

NEUTROSOPHIC VAGUE SOFT EXPERT SET THEORY

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Abstract. In this paper, we first introduce the concept of neutrosophic vague soft expert sets (NVSEs for short) which combines neutrosophic vague sets and soft expert sets to be more effective and useful. We also define its basic operation, namely complement, union, intersection, AND and OR along with illustrative examples, and study some related properties supporting proofs. In our model the user can know the opinion of all experts in one model.

Keywords. neutrosophic soft expert set; neutrosophic vague set; neutrosophic vague soft set; soft expert set.

1. Introduction

In reality, the limitation of precise research is increasingly being recognized in many fields, such as economics, social science, and management science, etc. It is well known that the real world is full of uncertainty, imprecision and vagueness, so researches on these areas are of great importance. In recent years, uncertain theories such as probability theory, fuzzy set theory [1], intuitionistic fuzzy set theory [2], vague set theory [3], rough set theory [4] and interval mathematics have developed greatly and achievements have been widely applied in lots of social fields. But as we know in some real life problems for proper description of an object in uncertain and ambiguous environment, we need to handle the indeterminate and incomplete information. But these theories do not handle the indeterminate and inconsistent information. Thus neutrosophic set (NS in short) is defined by Smarandache [5], as a new mathematical tool for dealing with problems involving incomplete, indeterminacy, inconsistent knowledge. In NS, the indeterminacy is quantified explicitly and truth-membership, indeterminacy membership, and false-membership are completely independent. Furthermore, many research and applications in the literature based on neutrosophic set are undertaken such as some aggregation operators of interval neutrosophic linguistic numbers for multiple attribute decision making [6], similarity measures between interval neutrosophic sets and their applications in

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multicriteria decision-making [7], multicriteria decision-making method using aggregation operators for simplified neutrosophic sets [8] and improved correlation coefficients of single valued neutrosophic sets and interval neutrosophic sets for multiple attribute decision making [9]. However, all of these theories have their inherent difficulties, in order to overcome these disadvantages, Molodtsov [10] firstly proposed a new mathematical tool called soft set theory to deal with uncertainty and imprecision and it has been demonstrated that this new theory brings about a rich potential for applications in decision making, measurement theory, game theory, etc.

Presently, research on the theoretical and application aspects of soft sets and its various generalizations are progressing rapidly. Thus After Molodtsov's work, some operations and application of soft sets were studied by Chen et al.[11] and Maji et al. [12,13] Also Maji et al.[14] have introduced the concept of fuzzy soft set, a more general concept, which is a combination of fuzzy set and soft set and studied its properties, and also Roy and Maji [15] used this theory to solve some decision-making problems. Vague soft set theory was provided by Xu [16], later on Alhazaymeh and Hassan [17] introduced the concept of generalized vague soft set and its application followed by possibility vague soft set along with a few applications in decision making [18], and interval-valued vague soft sets and its application. [19], they also introduced the concept of Possibility Interval-Valued Vague Soft Set [20]. By the concept of neutrosophic set and soft set, first time, Maji [21] introduced neutrosophic soft set. As a combination of neutrosophic set and vague set, Alkhazaleh [22] introduced the concept of neutrosophic vague set. Among the significant milestones in the development of the theory of soft sets and its generalizations is the element of expert sets which enables the users to know the opinion of all the experts in one model without the need for any operations. This new aspect has further improved the theory of soft sets and made it better suited to be used in solving decision making problems, especially when used with the more accurate generalizations of soft sets such as fuzzy soft sets, intuitionistic fuzzy soft sets, neutrosophic soft set and neutrosophic vague soft set. this aspect was established by Alkhazaleh and Salleh (see [23]). The duo of Alkhazaleh and Salleh then proceeded to introduce the notion of fuzzy soft expert sets [24] and gave the application of this concept in decision making and medical diagnosis problems. Hassan and Alhazaymeh introduced the theory of vague soft expert sets (see [25]) and used it to solve decision making and medical diagnosis problems. They also introduced the mapping on generalized vague soft expert set (see [26]).

In this paper we introduce the concept of neutrosophic vague soft expert set which is a combination of neutrosophic vague set and soft expert set, and then define its basic operation, namely complement, union, intersection, AND, and OR, and study their properties.

2. Preliminaries

In this section, we recall some basic notions in neutrosophic vague set, neutrosophic vague soft set, soft expert set and neutrosophic soft expert set.

DEFINITION 2.1. (see [22]) A neutrosophic vague set A_{NV} (NVS in short) on the universe of discourse X written as

$$A_{NV} = \{ \langle x; \widehat{T}_{A_{NV}}(x); \widehat{I}_{A_{NV}}(x); \widehat{F}_{A_{NV}}(x) \rangle; x \in X \}$$

whose truth-membership, indeterminacy - membership and falsity-membership functions is defined as $\widehat{T}_{A_{NV}}(x) = [T^-, T^+]$, $\widehat{I}_{A_{NV}}(x) = [I^-, I^+]$ and $\widehat{F}_{A_{NV}}(x) = [F^-, F^+]$ where

$$(1) T^+ = 1 - F^-, \quad (2) F^+ = 1 - T^- \text{ and } (3) -0 \leq T^- + I^- + F^- \leq 2^+.$$

DEFINITION 2.2. (see [22]) Let Ψ_{NV} be a NVS of the universe U where $\forall u_i \in U$, $\widehat{T}_{\Psi_{NV}}(x) = [1, 1]$, $\widehat{I}_{\Psi_{NV}}(x) = [0, 0]$, $\widehat{F}_{\Psi_{NV}}(x) = [0, 0]$, then Ψ_{NV} is called a unit NVS, where $1 \leq i \leq n$. Let Φ_{NV} be a NVS of the universe U where $\forall u_i \in U$, $\widehat{T}_{\Phi_{NV}}(x) = [0, 0]$, $\widehat{I}_{\Phi_{NV}}(x) = [1, 1]$, $\widehat{F}_{\Phi_{NV}}(x) = [1, 1]$, then Φ_{NV} is called a zero NVS, where $1 \leq i \leq n$.

DEFINITION 2.3. (see [22]) Let A_{NV} and B_{NV} be two NVSs of the universe U . If $\forall u_i \in U$,

$$(1) \widehat{T}_{A_{NV}}(u_i) = \widehat{T}_{B_{NV}}(u_i), \quad (2) \widehat{I}_{A_{NV}}(u_i) = \widehat{I}_{B_{NV}}(u_i) \text{ and } (3) \widehat{F}_{A_{NV}}(u_i) = \widehat{F}_{B_{NV}}(u_i).$$

then the NVS A_{NV} is equal to B_{NV} , denoted by $A_{NV} = B_{NV}$, where $1 \leq i \leq n$.

DEFINITION 2.4. (see [22]) Let A_{NV} and B_{NV} be two NVSs of the universe U . If $\forall u_i \in U$,

$$(1) \widehat{T}_{A_{NV}}(u_i) \leq \widehat{T}_{B_{NV}}(u_i), \quad (2) \widehat{I}_{A_{NV}}(u_i) \geq \widehat{I}_{B_{NV}}(u_i) \text{ and } (3) \widehat{F}_{A_{NV}}(u_i) \geq \widehat{F}_{B_{NV}}(u_i).$$

then the NVS A_{NV} is included by B_{NV} , denoted by $A_{NV} \subseteq B_{NV}$, where $1 \leq i \leq n$.

DEFINITION 2.5. (see [22]) The complement of a NVS A_{NV} is denoted by A^c and is defined by $\widehat{T}_{A_{NV}^c}(x) = [1 - T^+, 1 - T^-]$, $\widehat{I}_{A_{NV}^c}(x) = [1 - I^+, 1 - I^-]$ and $\widehat{F}_{A_{NV}^c}(x) = [1 - F^+, 1 - F^-]$.

DEFINITION 2.6. (see [22]) The union of two NVSs A_{NV} and B_{NV} is a NVS C_{NV} , written as $C_{NV} = A_{NV} \cup B_{NV}$, whose truth-membership, indeterminacy-membership and false-membership functions are related to those of A_{NV} and B_{NV} given by

$$\begin{aligned} \widehat{T}_{C_{NV}}(x) &= \left[\max \left(T_{A_{NV,x}}^-, T_{B_{NV,x}}^- \right), \max \left(T_{A_{NV,x}}^+, T_{B_{NV,x}}^+ \right) \right], \\ \widehat{I}_{C_{NV}}(x) &= \left[\min \left(I_{A_{NV,x}}^-, I_{B_{NV,x}}^- \right), \min \left(I_{A_{NV,x}}^+, I_{B_{NV,x}}^+ \right) \right] \text{ and} \\ \widehat{F}_{C_{NV}}(x) &= \left[\min \left(F_{A_{NV,x}}^-, F_{B_{NV,x}}^- \right), \min \left(F_{A_{NV,x}}^+, F_{B_{NV,x}}^+ \right) \right] \end{aligned}$$

DEFINITION 2.7. (see [22]) The intersection of two NVSs A_{NV} and B_{NV} is a NVS C_{NV} , written as $H_{NV} = A_{NV} \cap B_{NV}$, whose truth-membership, indeterminacy-membership and false-membership functions are related to those of A_{NV} and B_{NV} given by

$$\begin{aligned} \widehat{T}_{H_{NV}}(x) &= \left[\min \left(T_{A_{NV,x}}^-, T_{B_{NV,x}}^- \right), \min \left(T_{A_{NV,x}}^+, T_{B_{NV,x}}^+ \right) \right], \\ \widehat{I}_{H_{NV}}(x) &= \left[\max \left(I_{A_{NV,x}}^-, I_{B_{NV,x}}^- \right), \max \left(I_{A_{NV,x}}^+, I_{B_{NV,x}}^+ \right) \right] \text{ and} \\ \widehat{F}_{H_{NV}}(x) &= \left[\max \left(F_{A_{NV,x}}^-, F_{B_{NV,x}}^- \right), \max \left(F_{A_{NV,x}}^+, F_{B_{NV,x}}^+ \right) \right] \end{aligned}$$

DEFINITION 2.8. (see [22]) Let U be an initial universal set and let E be a set of parameters. Let $NV(U)$ denote the power set of all neutrosophic vague subsets of U and

let $A \subseteq E$. A collection of pairs (\widehat{F}, E) is called a *neutrosophic vague soft set* $\{NVSS\}$ over U where \widehat{F} is a mapping given by

$$\widehat{F} : A \rightarrow NV(U).$$

Let U be a universe, E a set of parameters, X a set of experts (agents), and O a set of opinions. Let $Z = E \times X \times O$ and $A \subseteq Z$.

DEFINITION 2.9. (see [23]) A pair (F, A) is called a soft expert set over U , where F is a mapping given by

$$F : A \rightarrow P(U)$$

where $P(U)$ denotes the power set of U .

Let U be a universe, E a set of parameters, X a set of experts (agents), and $O = \{1 = agree, 0 = disagree\}$ a set of opinions. Let $Z = E \times X \times O$ and $A \subseteq Z$.

DEFINITION 2.10. (see [27]) A pair (F, A) is called a neutrosophic soft expert set (NSES in short) over U , where F is a mapping given by

$$F : A \rightarrow P(U)$$

where $P(U)$ denotes the power neutrosophic set of U .

DEFINITION 2.11. (see [27]) Let (F, A) and (G, B) be two NSESs over the common universe U . (F, A) is said to be neutrosophic soft expert subset of (G, B) , if $A \widetilde{\subseteq} B$ and $T_{F(e)}(X) \widetilde{\leq} T_{G(e)}(X)$, $I_{F(e)}(X) \widetilde{\leq} I_{G(e)}(X)$, $F_{F(e)}(X) \widetilde{\geq} F_{G(e)}(X) \forall e \in A, X \in U$. We denote it by $(F, A) \widetilde{\subseteq} (G, B)$.

(F, A) is said to be neutrosophic soft expert superset of (G, B) if (G, B) is a neutrosophic soft expert subset of (F, A) . We denote by $(F, A) \widetilde{\supseteq} (G, B)$.

DEFINITION 2.12. (see [27]) Two (NSESs), (F, A) and (G, B) over the common universe U are said to be equal if (F, A) is neutrosophic soft expert subset of (G, B) and (G, B) is neutrosophic soft expert subset of (F, A) . We denote it by $(F, A) = (G, B)$.

DEFINITION 2.13. (see [27]) NOT set of set parameters. Let $E = \{-e_1, e_2, \dots, e_n\}$ be a set of parameters. The NOT set of E is denoted by $\neg E = \{\neg e_1, \neg e_2, \dots, \neg e_n\}$, where $\neg e_i = \text{not } e_i, \forall i$.

DEFINITION 2.14. (see [27]) The complement of a NSES (F, A) denoted by $(F, A)^c$ and is defined as $(F, A)^c = (F^c, \neg A)$ where $F^c : \neg A \rightarrow P(U)$ is mapping given by $F^c(x) = \text{neutrosophic soft expert complement with } T_{F^c(x)} = F_{F(x)}, I_{F^c(x)} = I_{F(x)}, F_{F^c(x)} = T_{F(x)}$.

DEFINITION 2.15. (see [27]) An agree-NSES $(F, A)_1$ over U is a neutrosophic soft expert subset of (F, A) defined as follow

$$(F,A)_1 = \{F_1(m) : m \in E \times X \times \{1\}\}$$

DEFINITION 2.16. (see [27]) A disagree-NSES $(F,A)_0$ over U is a neutrosophic soft expert subset of (F, A) defined as follow

$$(F,A)_0 = \{F_0(m) : m \in E \times X \times \{0\}\}$$

DEFINITION 2.17. (see [27]) Let (H, A) and (G, B) be two NSESs over the common universe U . Then the union of (H, A) and (G, B) is denoted by $(H,A)\widetilde{\cup}(G,B)$ and is defined by $(H,A)\widetilde{\cup}(G,B) = (K,C)$, where $C = A \cup B$ and the truthmembership, indeterminacy-membership and falsity-membership of (K,C) are as follows:

$$T_{H(e)}(m) = \begin{cases} T_{H(e)}(m), & \text{if } e \in A - B, \\ T_{G(e)}(m), & \text{if } e \in B - A, \\ \max(T_{H(e)}(m), T_{G(e)}(m)), & \text{if } e \in A \cap B, \end{cases}$$

$$I_{H(e)}(m) = \begin{cases} I_{H(e)}(m), & \text{if } e \in A - B, \\ I_{G(e)}(m), & \text{if } e \in B - A, \\ \frac{I_{H(e)}(m) + I_{G(e)}(m)}{2}, & \text{if } e \in A \cap B, \end{cases}$$

$$F_{H(e)}(m) = \begin{cases} F_{H(e)}(m), & \text{if } e \in A - B, \\ F_{G(e)}(m), & \text{if } e \in B - A, \\ \min(F_{H(e)}(m), F_{G(e)}(m)), & \text{if } e \in A \cap B. \end{cases}$$

DEFINITION 2.18. (see [27]) Let (H, A) and (G, B) be two NSESs over the common universe U . Then the intersection of (H, A) and (G, B) is denoted by $(H,A)\widetilde{\cap}(G,B)$ and is defined by $(H,A)\widetilde{\cap}(G,B) = (K,C)$, where $C = A \cap B$ and the truth membership, indeterminacy-membership and falsity-membership of (K,C) are as follows:

$$T_{K(e)}(m) = \min(T_{H(e)}(m), T_{G(e)}(m))$$

$$I_{K(e)}(m) = \frac{I_{H(e)}(m) + I_{G(e)}(m)}{2}$$

$$F_{K(e)}(m) = \max(F_{H(e)}(m), F_{G(e)}(m)), \text{ if } e \in A \cap B$$

DEFINITION 2.19. (see [27]) AND operation on two NSESs. Let (H, A) and (G, B) be two NSESs over the common universe U . Then AND operation on them is denoted by $(H,A)\widetilde{\wedge}(G,B)$ and is defined by $(H,A)\widetilde{\wedge}(G,B) = (K,A \times B)$, where the truth-membership, indeterminacy-membership and falsity-membership of $(K,A \times B)$ are as follows:

$$T_{K(\alpha,\beta)}(m) = \min(T_{H(\alpha)}(m), T_{G(\beta)}(m))$$

$$I_{K(\alpha,\beta)}(m) = \frac{I_{H(\alpha)}(m) + I_{G(\beta)}(m)}{2}$$

$$F_{K(\alpha,\beta)}(m) = \max(F_{H(\alpha)}(m), F_{G(\beta)}(m)), \forall \alpha \in A, \forall \beta \in B$$

DEFINITION 2.20. (see [27]) OR operation on two NSESs. Let (H, A) and (G, B) be two NSESs over the common universe U . Then OR operation on them is denoted by $(H, A) \tilde{\vee} (G, B)$ and is defined by $(H, A) \tilde{\vee} (G, B) = (O, A \times B)$, where the truth-membership, indeterminacy-membership and falsity-membership of $(O, A \times B)$ are as follows:

$$T_{O(\alpha, \beta)}(m) = \max(T_{H(\alpha)}(m), T_{G(\beta)}(m))$$

$$I_{O(\alpha, \beta)}(m) = \frac{I_{H(\alpha)}(m) + I_{G(\beta)}(m)}{2}$$

$$F_{O(\alpha, \beta)}(m) = \min(F_{H(\alpha)}(m), F_{G(\beta)}(m)), \forall \alpha \in A, \forall \beta \in B$$

3. Neutrosophic Vague Soft Expert Set

In this section, we introduce the definition of a neutrosophic vague soft expert set and give basic properties of this concept.

Let U be a universe, E a set of parameters, X a set of experts (agents), and $O = \{1 = \text{agree}, 0 = \text{disagree}\}$ a set of opinions. Let $Z = E \times X \times O$ and $A \subseteq Z$.

DEFINITION 3.1. A pair (F, A) is called a neutrosophic vague soft expert set over U , where F is a mapping given by

$$F : A \rightarrow NV^U$$

where NV^U denotes the power neutrosophic vague set of U .

Example 3.2. Suppose that a company produced new types of its products and wishes to take the opinion of some experts about concerning these products. Let $U = \{u_1, u_2, u_3, u_4\}$ be a set of products, $E = \{e_1, e_2\}$ a set of decision parameters where $e_i (i = 1, 2)$ denotes the decision easy to use, and quality, respectively, and let $X = \{p, q\}$ be a set of experts. Suppose that the company has distributed a questionnaire to three experts to make decisions on the company's products, and we get the following:

$$F(e_1, p, 1) = \left\{ \begin{array}{l} \frac{u_1}{\langle [0.2, 0.8]; [0.1, 0.3]; [0.2, 0.8] \rangle}, \frac{u_2}{\langle [0.1, 0.7]; [0.2, 0.5]; [0.3, 0.9] \rangle}, \\ \frac{u_3}{\langle [0.5, 0.6]; [0.3, 0.7]; [0.4, 0.5] \rangle}, \frac{u_4}{\langle [0.8, 1]; [0.1, 0.2]; [0, 0.2] \rangle} \end{array} \right\}$$

$$F(e_1, q, 1) = \left\{ \begin{array}{l} \frac{u_1}{\langle [0.8, 0.9]; [0.3, 0.4]; [0.1, 0.2] \rangle}, \frac{u_2}{\langle [0.2, 0.4]; [0.2, 0.4]; [0.6, 0.8] \rangle}, \\ \frac{u_3}{\langle [0, 0.5]; [0.5, 0.7]; [0.5, 1] \rangle}, \frac{u_4}{\langle [0.6, 0.7]; [0.2, 0.4]; [0.3, 0.4] \rangle} \end{array} \right\},$$

$$F(e_2, p, 1) = \left\{ \begin{array}{l} \frac{u_1}{\langle [0.3, 0.9]; [0.1, 0.3]; [0.1, 0.7] \rangle}, \frac{u_2}{\langle [0.2, 0.5]; [0.2, 0.5]; [0.5, 0.8] \rangle}, \\ \frac{u_3}{\langle [0.6, 0.9]; [0.1, 0.7]; [0.1, 0.4] \rangle}, \frac{u_4}{\langle [0.2, 0.4]; [0.2, 0.2]; [0.6, 0.8] \rangle} \end{array} \right\}$$

$$F(e_2, q, 1) = \left\{ \begin{array}{l} \frac{u_1}{\langle [0.4, 0.6]; [0.1, 0.4]; [0.4, 0.6] \rangle}, \frac{u_2}{\langle [0.1, 0.3]; [0.2, 0.4]; [0.7, 0.9] \rangle}, \end{array} \right\}$$

$$\begin{aligned}
& \left. \frac{u_3}{\langle [0.1,0.5];[0.5,0.7];[0.5,0.9] \rangle}, \frac{u_4}{\langle [0.2,0.7];[0.2,0.4];[0.3,0.8] \rangle} \right\}, \\
F(e_1, p, 0) &= \left\{ \frac{u_1}{\langle [0.1,0.7];[0.2,0.3];[0.3,0.9] \rangle}, \frac{u_2}{\langle [0.7,0.9];[0.5,0.5];[0.1,0.3] \rangle}, \right. \\
& \left. \frac{u_3}{\langle [0.1,0.2];[0.3,0.4];[0.8,0.9] \rangle}, \frac{u_4}{\langle [0.2,0.3];[0.2,0.4];[0.7,0.8] \rangle} \right\} \\
F(e_1, q, 0) &= \left\{ \frac{u_1}{\langle [0.2,0.3];[0.3,0.5];[0.7,0.8] \rangle}, \frac{u_2}{\langle [0.5,0.7];[0.3,0.4];[0.3,0.5] \rangle}, \right. \\
& \left. \frac{u_3}{\langle [0.5,0.8];[0.6,0.7];[0.2,0.5] \rangle}, \frac{u_4}{\langle [0.1,0.2];[0.4,0.5];[0.8,0.9] \rangle} \right\}, \\
F(e_2, p, 0) &= \left\{ \frac{u_1}{\langle [0.1,0.2];[0.3,0.3];[0.8,0.9] \rangle}, \frac{u_2}{\langle [0.7,0.8];[0.4,0.5];[0.2,0.3] \rangle}, \right. \\
& \left. \frac{u_3}{\langle [0.1,0.4];[0.5,0.7];[0.6,0.9] \rangle}, \frac{u_4}{\langle [0.5,0.8];[0.2,0.5];[0.2,0.5] \rangle} \right\} \\
F(e_2, q, 0) &= \left\{ \frac{u_1}{\langle [0.7,0.8];[0.3,0.4];[0.2,0.3] \rangle}, \frac{u_2}{\langle [0.5,0.6];[0.8,0.9];[0.4,0.5] \rangle}, \right. \\
& \left. \frac{u_3}{\langle [0.6,0.7];[0.6,0.7];[0.3,0.4] \rangle}, \frac{u_4}{\langle [0.8,0.9];[0.2,0.4];[0.1,0.2] \rangle} \right\}.
\end{aligned}$$

The neutrosophic vague soft expert set (F, Z) is a parameterized family $\{F(e_i), i = 1, 2, 3, \dots\}$ of all neutrosophic vague sets of U and describes a collection of approximation of an object.

DEFINITION 3.3. Let (F, A) and (G, B) be two neutrosophic vague soft expert sets over the common universe U . (F, A) is said to be neutrosophic vague soft expert subset of (G, B) if

1. $A \subseteq B$
2. $\forall \varepsilon \in A, F(\varepsilon)$ is a neutrosophic vague subset of $G(\varepsilon)$.

This relationship is denoted by $(F, A) \widetilde{\subseteq} (G, B)$. In this case (G, B) is called a neutrosophic vague soft expert superset of (F, A) .

DEFINITION 3.4. Two neutrosophic vague soft expert sets (F, A) and (G, B) over U are said to be equal if (F, A) is a neutrosophic vague soft expert subset of (G, B) and (G, B) is a neutrosophic vague soft expert subset of (F, A) .

Example 3.5. Consider Example 3.2. Suppose that the company takes the opinion of the experts once again after a month of using the products. Let

$$A = \{(e_1, p, 1), (e_1, q, 0)\} \text{ and } B = \{(e_1, p, 1), (e_1, q, 0), (e_2, p, 1)\}$$

Clearly $A \subseteq B$. Let (F, A) and (G, B) be defined as follows:

$$\begin{aligned}
(F, A) = & \left\{ \left((e_1, p, 1), \left\{ \frac{u_1}{\langle [0.2, 0.8]; [0.1, 0.3]; [0.2, 0.8] \rangle}, \frac{u_2}{\langle [0.1, 0.7]; [0.2, 0.5]; [0.3, 0.9] \rangle}, \right. \right. \\
& \left. \left. \frac{u_3}{\langle [0.1, 0.2]; [0.6, 0.7]; [0.8, 0.9] \rangle}, \frac{u_4}{\langle [0.4, 0.5]; [0.1, 0.2]; [0.5, 0.6] \rangle} \right\} \right), \\
& \left((e_1, q, 0), \left\{ \frac{u_1}{\langle [0.6, 0.9]; [0.2, 0.4]; [0.1, 0.4] \rangle}, \frac{u_2}{\langle [0.7, 0.8]; [0.3, 0.5]; [0.2, 0.3] \rangle}, \right. \right. \\
& \left. \left. \frac{u_3}{\langle [0.8, 0.9]; [0.3, 0.4]; [0.1, 0.2] \rangle}, \frac{u_4}{\langle [0.5, 0.6]; [0.5, 0.6]; [0.4, 0.5] \rangle} \right\} \right) \Big\}, \\
(G, B) = & \left\{ \left((e_1, p, 1), \left\{ \frac{u_1}{\langle [0.3, 0.9]; [0.1, 0.2]; [0.1, 0.7] \rangle}, \frac{u_2}{\langle [0.5, 0.7]; [0.2, 0.5]; [0.3, 0.5] \rangle}, \right. \right. \\
& \left. \left. \frac{u_3}{\langle [0.5, 0.6]; [0.3, 0.7]; [0.4, 0.5] \rangle}, \frac{u_4}{\langle [0.8, 1]; [0.1, 0.2]; [0.0, 2] \rangle} \right\} \right), \\
& \left((e_1, q, 0), \left\{ \frac{u_1}{\langle [0.7, 0.9]; [0.2, 0.3]; [0.1, 0.3] \rangle}, \frac{u_2}{\langle [0.8, 0.9]; [0.2, 0.4]; [0.1, 0.2] \rangle}, \right. \right. \\
& \left. \left. \frac{u_3}{\langle [0.9, 0.9]; [0.1, 0.3]; [0.1, 0.1] \rangle}, \frac{u_4}{\langle [0.8, 0.9]; [0.3, 0.4]; [0.1, 0.2] \rangle} \right\} \right), \\
& \left((e_2, p, 1), \left\{ \frac{u_1}{\langle [0.1, 0.2]; [0.2, 0.4]; [0.8, 0.9] \rangle}, \frac{u_2}{\langle [0.7, 0.8]; [0.3, 0.5]; [0.2, 0.3] \rangle}, \right. \right. \\
& \left. \left. \frac{u_3}{\langle [0.8, 0.8]; [0.1, 0.4]; [0.2, 0.2] \rangle}, \frac{u_4}{\langle [0.5, 0.6]; [0.5, 0.6]; [0.4, 0.5] \rangle} \right\} \right) \Big\}.
\end{aligned}$$

Therefore $(F, A) \widetilde{\subseteq} (G, B)$.

DEFINITION 3.6. An agree-neutrosophic vague soft expert set $(F, A)_1$ over U is a neutrosophic vague soft expert subset of (F, A) defined as follow

$$(F, A)_1 = \{F_1(m) : m \in E \times X \times \{1\}\}$$

Example 3.7. Consider Example 3.2. Then the agree-neutrosophic vague soft expert set $(F, A)_1$ over U is

$$\begin{aligned}
(F, A)_1 = & \left\{ \left((e_1, p, 1), \left\{ \frac{u_1}{\langle [0.2, 0.8]; [0.1, 0.3]; [0.2, 0.8] \rangle}, \frac{u_2}{\langle [0.1, 0.7]; [0.2, 0.5]; [0.3, 0.9] \rangle}, \right. \right. \\
& \left. \left. \frac{u_3}{\langle [0.5, 0.6]; [0.3, 0.7]; [0.4, 0.5] \rangle}, \frac{u_4}{\langle [0.8, 1]; [0.1, 0.2]; [0.0, 2] \rangle} \right\} \right), \\
& \left((e_1, q, 1), \left\{ \frac{u_1}{\langle [0.8, 0.9]; [0.3, 0.4]; [0.1, 0.2] \rangle}, \frac{u_2}{\langle [0.2, 0.4]; [0.2, 0.4]; [0.6, 0.8] \rangle}, \right. \right. \\
& \left. \left. \frac{u_3}{\langle [0.0, 5]; [0.5, 0.7]; [0.5, 1] \rangle}, \frac{u_4}{\langle [0.6, 0.7]; [0.2, 0.4]; [0.3, 0.4] \rangle} \right\} \right), \\
& \left((e_2, p, 1), \left\{ \frac{u_1}{\langle [0.3, 0.9]; [0.1, 0.3]; [0.1, 0.7] \rangle}, \frac{u_2}{\langle [0.2, 0.5]; [0.2, 0.5]; [0.5, 0.8] \rangle}, \right. \right. \\
& \left. \left. \frac{u_3}{\langle [0.6, 0.9]; [0.1, 0.7]; [0.1, 0.4] \rangle}, \frac{u_4}{\langle [0.2, 0.4]; [0.2, 0.2]; [0.6, 0.8] \rangle} \right\} \right), \\
& \left((e_2, q, 1), \left\{ \frac{u_1}{\langle [0.4, 0.6]; [0.1, 0.4]; [0.4, 0.6] \rangle}, \frac{u_2}{\langle [0.1, 0.3]; [0.2, 0.4]; [0.7, 0.9] \rangle}, \right. \right. \\
& \left. \left. \frac{u_3}{\langle [0.1, 0.5]; [0.5, 0.7]; [0.5, 0.9] \rangle}, \frac{u_4}{\langle [0.2, 0.7]; [0.2, 0.4]; [0.3, 0.8] \rangle} \right\} \right) \Big\}.
\end{aligned}$$

DEFINITION 3.8. A disagree-neutrosophic vague soft expert set $(F, A)_0$ over U is a neutrosophic vague soft expert subset of (F, A) defined as follows:

$$(F, A)_0 = \{F_0(m) : m \in E \times X \times \{0\}\}$$

Example 3.9. Consider Example 3.2. Then the disagree-neutrosophic vague soft expert set $(F, A)_0$ over U is

$$(F, A)_0 = \left\{ \left((e_1, p, 0), \left\{ \begin{array}{l} \frac{u_1}{\langle [0.1, 0.7]; [0.2, 0.3]; [0.3, 0.9] \rangle}, \frac{u_2}{\langle [0.7, 0.9]; [0.5, 0.5]; [0.1, 0.3] \rangle}, \\ \frac{u_3}{\langle [0.1, 0.2]; [0.3, 0.4]; [0.8, 0.9] \rangle}, \frac{u_4}{\langle [0.2, 0.3]; [0.2, 0.4]; [0.7, 0.8] \rangle} \end{array} \right\} \right), \right. \\ \left. \left((e_1, q, 0), \left\{ \begin{array}{l} \frac{u_1}{\langle [0.2, 0.3]; [0.3, 0.5]; [0.7, 0.8] \rangle}, \frac{u_2}{\langle [0.5, 0.7]; [0.3, 0.4]; [0.3, 0.5] \rangle}, \\ \frac{u_3}{\langle [0.5, 0.8]; [0.6, 0.7]; [0.2, 0.5] \rangle}, \frac{u_4}{\langle [0.1, 0.2]; [0.4, 0.5]; [0.8, 0.9] \rangle} \end{array} \right\} \right), \right. \\ \left. \left((e_2, p, 0), \left\{ \begin{array}{l} \frac{u_1}{\langle [0.1, 0.2]; [0.3, 0.3]; [0.8, 0.9] \rangle}, \frac{u_2}{\langle [0.7, 0.8]; [0.4, 0.5]; [0.2, 0.3] \rangle}, \\ \frac{u_3}{\langle [0.1, 0.4]; [0.5, 0.7]; [0.6, 0.9] \rangle}, \frac{u_4}{\langle [0.5, 0.8]; [0.2, 0.5]; [0.2, 0.5] \rangle} \end{array} \right\} \right), \right. \\ \left. \left((e_2, q, 0), \left\{ \begin{array}{l} \frac{u_1}{\langle [0.7, 0.8]; [0.3, 0.4]; [0.2, 0.3] \rangle}, \frac{u_2}{\langle [0.5, 0.6]; [0.8, 0.9]; [0.4, 0.5] \rangle}, \\ \frac{u_3}{\langle [0.6, 0.7]; [0.6, 0.7]; [0.3, 0.4] \rangle}, \frac{u_4}{\langle [0.8, 0.9]; [0.2, 0.4]; [0.1, 0.2] \rangle} \end{array} \right\} \right) \right\}.$$

4. Basic Operations on Neutrosophic Vague Soft Expert Sets

In this section, we introduce some basic operations on neutrosophic vague soft expert sets, namely the complement, union and intersection of neutrosophic vague soft expert sets, derive their properties, and give some examples.

We define the complement operation for neutrosophic vague soft expert set and give an illustrative example and proved proposition.

DEFINITION 4.1 The complement of a neutrosophic vague soft expert set (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$ where $F^c : A \rightarrow NV^U$ is a mapping given by

$$F^c(\alpha) = \tilde{c}(F(\alpha)), \forall \alpha \in A$$

where \tilde{c} is a neutrosophic vague complement.

Example 4.2 Consider Example 3.2. By using the basic neutrosophic vague complement, we have

$$\begin{aligned}
(F, Z)^c = & \left\{ \left((e_1, p, 1), \left\{ \frac{u_1}{\langle [0.2, 0.8]; [0.7, 0.9]; [0.2, 0.8] \rangle}, \frac{u_2}{\langle [0.3, 0.9]; [0.5, 0.8]; [0.1, 0.7] \rangle}, \right. \right. \\
& \left. \left. \frac{u_3}{\langle [0.4, 0.5]; [0.3, 0.7]; [0.5, 0.6] \rangle}, \frac{u_4}{\langle [0.0, 0.2]; [0.8, 0.9]; [0.8, 1] \rangle} \right\} \right), \\
& \left((e_1, q, 1), \left\{ \frac{u_1}{\langle [0.1, 0.2]; [0.6, 0.7]; [0.8, 0.9] \rangle}, \frac{u_2}{\langle [0.6, 0.8]; [0.6, 0.8]; [0.2, 0.4] \rangle}, \right. \right. \\
& \left. \left. \frac{u_3}{\langle [0.5, 1]; [0.3, 0.5]; [0, 0.5] \rangle}, \frac{u_4}{\langle [0.3, 0.4]; [0.6, 0.8]; [0.6, 0.7] \rangle} \right\} \right), \\
& \left((e_2, p, 1), \left\{ \frac{u_1}{\langle [0.1, 0.7]; [0.7, 0.9]; [0.3, 0.9] \rangle}, \frac{u_2}{\langle [0.5, 0.8]; [0.5, 0.8]; [0.2, 0.5] \rangle}, \right. \right. \\
& \left. \left. \frac{u_3}{\langle [0.1, 0.4]; [0.3, 0.9]; [0.6, 0.9] \rangle}, \frac{u_4}{\langle [0.6, 0.8]; [0.8, 0.8]; [0.2, 0.4] \rangle} \right\} \right), \\
& \left((e_2, q, 1), \left\{ \frac{u_1}{\langle [0.4, 0.6]; [0.6, 0.9]; [0.4, 0.6] \rangle}, \frac{u_2}{\langle [0.7, 0.9]; [0.6, 0.8]; [0.1, 0.3] \rangle}, \right. \right. \\
& \left. \left. \frac{u_3}{\langle [0.5, 0.9]; [0.3, 0.5]; [0.1, 0.5] \rangle}, \frac{u_4}{\langle [0.3, 0.8]; [0.6, 0.8]; [0.2, 0.7] \rangle} \right\} \right), \\
& \left((e_1, p, 0), \left\{ \frac{u_1}{\langle [0.3, 0.9]; [0.7, 0.8]; [0.1, 0.7] \rangle}, \frac{u_2}{\langle [0.1, 0.3]; [0.5, 0.5]; [0.7, 0.9] \rangle}, \right. \right. \\
& \left. \left. \frac{u_3}{\langle [0.8, 0.9]; [0.6, 0.7]; [0.1, 0.2] \rangle}, \frac{u_4}{\langle [0.7, 0.8]; [0.6, 0.7]; [0.2, 0.3] \rangle} \right\} \right), \\
& \left((e_1, q, 0), \left\{ \frac{u_1}{\langle [0.7, 0.8]; [0.5, 0.7]; [0.2, 0.3] \rangle}, \frac{u_2}{\langle [0.3, 0.5]; [0.6, 0.7]; [0.5, 0.7] \rangle}, \right. \right. \\
& \left. \left. \frac{u_3}{\langle [0.2, 0.5]; [0.3, 0.4]; [0.5, 0.8] \rangle}, \frac{u_4}{\langle [0.8, 0.9]; [0.5, 0.6]; [0.1, 0.2] \rangle} \right\} \right), \\
& \left((e_2, p, 0), \left\{ \frac{u_1}{\langle [0.8, 0.9]; [0.7, 0.7]; [0.1, 0.2] \rangle}, \frac{u_2}{\langle [0.2, 0.3]; [0.5, 0.6]; [0.7, 0.8] \rangle}, \right. \right. \\
& \left. \left. \frac{u_3}{\langle [0.6, 0.9]; [0.3, 0.5]; [0.1, 0.4] \rangle}, \frac{u_4}{\langle [0.2, 0.5]; [0.5, 0.8]; [0.5, 0.8] \rangle} \right\} \right), \\
& \left. \left((e_2, q, 0), \left\{ \frac{u_1}{\langle [0.2, 0.3]; [0.6, 0.7]; [0.7, 0.8] \rangle}, \frac{u_2}{\langle [0.4, 0.5]; [0.1, 0.2]; [0.5, 0.6] \rangle}, \right. \right. \right. \\
& \left. \left. \frac{u_3}{\langle [0.3, 0.4]; [0.3, 0.4]; [0.6, 0.7] \rangle}, \frac{u_4}{\langle [0.1, 0.2]; [0.6, 0.8]; [0.8, 0.9] \rangle} \right\} \right) \Big\}.
\end{aligned}$$

Proposition 4.3. If (F, A) is a neutrosophic vague soft expert set over U , then

$$((F, A)^c)^c = (F, A)$$

Proof. From Definition 4.1. we have $(F, A)^c = (F^c, A)$ where $F^c(\alpha) = \bar{1} - F(\alpha)$, $\forall \alpha \in A$. Now, $((F, A)^c)^c = ((F^c)^c, A)$ where $(F^c)^c(\alpha) = \bar{1} - (\bar{1} - F(\alpha))$, $\forall \alpha \in A$
 $= F(\alpha)$, $\forall \alpha \in A$.

We define the union of two neutrosophic vague soft expert sets and give an illustrative example.

DEFINITION 4.4 The union of two neutrosophic vague soft expert sets (F, A) and (G, B) over U , denoted by $(F, A) \tilde{\cup} (G, B)$, is a neutrosophic vague soft expert set (H, C) where $C = A \cup B$ and $\forall \mathcal{E} \in C$,

$$(H, C) = \begin{cases} F(\varepsilon), & \text{if } \varepsilon \in A - B, \\ G(\varepsilon), & \text{if } \varepsilon \in B - A, \\ F(\varepsilon) \tilde{\cup} G(\varepsilon), & \text{if } \varepsilon \in A \cap B. \end{cases}$$

where $\tilde{\cup}$ denote the neutrosophic vague set union.

Example 4.5 Consider Example 3.2. Let

$$A = \{(e_1, p, 1), (e_1, q, 0), (e_1, p, 0)\} \text{ and } B = \{(e_1, p, 1), (e_1, q, 0), (e_2, p, 1)\}$$

Suppose (F, A) and (G, B) are two neutrosophic vague soft expert sets over U such that:

$$(F, A) = \left\{ \left((e_1, p, 1), \left\{ \begin{array}{l} \frac{u_1}{\langle\langle 0.7, 0.8 \rangle; \langle 0.1, 0.3 \rangle; \langle 0.2, 0.3 \rangle\rangle}, \frac{u_2}{\langle\langle 0.4, 0.7 \rangle; \langle 0.2, 0.5 \rangle; \langle 0.3, 0.6 \rangle\rangle}, \\ \frac{u_3}{\langle\langle 0.1, 0.2 \rangle; \langle 0.6, 0.7 \rangle; \langle 0.8, 0.9 \rangle\rangle}, \frac{u_4}{\langle\langle 0.4, 0.5 \rangle; \langle 0.1, 0.2 \rangle; \langle 0.5, 0.6 \rangle\rangle} \end{array} \right\} \right), \right. \\ \left. \left((e_1, q, 0), \left\{ \begin{array}{l} \frac{u_1}{\langle\langle 0.1, 0.9 \rangle; \langle 0.2, 0.4 \rangle; \langle 0.1, 0.9 \rangle\rangle}, \frac{u_2}{\langle\langle 0.4, 0.8 \rangle; \langle 0.3, 0.5 \rangle; \langle 0.2, 0.6 \rangle\rangle}, \\ \frac{u_3}{\langle\langle 0.8, 0.9 \rangle; \langle 0.3, 0.4 \rangle; \langle 0.1, 0.2 \rangle\rangle}, \frac{u_4}{\langle\langle 0.5, 0.6 \rangle; \langle 0.5, 0.6 \rangle; \langle 0.4, 0.5 \rangle\rangle} \end{array} \right\} \right), \right. \\ \left. \left((e_1, p, 0), \left\{ \begin{array}{l} \frac{u_1}{\langle\langle 0.5, 0.8 \rangle; \langle 0.1, 0.4 \rangle; \langle 0.2, 0.5 \rangle\rangle}, \frac{u_2}{\langle\langle 0.4, 0.9 \rangle; \langle 0.8, 0.9 \rangle; \langle 0.1, 0.6 \rangle\rangle}, \\ \frac{u_3}{\langle\langle 0.2, 0.3 \rangle; \langle 0.4, 0.6 \rangle; \langle 0.7, 0.8 \rangle\rangle}, \frac{u_4}{\langle\langle 0.5, 0.7 \rangle; \langle 0.4, 0.6 \rangle; \langle 0.3, 0.5 \rangle\rangle} \end{array} \right\} \right) \right\},$$

$$(G, B) = \left\{ \left((e_1, p, 1), \left\{ \begin{array}{l} \frac{u_1}{\langle\langle 0.3, 0.6 \rangle; \langle 0.1, 0.4 \rangle; \langle 0.4, 0.7 \rangle\rangle}, \frac{u_2}{\langle\langle 0.5, 0.7 \rangle; \langle 0.2, 0.5 \rangle; \langle 0.3, 0.5 \rangle\rangle}, \\ \frac{u_3}{\langle\langle 0.5, 0.8 \rangle; \langle 0.3, 0.7 \rangle; \langle 0.2, 0.5 \rangle\rangle}, \frac{u_4}{\langle\langle 0.8, 1 \rangle; \langle 0.1, 0.2 \rangle; \langle 0.2, 0.2 \rangle\rangle} \end{array} \right\} \right), \right. \\ \left. \left((e_1, q, 0), \left\{ \begin{array}{l} \frac{u_1}{\langle\langle 0.1, 0.9 \rangle; \langle 0.2, 0.3 \rangle; \langle 0.1, 0.9 \rangle\rangle}, \frac{u_2}{\langle\langle 0.8, 0.9 \rangle; \langle 0.2, 0.4 \rangle; \langle 0.1, 0.2 \rangle\rangle}, \\ \frac{u_3}{\langle\langle 0.9, 0.9 \rangle; \langle 0.1, 0.3 \rangle; \langle 0.1, 0.1 \rangle\rangle}, \frac{u_4}{\langle\langle 0.8, 0.9 \rangle; \langle 0.3, 0.4 \rangle; \langle 0.1, 0.2 \rangle\rangle} \end{array} \right\} \right), \right. \\ \left. \left((e_2, p, 1), \left\{ \begin{array}{l} \frac{u_1}{\langle\langle 0.1, 0.4 \rangle; \langle 0.2, 0.4 \rangle; \langle 0.6, 0.9 \rangle\rangle}, \frac{u_2}{\langle\langle 0.5, 0.8 \rangle; \langle 0.3, 0.5 \rangle; \langle 0.2, 0.5 \rangle\rangle}, \\ \frac{u_3}{\langle\langle 0.8, 0.8 \rangle; \langle 0.1, 0.4 \rangle; \langle 0.2, 0.2 \rangle\rangle}, \frac{u_4}{\langle\langle 0.5, 0.6 \rangle; \langle 0.5, 0.6 \rangle; \langle 0.4, 0.5 \rangle\rangle} \end{array} \right\} \right) \right\}.$$

By using basic neutrosophic vague union, we have $(F, A) \tilde{\cup} (G, B) = (H, C)$ where

$$(H, C) = \left\{ \left((e_1, p, 1), \left\{ \begin{array}{l} \frac{u_1}{\langle\langle 0.7, 0.8 \rangle; \langle 0.1, 0.3 \rangle; \langle 0.2, 0.3 \rangle\rangle}, \frac{u_2}{\langle\langle 0.5, 0.7 \rangle; \langle 0.2, 0.5 \rangle; \langle 0.3, 0.5 \rangle\rangle}, \\ \frac{u_3}{\langle\langle 0.5, 0.8 \rangle; \langle 0.3, 0.7 \rangle; \langle 0.2, 0.5 \rangle\rangle}, \frac{u_4}{\langle\langle 0.8, 1 \rangle; \langle 0.1, 0.2 \rangle; \langle 0.2, 0.2 \rangle\rangle} \end{array} \right\} \right), \right.$$

$$\left((e_1, q, 0), \left\{ \begin{array}{l} \overline{\langle \frac{u_1}{[0.1, 0.9]; [0.2, 0.3]; [0.1, 0.9]} \rangle}, \overline{\langle \frac{u_2}{[0.8, 0.9]; [0.2, 0.4]; [0.1, 0.2]} \rangle}, \\ \overline{\langle \frac{u_3}{[0.9, 0.9]; [0.1, 0.3]; [0.1, 0.1]} \rangle}, \overline{\langle \frac{u_4}{[0.8, 0.9]; [0.3, 0.4]; [0.1, 0.2]} \rangle} \end{array} \right\} \right),$$

$$\left((e_1, p, 0), \left\{ \begin{array}{l} \overline{\langle \frac{u_1}{[0.5, 0.8]; [0.1, 0.4]; [0.2, 0.5]} \rangle}, \overline{\langle \frac{u_2}{[0.4, 0.9]; [0.8, 0.9]; [0.1, 0.6]} \rangle}, \\ \overline{\langle \frac{u_3}{[0.2, 0.3]; [0.4, 0.6]; [0.7, 0.8]} \rangle}, \overline{\langle \frac{u_4}{[0.5, 0.7]; [0.4, 0.6]; [0.3, 0.5]} \rangle} \end{array} \right\} \right),$$

$$\left((e_2, p, 1), \left\{ \begin{array}{l} \overline{\langle \frac{u_1}{[0.1, 0.4]; [0.2, 0.4]; [0.6, 0.9]} \rangle}, \overline{\langle \frac{u_2}{[0.5, 0.8]; [0.3, 0.5]; [0.2, 0.5]} \rangle}, \\ \overline{\langle \frac{u_3}{[0.8, 0.8]; [0.1, 0.4]; [0.2, 0.2]} \rangle}, \overline{\langle \frac{u_4}{[0.5, 0.6]; [0.5, 0.6]; [0.4, 0.5]} \rangle} \end{array} \right\} \right).$$

We define the intersection of two neutrosophic vague soft expert sets and give an illustrative example.

DEFINITION 4.6 The *intersection* of two neutrosophic vague soft expert sets (F, A) and (G, B) over a universe U , is a neutrosophic vague soft expert set (H, C) , denoted by $(F, A) \tilde{\cap} (G, B)$, such that $C = A \cup B$ and $\forall e \in c$

$$(H, C) = \begin{cases} F(e), & \text{if } e \in A - B, \\ G(e), & \text{if } e \in B - A, \\ F(e) \tilde{\cap} G(e), & \text{if } e \in A \cap B. \end{cases}$$

where $\tilde{\cap}$ denoted the neutrosophic vague set intersection.

Example 4.7 Consider Example 4.5. By using basic neutrosophic vague intersection, we have $(F, A) \tilde{\cap} (G, B) = (H, C)$ where

$$(H, C) = \left\{ \left((e_1, p, 1), \left\{ \begin{array}{l} \overline{\langle \frac{u_1}{[0.3, 0.6]; [0.1, 0.4]; [0.4, 0.7]} \rangle}, \overline{\langle \frac{u_2}{[0.4, 0.7]; [0.2, 0.5]; [0.3, 0.6]} \rangle}, \\ \overline{\langle \frac{u_3}{[0.1, 0.2]; [0.6, 0.7]; [0.8, 0.9]} \rangle}, \overline{\langle \frac{u_4}{[0.4, 0.5]; [0.1, 0.2]; [0.5, 0.6]} \rangle} \end{array} \right\} \right),$$

$$\left((e_1, q, 0), \left\{ \begin{array}{l} \overline{\langle \frac{u_1}{[0.1, 0.9]; [0.2, 0.4]; [0.1, 0.9]} \rangle}, \overline{\langle \frac{u_2}{[0.4, 0.8]; [0.3, 0.5]; [0.2, 0.6]} \rangle}, \\ \overline{\langle \frac{u_3}{[0.8, 0.9]; [0.3, 0.4]; [0.1, 0.2]} \rangle}, \overline{\langle \frac{u_4}{[0.5, 0.7]; [0.4, 0.6]; [0.3, 0.5]} \rangle} \end{array} \right\} \right),$$

$$\left((e_1, p, 0), \left\{ \begin{array}{l} \overline{\langle \frac{u_1}{[0.5, 0.8]; [0.1, 0.4]; [0.2, 0.5]} \rangle}, \overline{\langle \frac{u_2}{[0.4, 0.9]; [0.8, 0.9]; [0.1, 0.6]} \rangle}, \\ \overline{\langle \frac{u_3}{[0.2, 0.3]; [0.4, 0.6]; [0.7, 0.8]} \rangle}, \overline{\langle \frac{u_4}{[0.5, 0.7]; [0.4, 0.6]; [0.3, 0.5]} \rangle} \end{array} \right\} \right),$$

$$\left((e_2, p, 1), \left\{ \begin{array}{l} \overline{\langle \frac{u_1}{[0.1, 0.4]; [0.2, 0.4]; [0.6, 0.9]} \rangle}, \overline{\langle \frac{u_2}{[0.5, 0.8]; [0.3, 0.5]; [0.2, 0.5]} \rangle}, \\ \overline{\langle \frac{u_3}{[0.8, 0.8]; [0.1, 0.4]; [0.2, 0.2]} \rangle}, \overline{\langle \frac{u_4}{[0.5, 0.6]; [0.5, 0.6]; [0.4, 0.5]} \rangle} \end{array} \right\} \right).$$

5. AND and OR Operations

In this section, we introduce the definitions of AND and OR operations for neutrosophic vague soft expert set, and derive their properties.

DEFINITION 5.1 Let (F, A) and (G, B) be any two neutrosophic vague soft expert sets over a soft universe (U, Z) .

Then " (F, A) AND (G, B) " denoted $(F, A) \tilde{\wedge} (G, B)$ is defined by :

$$(F, A) \tilde{\wedge} (G, B) = (H, A \times B)$$

where $(H, A \times B) = H(\alpha, \beta)$, such that $H(\alpha, \beta) = F(\alpha) \cap G(\beta)$, for all $(\alpha, \beta) \in A \times B$. And \cap represent the basic intersection.

DEFINITION 5.2 Let (F, A) and (G, B) be any two neutrosophic vague soft expert sets over a soft universe (U, Z) .

Then " (F, A) OR (G, B) " denoted $(F, A) \tilde{\vee} (G, B)$ is defined by :

$$(F, A) \tilde{\vee} (G, B) = (H, A \times B)$$

where $(H, A \times B) = H(\alpha, \beta)$, such that $H(\alpha, \beta) = F(\alpha) \cup G(\beta)$, for all $(\alpha, \beta) \in A \times B$. And \cup represent the basic union.

Proposition 5.3 If (F, A) and (G, B) are two neutrosophic vague soft expert sets over a soft universe (U, Z) . Then,

1. $((F, A) \tilde{\wedge} (G, B))^c = (F, A)^c \tilde{\vee} (G, B)^c$
2. $((F, A) \tilde{\vee} (G, B))^c = (F, A)^c \tilde{\wedge} (G, B)^c$

Proof (1) Suppose that (F, A) and (G, B) are two neutrosophic vague soft expert sets over a soft universe (U, Z) defined as:

$(F, A) = F(\alpha)$ for all $\alpha \in A \subseteq Z$ and $(G, B) = G(\beta)$ for all $\beta \in B \subseteq Z$. Then by definitions 4.8 and 4.9 it follows that:

$$\begin{aligned} ((F, A) \tilde{\wedge} (G, B))^c &= ((F(\alpha) \tilde{\wedge} G(\beta))^c \\ &= ((F(\alpha) \cap G(\beta))^c \\ &= (\tilde{c}(F(\alpha) \cap G(\beta))) \\ &= (\tilde{c}(F(\alpha) \cup \tilde{c}G(\beta))) \\ &= (F(\alpha))^c \tilde{\vee} (G(\beta))^c \\ &= (F, A)^c \tilde{\vee} (G, B)^c. \end{aligned}$$

(2) The proof is similar to that in part(1) and therefore is omitted.

6. Conclusion

In this paper, we reviewed the basic concepts of neutrosophic vague set and neutrosophic soft expert set, and gave some basic operations on both neutrosophic vague set and neutrosophic soft expert set, the concept of neutrosophic vague soft expert set was established. The basic operations on neutrosophic vague soft expert set, namely complement, union, intersection, AND, and OR operations, were defined. Subsequently, the basic properties of these operations such as De Morgans laws and other relevant laws pertaining to the concept of neutrosophic vague soft expert set are proved. Finally, we intend to further explore the applications of neutrosophic vague soft expert set approach to solve certain decision making problems. This new extension will provide a significant addition to existing theories for handling uncertainties, and lead to potential areas of further research and pertinent applications.

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