[0,1]$. Their pointwise max or min values remain in $[0,1]$. Checking the sum condition

$$T + I + F \ \ \leq \ 3$$

follows the same argument used in Theorem 3.4, showing that $\mathbf{F}^{\cup}(X)$ and $\mathbf{F}^{\cap}(X)$ are valid Neutrosophic Sets. One can interpret $\mathbf{F}^{\cup}$ and $\mathbf{F}^{\cap}$ as "logical union" and "logical intersection" of the two Neutrosophic Forestsoft Sets. □

3.3 Neutrosophic ForestSoft Expert Set

The Neutrosophic ForestSoft Expert Set is a concept that combines the principles of the ForestSoft Set, Neutrosophic Set, and Soft Expert Set. Its definition is provided below.

Definition 3.7 (Neutrosophic ForestSoft Expert Set (NFS-ES))**.** Let:

- $H \subseteq U$ be a non-empty subset of a universe U.
- $\{\text{Tree}(A^{(t)})\}_{t\in T}$ be an indexed family of trees (each a hierarchical attribute structure). Their disjoint union is

$$\text{Forest}\big(\{A^{(t)}\}_{t\in T}\big) \;=\; \bigsqcup_{t\in T} \text{Tree}(A^{(t)}).$$

- X be a set of experts.
- $O = \{1, 0\}$ a set of opinions, where 1 indicates *agreement* and 0 indicates *disagreement*.

Define

$$Z \;=\; P\big(\text{Forest}(\{A^{(t)}\}_{t\in T})\big) \;\times\; X \;\times\; O.$$

A *Neutrosophic ForestSoft Expert Set (NFS-ES)* over H is a pair $\big(\mathbf{F}, S\big)$ where $S \subseteq Z$ and

$$\mathbf{F} : S \;\longrightarrow\; N(H),$$

assigns to each $\big(Y, x, o\big) \in S$ a Neutrosophic Set $\mathbf{F}(Y, x, o)$ on H. Concretely, for

$$\mathbf{F}(Y, x, o) \;=\; \big(T_{Y,x,o},\; I_{Y,x,o},\; F_{Y,x,o}\big),$$

we require

$$0 \;\le\; T_{Y,x,o}(h) \;+\; I_{Y,x,o}(h) \;+\; F_{Y,x,o}(h) \;\le\; 3, \quad \forall\, h \in H.$$

Remark 3.8. In words, for each:

- Subset $Y \subseteq \text{Forest}(\{A^{(t)}\})$ (possibly spanning multiple trees),
- Expert $x \in X$,
- Opinion $o \in \{1, 0\}$,

the NFSES structure $\mathbf{F}(Y, x, o)$ returns a triple $\big(T, I, F\big)$, describing the truth, indeterminacy, and falsity degrees of every element $h \in H$. This merges the multi-tree, multi-expert, and neutrosophic membership perspectives into a single formalism.

Theorem 3.9 (Generalization of Neutrosophic TreeSoft Expert Set)**.** *A Neutrosophic ForestSoft Expert Set (NFS-ES) generalizes the Neutrosophic TreeSoft Expert Set (NTSES). Specifically, if the forest consists of* $|T| = 1$ *tree, then the NFS-ES is precisely an NTSES.*

Proof. Suppose there is only a single tree $\text{Tree}(A^{(t_0)})$. Then

$$\text{Forest}\big(\{A^{(t_0)}\}\big) \;=\; \text{Tree}(A^{(t_0)}),$$

and

$$Z = P\Big(\text{Forest}(\{A^{(t_0)}\})\Big) \;\times\; X \times O = P\big(\text{Tree}(A^{(t_0)})\big) \;\times\; X \times O.$$

Hence a Neutrosophic ForestSoft Expert Set $\big(\mathbf{F}, S\big)$ is merely the assignment

$$\mathbf{F} : S \;\longrightarrow\; N(H),$$

where $S \subseteq P(\text{Tree}(A^{(t_0)})) \times X \times O$. But this is exactly the definition of a Neutrosophic TreeSoft Expert Set in NTSES. Therefore, NFS-ES reduces to NTSES when there is only one tree in the forest. □

Theorem 3.10 (Generalization of ForestSoft Set)**.** *A Neutrosophic ForestSoft Expert Set generalizes the (classical) ForestSoft Set. If we* trivialize *the neutrosophic membership into crisp subsets (e.g., choose a threshold* α *for truth and interpret "membership" above that threshold as 1, else 0), and collapse the expert-opinion dimension, the structure becomes a standard ForestSoft Set.*

Proof. Consider a Neutrosophic ForestSoft Expert Set $(\mathbf{F}, S)$ on $\mathrm{Forest}(\{A^{(t)}\})$. If we fix a single expert $x_0 \in X$ and a single opinion $o_0 \in O = \{1, 0\}$, then we only look at

$$S' = \{(Y, x_0, o_0) \mid Y \subseteq \mathrm{Forest}(\{A^{(t)}\})\} \subseteq S.$$

For each $Y \subseteq \mathrm{Forest}(\{A^{(t)}\})$, $\mathbf{F}(Y, x_0, o_0)$ is a Neutrosophic Set (T_Y, I_Y, F_Y). By imposing a crisping procedure (e.g., "include h if $T_Y(h) \geq \alpha$ and $F_Y(h) \leq \gamma$, etc."), we get a subset of H. Concretely, define

$$\widetilde{\mathbf{F}}(Y) = \{h \in H \mid T_Y(h) \geq \alpha,\ I_Y(h) \leq \beta,\ F_Y(h) \leq \gamma\},$$

for fixed thresholds α, β, γ. Then $\widetilde{\mathbf{F}} : P(\mathrm{Forest}(\{A^{(t)}\})) \to P(H)$ is precisely a *ForestSoft Set*, since each Y is mapped to a crisp subset of H. Thus, by ignoring additional experts/opinions and converting neutrosophic degrees into classical membership, we recover a standard ForestSoft Set structure. □

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Data Availability

This research is purely theoretical, involving no data collection or analysis. We encourage future researchers to pursue empirical investigations to further develop and validate the concepts introduced here.

Ethical Approval

As this research is entirely theoretical in nature and does not involve human participants or animal subjects, no ethical approval is required.

Conflicts of Interest

The authors confirm that there are no conflicts of interest related to the research or its publication.

Disclaimer

This work presents theoretical concepts that have not yet undergone practical testing or validation. Future researchers are encouraged to apply and assess these ideas in empirical contexts. While every effort has been made to ensure accuracy and appropriate referencing, unintentional errors or omissions may still exist. Readers are advised to verify referenced materials on their own. The views and conclusions expressed here are the authors' own and do not necessarily reflect those of their affiliated organizations.

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