



New Concepts in Partner Multineutrosophic Topological Space

Tuqa A. H. Al-Tamimi^{1*}, Luay A. A. Al-Swidi², Ali H. M. Al-Obaidi³

^{1, 2, 3}Department of Mathematics, College of Education for Pure Science, University of Babylon, Hillah, Iraq.

Emails: tuqa.hasan.pure442@student.uobabylon.edu.iq; pure.leal.abd@uobabylon.edu.iq; aalobaidi@uobabylon.edu.iq

*Correspondence: tuqa.hasan.pure442@student.uobabylon.edu.iq

Abstract

We have new introduced two fundamental new concept that characterize partner multineutrosophic sets, the first of which we call the locally partner multineutrosophic interior and the second the locally partner fuzzy exterior, in addition to a concept related to set composition that we call detachable set, with the most important relationships to which these concept are linked and the properties that we have reached with an analytical view of them.

Keyword: Partner multineutrosophic sets; Locally partner multineutrosophic interior and exterior set; detachable set.

1. Introduction

Among the transformations that the scientific and engineering fields witnessed in last century, with the applied sciences, is what zadeh [1] presented through the introduction of fuzzy sets. This illumination has attracted many researchers to transfer their science, mathematical concepts, and other to the $X \times I (I=[0,1])$ space in which the fuzzy sets move, what are the capabilities that characterize this space and the conditions that must be to a chi eve similarity between them. For example, the researcher Raghad [5] studied the local function with the space $X \times I$ and obtained different and varied types. But Florentin [7] brought a new and bright idea to the eyes of the world and researcher, and it resonated in the applied fields. He circulated the idea of Zadeh in a unique style with in the space $X \times I \times I \times I$, and it included the foundations for new sets called Neutrosophic sets. But in our paper for new sets called in 2021 [4], both fuzzy sets and soft sets were combined with Neutrosophic crisp sets. But in 2022 [6] we developed a generalized formula to neutrosophic crisp and called Neutrosophic axiol set. In 2023[9], Smarandache, F. generalized the Neutrosophic sets and called then multineutrosophic sets. Imran et al. [12-14] gave the view of some types of neutrosophic topological groups with respect to neutrosophic alpha open sets, new types of weakly neutrosophic crisp continuity, and new concepts of neutrosophic crisp open sets. Finally, the senses of new types of weakly neutrosophic crisp open mappings and new types of weakly neutrosophic crisp closed functions were informed by Al-Obaidi et al. [15,16]. The binary operation $\varepsilon, \cup, =$, the classical operation.

2. Primitive

There are some basic definition of partner sets that serve as an introduction to building the mathematical concepts of our research poth.

Definition 2-1: Let X be nonempty universal set and A be any nonempty subset of X . Then $\{ \langle x, T, I, F \rangle : x \in X \}$ is called neutrosophic set and denoted by briefly by A if $T, I, F: x \rightarrow [0,1]$ are degree of truth, indeterminacy, falsehood membership of x corresponding with subset A , respectively. It is clear that $0 \leq \inf T + \inf I + \inf F \leq \sup T + \sup I + \sup F \leq 3$

Moreover, if the values of T, I, F are numbers from $[0,1]$, then all of them have single – valued neutrosophic set (SVNS).

Definition 2-2: Let X be universal set and M be any nonempty subset of X , then $\{ \langle x, T_1, \dots, T_r, I_1, \dots, I_s, F_1, \dots, F_t \rangle : x \in X \}$ is called multineutrosophic set and denoted briefly by M $T_1, \dots, T_r: X \rightarrow [0,1]$ are the degree of truth membership functions of x corresponding with the subset $M, I_1, \dots, I_s: X \rightarrow [0,1]$ are

the degree of indeterminacy membership function of x corresponding with the subset M , and $F_1, \dots, F_t: X \rightarrow [0,1]$ are the degree of falsehood membership function of x corresponding with the subset M where $r+s+t=n$. It is obvious that $0 \leq \sum_{i=1}^r \inf T_i + \sum_{j=1}^s \inf I_j + \sum_{k=1}^t \inf F_k \leq \sum_{i=1}^r \sup T_i + \sum_{j=1}^s \sup I_j + \sum_{k=1}^t \sup F_k \leq n$

Definition 2-3: Let \wedge_N, \wedge_F be the neutrosophic intersection and t-norm, respectively. Moreover, let \vee_N, \vee_F be the neutrosophic and t-conorm, respectively. Let M_1, M_2 be neutrosophic sets with same (r, s, t) -form. Then $M_1 = \{ \langle x, T_1, \dots, T_r, I_1, \dots, I_s, F_1, \dots, F_t \rangle : x \in X \}$ and $M_2 = \{ \langle x, T'_1, \dots, T'_r, I'_1, \dots, I'_s, F'_1, \dots, F'_t \rangle : x \in X \}$

(i) The neutrosophic intersection

$$M_1 \wedge_F M_2 = \{ \langle x, T_1 \wedge_F T'_1, \dots, T_r \wedge_F T'_r, I_1 \vee_F I'_1, \dots, I_s \vee_F I'_s, F_t \vee_F F'_t, \dots, F_t \vee_F F'_t \rangle : x \in X \}$$

(ii) The multilineutrosophic union

$$M_1 \vee_n M_2 = \{ \langle x, T_1 \vee_n T'_1, \dots, T_r \vee_n T'_r, I_1 \wedge_N I'_1, \dots, I_s \wedge_N I'_s, F_t \wedge_N F'_t, \dots, F_t \wedge_N F'_t \rangle, x \in X \}$$

3. The Partner Set of Multineutrosophic Set

Definition 3-1: Let X be non-empty universal set and let $M^n = \{ \langle x, T_1, \dots, T_r, I_1, \dots, I_s, F_1, \dots, F_t \rangle : x \in X, r + s + t = n \}$ be a multilineutrosophic set in X . The fuzzy set M_n is called a **partner set to the multilineutrosophic set** $M^n = \{ \langle x, T_1, \dots, T_r, I_1, \dots, I_s, F_1, \dots, F_t \rangle : x \in X, r + s + t = n \}$ and it has the following form

$$M_n = \{ \langle x, f_{M_n}(x) \rangle : x \in X \}$$

where

$$f_{M_n}(x) = \frac{1}{n} \left(\sum_{i=1}^r T_i + \sum_{j=1}^s I_j + \sum_{k=1}^t F_k \right), \forall x \in X.$$

Definition 3-2[2,8]: Let X by a nonempty universal set. For any multilineutrosophic sets A^n and B^m , we defined the union and intersection of them concerning partner A_p, B_p .

1- $A_p \wedge B_p = \{ \langle x, \min(f_{A_n}(x), g_{B_m}(x)) \rangle, x \in X \}$

2- $A_p \vee B_p = \{ \langle x, \max(f_{A_n}(x), g_{B_m}(x)) \rangle, x \in X \}$

3- $A_p^c = \{ \langle x, 1 - f_{A_n}(x) \rangle, x \in X \}$, the partner complement of partner set A_n , the partner zero set or partner empty fuzzy set

4- $0_p = \{ \langle x, 0 \rangle, \forall x \in X \}$, the partner universal set or partner universal fuzzy set

5- $1_p = \{ \langle x, 1 \rangle, \forall x \in X \}$,

6- the partner multilineutrosophic point PMP is of the form $P_x^\lambda(y) = \begin{cases} \lambda, & y = x \\ 0, & y = 0 \end{cases}, \forall y \in X$

Definition 3-3 [2]: The nonempty collection of partner multilineutrosophic sets PMSs, is called partner multilineutrosophic topology PMt, if satisfying the following

1- $0_p, 1_p \in \tau_p$

2- if $M_p, N_p \in \tau_p$ then $M_p \wedge N_p \in \tau_p$ 3- For any index $\Delta, H_{\lambda p} \in \tau_p, \forall \Delta$, then $\vee_{\lambda \in \Delta} H_{\lambda p} \in \tau_p$

Any $PMS M_p \in \tau_p$, is called partner multilineutrosophic open set PMOS, and the complement is called partner multilineutrosophic closed set PMCS where ε is classical belong to.

4. Locally Partner Multineutrosophic Sets LPMIS Interior

The PIM point of PMSs are important tools in diagnosing the form and attributes of the topology derived from it, because through them all topological concepts can be constructed. We will give a special type for partner interior points, which are special point in the universal set with its properties.

Definition 4-1: Let $(1_p, \tau_p)$ be (PMTS) a partner fuzzy neutrosophic point PMP P_x^λ is called locally partner partner multilineutrosophic interior of PMTS M^m is defined by

$$\ell_i(M_p)_x = \vee_x \{ G_p \in \tau_p; \lambda \leq f_{G_p}(x) \leq h_{M_p}(x) \} \text{ where } \vee_x H_\lambda^p = \sup_{\lambda \in \Pi} \{ H_{\lambda p}(x) \}; \text{ for any index } \Pi$$

Example 4-2: Let $X = \{1,2,3\}$,

$$A^7 = \{ (1, (0.3,0.3), (0.3,0.1,0.1), (0.2,0.21), 2, (0.1,0.2), (0.1,0.0,0.1), (0.1,0.1), 3, (0.0,0.1), (0.0,0.1,0.0), (0.2,0.0)) \}$$

$$B^7 = \{ (1, (0.1,0.2,0.1), (0.1,0.3), (0.2,0.1), (2, (0.1,0.0,0.4), (0.2,0.2), (0.2,0.3),$$

$$(3, (0.11,0.4,0.3), (0.1,0.3), (0.1,0.2)) \}, M^7 = \{ (1, (0.0,0.0), (0.1,0.0), (0.0,0.1,0.0),$$

$$(2, (0.1,0.1), (0.2,0.0), (0.16,0.1,0.1), (3, (0.2,0.1), (0.0,0.3), (0.2,0.0,0.2)) \}$$

$$A_p = \{ (1,0.75), (2,0.35), (3,0.2) \}, \quad B_p = \{ (1,0.4), (2,0.46), (3,0.5) \}$$

$$A_p \wedge B_p = \{ \langle x, \min(A_p(x), g_{B_p}(x)) \rangle, \forall x \in X \}, A_p \wedge B_p = \{ (1,0.4), (2,0.35), (3,0.2) \}$$

$$A_p \vee B_p = \{ \langle x, \max(A_p(x), g_{B_p}(x)) \rangle, \forall x \in X \}, A_p \vee B_p = \{ (1,0.75), (2,0.46), (3,0.5) \},$$

$$\tau = \{ 0_p, 1_p, A_p, B_p, A_p \vee B_p, A_p \wedge B_p \}, \lambda = 0.2, \text{ Let } M_p = \{ (1,0.1), (2,0.38), (3,0.5) \},$$

$$\ell_i(M_p)_x = \vee_x \{G_p \in \tau; \lambda \leq f_{G_p}(x) \leq h_{M_p}(x)\}, \ell_i(M_p)_1 = 0, \ell_i(M_p)_2 = 0.35, \ell_i(M_p)_3 = 0.5$$

From this we conclude that totally partner set does not necessarily indicate that it is an PMOS set, see example 4-2, also is not PMCS

Definition 4-3: The multineutrosophic interior set is of all LPMIS at PMS M_r , $\forall x \in X$ is called the totally locally fuzzy interior is defined by

$$t\ell_i(M_r) = \cup_{x \in X} \ell_i(M_p)_x, \text{ where } \cup \text{ the classical union}$$

it is simply to see that $p_y^\lambda \in t\ell_i(M_p)$ iff $p_y^\lambda \in \vee_{x \in X} \ell_i(M_p)_x$, iff $P_y^\lambda \in \ell_i(M_p)_y = \vee \{G_p \in \tau; \lambda \leq f_G(y) \leq h_{M_p}(y)\}$, $\lambda \leq \sup f_{G_p}(y) \vee G_p \in \tau \exists \lambda \leq f_{G_p}(y) \leq h_{M_p}(y)$. Also to see that $t\ell_i(1_p) = 1_p$ and $t\ell_i(0_p) = 0_p$

Example : from example 2-3 we have

$$t\ell_i(M_p) = \{(1,0), (2,0.35), (3,0.6)\}$$

From this we conclude that TLPMS does not necessarily indicate that it is a PMOS, see example 2-4 at that $t\ell_i(M_p)_x$ is not PMOS.

There are several properties that locally partner pinks have, as show in following theorem.

Theorem 4-4: Let is $(1_p, \tau_p)$ a PMTS, then

$$1 - \text{int}(M_p) \leq t\ell_i(M_p)$$

$$2- \text{if } M_r \text{ is PMS then } t\ell_i(M_p) \leq M_p$$

$$3- \text{if } M_p \leq N_p \rightarrow t\ell_i(M_p) \leq t\ell_i(N_p)$$

$$4- t\ell_i(M_p \wedge N_p) = t\ell_i(M_p) t\ell_i(N_p)$$

$$5- t\ell_i(M_p) \vee t\ell_i(N_p) = t\ell_i(M_p \vee N_p)$$

Proof (1): let $P_x^\lambda \in \text{Int}(M_p)$

$$\Rightarrow \exists G_p \in \tau_p \exists P_x^\lambda \in G_p \leq M_p$$

$$\Rightarrow \lambda \leq f_{G_p}(x) \leq \sup f_{U_p}(x) \leq h_{M_p}(x) ,$$

$$\forall G_p \in \tau_p \exists \lambda \leq f_{U_p}(x) \leq h_{M_p}(x)$$

$$\text{iff } P_x^\lambda \in \vee_x \{U_p \in \tau_p; \lambda \leq f_{U_p}(x) \leq h_{M_p}(x)\} = \ell_i(M_p) \leq \vee_{x \in X} \ell_i(M_p)_x = t\ell_i(M_p)$$

Proof (2): let $P_x^\lambda \in t\ell_i(M_p)$

$$\Rightarrow P_x^\lambda \in \ell_i(M_p)_x \Rightarrow P_x^\lambda \in \ell_i(M_p)_x$$

$$\Rightarrow \lambda \leq f_{G_p}(x) \leq h_{M_p}(x)$$

$$\Rightarrow P_x^\lambda \in M_p$$

$$\Rightarrow t\ell_i(M_p) \leq M_p$$

$$\Rightarrow P_x^\lambda \in \ell_i(M_p)_x,$$

Proof(3): let $P_x^\lambda \in t\ell_i(M_p) \Rightarrow P_x^\lambda \in \vee \ell_i(M_p)_x$

$$\Rightarrow \exists G_p \in \tau_p \exists \lambda \leq f_{G_p}(x) \leq h_{M_p}(x) \leq g_{N_p}(x)$$

$$\Rightarrow P_x^\lambda \in \ell_i(N_p)_x \Rightarrow P_x^\lambda \in t\ell_i(N_p)_x, \Rightarrow \lambda \leq f_{G_p}(x) \leq g_{N_p}, \Rightarrow t\ell_i(M_p) \leq t\ell_i(N_p)$$

Proof(4): since

$$M_p \wedge N_p \leq M_p \Rightarrow t\ell_i(M_p \wedge N_p) \leq t\ell_i(M_p)$$

$$M_p \wedge N_p \leq N_p \Rightarrow t\ell_i(M_p \wedge N_p) \leq t\ell_i(N_p)$$

$$t\ell_i(M_p \wedge N_p) \leq t\ell_i(M_p) \wedge t\ell_i(N_p)$$

\Leftarrow conversely

$$\text{Let } P_x^\lambda \in t\ell_i(M_p) \wedge t\ell_i(N_p) \Rightarrow P_x^\lambda \in t\ell_i(M_p) \& P_x^\lambda \in t\ell_i(N_p)$$

$$\Rightarrow \exists G_{1p} \in \tau_p \exists P_x^\lambda \in G_p \leq M_p \& G_{2p} \in \tau_p \exists P_x^\lambda \in G_{2p} \leq N_p$$

$$\Rightarrow \lambda \leq f_{G_{1p}}(x) \leq h_{M_p}(x) \& \lambda \leq f_{G_{2p}}(x) \leq g_{N_p}(x)$$

$$\Rightarrow \lambda \leq \min(f_{G_{1p}}(x), f_{G_{2p}}(x)) \leq \min(h_{M_p}(x), g_{N_p}(x))$$

$$\Rightarrow P_x^\lambda \in \ell_i(M_p \wedge N_p) \Rightarrow P_x^\lambda \in \ell_i(M_p \wedge N_p) \Rightarrow P_x^\lambda \in t\ell_i(M_p \wedge N_p)$$

$$M_p \leq M_p \vee N_p \Rightarrow t\ell_i(M_p) \leq t\ell_i(M_p \vee N_p) \text{ proof(5):}$$

$$\text{And } N_p \leq M_p \vee N_p \Rightarrow t\ell_i(N_p) \leq t\ell_i(M_p \vee N_p)$$

$$\Rightarrow t\ell_i(M_p) \vee t\ell_i(N_p) \leq t\ell_i(M_p \vee N_p)$$

$$\Leftarrow \text{let } P_x^\lambda \in t\ell_i(M_p \vee N_p) \Rightarrow P_x^\lambda \in \vee_x \ell_i(M_p \vee N_p) \Rightarrow P_x^\lambda \in \ell_i(M_p \vee N_p) = \vee_x \{G_p \in \tau_p \exists \lambda \leq f_{G_p}(x)$$

$$\leq h_{(M_p \vee N_p)}(x) = \max(g_{M_p}(x), k_{N_p}(x))\}$$

$$= \vee_x \{G_p \in \tau_p \exists \lambda \leq f_{G_p}(x) \leq g_{M_p}(x) \text{ or } \lambda \leq f_{G_p}(x) \leq k_{N_p}(x)\}$$

$$\Rightarrow \text{either } P_x^\lambda \in \vee_x \{G_p \in \tau_p \exists \lambda \leq f_{G_p}(x) \leq g_{M_p}(x)\} \text{ or } P_x^\lambda \in \vee_x \{G_p \in \tau_p \exists \lambda \leq f_{G_p}(x) \leq k_{N_p}(x)\}$$

\Rightarrow either $P_x^\lambda \in \ell_i(M_p)$ or $P_x^\lambda \in \ell_i(N_p)$ either $P_x^\lambda \in t\ell_i(M_p)$ or $P_x^\lambda \in t\ell_i(N_p) \Rightarrow P_x^\lambda \in t\ell_i(M_p) \vee t\ell_i(N_p)$

The converse of part (i), (ii) and (iii) are incorrect, as show in examples 4-5

Example 4-5: Let $X=\{1,2,3,4,5\}$

A^8

$= \{(1, (0.1,0.0,0.2), (0.0,0.2,0.1), (0.3,0.1), (2, (0.0,0.0,0.1), (0.1,0.2,0.0), (0.2,0.1), (3, (0.08,0.2,0.1), (0.2,0.0,0.1), (0.1,0.1), (4, (0.11,0.0,0.1), (0.1,0.2,0.1), (0.0,0.1), (5, (0.0,0.1,0.0), (0.0,0.2,0.1), (0.1,0.0)\}$

$B^8 = \{(1, (0.2,0.0,0.3), (0.1,0.4,0.1), (0.2,0.2), (2, (0.13,0.2,0.5), (0.1,0.0,0.0), (0.1,0.2), (3, (0.1,0.18,0.2), (0.0,0.2,0.0), (0.1,0.1), (4, (0.1,0.1,0.0), (0.2,0.11,0.0), (0.2,0.0), (5, (0.08,0.01,0.0), (0.03,0.07,0.1), (0.19,0.0)\}$

$M^8 = \{(1, (0.1,0.1,0.2), (0.0,0.2,0.0), (0.3,0.1), (2, (0.2,0.11,0.0), (0.1,0.1,0.2), (0.1,0.2), (3, (0.15,0.2,0.1), (0.09,0.11,0.03), (0.17,0.1), (4, (0.19,0.18,0.02), (0.1,0.2,0.1), (0.28,0.02), (5, (0.2,0.14,0.09), (0.11,0.1,0.07), (0.13,0.2)\}$

$N^5 = \{(1, (0.1,0.3), (0.2,0.1), (0.3), (2, (0.025,0.2), (0.1,0.01), (0.09), (3, (0.072,0.1), (0.3,0.4), (0.2), (4,0.06), (0.19,0.01), (0.2), (5, (0.05,0.03), (0.07,0.05), (0.05)\}$

$S^4 = \{(1, (0.1,0.2), (0.08), (0.22), (2, (0.03,0.08), (0.12), (0.2), (3, (0.51,0.2), (0.1), (0.2), (4, (0.05,0.06), (0.07), (0.17), (5, (0.04,0.1), (0.1), (0.1)\}$

$A_p = \{(1,0.5,0.5), (2,0.5,0.2), (3,0.33,0.1), (4,0.25,0.05), (5,0.2,0.3)\}$

$B_p = \{(1, 0.5,0.5,0.5), (2,0.33,0.4,0.4), (3,0.25,0.33,0.3), (4,0.2,0.28,0.23), (5,0.16,0.25,0.19)\}$

$A_p = \{(1,0.5), (2,0.35), (3,0.215), (4,0.15), (5,0.25)\}$

$B_p = \{(1,0.5), (2,0.37), (3,0.19), (4,0.23), (5,0.2)\}$

$B_p \wedge B_p = \{(x, \min(f_A(x), g_b(x), \forall x \in X) = \{(1,0.5), (2,0.35), (3,0.19), (4,0.15), (5,0.2)\}$

$A_p \vee B_p = \{(x, \max(f_A(x), g_B(x), \forall x \in X) = \{(1,0.5), (2,0.37), (3,0.215), (4,0.23), (5,0.25)\}$

$\tau_p = \{0_p, 1_p, A_p, B_p, A_p \wedge B_p, A_p \vee B_p\}$

$M_p = \{(1,0.5), (2,0.505), (3,0.525), (4,0.545), (5,0.75)\}$

$N_p = \{(1,0.415), (2,0.2125), (3,0.161), (4,0.1405), (5,0.125)\}$

$\ell_i(M_p)_1 = 0.5, \ell_i(M_p)_2 = 0.37, \ell_i(M_p)_3 = 0.215, \ell_i(M_p)_4 = 0.23, \ell_i(M_p)_5 = 0.25$

$t\ell_i(M_p)_x = \vee_x \ell_i(M_p)_x$

$t\ell_i(M_p)_x = \{(1,5), (2,0.37), (3,0.215), (4,0.23), (5,0.25)\}$

$int(M_p) = \vee \{G_p \in \tau_p; G_p \leq M_p\} = N_p \vee (N_p \wedge M_p) = \{(1,0.5), (2,0.35), (3,0.215), (4,0.23), (5,0.2)\} \leq t\ell_i(M_p)_x$

$\Rightarrow t\ell_i(M_p) \not\leq int(M_p)$ and $M_p \not\leq t\ell_i(M_p)$

The converse of the part (iii) is not true from the example

$S_p = \{(1,0.25), (2,0.215), (3,0.19), (4,0.175), (5,0.17)\}$

$\ell_i(S_p)_1 = 0, \ell_i(S_p)_2 = 0, \ell_i(S_p)_3 = 0.19, \ell_i(S_p)_4 = 0.15, \ell_i(S_p)_5 = 0$

$t\ell_i(S_p) = \{(1,0), (2,0), (3,0.19), (4,0.15), (5,0)\}$

$S_p \leq M_p$ but $t\ell_i(M_p)_x \not\leq t\ell_i(S_p)_x$.

5. Locally Partner Multineutrosophic Exterior Set LPMES

Now in this section, we will give the form of the description of point outside the partner set, its relationship with the points inside the partner set, and the relationships that connect them together, while examining its properties.

Definition 5-1: A fuzzy point P_x^λ is called locally fuzzy exterior of a partner set M^p at the point $x \in X$ is defined by

$\ell_{ex}(M_p)_x = \vee_x \{G_p \in \tau_p; \lambda \leq f_{G_p}(x) \leq h_{1-M_p}(x) \text{ and } t\ell_e(M_p) = \vee_{x \in X} \ell_e(M_p)_x$ is called totally partner exterior of partner set M_p

Example 5-2: $A^6 = \{(1, (0.5,0.3), (0.1,0.2), (0.2,0.2), (2, (0.05,0.1), (0.2,0.2), (0.3,0.1)$

$(3, (0.03,0.08), (0.12,0.03), (0.17,0.1), (4, (0.1,0.2), (0.1,0.1), (0.08,0.12), (5, (0.4,0.2), (0.3,0.2), (0.28,0.12))\}$

B^6

$= \{(1, (0.1,0.2), (0.02,0.08,0.1), (0.1,0.1), (2, (0.2,0.2), (0.3,0.1,0.1), (0.4,0.2), (3, (0.6,0.3), 0.3,0.3,0.1), (0.5,0.4), (4, (0.1,0.3), (0.20,0.27,0.1), (0.1,0.1), (5, (0.05,0.05), (0.2,0.2,0.08), 90.2,0.02)\}$

$A^6 = \{(1,1,0.5), (2,0.5,0.25), (3,0.33,0.2), (4,0.7), (5,0.9,0.5)\}$

$B^7 = \{(1,0.7), (2,0.5,1), (3,1,0.15), (4,0.9,0.2), (5,0.2,0.6)\}$

$A_p = \{(x, \frac{\sum f(x)}{n}), \forall x \in X\}, A_p = \{(1,0.75), (2,0.375), (3,0.265), (4,0.35), (5,0.7)\}$

$B_p = \{(1,0.35), (2,0.75), (3,0.31), (4,0.55), (5,0.4)\}$

$A_p \wedge B_p = \{(x, \min(f(x), g(x)), \forall x \in X) = \{(1,0.35), (2,0.75), (3,0.31), (4,0.35), (5,0.4)\}$

$$A_p \vee B_p = \{(x, \max(f(x), g(x)), \forall x \in X\} = \{(1,0.75), (2,0.357), (3,0.265), (4,0.55), (5,0.7)\}$$

$$\tau_p = \{0_p, 1_p, A_p, B_p, A_p \vee B_p, A_p \wedge B_p\}$$

$$\text{Let } M_p = \{(1,0.52), (2,0.81), (3,0.43), (4,0.7), (5,0.33)\}$$

$$M_p^{\#c} = \{(x, 1 - h_{M^{\sim p}}(x), \forall x \in X\} = \{(1, 0.48), (2,0.643), (3,0.735), (4,0.45), (5,0.67)\}$$

$$\ell_e(M_p)_1 = 0.35, \ell_e(M_p)_2 = 0.375, \ell_e(M_p)_3 = 0.265, \ell_e(M_p)_4 = 0.35, \ell_e(M_p)_5 = 0.7$$

$$t\ell_e(M_p) = \{(1,0.35), (2,0.375), (3,0.265), (4,0.35), (5, (0.7))\}$$

1- By carefully analysing the above definition logically, we can conclude the following, $t\ell_e(0_p) = 1_p$ and $t\ell_e(1_p) = 0_p$

2- we can also notice $t\ell_e(M_p) \wedge M_p = 0_p$ for any PMSs M_p

Theorem 5-3: Let $(1_p, \tau_p)$ be partner fuzzy topological space and let A_p and B_p are PMSs then

1- if A_p nonzero PMCS then $t\ell_e(A_p) \neq 0$

2- $t\ell_i(M_p) \leq t\ell_e(t\ell_e(M_p))$

3- $\ell_{ex}(B_p \wedge A_p)_x = \ell_{ex}(A_p)_x \vee \ell_{ex}(B_p)_x$

4- $\ell_{ex}(A_p \vee B_p)_x = \ell_{ex}(A_p)_x \wedge \ell_{ex}(B_p)_x$

5- $t\ell_{ex}(A_p \wedge B_p)_x = t\ell_{ex}(A_p)_x \vee t\ell_{ex}(B_p)_x$

6- $t\ell_{ex}(A_p \vee B_p)_x \leq t\ell_{ex}(A_p)_x \wedge t\ell_{ex}(B_p)_x$

7- $\ell_{ex}(1_p - A_p) = \ell_i(A_p)$ and $\ell_i(1_p - A_p) = \ell_{ex}(A_p)$

Proof:

$$3- \ell_e(A_p \wedge B_p)_x = \vee_x \{V_p \varepsilon \tau_p, f_{V_p}(x) \leq 1 - \min(g_{A_p}(x), h_{B_p}(x))\} = \max(1 - f_{A_p}(x), 1 - g_{B_p}(x)) \\ = (\vee_x |V_p \varepsilon \tau_p, f_{A_p}(x) \leq 1 - h_{B_p}(x)\} = \ell_e(A_p)_x \vee \ell_e(B_p)_x$$

$$4- \ell_{ex}(A_p \vee B_p)_x = \vee \{V_p \varepsilon \tau_p, f_{V_p}(x) \leq 1 - \max(g_{A_p}(x), h_{B_p}(x))\} = \min(1 - g_{A_p}(x), 1 - h_{B_p}(x)) \\ = [\vee \{V_p \varepsilon \tau_p, f_{V_p}(x) \leq 1 - g_{A_p}(x)\}] \wedge [\vee \{V_p \varepsilon \tau_p, f_{V_p}(x) \leq 1 - h_{B_p}(x)\}] = \ell_{ex}(B_p)_x \wedge \ell_{ex}(A_p)_x$$

$$5- A_p \leq A_p \vee B_p \text{ \& } B_p \leq A_p \vee B_p$$

$$\Rightarrow t\ell_e(A_p \vee B_p) \leq t\ell_e(A_p) \text{ \& } t\ell_e(A_p \vee B_p) \leq t\ell_e(B_p)$$

$$t\ell_e(A_p \vee B_p) \leq t\ell_{ex}(A_p) \wedge t\ell_{ex}(B_p)$$

$$6- \ell_e(1_p - A_p)_x = \vee_x \{V_p \varepsilon \tau_p, f_{V_p}(x) \leq 1 - g_{1_p - A_p}(x) = g_{1_p - (1_p - A_p)}(x) = g_{A_p}(x)\} = \ell_i(A_p)$$

The converse of part (6) is not true

Example 5-4: Let $X = \{1,2,3,4\}$

$$A^5 = \{(1, (0.2,0.3), (0.3), (0.23,0.27), (2, (0.05,0.1), (0.3), (0.2,0.2), (3, (0.06,0.07), (0.03), (0.27,0.13), (4, (0.05,0.025), (0.05), (0.8,0.12))\}$$

$$B^5 = \{(1,0.03,0.05), (0.06), (0.04,0.15), (2, (0.08,0.1), (0.13), (0.07,0.2), (3, (0.021,0.1), (0.2), (0.2,0.1) (4, (0.076,0.2), (0.2), (0.3,0.1))\}$$

$$M^5 = \{(1, (0.05,0.1), (0.2), (0.3,0.1), (2, (0.04,0.1), (0.1), (0.23,0.17), (3, (0.014,0.2), (0.1), (0.23,0.17) (4, (0.01,0.1), (0.08), (0.02,0.2))\}, N^5 = \{(1, (0.03,0.08), (0.02), (0.1,0.2), (2, (0.03,0.08), (0.1), (0.1,0.2) (3, (0.2,0.3), (0.2), (0.1,0.1), (4, (0.06,0.3), (0.2), (0.4,0.1))\}$$

$$A_p \sim = \{(1,0.65), (2,0.425), (3,0.56), (4,0.1625)\}, B_p \sim = \{(1,0.34), (2,0.29), (3,0.31), (4,0.348)\}$$

$$A_p \sim \vee B_p \sim = \{(1,0.65), (2,0.425), (3,0.56), (4,0.348)\}, A_p \sim \wedge B_p \sim = \{(1,0.34), (2,0.29), (3,0.31), (4,0.1625)\}$$

$$\tau_p \sim = \{0_p \sim, 1_p \sim, A_p \sim, B_p \sim, A_p \sim \vee B_p \sim, A_p \sim \wedge B_p \sim\}$$

$$M_p \sim = \{(1,0.325), (2,0.31), (3,0.257), (4,0.205)\}, N_p \sim = \{(1,0.215), (2,0.35), (3,0.45), (0.53)\}$$

$$(M_p \sim)^c = \{(1,0.675), (2,0.69), (3,0.743), (4,0.795)\}, (N_p \sim)^c = \{(1,0.785), (2,0.65), (3,0.55), (4,0.46)\}$$

$$\ell_{ex}(M_p \sim)_1 = 0.65, \ell_{ex}(M_p \sim)_2 = 0.425, \ell_{ex}(M_p \sim)_3 = 0.56, \ell_{ex}(M_p \sim)_4 = 0.348$$

$$t\ell_{ex}(M_p \sim)_x = \{(1,0.65), (2,0.425), (3,0.56), (4,0.348)\}, t\ell_{ex}(N_p \sim)_x = \{(1,0.65), (2,0.425), (3,0.31), (4,0.348)\}$$

$$t\ell_{ex}(M_p \sim)_x \wedge t\ell_{ex}(N_p \sim)_x = \{(1,0.65), (2,0.425), (3, (0.31))\}, M_p \sim \vee N_p \sim = \{(1,0.325), (2,0.35), (3,0.45)\}$$

$$M_p \sim \vee N_p \sim = \{(1,0.675), (2,0.65), (3,0.55)\}$$

Theorem 5-5: Each of the following properties are equivalent, for any PMS in PMTS $(1_p \sim, \tau_p \sim)$

a - $t\ell_{ex}(A_p \sim) = 0_p \sim$

b- the only PMOS set contain in $1_p \sim - A_p \sim$ is PM empty set

c- the only PMCS set contain in $A_p \sim$ is the PM universal set

proof: $a \leftrightarrow b$

let $\forall x \in X, 0_{\tilde{p}} = \ell_e(A_{\tilde{p}})_x = \vee \ell_e(A_{\tilde{p}})_x$
 iff $\forall x \in X, 0_{\tilde{p}} = \ell_e(A_{\tilde{p}})_x = \wedge_x \{G_{\tilde{p}} \varepsilon \tau_{\tilde{p}} \ni f_{G_{\tilde{p}}}(x) = 1_{\tilde{p}} - g_{A_{\tilde{p}}}(x)\}$
 iff $\forall x \in X, G_{\tilde{p}} \varepsilon \tau_{\tilde{p}} \ni f_{G_{\tilde{p}}}(x) = 0_{\tilde{p}}$
 Iff the only PMOS contain in $1_{\tilde{p}} - A_{\tilde{p}}$ is PM empty set

Corollary5-6: If $t\ell_{ex}(A_{\tilde{p}}) = 0_{\tilde{p}}$ then $\forall U_{\tilde{p}} \varepsilon \tau_{\tilde{p}} \exists x \in X$ sch that $\min(f_{U_{\tilde{p}}}(x), g_{A_{\tilde{p}}}(x)) \neq 0_{\tilde{p}}$

Proof: if possible $\exists 0_{\tilde{p}} \neq U_{\tilde{p}} \varepsilon \tau_{\tilde{p}}$ and $\forall y \in X \min(f_{U_{\tilde{p}}}(y), g_{A_{\tilde{p}}}(y)) = 0_{\tilde{p}}$

$U_{\tilde{p}} \wedge A_{\tilde{p}} = 0_{\tilde{p}}$ then $U_{\tilde{p}} < 1_{\tilde{p}} - A_{\tilde{p}}$

Then the PMOS $U_{\tilde{p}}$ contain in $1_{\tilde{p}} - A_{\tilde{p}}$ which contradiction by theorem 5-5

For any a PMTS $(1_{\tilde{p}}, \tau_{\tilde{p}})$ and PMS $M_{\tilde{p}}$, we get $t\ell_i(M_{\tilde{p}}) = t\ell_i(t\ell_i(M_{\tilde{p}}))$, from this characteristic we conclude that $t\ell_e(M_{\tilde{p}}) = t\ell_e(1_{\tilde{p}} - t\ell_e(M_{\tilde{p}}))$

Through definition 2-1 and 3-1, we conclude that is not necessarily $t\ell_i(M_{\tilde{p}}) \wedge t\ell_e(M_{\tilde{p}}) = M_{\tilde{p}}$, and as in example 2-1, where $t\ell_i(M_{\tilde{p}}) = \{(1,0), (2,0.55), (3,0.5)\}$ and $t\ell_e(M_{\tilde{p}}) = \{(1,0.75), (2,0.46), (3,0.5)\}$, so $t\ell_i(M_{\tilde{p}}) \wedge t\ell_e(M_{\tilde{p}}) = t\ell_i(M_{\tilde{p}}) \neq 0_{\tilde{p}}$

Definition 5-7: A PMS $M_{\tilde{p}}$ is detachable set if $t\ell_i(M_{\tilde{p}}) \wedge t\ell_e(M_{\tilde{p}}) = 0_{\tilde{p}}$

Example 5-8: Let $X=\{1,2,3\}$,

$A^6 = \{(0.3,0.2), (0.3), (0.1,0.08,0.02), (2, (0.1,0.06), (0.2), (0.3,0.1,0.1), (3, (0.05,0.1), (0.1), (0.1,0.02,0.42))\}$

$B^6 = \{(1, (0.1,0.1), (0.1,0.2), (0.1), (2,)0.09,0.18), (0.12,0.3), (0.3), (3, (0.05,0.33), (0.27,0.3), (0.2))\}$

$M^5 = \{(1, (0.2), (0.1,0.2), (0.3,0.1), (2, (0.1), (0.1,0.08), (0.1,0.2), (3(0.17), (0.2,0.2), (0.21,0.19))\}$

$A_{\tilde{p}} = \{(1,0.5), (2,0.43), (3,0.425)\}$, $B_{\tilde{p}} = \{(1,0.75), (2,0.495), (3,0.375)\}$,

$A_{\tilde{p}} \wedge B_{\tilde{p}} = \{(1,0.5), (2,0.43), (3,0.375)\}$, $A_{\tilde{p}} \vee B_{\tilde{p}} = \{(1,0.75), (2,0.495), (3,0.425)\}$,

$\tau_{\tilde{p}} = \{0_{\tilde{p}}, 1_{\tilde{p}}, A_{\tilde{p}}, B_{\tilde{p}}, A_{\tilde{p}} \wedge B_{\tilde{p}}, A_{\tilde{p}} \vee B_{\tilde{p}}\}$, $M_{\tilde{p}} = \{(1,0.45), (2,0.25), (3,0.17)\}$,

$(M_{\tilde{p}})^c = \{(1,0.55), (2,0.75), (3,0.83)\}$

$\ell_i(M_{\tilde{p}})_1 = 0_{\tilde{p}}, \ell_i(M_{\tilde{p}})_2 = 0_{\tilde{p}}, \ell_i(M_{\tilde{p}})_3 = 0_{\tilde{p}}, t\ell_i(M_{\tilde{p}}) = \{(1, 0_{\tilde{p}}), (2, 0_{\tilde{p}}), (3, 0_{\tilde{p}})\}$

$\ell_e(M_{\tilde{p}})_1 = 0.5, \ell_e(M_{\tilde{p}})_2 = 0.495, \ell_e(M_{\tilde{p}})_3 = 0.425, t\ell_e(M_{\tilde{p}}) = \{(1,0.5), (2,0.495), (3,0.425)\}$

$t\ell_i(M_{\tilde{p}}) \wedge t\ell_e(M_{\tilde{p}}) = 0_{\tilde{p}}$

Though above definition, we conclude that $\forall x \in X, \ell_i(M_{\tilde{p}})_x \wedge \ell_e(M_{\tilde{p}})_x = 0_{\tilde{p}}$

Definition 5-9: For any PMS $M_{\tilde{p}}$ in PMTS $(1_{\tilde{p}}, \tau_{\tilde{p}})$ put their partner multineutrosophic conducted PMCOS of is defined by form $\forall_p(M_p)_x = \ell_i(M_p)_x \vee \ell_e(M_p)_x$ and the totally partner multineutrosophic conducted YPMCOS of is defined by $\forall^{~p} = \cup_{x \in X} \forall_x^{~p}$

Example 5-10: Let $X=\{1,2,3\}$

$A^6 = \{(1, (0.03,0.3), (0.17,0.13), (0.2,0.2), (2, (0.11,0.09), (0.23,0.07), (0.22,0.08),$

$(3, (0.01,0.1), (0.1,0.1), (0.1,0.1))\}$, $B^6 = \{(1, (0.01,0.09), (0.03,0.17), (0.1,0.2), (2, (0.05,0.04), (0.06,0.1), (0.2,0.2), (3, (0.06,0.3), (0.3,0.2), (0.1,0.1))\}$,

$M^5 = \{(1, (0.05,0.05), (0.06,0.04), (0.05), (2, (0.07,0.08), (0.2,0.2), (0.2),$

$(3, (0.034,0.2), (0.3,0.1), (0.2))\}$

$A_{\tilde{p}} = \{(1,0.65), (2,0.4), (3,0.255)\}$, $B_{\tilde{p}} = \{(1,0.3), (2,0.325), (3,0.53)\}$, $A_{\tilde{p}} \vee B_{\tilde{p}}$

$= \{(1,0.65), (2,0.325), (3,0.255)\}$

$A_{\tilde{p}} \wedge B_{\tilde{p}} = \{(1,0.3), (2,0.325), (3,0.255)\}$, $\tau_{\tilde{p}} = \{1_{\tilde{p}}, 0_{\tilde{p}}, A_{\tilde{p}}, B_{\tilde{p}}, A_{\tilde{p}} \wedge B_{\tilde{p}}, A_{\tilde{p}} \vee B_{\tilde{p}}\}$

Let $M = \{1,2,3\}$, $M_{\tilde{2}} = \{(1,0.125), (2,0.36), (3,0.437)\}$, $\ell_i(M_{\tilde{2}})_1 = 0_{\tilde{p}}, \ell_i(M_{\tilde{2}})_2 = 0.325, \ell_i(M_{\tilde{2}})_3 = 0.255$

$M_{\tilde{p}}^c = \{(1,0.875), (2,0.64), (3,0.563)\}$, $\ell_i(M_{\tilde{p}})_1 = 0.65, \ell_i(M_{\tilde{p}})_2 = 0.4, \ell_i(M_{\tilde{p}})_3 = 0.53$

$\forall_p(M_p)_x = \ell_e(M_{\tilde{p}})_x \vee \ell_i(M_{\tilde{p}})_x$, $\forall_p^1 = 0.65, \forall_p^2 = 0.4, \forall_p^3 = 0.53$

5. Conclusion

Throughout definition of LPMIS and LPMES, we have made clear that they are not necessarily TLPMISs, and in the same context we have also shown that TLPMIS and TLPMES are not necessarily PMOSs.

It is possible to generalize the concepts presented by both Abdulsada [10] and Al-Swidi[11] and study them on the PMSs presented.

References

[1] Zadeh L.A.(1965), Fuzzy set, Information and Control 8(142-150)

- [2] Tuqa A. hassen, L.A.A.Al.Swidi, Ali H. M. Al-Obaidi, (2024), partner Sets for Generalization of Neutrosophic Sets, International Journal of Neutrosophic Since.
- [3] Smarandache, F. (1999), A unifying field in Logics Neutrosophic Logic, In philosophy (pp. 1-141) American Research Press.
- [4] Al-Nafee, A.B, Broumi, s., Al-Swidi, L.A. (2021), n. Valued Refined Neutrosophic crisp Sets, International Journal of Neutrosophic Science, 17(2), (87-95).
- [5] Almohammed, B. , Ali Swidi, L.A., (2022), New concept of fuzzy local function, Baghdad science Journal, 17(2) (515-522).
- [6] Hadi, M.H, Al-Swidi, L.A.A., (2022), Neutrosophic Axial set theory, Neutrosophic Sets and Systems, 51(295-302).
- [7] Florentin Smarandache, (2015), Neutrosophic Precalculus Pons Publishing House Brussels, 2017.
- [8] N. Palaniappan, Fuzzy Topology, Alpha Science Intre. Ltd. 2002.
- [9] Smarandache, F. (2023), Introduction to the multineutrosophic set, neutrosophic sets and System, 61(I), pp.89-99.
- [10] Abdulsada, D.A., Al-Swidi, L.A.A., (2022), separation axioms of center topological space, Journal of Advanced Research in Dynamical and control Systems, 12(2), pp.186-192.
- [11] Al-Swidi, L.A. Al-Ethary, M.A. (2014), Compactness with Gem-set, International Journal of Mathematical Analysis, 8(21-24), pp.1105-1117 .
- [12] Q. H. Imran, A. H. M. Al-Obaidi, & Smarandache, F., On Some Types of Neutrosophic Topological Groups with Respect to Neutrosophic Alpha Open Sets. Neutrosophic Sets and Systems, 32, (2020), 425-434.
- [13] Q. H. Imran, R. K. Al-Hamido and A. H. M. Al-Obaidi, On new types of weakly neutrosophic crisp continuity. Neutrosophic Sets and Systems, 38(2020), 179-187.
- [14] Q. H. Imran, K. S. Tanak and A. H. M. Al-Obaidi, On new concepts of neutrosophic crisp open sets. Journal of Interdisciplinary Mathematics, 25(2)(2022), 563-572.
- [15] A. H. M. Al-Obaidi and Q. H. Imran, On new types of weakly neutrosophic crisp open mappings. Iraqi Journal of Science, 62(8) (2021), 2660-2666.
- [16] A. H. M. Al-Obaidi, Q. H. Imran and M. M. Abdulkadhim, On new types of weakly neutrosophic crisp closed functions. Neutrosophic Sets and Systems, 50(2022), 239-247.