

Paper Type: Original Article

Note for Neutrosophic Incidence and Threshold Graph

Takaaki Fujita ^{1,*} 

¹ Independent Researcher, Shinjuku, Shinjuku-ku, Tokyo, Japan; t171d603@gunma-u.ac.jp.

Received: 30 Sep 2024

Revised: 05 Nov 2024

Accepted: 15 Nov 2024

Published: 17 Nov 2024

Abstract

Uncertain Graph Theory has emerged to model the uncertainties present in real-world networks. An Incidence Graph represents the connections between vertices and edges using incidence pairs to illustrate relationships. A Threshold Graph is defined by vertex weights and thresholds, forming cliques or independent sets. We explore the concepts of the Pentapartitioned Neutrosophic Incidence Graph, Turiyam Neutrosophic Incidence Graph, Pentapartitioned Neutrosophic Fuzzy Threshold Graph, and Turiyam Neutrosophic Fuzzy Threshold Graph.

Keywords: Neutrosophic Graph; Fuzzy Graph; Turiyam Neutrosophic Graph; Threshold Graph; Incidence Graph.

1 | Introduction

1.1 | Uncertain Graph Theory

Graph theory is a fundamental branch of mathematics that uses vertices (nodes) and edges (connections) to model networks, effectively capturing relationships within various systems [34, 38, 63, 154, 162, 229].

This paper explores various models of uncertain graphs, such as Fuzzy, Intuitionistic Fuzzy, Neutrosophic, Turiyam, and Plithogenic Graphs. These models extend classical graph theory by incorporating different levels of uncertainty, enabling the analysis of complex and ambiguous relationships. The development of uncertain graph models has led to a wide range of applications in real-world contexts and has inspired the creation of many related graph classes [72, 73, 75-78, 80, 81, 83-87]. Core concepts like Fuzzy Sets and Neutrosophic Sets form the foundation of these models and are well-documented in the literature [20-24, 56, 65-67, 70, 125, 129, 159, 190, 219-224, 227].

Given the extensive literature and diverse applications, the study of uncertain graphs is highly significant for advancing our understanding of uncertain networks. For an in-depth exploration of these concepts, readers are encouraged to refer to the existing survey papers [79, 81, 83].

1.2 | Uncertain Incidence and Threshold Graph

Among the graph classes in Uncertain Graph Theory, Fuzzy Incidence Graph [1, 112, 140, 141, 148, 151, 152, 163, 170, 174, 179], Fuzzy Threshold Graph [3, 10, 104, 139, 155, 214], Neutrosophic Incidence Graph [11, 18, 110, 143, 144, 196], and Neutrosophic Fuzzy Threshold Graph [123] have been extensively studied. Incidence Graph Represents connections between vertices and edges, using incidence pairs to show



Corresponding Author: t171d603@gunma-u.ac.jp



<https://doi.org/10.61356/j.scin.2024.1421>



Licensee SciNexuses. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<http://creativecommons.org/licenses/by/4.0>).

relationships. The Incidence Graph in classical graph theory is a graph concept extended to Uncertain Graphs [201]. Threshold Graph is characterized by vertex weights and thresholds, forming cliques or independent sets. Threshold Graph in classical graph theory is a graph concept extended to Uncertain Graphs [52, 60, 113, 213]. These graphs are explored from various perspectives, including their applications and mathematical structures.

1.3 | Contributions

We explore the concepts of the Pentapartitioned Neutrosophic Incidence Graph and the Turiyam Neutrosophic Incidence Graph, as well as the Pentapartitioned Neutrosophic Fuzzy Threshold Graph and the Turiyam Neutrosophic Fuzzy Threshold Graph. Turiyam Neutrosophic Graphs enhance the traditional graph framework by integrating four membership values-truth, indeterminacy, falsity, and liberal state-at each vertex and edge, offering a more detailed representation of complex relationships [79, 81, 88, 187]. Meanwhile, the Pentapartitioned Neutrosophic Graph assigns five degrees (truth, contradiction, ignorance, unknown, falsity) to each vertex and edge, effectively capturing complex uncertainty [54, 109, 111, 167]. Related concepts include the Turiyam Neutrosophic Set [27, 81, 89, 90, 183, 186, 188] and the Pentapartitioned Neutrosophic Set [33, 55, 138, 168].

2 | Preliminaries and Definitions

This section provides an overview of the fundamental definitions and notations used throughout the paper.

2.1 | Basic Graph Concepts

Below are some of the foundational concepts in graph theory. For more comprehensive information on graph theory and its notations, refer to [61 63, 100 206].

Definition 1 (Graph). [63] A graph G is a mathematical structure that represents relationships between objects. It consists of a set of vertices $V(G)$ and a set of edges $E(G)$, where each edge connects a pair of vertices. Formally, a graph is represented as $G = (V, E)$, where V is the set of vertices and E is the set of edges.

Definition 2 (Degree). [63] Let $G = (V, E)$ be a graph. The degree of a vertex $v \in V$, denoted $\deg(v)$, is defined as the number of edges connected to v . For undirected graphs, the degree is given by:

$$\deg(v) = |\{e \in E \mid v \in e\}|$$

For directed graphs, the in-degree $\deg^-(v)$ refers to the number of edges directed towards v , while the out-degree $\deg^+(v)$ represents the number of edges directed away from v .

2.2 | Uncertain Graph

This paper addresses Fuzzy, Intuitionistic Fuzzy, Neutrosophic, Turiyam, and Plithogenic concepts. Note that Turiyam Neutrosophic Set is actually a particular case of the Quadruple Neutrosophic Set, by replacing "Contradiction" with "Liberal" [189].

Definition 3 (Unified Uncertain Graphs Framework). (cf. [82]) Let $G = (V, E)$ be a classical graph with a set of vertices V and a set of edges E . Depending on the type of graph, each vertex $v \in V$ and edge $e \in E$ is assigned membership values to represent various degrees of truth, indeterminacy, falsity, and other nuanced measures of uncertainty.

1. Fuzzy Graph [32, 74, 91, 95, 122, 147, 156, 175, 176, 197, 205]:
 - Each vertex $v \in V$ is assigned a membership degree $\sigma(v) \in [0, 1]$.
 - Each edge $e = (u, v) \in E$ is assigned a membership degree $\mu(u, v) \in [0, 1]$.
2. Intuitionistic Fuzzy Graph (IFG) [4, 31, 53, 114, 149, 199, 200, 231]:

- Each vertex $v \in V$ is assigned two values: $\mu_A(v) \in [0,1]$ (degree of membership) and $\nu_A(v) \in [0,1]$ (degree of non-membership), such that $\mu_A(v) + \nu_A(v) \leq 1$.
 - Each edge $e = (u, v) \in E$ is assigned two values: $\mu_B(u, v) \in [0,1]$ and $\nu_B(u, v) \in [0,1]$, with $\mu_B(u, v) + \nu_B(u, v) \leq 1$.
3. Neutrosophic Graph [10, 12, 39, 79, 84, 102, 108, 119, 180, 192, 195]:
- Each vertex $v \in V$ is assigned a triplet $\sigma(v) = (\sigma_T(v), \sigma_I(v), \sigma_F(v))$, where $\sigma_T(v), \sigma_I(v), \sigma_F(v) \in [0,1]$ and $\sigma_T(v) + \sigma_I(v) + \sigma_F(v) \leq 3$.
 - Each edge $e = (u, v) \in E$ is assigned a triplet $\mu(e) = (\mu_T(e), \mu_I(e), \mu_F(e))$.
4. Turiyam Neutrosophic Graph [88-90]:
- Each vertex $v \in V$ is assigned a quadruple $\sigma(v) = (t(v), iv(v), fv(v), lv(v))$, where each component is in $[0,1]$ and $t(v) + iv(v) + fv(v) + lv(v) \leq 4$.
 - Each edge $e = (u, v) \in E$ is similarly assigned a quadruple.
5. Vague Graph [7, 8, 35-37, 172, 173, 181]:
- Each vertex $v \in V$ is assigned a pair $(\tau(v), \phi(v))$, where $\tau(v) \in [0,1]$ is the degree of truth membership and $\phi(v) \in [0,1]$ is the degree of false-membership, with $\tau(v) + \phi(v) \leq 1$.
 - The grade of membership is characterized by the interval $[\tau(v), 1 - \phi(v)]$.
 - Each edge $e = (u, v) \in E$ is assigned a pair $(\tau(e), \phi(e))$, satisfying:

$$\tau(e) \leq \min\{\tau(u), \tau(v)\}, \phi(e) \geq \max\{\phi(u), \phi(v)\}$$
6. Hesitant Fuzzy Graph [26 96 157, 158, 210]:
- Each vertex $v \in V$ is assigned a hesitant fuzzy set $\sigma(v)$, represented by a finite subset of $[0,1]$, denoted $\sigma(v) \subseteq [0,1]$.
 - Each edge $e = (u, v) \in E$ is assigned a hesitant fuzzy set $\mu(e) \subseteq [0,1]$.
 - Operations on hesitant fuzzy sets (e.g., intersection, union) are defined to handle the hesitation in membership degrees.
7. Single-Valued Pentapartitioned Neutrosophic Graph [54, 109, 111, 167]:
- Each vertex $v \in V$ is assigned a quintuple $\sigma(v) = (T(v), C(v), R(v), U(v), F(v))$, where:
 - $T(v) \in [0,1]$ is the truth-membership degree.
 - $C(v) \in [0,1]$ is the contradiction-membership degree.
 - $R(v) \in [0,1]$ is the ignorance-membership degree.
 - $U(v) \in [0,1]$ is the unknown-membership degree.
 - $F(v) \in [0,1]$ is the false-membership degree.
 - $T(v) + C(v) + R(v) + U(v) + F(v) \leq 5$.
 - Each edge $e = (u, v) \in E$ is assigned a quintuple $\mu(e) = (T(e), C(e), R(e), U(e), F(e))$, satisfying:

$$\begin{cases} T(e) \leq \min\{T(u), T(v)\} \\ C(e) \leq \min\{C(u), C(v)\} \\ R(e) \geq \max\{R(u), R(v)\} \\ U(e) \geq \max\{U(u), U(v)\} \\ F(e) \geq \max\{F(u), F(v)\} \end{cases}$$

2.3 | Fuzzy Incidence Graph and Single-Valued Neutrosophic Incidence Graph

The definition of the already known Incidence Graph is described as follows.

Definition 4 (Fuzzy Incidence Graph). [1, 140] Let $G' = (V, E, I)$ be an incidence graph, where:

- V is a non-empty set of vertices,
- E is a set of edges,
- $I \subseteq V \times E$ is a set of incidence pairs.

A fuzzy incidence graph of G' , denoted as $\tilde{G} = (\mu, \lambda, \psi)$, is defined as an ordered triplet where:

- $\mu: V \rightarrow [0,1]$ is a fuzzy subset of the vertex set V ,
- $\lambda: E \rightarrow [0,1]$ is a fuzzy relation on the edge set E ,
- $\psi: I \rightarrow [0,1]$ is a fuzzy subset of the incidence set I .

The fuzzy incidence graph $\tilde{G}$ satisfies the following condition:

$$\psi(x, xy) \leq \min\{\mu(x), \lambda(xy)\}, \forall x \in V, xy \in E$$

where $\psi(x, xy)$ represents the degree of incidence between a vertex x and an edge xy .

Definition 5 (Single-Valued Neutrosophic Incidence Graph). [11] Let $G' = (V, E, I)$ be an incidence graph, where:

- V is a non-empty set of vertices,
- E is a set of edges,
- $I \subseteq V \times E$ is a set of incidence pairs.

A single-valued neutrosophic incidence graph (SVNIG) of G' , denoted as $\tilde{G} = (A, B, C)$, is defined as an ordered triplet where:

- A is a single-valued neutrosophic set on the vertex set V , with $A(x) = (T_A(x), I_A(x), F_A(x))$, where:
 - $T_A(x) \in [0,1]$: Truth-membership of vertex x ,
 - $I_A(x) \in [0,1]$: Indeterminacy-membership of vertex x ,
 - $F_A(x) \in [0,1]$: Falsity-membership of vertex x .
- B is a single-valued neutrosophic relation on the edge set E , with $B(xy) = (T_B(xy), I_B(xy), F_B(xy))$, where:
 - $T_B(xy) \in [0,1]$: Truth-membership of edge xy ,
 - $I_B(xy) \in [0,1]$: Indeterminacy-membership of edge xy ,
 - $F_B(xy) \in [0,1]$: Falsity-membership of edge xy .
- C is a single-valued neutrosophic subset of the incidence set I , with $C(x, xy) = (T_C(x, xy), I_C(x, xy), F_C(x, xy))$, where:

- $T_C(x, xy) \leq \min\{T_A(x), T_B(xy)\}$: Truth-membership condition,
 - $I_C(x, xy) \leq \min\{I_A(x), I_B(xy)\}$: Indeterminacy-membership condition,
 - $F_C(x, xy) \geq \max\{F_A(x), F_B(xy)\}$: Falsity-membership condition,
- for all $x \in V$ and $xy \in E$.

Example 6 (Single-Valued Neutrosophic Incidence Graph). [11] Consider an incidence graph $G = (V, E, I)$, where:

- The vertex set is $V = \{a, b, c, d\}$.
- The edge set is $E = \{ab, bc, bd, cd, ad\}$.
- The incidence set is

$$I = \{(a, ab), (b, ab), (b, bc), (c, bc), (b, bd), (d, bd), (c, cd), (d, cd), (d, ad), (a, ad)\}.$$

Let $\tilde{G} = (A, B, C)$ be a Single-Valued Neutrosophic Incidence Graph (SVNIG) associated with G , where:

- The single-valued neutrosophic set A on the vertex set V is defined as:

$$A = \{(a, 0.2, 0.5, 0.8), (b, 0.3, 0.5, 0.1), (c, 0.9, 0.9, 0.1), (d, 0.8, 0.1, 0.2)\}$$

Here, each vertex $x \in V$ is represented by a triplet $(T_A(x), I_A(x), F_A(x))$, indicating the truth, indeterminacy, and falsity memberships, respectively.

- The single-valued neutrosophic relation B on the edge set E is defined as:

$$B = \{(ab, 0.2, 0.4, 0.7), (bc, 0.3, 0.4, 0.1), (bd, 0.1, 0.1, 0.1), (cd, 0.7, 0.1, 0.2), (ad, 0.1, 0.1, 0.5)\}$$

Each edge $xy \in E$ is represented by a triplet $(T_B(xy), I_B(xy), F_B(xy))$, indicating the truth, indeterminacy, and falsity memberships, respectively.

- The single-valued neutrosophic subset C of the incidence set I is defined as:

$$C = \{((a, ab), 0.2, 0.3, 0.7), ((b, ab), 0.1, 0.4, 0.6), ((b, bc), 0.3, 0.3, 0.1), ((c, bc), 0.2, 0.3, 0.1), ((b, bd), 0.1, 0.1, 0.1), ((d, bd), 0.1, 0.1, 0.2), ((c, cd), 0.7, 0.1, 0.2), ((d, cd), 0.7, 0.1, 0.2), ((d, ad), 0.1, 0.1, 0.4), ((a, ad), 0.1, 0.1, 0.5)\}$$

Each incidence pair $(x, xy) \in I$ is represented by a triplet $(T_C(x, xy), I_C(x, xy), F_C(x, xy))$, indicating the truth, indeterminacy, and falsity memberships, respectively.

Thus, the Single-Valued Neutrosophic Incidence Graph $\tilde{G} = (A, B, C)$ accurately represents the uncertain relationships among the vertices, edges, and incidence pairs in the original incidence graph G .

The following holds obviously.

Proposition 7. Every Single-Valued Neutrosophic Incidence Graph (SVNIG) can be transformed into a Fuzzy Incidence Graph (FIG).

Proof. Let $\tilde{G} = (A, B, C)$ be a given Single-Valued Neutrosophic Incidence Graph. To transform $\tilde{G}$ into a Fuzzy Incidence Graph $\tilde{G}' = (\mu, \lambda, \psi)$, we define the following mappings:

- For the vertex set V , define:

$$\mu(x) = T_A(x), \forall x \in V$$

This mapping assigns the truth-membership of each vertex in the SVNIG to the fuzzy membership of the corresponding vertex in the FIG.

- For the edge set E , define:

$$\lambda(xy) = T_B(xy), \forall xy \in E.$$

This mapping assigns the truth-membership of each edge in the SVNIG to the fuzzy membership of the corresponding edge in the FIG.

- For the incidence set I , define:

$$\psi(x, xy) = T_C(x, xy), \forall (x, xy) \in I.$$

This mapping assigns the truth-membership of each incidence pair in the SVNIG to the fuzzy membership of the corresponding incidence pair in the FIG.

To verify that $\tilde{G}' = (\mu, \lambda, \psi)$ satisfies the conditions of a Fuzzy Incidence Graph, we need to check the incidence constraint:

$$\psi(x, xy) \leq \min\{\mu(x), \lambda(xy)\}, \forall x \in V, xy \in E$$

From the definition of $\tilde{G}$, we know that:

$$T_C(x, xy) \leq \min\{T_A(x), T_B(xy)\}, \forall x \in V, xy \in E.$$

By substituting the mappings for $\mu(x)$, $\lambda(xy)$, and $\psi(x, xy)$, we get:

$$\psi(x, xy) = T_C(x, xy) \leq \min\{T_A(x), T_B(xy)\} = \min\{\mu(x), \lambda(xy)\}, \forall x \in V, xy \in E.$$

Thus, the transformed graph $\tilde{G}' = (\mu, \lambda, \psi)$ satisfies the conditions of a Fuzzy Incidence Graph.

Therefore, every Single-Valued Neutrosophic Incidence Graph can be transformed into a Fuzzy Incidence Graph.

2.4 | Intuitionistic Fuzzy Threshold Graph and Neutrosophic Fuzzy Threshold Graph

The definition of the already known Threshold Graph is described as follows.

Definition 8. [214] An Intuitionistic Fuzzy Threshold Graph (IFTG) is a special type of Intuitionistic Fuzzy Graph (IFG) defined by two threshold parameters t_1 and t_2 . An IFTG is represented as $G = (A, B; t_1, t_2)$, where:

- $G = (A, B)$ is an Intuitionistic Fuzzy Graph, where:
 - $A = (\mu_A, \nu_A)$ is an Intuitionistic Fuzzy Set (IFS) on the set of vertices V^* , with:

$$\begin{aligned} \mu_A: V^* &\rightarrow [0,1] \\ \nu_A: V^* &\rightarrow [0,1] \end{aligned}$$

satisfying:

$$\mu_A(u) + \nu_A(u) \leq 1, \forall u \in V^*$$

- $B = (\mu_B, \nu_B)$ is an Intuitionistic Fuzzy Relation (IFR) on the set of edges E^* , with:

$$\begin{aligned} \mu_B: E^* &\rightarrow [0,1], \\ \nu_B: E^* &\rightarrow [0,1], \end{aligned}$$

satisfying:

$$\mu_B(u, v) + \nu_B(u, v) \leq 1, \forall (u, v) \in E^*$$

- $t_1 > 0$ and $t_2 > 0$ are the threshold parameters.

The graph G is called an IFTG if and only if the following conditions hold:

$$\sum_{u \in U} \mu_A(u) \leq t_1 \text{ and } \sum_{u \in U} (1 - v_A(u)) \leq t_2$$

for any subset $U \subseteq V^*$, where U is an independent set in the underlying graph G^* of G .

Example 9 (Example of an Intuitionistic Fuzzy Threshold Graph (IFTG)). [214] Consider a graph $G^* = (V^*, E^*)$ with the following sets of vertices and edges:

$$V^* = \{a, b, c, d, e\}$$

$$E^* = \{(a, b), (b, e), (e, d), (b, d), (b, c), (c, d)\}$$

Let A be an intuitionistic fuzzy subset on V^* and B be an intuitionistic fuzzy subset on E^* . The intuitionistic fuzzy membership degrees for vertices and edges are as follows:

Vertices:

Vertex	μ_A	v_A
a	0.1	0.9
b	0.5	0.4
c	0.2	0.7
d	0.4	0.6
e	0.2	0.8

Edges:

Edge	μ_B	v_B
(a, b)	0.1	0.9
(b, c)	0.2	0.7
(b, d)	0.4	0.6
(b, e)	0.2	0.8
(e, d)	0.2	0.7
(c, d)	0.2	0.6

Now, based on the intuitionistic fuzzy sets A and B , we define the Intuitionistic Fuzzy Threshold Graph $G = (A, B; t_1, t_2)$ with thresholds:

$$t_1 = 0.5, t_2 = 0.6$$

The graph G is called an Intuitionistic Fuzzy Threshold Graph (IFTG) if, for any subset $U \subseteq V^*$ that is an independent set in G^* , the following conditions hold:

$$\sum_{u \in U} \mu_A(u) \leq t_1 \text{ and } \sum_{u \in U} (1 - v_A(u)) \leq t_2$$

Example Calculation Let $U = \{b, e\}$ be an independent set in G^* .

- Sum of membership degrees μ_A :

$$\mu_A(b) + \mu_A(e) = 0.5 + 0.2 = 0.7 \text{ (which exceeds } t_1 = 0.5)$$

Thus, this subset does not satisfy the first condition.

- Sum of non-membership complements $1 - v_A$:

$$(1 - v_A(b)) + (1 - v_A(e)) = (1 - 0.4) + (1 - 0.8) = 0.6 + 0.2 = 0.8 \text{ (which exceeds } t_2 = 0.6)$$

Thus, this subset does not satisfy the second condition.

Therefore, the set $U = \{b, e\}$ is not an independent set in the IFTG G .

Definition 10. [123] A Neutrosophic Fuzzy Threshold Graph (NFTG) is an extension of neutrosophic fuzzy graphs that incorporates three threshold parameters to define its independent sets. An NFTG is denoted as $(P, Q; \tau_1, \tau_2, \tau_3)$, where:

- $G = (P, Q)$ is a Neutrosophic Fuzzy Graph (NFG) defined on a vertex set V^* and an edge set E^* .
 - The neutrosophic fuzzy set P on the vertices V^* is defined by three functions:

$$\begin{aligned} \mu_P: V^* &\rightarrow [0,1], \text{ (truth-membership)} \\ v_P: V^* &\rightarrow [0,1], \text{ (falsity-membership)} \\ \sigma_P: V^* &\rightarrow [0,1], \text{ (indeterminacy-membership)} \end{aligned}$$

- The neutrosophic fuzzy set Q on the edges E^* is defined by three functions:

$$\begin{aligned} \mu_Q: E^* &\rightarrow [0,1], \\ v_Q: E^* &\rightarrow [0,1], \\ &\text{(truth-membership)} \\ \sigma_Q: E^* &\rightarrow [0,1], \\ &\text{(falsity-membership)} \\ &\text{(indeterminacy-membership)} \end{aligned}$$

The graph $G = (P, Q)$ is defined as an *NFTG* if there exist three positive thresholds $\tau_1 > 0, \tau_2 > 0$, and $\tau_3 > 0$ such that, for any subset $U \subseteq V^*$ that is an independent set in G , the following conditions hold:

$$\sum_{u \in U} \mu_P(u) \leq \tau_1, \quad \sum_{u \in U} (1 - v_P(u)) \leq \tau_2, \quad \text{and} \quad \sum_{u \in U} \sigma_P(u) \leq \tau_3$$

Remark

- The notion of an independent set in an *NFTG* is the same as in its underlying classical graph G^* .
- If $G = (P, Q; \tau_1, \tau_2, \tau_3)$ and $U \subseteq V^*$ is a dependent set in G , then at least one of the following inequalities must hold:

$$\sum_{u \in U} \mu_P(u) > \tau_1, \quad \sum_{u \in U} (1 - v_P(u)) > \tau_2, \quad \text{or} \quad \sum_{u \in U} \sigma_P(u) > \tau_3$$

Example 11. (Example of a Neutrosophic Fuzzy Threshold Graph (NFTG)). Consider a graph $G^* = (V^*, E^*)$ with the following sets of vertices and edges:

$$\begin{aligned} V^* &= \{m, n, o, p, q\} \\ E^* &= \{(m, n), (n, o), (o, p), (p, n), (n, q)\} \end{aligned}$$

Let P be the neutrosophic fuzzy subset defined on V^* and Q be the neutrosophic fuzzy subset defined on E^* . The degrees of truth-membership, falsity-membership, and indeterminacy-membership for vertices and edges are provided in the following tables:

Vertices:

Vertex	μ_P	v_P	σ_P
m	0.6	0.2	0.1
n	0.2	0.3	0.1
o	0.2	0.4	0.2
p	0.1	0.2	0.3
q	0.3	0.4	0.2

Edges:

Edge	μ_Q	v_Q	σ_Q
(m, n)	0.2	0.5	0.2
(n, o)	0.1	0.6	0.2
(o, p)	0.1	0.4	0.3
(p, n)	0.1	0.5	0.3
(n, q)	0.2	0.5	0.2

Based on the neutrosophic fuzzy subsets P and Q , we define the Neutrosophic Fuzzy Threshold Graph (NFTG) as:

$$G = (P, Q; \tau_1, \tau_2, \tau_3)$$

with the thresholds:

$$\tau_1 = 0.6, \tau_2 = 0.8, \tau_3 = 0.5$$

The graph G is called a Neutrosophic Fuzzy Threshold Graph (NFTG) if, for any subset $U \subseteq V^*$ that is an independent set in the underlying graph G^* , the following conditions hold:

$$\sum_{u \in U} \mu_P(u) \leq \tau_1, \sum_{u \in U} (1 - v_P(u)) \leq \tau_2, \sum_{u \in U} \sigma_P(u) \leq \tau_3$$

Example Calculation. Let $U = \{m, o\}$ be an independent set in G^* .

- Sum of truth-membership degrees μ_P :

$$\mu_P(m) + \mu_P(o) = 0.6 + 0.2 = 0.8 \text{ (which exceeds } \tau_1 = 0.6 \text{)}$$

Thus, this subset does not satisfy the first condition.

- Sum of non-membership complements $1 - v_P$:

$$(1 - v_P(m)) + (1 - v_P(o)) = (1 - 0.2) + (1 - 0.4) = 0.8 + 0.6 = 1.4 \text{ (which exceeds } \tau_2 = 0.8 \text{)}$$

Thus, this subset does not satisfy the second condition.

- Sum of indeterminacy-membership degrees σ_P :

$$\sigma_P(m) + \sigma_P(o) = 0.1 + 0.2 = 0.3 \text{ (which is within } \tau_3 = 0.5 \text{)}$$

This subset satisfies the third condition.

Therefore, the set $U = \{m, o\}$ is not an independent set in the NFTG G .

3 | Results

State the results of this paper.

3.1 | Result: Incidence Graph

Define the Single-Valued Turiyam Neutrosophic Incidence Graph and the Single-Valued Pentapartitioned Neutrosophic Incidence Graph, and then prove that they can be transformed into other graph classes.

Definition 12 (Single-Valued Turiyam Neutrosophic Incidence Graph). Let $G' = (V, E, I)$ be an incidence graph, where:

- V is a non-empty set of vertices.
- E is a set of edges.
- $I \subseteq V \times E$ is a set of incidence pairs.

A Single-Valued Turiyam Neutrosophic Incidence Graph (SVTIG) of G' , denoted as $\tilde{G} = (A, B, C)$, is defined as an ordered triplet where:

1. A is a single-valued Turiyam Neutrosophic set on the vertex set V , with $A(x) = (t_A(x), iv_A(x), fv_A(x), lv_A(x))$, where:
 - $t_A(x) \in [0,1]$: Truth-membership degree of vertex x .
 - $iv_A(x) \in [0,1]$: Indeterminacy-membership degree of vertex x .
 - $fv_A(x) \in [0,1]$: Falsity-membership degree of vertex x .
 - $lv_A(x) \in [0,1]$: Liberation-membership degree of vertex x .
 - Sum condition: $t_A(x) + iv_A(x) + fv_A(x) + lv_A(x) \leq 4$.
2. B is a single-valued Turiyam Neutrosophic relation on the edge set E , with $B(xy) = (t_B(xy), iv_B(xy), fv_B(xy), lv_B(xy))$, where:
 - $t_B(xy) \in [0,1]$: Truth-membership degree of edge xy .
 - $iv_B(xy) \in [0,1]$: Indeterminacy-membership degree of edge xy .
 - $fv_B(xy) \in [0,1]$: Falsity-membership degree of edge xy .
 - $lv_B(xy) \in [0,1]$: Liberation-membership degree of edge xy .
 - Sum condition: $t_B(xy) + iv_B(xy) + fv_B(xy) + lv_B(xy) \leq 4$.
3. C is a single-valued Turiyam Neutrosophic subset of the incidence set I , with $C(x, xy) = (t_C(x, xy), iv_C(x, xy), fv_C(x, xy), lv_C(x, xy))$ satisfying:

$$\begin{aligned} t_C(x, xy) &\leq \min\{t_A(x), t_B(xy)\} \\ iv_C(x, xy) &\leq \min\{iv_A(x), iv_B(xy)\} \\ fv_C(x, xy) &\geq \max\{fv_A(x), fv_B(xy)\} \\ lv_C(x, xy) &\geq \max\{lv_A(x), lv_B(xy)\} \end{aligned}$$

for all $x \in V$ and $xy \in E$.

Definition 13 (Single-Valued Pentapartitioned Neutrosophic Incidence Graph). Let $G' = (V, E, I)$ be an incidence graph, where:

- V is a non-empty set of vertices.

- E is a set of edges.
- $I \subseteq V \times E$ is a set of incidence pairs.

A Single-Valued Pentapartitioned Neutrosophic Incidence Graph (SVPPNIG) of G' , denoted as $\tilde{G} = (A, B, C)$, is defined as an ordered triplet where:

1. A is a single-valued pentapartitioned neutrosophic set on the vertex set V , with

$$A(x) = (T_A(x), C_A(x), R_A(x), U_A(x), F_A(x))$$

, where:

- $T_A(x) \in [0,1]$: Truth-membership degree of vertex x .
- $C_A(x) \in [0,1]$: Contradiction-membership degree of vertex x .
- $R_A(x) \in [0,1]$: Ignorance-membership degree of vertex x .
- $U_A(x) \in [0,1]$: Unknown-membership degree of vertex x .
- $F_A(x) \in [0,1]$: Falsity-membership degree of vertex x .
- Sum condition: $T_A(x) + C_A(x) + R_A(x) + U_A(x) + F_A(x) \leq 5$.

2. B is a single-valued Pentapartitioned neutrosophic relation on the edge set E , with

$$B(xy) = (T_B(xy), C_B(xy), R_B(xy), U_B(xy), F_B(xy))$$

, where:

- $T_B(xy) \in [0,1]$: Truth-membership degree of edge xy .
- $C_B(xy) \in [0,1]$: Contradiction-membership degree of edge xy .
- $R_B(xy) \in [0,1]$: Ignorance-membership degree of edge xy .
- $U_B(xy) \in [0,1]$: Unknown-membership degree of edge xy .
- $F_B(xy) \in [0,1]$: Falsity-membership degree of edge xy .
- Sum condition: $T_B(xy) + C_B(xy) + R_B(xy) + U_B(xy) + F_B(xy) \leq 5$.

3. C is a single-valued pentapartitioned neutrosophic subset of the incidence set I , with

$$C(x, xy) = (T_C(x, xy), C_C(x, xy), R_C(x, xy), U_C(x, xy), F_C(x, xy))$$

, satisfying:

$$\begin{aligned} T_C(x, xy) &\leq \min\{T_A(x), T_B(xy)\} \\ C_C(x, xy) &\leq \min\{C_A(x), C_B(xy)\} \\ R_C(x, xy) &\leq \min\{R_A(x), R_B(xy)\} \\ U_C(x, xy) &\leq \min\{U_A(x), U_B(xy)\} \\ F_C(x, xy) &\geq \max\{F_A(x), F_B(xy)\} \end{aligned}$$

for all $x \in V$ and $xy \in E$.

Theorem 14. A Single-Valued Pentapartitioned Neutrosophic Incidence Graph (SVPPNIG) can be transformed into:

1. A Single-Valued Turiyam Neutrosophic Incidence Graph (SVTIG) by merging specific membership degrees and adjusting the sum conditions accordingly.

2. A Neutrosophic Incidence Graph by setting certain membership degrees to zero and reinterpreting others.
3. A Fuzzy Incidence Graph by simplifying the membership degrees from the Neutrosophic Incidence Graph.

Proof. 1. Transformation from SVPPNIG to SVTIG

Let $\tilde{G} = (A, B, C)$ be an SVPPNIG. For each vertex $x \in V$ and edge $xy \in E$, the membership degrees are:

- Vertices:

$$A(x) = (T_A(x), C_A(x), R_A(x), U_A(x), F_A(x)), T_A(x) + C_A(x) + R_A(x) + U_A(x) + F_A(x) \leq 5$$

- Edges:

$$B(xy) = (T_B(xy), C_B(xy), R_B(xy), U_B(xy), F_B(xy)), T_B(xy) + C_B(xy) + R_B(xy) + U_B(xy) + F_B(xy) \leq 5.$$

To transform $\tilde{G}$ into an SVTIG, proceed as follows:

1. Merge Contradiction-membership into Truth-membership:

$$t_A(x) = T_A(x) + C_A(x), t_B(xy) = T_B(xy) + C_B(xy)$$

2. Rename Ignorance and Unknown-membership degrees:

$$iv_A(x) = R_A(x), lv_A(x) = U_A(x); iv_B(xy) = R_B(xy), lv_B(xy) = U_B(xy)$$

3. Keep Falsity-membership degrees unchanged:

$$fv_A(x) = F_A(x), fv_B(xy) = F_B(xy).$$

4. Sum Conditions:

$$t_A(x) + iv_A(x) + fv_A(x) + lv_A(x) = T_A(x) + C_A(x) + R_A(x) + F_A(x) + U_A(x) \leq 5.$$

5. Normalization: To conform to the SVTIG sum condition (≤ 4), normalize the membership degrees:

$$\tilde{t}_A(x) = \frac{t_A(x)}{5} \times 4, \tilde{iv}_A(x) = \frac{iv_A(x)}{5} \times 4, \tilde{fv}_A(x) = \frac{fv_A(x)}{5} \times 4, \tilde{lv}_A(x) = \frac{lv_A(x)}{5} \times 4.$$

Similarly

for

edges.

The sum condition becomes:

$$\tilde{t}_A(x) + \tilde{iv}_A(x) + \tilde{fv}_A(x) + \tilde{lv}_A(x) \leq 4.$$

6. Incidence Conditions: The incidence membership degrees $C(x, xy)$ transform accordingly, satisfying the SVTIG conditions.

Thus, the SVPPNIG is transformed into an SVTIG.

2. Transformation from SVPPNIG to Neutrosophic Incidence Graph Proceed as follows:

1. Set Contradiction and Unknown-membership degrees to zero:

$$C_A(x) = U_A(x) = 0, C_B(xy) = U_B(xy) = 0.$$

2. Rename Ignorance-membership degree as Indeterminacy-membership degree:

$$I_A(x) = R_A(x), I_B(xy) = R_B(xy).$$

3. Define Remaining Membership Degrees:

$$A(x) = (T_A(x), I_A(x), F_A(x)), B(xy) = (T_B(xy), I_B(xy), F_B(xy)).$$

4. Sum Conditions:

$$T_A(x) + I_A(x) + F_A(x) = T_A(x) + R_A(x) + F_A(x) \leq 5.$$

5. Normalization: Normalize the membership degrees to satisfy the Neutrosophic sum condition (≤ 3):

$$\tilde{T}_A(x) = \frac{T_A(x)}{5} \times 3, \tilde{I}_A(x) = \frac{I_A(x)}{5} \times 3, \tilde{F}_A(x) = \frac{F_A(x)}{5} \times 3.$$

Similarly

for

edges.

The sum condition becomes:

$$\tilde{T}_A(x) + \tilde{I}_A(x) + \tilde{F}_A(x) \leq 3$$

6. Incidence Conditions: The incidence membership degrees $\mathcal{C}(x, xy)$ adjust accordingly, satisfying the Neutrosophic incidence graph conditions.

Thus, the SVPPNIG is transformed into a Neutrosophic Incidence Graph.

3. Transformation from Neutrosophic Incidence Graph to Fuzzy Incidence Graph Proceed as follows:

1. Simplify Membership Degrees:

$$\mu(x) = \tilde{T}_A(x), \lambda(xy) = \tilde{T}_B(xy)$$

2. Incidence Membership Degrees:

$$\psi(x, xy) = \min\{\mu(x), \lambda(xy)\}, \forall x \in V, xy \in E.$$

3. Sum Conditions: Since $\mu(x) \in [0,1]$, the fuzzy incidence graph conditions are satisfied.

Thus, the Neutrosophic Incidence Graph simplifies to a Fuzzy Incidence Graph.

3.2 | Result: Threshold Graph

We will provide the definitions of Threshold Graphs extended to Turiyam Neutrosophic Graphs and Single-Valued Pentapartitioned Neutrosophic Graphs, and examine their relationships with other graph classes. The definitions are described as follows.

Definition 15. A Turiyam Neutrosophic Fuzzy Threshold Graph (TFTG) is a graph $G = (V, E)$ where:

- Each vertex $v \in V$ is associated with four membership degrees:

$$\begin{aligned} t(v) &\in [0,1], && \text{(truth-membership)} \\ iv(v) &\in [0,1], && \text{(indeterminacy due to indeterminacy)} \\ fv(v) &\in [0,1], && \text{(falsity-membership)} \\ lv(v) &\in [0,1], && \text{(latent-membership)} \end{aligned}$$

satisfying:

$$t(v) + iv(v) + fv(v) + lv(v) \leq 4$$

- For any independent set $U \subseteq V$, the threshold conditions are:

$$\sum_{v \in U} t(v) \leq \tau_1, \sum_{v \in U} iv(v) \leq \tau_2, \sum_{v \in U} fv(v) \leq \tau_3, \sum_{v \in U} lv(v) \leq \tau_4,$$

where $\tau_1, \tau_2, \tau_3, \tau_4 > 0$.

Definition 16. A Pentapartitioned Neutrosophic Fuzzy Threshold Graph (PNFTG) is a graph $G = (V, E)$ where:

- Each vertex $v \in V$ is associated with five membership degrees:

$$\begin{aligned} T(v) &\in [0,1], (\text{truth} - \text{membership}) \\ C(v) &\in [0,1], (\text{contradiction} - \text{membership}) \\ R(v) &\in [0,1], (\text{ignorance} - \text{membership}) \\ U(v) &\in [0,1], (\text{unknown} - \text{membership}) \\ F(v) &\in [0,1], (\text{falsity} - \text{membership}) \end{aligned}$$

satisfying:

$$T(v) + C(v) + R(v) + U(v) + F(v) \leq 5$$

- For any independent set $U \subseteq V$, the threshold conditions are:

$$\sum_{v \in U} T(v) \leq \theta_1, \sum_{v \in U} C(v) \leq \theta_2, \sum_{v \in U} R(v) \leq \theta_3, \sum_{v \in U} U(v) \leq \theta_4, \sum_{v \in U} F(v) \leq \theta_5,$$

where $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5 > 0$.

Theorem 17. Every Turiyam Neutrosophic Fuzzy Threshold Graph can be transformed into a Neutrosophic Fuzzy Threshold Graph.

Proof. We define the mappings:

$$\begin{aligned} \mu_P(v) &= t(v) \\ \sigma_P(v) &= iv(v) + lv(v) \\ v_P(v) &= fv(v) \end{aligned}$$

Since:

$$\mu_P(v) + \sigma_P(v) + v_P(v) \leq 4$$

We normalize:

$$\begin{aligned} \mu'_P(v) &= \frac{\mu_P(v)}{4} \times 3 \\ \sigma'_P(v) &= \frac{\sigma_P(v)}{4} \times 3 \\ v'_P(v) &= \frac{v_P(v)}{4} \times 3 \end{aligned}$$

Adjust the thresholds:

$$\tau'_i = \frac{\tau_i}{4} \times 3, i = 1, 2, 3$$

Thus, we obtain a Neutrosophic Fuzzy Threshold Graph $G' = (P', Q'; \tau'_1, \tau'_2, \tau'_3)$.

Theorem 18. Every Pentapartitioned Neutrosophic Fuzzy Threshold Graph can be transformed into a Neutrosophic Fuzzy Threshold Graph and an Intuitionistic Fuzzy Threshold Graph.

Proof. We define the mappings:

$$\begin{aligned} \mu_P(v) &= T(v) \\ \sigma_P(v) &= C(v) + R(v) + U(v) \\ v_P(v) &= F(v) \end{aligned}$$

Since:

$$\mu_P(v) + \sigma_P(v) + v_P(v) \leq 5$$

We normalize:

$$\begin{aligned}\mu'_P(v) &= \frac{\mu_P(v)}{5} \times 3 \\ \sigma'_P(v) &= \frac{\sigma_P(v)}{5} \times 3 \\ v'_P(v) &= \frac{v_P(v)}{5} \times 3\end{aligned}$$

Adjust the thresholds:

$$\tau'_i = \frac{\theta_i}{5} \times 3, \quad i = 1, 2, 3$$

To transform into an Intuitionistic Fuzzy Threshold Graph, we map:

$$\begin{aligned}\mu_A(v) &= \frac{T(v)}{T(v) + F(v)} \\ v_A(v) &= \frac{F(v)}{T(v) + F(v)}\end{aligned}$$

This ensures $\mu_A(v) + v_A(v) = 1$.

Theorem 19. Every Pentapartitioned Neutrosophic Fuzzy Threshold Graph can be transformed into a Turiyam Neutrosophic Fuzzy Threshold Graph.

Proof. Define the mappings for each vertex $\in V$:

$$\begin{aligned}t(v) &= T(v) \\ iv(v) &= C(v) + R(v) \\ fv(v) &= F(v) \\ lv(v) &= U(v)\end{aligned}$$

Then,

$$\begin{aligned}t(v) + iv(v) + fv(v) + lv(v) &= T(v) + [C(v) + R(v)] + F(v) + U(v) \\ &= T(v) + C(v) + R(v) + U(v) + F(v) \\ &\leq 5\end{aligned}$$

Normalize the membership degrees:

$$\begin{aligned}t'(v) &= \frac{t(v)}{5} \times 4 \\ iv'(v) &= \frac{iv(v)}{5} \times 4 \\ fv'(v) &= \frac{fv(v)}{5} \times 4 \\ lv'(v) &= \frac{lv(v)}{5} \times 4\end{aligned}$$

Adjust the thresholds:

$$\tau_i = \frac{\theta_i}{5} \times 4, \quad \text{for } i = 1, 2, 3, 4$$

Thus, we obtain a Turiyam Neutrosophic Fuzzy Threshold Graph $G' = (V, E)$.

Theorem 20. In a Pentapartitioned Neutrosophic Fuzzy Threshold Graph G , the sum of all membership degrees for any independent set $U \subseteq V$ satisfies:

$$\sum_{v \in U} [T(v) + C(v) + R(v) + U(v) + F(v)] \leq 5 \times |U|$$

Proof. For each $v \in U$:

$$T(v) + C(v) + R(v) + U(v) + F(v) \leq 5$$

Summing over :

$$\sum_{v \in U} [T(v) + C(v) + R(v) + U(v) + F(v)] \leq 5 \times |U|$$

Theorem 21. The maximum cardinality of an independent set $U \subseteq V$ in a Pentapartitioned Neutrosophic Fuzzy Threshold Graph G is bounded by:

$$|U| \leq \min \left\{ \frac{\theta_1}{\delta_T}, \frac{\theta_2}{\delta_C}, \frac{\theta_3}{\delta_R}, \frac{\theta_4}{\delta_U}, \frac{\theta_5}{\delta_F} \right\}$$

where $\delta_i = \min\{d_i(v) \mid v \in V\}$ for $i = T, C, R, U, F$.

Proof. Since $d_i(v) \geq \delta_i$ for all $v \in U$:

$$\delta_i \times |U| \leq \theta_i \Rightarrow |U| \leq \frac{\theta_i}{\delta_i}$$

Thus,

$$|U| \leq \min \left\{ \frac{\theta_1}{\delta_T}, \frac{\theta_2}{\delta_C}, \frac{\theta_3}{\delta_R}, \frac{\theta_4}{\delta_U}, \frac{\theta_5}{\delta_F} \right\}.$$

Theorem 22. A Pentapartitioned Neutrosophic Fuzzy Threshold Graph does not contain any Pentapartitioned neutrosophic fuzzy alternating 4-cycles.

Proof. Assuming the existence of such a cycle leads to a violation of the threshold conditions due to the cumulative membership degrees exceeding the thresholds. This contradicts the definition of a PNFTG.

Theorem 23. If G is a Pentapartitioned Neutrosophic Fuzzy Threshold Graph, then its complement $\bar{G}$ is also a Pentapartitioned Neutrosophic Fuzzy Threshold Graph under complementary membership degrees.

Proof. Define:

$$\begin{aligned} T'(v) &= F(v) \\ C'(v) &= U(v) \\ R'(v) &= R(v) \\ U'(v) &= C(v) \\ F'(v) &= T(v) \end{aligned}$$

Since:

$$T'(v) + C'(v) + R'(v) + U'(v) + F'(v) = 5$$

Adjust thresholds θ'_i to maintain the PNFTG conditions in $\bar{G}$.

Theorem 24. In a Pentapartitioned Neutrosophic Fuzzy Threshold Graph, the vertex set can be partitioned into a clique and an independent set based on the membership degrees and thresholds.

Proof. Using the threshold conditions, vertices can be partitioned into a clique K and an independent set I , satisfying the PNFTG properties.

4 | Future Tasks: Some Uncertain Graph and Linguistic Graphs

Future research aims to extend the aforementioned graphs to hypergraphs [40, 71, 97-99] and superhypergraphs [105, 191-194]. In the context of Uncertain Graphs, hypergraphs and superhypergraphs are considered generalizations of traditional graphs and have been studied extensively [5, 6, 9, 153].

And I would like to study Some Uncertain Graph and Linguistic Graphs. Although it is still at the conceptual stage, I aim to provide a clear definition, including related concepts.

4.1 | Z-Graph

We plan to extend the concepts of Z-Number [16, 25, 120, 121, 142, 150, 212, 225, 226] and Z-Numbers Soft Set [128, 230] to graph theory in the future. Z-Number and Z-Numbers Soft Set are well-studied in areas like Uncertain Set Theory. Additionally, the Soft Set [13, 14, 136, 215] is known as a related concept of the Z-Numbers Soft Set. Although it is still at the conceptual stage, the definitions are described as follows.

Definition 25. [226] A Z-number is an ordered pair $Z = (A, B)$, where:

- A is a fuzzy restriction on the possible values of a real-valued uncertain variable X .
- B is a measure of reliability or certainty of the information described by A .

In other words, A represents the uncertain value of X , and B quantifies the confidence in that uncertainty. Z-numbers provide a formal framework to handle both the uncertainty and the reliability of information simultaneously.

Definition 26. Let $G = (V, E)$ be a classical graph, where:

- V is the set of vertices.
- $E \subseteq V \times V$ is the set of edges.

A Z-Graph is a graph where each vertex and/or edge is associated with a Z-number to model uncertainty and reliability in the graph's structure. Formally, a Z-Graph $G_Z = (V, E, \sigma_V, \sigma_E)$ is defined as:

- $\sigma_V: V \rightarrow \mathcal{Z}$, a function assigning a Z-number to each vertex.
- $\sigma_E: E \rightarrow \mathcal{Z}$, a function assigning a Z-number to each edge.

Here, $\mathcal{Z}$ denotes the set of all possible Z-numbers. For each vertex $v \in V$ and edge $e \in E$:

$$\begin{aligned}\sigma_V(v) &= Z_v = (A_v, B_v), \\ \sigma_E(e) &= Z_e = (A_e, B_e),\end{aligned}$$

where:

- A_v and A_e are fuzzy restrictions representing uncertainty about vertex v and edge e , respectively.
- B_v and B_e are measures of reliability for A_v and A_e , respectively.

Definition 27. [230] Let U be a universe of discourse, E be a set of parameters, and $A \subseteq E$ be a non-empty set of attributes.

A Z-Numbers Soft Set over U is a pair $(\tilde{F}, A)$, where:

- $\tilde{F}: A \rightarrow \mathcal{P}_{\mathcal{Z}}(U)$ is a mapping from parameters to the power set of Z-numbers over U .

For each parameter $e \in A$, $\tilde{F}(e)$ is defined as:

$$\tilde{F}(e) = \{(x, Z_{x,e}) \mid x \in U\}$$

where $Z_{x,e} = (A_{x,e}, B_{x,e})$ is a Z-number associated with the element x under the parameter e :

- $A_{x,e}$ is the fuzzy restriction representing the degree to which x satisfies the parameter e .
- $B_{x,e}$ is the reliability of the information $A_{x,e}$.

Definition 28. A Z-Numbers Soft Graph is a graph that incorporates Z-numbers soft sets to model uncertainty and reliability in both its vertices and edges. Formally, a Z-Numbers Soft Graph $G_{ZSS} = (V, E, \tilde{F}_V, \tilde{F}_E)$ consists of:

- V , a non-empty set of vertices.
- $E \subseteq V \times V$, the set of edges.
- $\tilde{F}_V: A \rightarrow \mathcal{P}_Z(V)$, a mapping assigning Z-numbers soft sets to vertices.
- $\tilde{F}_E: A \rightarrow \mathcal{P}_Z(E)$, a mapping assigning Z-numbers soft sets to edges.

For each parameter $e \in A$:

$$\begin{aligned}\tilde{F}_V(e) &= \{(v, Z_{v,e}) \mid v \in V\} \\ \tilde{F}_E(e) &= \{(e', Z_{e',e}) \mid e' \in E\}\end{aligned}$$

where:

- $Z_{v,e} = (A_{v,e}, B_{v,e})$ represents the fuzzy restriction and reliability for vertex v under parameter e .
- $Z_{e',e} = (A_{e',e}, B_{e',e})$ represents the fuzzy restriction and reliability for edge e' under parameter e .

A related concept is the Linguistic Z-Graph [135], which is defined as follows. Additionally, concepts like the Linguistic Set [48, 116, 164] are known to be related to the Linguistic Z-Graph.

Definition 29. [135] Let V be a non-empty set and R be a relation on $V \times V$. A Linguistic Z-Graph $G = (V, \sigma, \mu)$ is defined as follows:

- V is the set of vertices.
- $\sigma: V \rightarrow \theta(z)$ is a function that maps each vertex to a linguistic Z-number.
- $\mu: V \times V \rightarrow \theta(z)$ is a function that maps each pair of vertices to a linguistic Z-number.

Here, $\theta(z)$ represents a set of linguistic Z-numbers, where each Z-number is denoted as $z = (A, B) = (h_\alpha, g_\beta)$. The membership value $\sigma(x)$ of a vertex x is given by $\theta(z_x) = (h_\alpha, g_\beta)$, and the edge membership value $\mu(x, y)$ for an edge (x, y) is calculated as:

$$\mu(x, y) = \sigma(x) * \sigma(y) = (h_r, g_\gamma)$$

where:

- $h_r \leq \min\{h_\alpha, h'_\alpha\}$, for all $x, y \in V$,
- $\beta \leq \gamma \leq \beta'$, and
- $\sigma(x) = \theta(z_x) = (h_\alpha, g_\beta), \sigma(y) = \theta(z_y) = (h'_\alpha, g'_\beta)$.

4.2 | Neutrosophic Linguistic Graph

One of the future prospects is to define the concepts of Single-Valued Neutrosophic Linguistic Set [116, 165, 204, 217], Interval-Valued Neutrosophic Linguistic Set [47], and Multi-Valued Neutrosophic Linguistic Set [118] extended to graphs. This will involve examining their mathematical structures, graph parameters, and

various applications. Although it is still at the conceptual stage, the definitions are described as follows.

Definition 30. Let U be a universe of discourse, and $V \subseteq U$ be a non-empty set of vertices. Let $E \subseteq V \times V$ be the set of edges. Let Θ be an ordered set of linguistic terms.

A Single-Valued Neutrosophic Linguistic Graph (*SVNLG*) is a quadruple $G = (V, E, \sigma_V, \sigma_E)$, where:

- $\sigma_V: V \rightarrow \Theta \times [0,1]^3$ assigns to each vertex $v \in V$ a linguistic term $G_q(v) \in \Theta$ and a neutrosophic triplet $(t_Q(v), d_Q(v), l_Q(v))$, with $t_Q(v), d_Q(v), l_Q(v) \in [0,1]$ satisfying:

$$0 \leq t_Q(v) + d_Q(v) + l_Q(v) \leq 3$$

Here, $t_Q(v)$ is the degree of truth-membership, $d_Q(v)$ is the degree of indeterminacy-membership, and $l_Q(v)$ is the degree of falsity-membership for vertex v .

- $\sigma_E: E \rightarrow \Theta \times [0,1]^3$ assigns to each edge $e = (u, v) \in E$ a linguistic term $G_q(e) \in \Theta$ and a neutrosophic triplet $(t_Q(e), d_Q(e), l_Q(e))$, satisfying:

$$0 \leq t_Q(e) + d_Q(e) + l_Q(e) \leq 3$$

Definition 31. Let U be a universe of discourse.

An Interval-Valued Neutrosophic Linguistic Graph (*IVNLG*) is a quadruple $G = (V, E, \sigma_V, \sigma_E)$, where:

- $V \subseteq U$ is the set of vertices.
- $E \subseteq V \times V$ is the set of edges.
- $\sigma_V: V \rightarrow \Theta \times [0,1]^6$ assigns to each vertex $v \in V$ a linguistic term $G_q(v) \in \Theta$ and interval-valued neutrosophic triplets:

$$([t_Q^-(v), t_Q^+(v)], [d_Q^-(v), d_Q^+(v)], [l_Q^-(v), l_Q^+(v)])$$

where $t_Q^-(v), t_Q^+(v), d_Q^-(v), d_Q^+(v), l_Q^-(v), l_Q^+(v) \in [0,1]$ satisfy:

$$0 \leq t_Q^-(v) + d_Q^-(v) + l_Q^-(v) \leq 3, 0 \leq t_Q^+(v) + d_Q^+(v) + l_Q^+(v) \leq 3$$

- $\sigma_E: E \rightarrow \Theta \times [0,1]^6$ assigns interval-valued neutrosophic linguistic values to edges similarly.

Definition 32. Let U be a universe of discourse, and $\Theta = \{G_0, G_1, \dots, G_t\}$ be an ordered set of linguistic terms, where t is an odd integer.

A Multi-Valued Neutrosophic Linguistic Graph (*MVNLG*) is a quadruple $G = (V, E, \sigma_V, \sigma_E)$, where:

- $V \subseteq U$ is the set of vertices.
- $E \subseteq V \times V$ is the set of edges.
- $\sigma_V: V \rightarrow \Theta \times [0,1]^3$ assigns to each vertex $v \in V$ a linguistic term $G_q(v) \in \Theta$ and a multi-valued neutrosophic triplet $(\tilde{t}_Q(v), \tilde{d}_Q(v), \tilde{l}_Q(v))$, with $\tilde{t}_Q(v), \tilde{d}_Q(v), \tilde{l}_Q(v) \in [0,1]$ satisfying:

$$0 \leq \tilde{t}_Q(v) + \tilde{d}_Q(v) + \tilde{l}_Q(v) \leq 3$$

- $\sigma_E: E \rightarrow \Theta \times [0,1]^3$ assigns multi-valued neutrosophic linguistic values to edges similarly.

4.3 | Linguistic Soft Graph

One of the future prospects is to define the concepts of Fuzzy Linguistic Soft Set [2, 137], Intuitionistic Fuzzy Linguistic Soft Set [101], and Multi-Valued Neutrosophic Linguistic Soft Set [118] extended to graphs. This

will involve examining their mathematical structures, graph parameters, and various applications. Although it is still at the conceptual stage, the definitions are described as follows.

Definition 33. Let U be a universe of discourse, and $\Theta = \{G_0, G_1, \dots, G_t\}$ be a linguistic assessment set. Let $\text{FLSS}(U)$ denote the set of all fuzzy subsets of U .

A Fuzzy Linguistic Soft Set (FLSS) over U is a pair (K, A) , where:

- $A \subseteq \Theta$ is a non-empty set of parameters.
- $K: A \rightarrow \text{FLSS}(U)$ assigns to each linguistic term $G_g \in A$ a fuzzy subset of U .

A Fuzzy Linguistic Soft Graph (FLSG) is a graph $G = (V, E, K_V, K_E)$, where:

- V is the set of vertices.
- E is the set of edges.
- $K_V: A \rightarrow \text{FLSS}(V)$ assigns fuzzy linguistic soft sets to vertices.
- $K_E: A \rightarrow \text{FLSS}(E)$ assigns fuzzy linguistic soft sets to edges.

Definition 34. Let U be a universe of discourse, and $\Theta = \{G_0, G_1, \dots, G_t\}$ be a linguistic assessment set. Let $\text{IFLSS}(U)$ denote the set of all intuitionistic fuzzy subsets of U .

An Intuitionistic Fuzzy Linguistic Soft Set (IFLSS) over U is a pair (P, A) , where:

- $A \subseteq \Theta$ is a non-empty set of parameters.
- $P: A \rightarrow \text{IFLSS}(U)$ such that for each $G_g \in A$:

$$P(G_g) = \{\langle m_P(G_g)(y), n_P(G_g)(y) \rangle \mid y \in U\},$$

where $m_P(G_g)(y), n_P(G_g)(y) \in [0,1]$ and $m_P(G_g)(y) + n_P(G_g)(y) \leq 1$.

An Intuitionistic Fuzzy Linguistic Soft Graph (IFLSG) is a graph $G = (V, E, P_V, P_E)$, where:

- V is the set of vertices.
- E is the set of edges.
- $P_V: A \rightarrow \text{IFLSS}(V)$ assigns intuitionistic fuzzy linguistic soft sets to vertices.
- $P_E: A \rightarrow \text{IFLSS}(E)$ assigns intuitionistic fuzzy linguistic soft sets to edges.

Definition 35. Let U be a universe of discourse, and $\Theta = \{G_0, G_1, \dots, G_t\}$ be an ordered set of linguistic terms. A Multi-Valued Neutrosophic Linguistic Soft Set (MVNLSS) over U is a pair (Q, A) , where:

- $A \subseteq \Theta$ is a non-empty set of parameters.
- $Q: A \rightarrow \text{MVNLS}(U)$, with $\text{MVNLS}(U)$ denoting the set of all multi-valued neutrosophic linguistic subsets of U .
- For each $G_q \in A$:

$$Q(G_q) = \{\langle G_q(y), (\tilde{t}_Q(y), \tilde{d}_Q(y), \tilde{l}_Q(y)) \rangle \mid y \in U\},$$

where $\tilde{t}_Q(y), \tilde{d}_Q(y), \tilde{l}_Q(y) \in [0,1]$ satisfy:

$$0 \leq \tilde{t}_Q(y) + \tilde{d}_Q(y) + \tilde{l}_Q(y) \leq 3.$$

A Multi-Valued Neutrosophic Linguistic Soft Graph (MVNLSSG) is a graph $G = (V, E, Q_V, Q_E)$, where:

- V is the set of vertices.
- E is the set of edges.
- $Q_V: A \rightarrow \text{MVNLS}(V)$ assigns multi-valued neutrosophic linguistic soft sets to vertices.
- $Q_E: A \rightarrow \text{MVNLS}(E)$ assigns multi-valued neutrosophic linguistic soft sets to edges.

4.4 | Extending Other Sets to Graph Theory

In set theory, many other sets and related concepts are known. In the future, we plan to explore the mathematical characteristics of these extended graph concepts. For example, we would like to consider the following concepts (cf. [80]).

- Extend Genuine Sets [57, 106] to Genuine graph.
- Extend Tolerance Rough Fuzzy Sets [17, 228] to Tolerance Rough Fuzzy graph.
- Extend Hybrid Fuzzy Sets [44, 161] to Hybrid Fuzzy graph.
- Extend Level Fuzzy Sets [131, 169] to Level Fuzzy graph.
- Extend the Bell-Shaped Fuzzy Set [49, 51] to Bell-Shaped Fuzzy Graph.
- Extend the Hyperbolic Fuzzy Set [64, 68, 69] to Hyperbolic Fuzzy Graph.
- Extend the Probabilistic Fuzzy Set [43, 103, 115, 133] to Probabilistic Fuzzy Graph.
- Extend Conditional Fuzzy Set [203] to graph theory.
- Extend the Hexagonal Fuzzy Set [45, 46, 145, 182] to Hexagonal Fuzzy Graph.
- Extend the Sigmoid Fuzzy Set [59] to Sigmoid Fuzzy Graph.
- Extend the Convex Fuzzy Set [124, 132, 178] to Convex Fuzzy Graph.
- Extend Atanassov intuitionistic fuzzy sets [15, 30, 92, 146] to graph theory.
- Extend the Gray Fuzzy Set [19, 107, 198, 211] to Gray Fuzzy Graph.
- Extend the Granular Fuzzy Set [127, 130, 134, 177, 202, 209, 216] to Granular Fuzzy Graph.
- Extend the Continuous Fuzzy Set [126, 171, 218] to Continuous Fuzzy Graph.
- Extend Symmetric Fuzzy Set [28, 166] to graph theory.
- Extend shadowed fuzzy set [41, 42, 160, 207] to graph theory.
- Extend Stochastic Fuzzy Set [93] to graph theory.
- Extend Fuzzy Power Set [29, 58, 208] to Fuzzy Power graph.
- Extend Hyperfuzzy Sets [94, 117] to graph theory.

Acknowledgments

The author is grateful to the editorial and reviewers, as well as the correspondent author, who offered assistance in the form of advice, assessment, and checking during the study period.

Funding

This research has no funding source.

Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

References

- [1] S Afsharmanesh and Rajab Ali Borzooei. Domination in fuzzy incidence graphs based on valid edges. *Journal of Applied Mathematics and Computing*, 68(1):101-124, 2022.
- [2] Zhao Aiwu and Guan Hongjun. Fuzzy-valued linguistic soft set theory and multi-attribute decision-making application. *Chaos Solitons & Fractals*, 89:2-7, 2016.
- [3] Muhammad Akram, Uzma Ahmad, Rukhsar, and Faruk Karaaslan. Complex pythagorean fuzzy threshold graphs with application in petroleum replenishment. *Journal of applied mathematics and computing*, 68(3):2125-2150, 2022.
- [4] Muhammad Akram and Noura Omair Alshehri. Intuitionistic fuzzy cycles and intuitionistic fuzzy trees. *The Scientific World Journal*, 2014(1):305836, 2014.
- [5] Muhammad Akram and Noura Omair Alshehri. Tempered interval-valued fuzzy hypergraphs. *University politehnica of bucharest scientific bulletin-series a-applied mathematics and physics*, 77(1):39-48, 2015.
- [6] Muhammad Akram and Wieslaw A. Dudek. Intuitionistic fuzzy hypergraphs with applications. *Inf. Sci.*, 218:182-193, 2013.
- [7] Muhammad Akram, Feng Feng, Shahzad Sarwar, and Youne Bae Jun. Certain types of vague graphs. *University Politehnica of Bucharest Scientific Bulletin Series A*, 76(1):141-154, 2014.
- [8] Muhammad Akram, A Nagoor Gani, and A Borumand Saeid. Vague hypergraphs. *Journal of Intelligent & Fuzzy Systems*, 26(2):647-653, 2014.
- [9] Muhammad Akram and Anam Luqman. *Fuzzy hypergraphs and related extensions*. Springer, 2020.
- [10] Muhammad Akram, Hafsa M Malik, Sundas Shahzadi, and Florentin Smarandache. Neutrosophic soft rough graphs with application. *Axioms*, 7(1):14, 2018.
- [11] Muhammad Akram, Sidra Sayed, and Florentin Smarandache. Neutrosophic incidence graphs with application. *Axioms*, 7(3):47, 2018.
- [12] Muhammad Akram and Gulfam Shahzadi. *Operations on single-valued neutrosophic graphs*. Infinite Study, 2017.
- [13] José Carlos R Alcantud, Azadeh Zahedi Khameneh, Gustavo Santos-García, and Muhammad Akram. A systematic literature review of soft set theory. *Neural Computing and Applications*, 36(16):8951-8975, 2024.
- [14] M Irfan Ali, Feng Feng, Xiaoyan Liu, Won Keun Min, and Muhammad Shabir. On some new operations in soft set theory. *Computers & Mathematics with Applications*, 57(9):1547-1553, 2009.
- [15] Muhammad Irfan Ali, Jianming Zhan, Muhammad Jabir Khan, Tahir Mahmood, and Haider Faizan. Another view on knowledge measures in atanassov intuitionistic fuzzy sets. *Soft Computing*, 26:6507 - 6517, 2022.
- [16] Rafik A Aliev, Witold Pedrycz, Oleg H Huseynov, and Serife Zihni Eyupoglu. Approximate reasoning on a basis of z-number-valued if-then rules. *IEEE Transactions on Fuzzy Systems*, 25(6):1589-1600, 2016.
- [17] Torki A. Altameem and Mohammed Amoon. Hybrid tolerance rough fuzzy set with improved monkey search algorithm based document clustering. *Journal of Ambient Intelligence and Humanized Computing*, pages 1-11, 2018.
- [18] P Anitha and P Chitra Devi. Neutrosophic bipolar vague incidence graph. *Gorteria UGC-CARE (Group-II) Journal*, 34(8):13, 2021.
- [19] Abdollah Arasteh, Alireza Aliahmadi, and Mohammad Mohammadpour Omran. Application of gray systems and fuzzy sets in combination with real options theory in project portfolio management. *Arabian Journal for Science and Engineering*, 39:6489-6506, 2014.
- [20] Krassimir Atanassov. Intuitionistic fuzzy sets. *International journal bioautomation*, 20:1, 2016.
- [21] Krassimir T. Atanassov. Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 20:87-96, 1986.

- [22] Krassimir T Atanassov. On intuitionistic fuzzy sets theory, volume 283. Springer, 2012.
- [23] Krassimir T. Atanassov. On interval valued intuitionistic fuzzy sets. *Interval-Valued Intuitionistic Fuzzy Sets*, 2019.
- [24] Krassimir T Atanassov and Krassimir T Atanassov. *Intuitionistic fuzzy sets*. Springer, 1999.
- [25] Ali Azadeh, Morteza Saber, Nasim Zandi Atashbar, Elizabeth Chang, and Peiman Pazhoheshfar. Z-ahp: A z-number extension of fuzzy analytical hierarchy process. In *2013 7th IEEE International Conference on Digital Ecosystems and Technologies (DEST)*, pages 141-147. IEEE, 2013.
- [26] Wenhui Bai, Juanjuan Ding, and Chao Zhang. Dual hesitant fuzzy graphs with applications to multi-attribute decision making. *International Journal of Cognitive Computing in Engineering*, 1:18-26, 2020.
- [27] Mikail Bal, PK Singh, and D Ahmad Katy. A short introduction to the symbolic turijam vector spaces and complex numbers. *Journal of Neutrosophic and Fuzzy Systems*, 2(1):76-87, 2022.
- [28] James F. Baldwin and Sachin Baban Karale. Asymmetric triangular fuzzy sets for classification models. In *International Conference on Knowledge-Based Intelligent Information & Engineering Systems*, 2003.
- [29] Wyllis Bandler and Ladislav J. Kohout. Fuzzy power sets and fuzzy implication operators. *Fuzzy Sets and Systems*, 4:13-30, 1980.
- [30] Gleb Beliakov, Humberto Bustince, Debdipta Goswami, U. K. Mukherjee, and Nikhil Ranjan Pal. On averaging operators for atanassov's intuitionistic fuzzy sets. *Inf. Sci.*, 181:1116-1124, 2011.
- [31] T Bharathi, S Felixia, and S Leo. Intuitionistic felicitous fuzzy graphs.
- [32] Anushree Bhattacharya and Madhumangal Pal. A fuzzy graph theory approach to the facility location problem: A case study in the indian banking system. *Mathematics*, 11(13):2992, 2023.
- [33] Pranab Biswas, Surapati Pramanik, and Bibhas Chandra Giri. Single valued bipolar pentapartitioned neutrosophic set and its application in madm strategy. 2022.
- [34] John Adrian Bondy, Uppaluri Siva Ramachandra Murty, et al. *Graph theory with applications*, volume 290. Macmillan London, 1976.
- [35] RA Borzooei and HOSSEIN RASHMANLOU. Degree of vertices in vague graphs. *Journal of applied mathematics & informatics*, 33(5_6):545-557, 2015.
- [36] RA Borzooei and Hossein Rashmanlou. More results on vague graphs. *UPB Sci. Bull. Ser. A*, 78(1):109-122, 2016.
- [37] Rajab Ali Borzooei, Hossein Rashmanlou, Sovan Samanta, and Madhumangal Pal. Regularity of vague graphs. *Journal of Intelligent & Fuzzy Systems*, 30(6):3681-3689, 2016.
- [38] Andreas Brandstädt, Van Bang Le, and Jeremy P Spinrad. *Graph classes: a survey*. SIAM, 1999.
- [39] Said Broumi, Mohamed Talea, Assia Bakali, and Florentin Smarandache. Interval valued neutrosophic graphs. *Critical Review*, XII, 2016:5-33, 2016.
- [40] Derun Cai, Moxian Song, Chenxi Sun, Baofeng Zhang, Shenda Hong, and Hongyan Li. Hypergraph structure learning for hypergraph neural networks. In *IJCAI*, pages 1923-1929, 2022.
- [41] Mingjie Cai, Qingguo Li, and Guangming Lang. Shadowed sets of dynamic fuzzy sets. *Granular Computing*, 2:85-94, 2016.
- [42] Gianpiero Cattaneo and Davide Ciucci. Shadowed sets and related algebraic structures. *Fundamenta Informaticae*, 55(3-4):255-284, 2003.
- [43] Hung-Chi Chang, Jing-Shing Yao, and Liang-Yuh Ouyang. Fuzzy mixture inventory model with variable lead-time based on probabilistic fuzzy set and triangular fuzzy number. *Mathematical and computer modelling*, 39(2-3):287-304, 2004.
- [44] Kuei-Hu Chang, Hsiang-Yu Chung, Chia Nan Wang, Yuguang Lai, and Chi-Hung Wu. A new hybrid fermatean fuzzy set and entropy method for risk assessment. *Axioms*, 12:58, 2023.
- [45] Yu-Chuan Chang, Shyi-Ming Chen, and Churn-Jung Liao. A new fuzzy interpolative reasoning method based on the areas of fuzzy sets. In *2007 IEEE International Conference on Systems, Man and Cybernetics*, pages 320-325. IEEE, 2007.
- [46] Yu-Chuan Chang, Shyi-Ming Chen, and Churn-Jung Liao. Fuzzy interpolative reasoning for sparse fuzzy-rule-based systems based on the areas of fuzzy sets. *IEEE Transactions on Fuzzy Systems*, 16(5):1285-1301, 2008.
- [47] Huakun Chen, Jingping Shi, Yongxi Lyu, and Qianlei Jia. A decision-making model with cloud model, z-numbers, and interval-valued linguistic neutrosophic sets. *Entropy*, 2024.
- [48] Liuxin Chen, Yutai Wang, and Dongmei Yang. Picture fuzzy z-linguistic set and its application in multiple attribute group decision-making. *J. Intell. Fuzzy Syst.*, 43:5997-6011, 2022.
- [49] Shyi-Ming Chen and Yu-Chuan Chang. A new method for weighted fuzzy interpolative reasoning based on weights learning techniques. In *International Conference on Fuzzy Systems*, pages 1-6. IEEE, 2010.
- [50] Shyi-Ming Chen and Yu-Chuan Chang. Weighted fuzzy interpolative reasoning for sparse fuzzy rule-based systems. *Expert Systems with Applications*, 38(8):9564-9572, 2011.
- [51] Shyi-Ming Chen and Yu-Chuan Chang. Weights-learning for weighted fuzzy rule interpolation in sparse fuzzy rulebased systems. In *2011 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE 2011)*, pages 346-351. IEEE, 2011.
- [52] Hans Christianson and Victor Reiner. The critical group of a threshold graph. *Linear algebra and its applications*, 349(1-3):233-244, 2002.
- [53] Sujit Das and Samarjit Kar. Intuitionistic multi fuzzy soft set and its application in decision making. In *Pattern Recognition and Machine Intelligence: 5th International Conference, PReMI 2013, Kolkata, India, December 10-14, 2013. Proceedings 5*, pages 587-592. Springer, 2013.
- [54] Suman Das, Rakhal Das, and Surapati Pramanik. Single valued pentapartitioned neutrosophic graphs. *Neutrosophic Sets and Systems*, 50(1):225-238, 2022.

- [55] Suman Das, Rakhal Das, and Binod Chandra Tripathy. Topology on rough pentapartitioned neutrosophic set. *Iraqi Journal of Science*, 2022.
- [56] Supriya Kumar De, Ranjit Biswas, and Akhil Ranjan Roy. Some operations on intuitionistic fuzzy sets. *Fuzzy sets and Systems*, 114(3):477-484, 2000.
- [57] Mustafa Demirci. Genuine sets, various kinds of fuzzy sets and fuzzy rough sets. *Int. J. Uncertain. Fuzziness Knowl. Based Syst.*, 11:467-494, 2003.
- [58] Mustafa Demirci. Partially ordered fuzzy power set monads on the category of 1-sets and their associated categories of topological space objects. *Fuzzy Sets Syst.*, 460:1-32, 2022.
- [59] Ferdinando Di Martino, Salvatore Sessa, Ferdinando Di Martino, and Salvatore Sessa. Fuzzy transform concepts. *Fuzzy Transforms for Image Processing and Data Analysis: Core Concepts, Processes and Applications*, pages 1-14, 2020.
- [60] Persi Diaconis, Susan Holmes, and Svante Janson. Threshold graph limits and random threshold graphs. *Internet Mathematics*, 5(3):267-320, 2008.
- [61] Reinhard Diestel. Graduate texts in mathematics: Graph theory.
- [62] Reinhard Diestel. Graph theory 3rd ed. Graduate texts in mathematics, 173(33):12, 2005.
- [63] Reinhard Diestel. Graph theory. Springer (print edition); Reinhard Diestel (eBooks), 2024.
- [64] Mehdi Divsalar, Marzieh Ahmadi, Maryam Ghaedi, and Alessio Ishizaka. An extended todim method for hyperbolic fuzzy environments. *Computers & Industrial Engineering*, 185:109655, 2023.
- [65] Didier Dubois and Henri Prade. Fuzzy sets and systems: theory and applications. In *Mathematics in Science and Engineering*, 2011.
- [66] Didier Dubois and Henri Prade. Fundamentals of fuzzy sets, volume 7. Springer Science & Business Media, 2012.
- [67] Didier Dubois, Henri Prade, and Lotfi A. Zadeh. Fundamentals of fuzzy sets. 2000.
- [68] Palash Dutta and Gourangajit Borah. Construction of hyperbolic fuzzy set and its applications in diverse covid-19 associated problems. *New Mathematics and Natural Computation*, 19(01):217-288, 2023.
- [69] Palash Dutta and Gourangajit Borah. Erratum: Construction of hyperbolic fuzzy set and its applications in diverse covid-19 associated problems. *New Mathematics and Natural Computation*, 19(01):1-2, 2023.
- [70] PA Ejegwa, SO Akowe, PM Otene, and JM Ikyule. An overview on intuitionistic fuzzy sets. *Int. J. Sci. Technol. Res*, 3(3):142-145, 2014.
- [71] Yifan Feng, Haoxuan You, Zizhao Zhang, Rongrong Ji, and Yue Gao. Hypergraph neural networks. In *Proceedings of the AAAI conference on artificial intelligence*, volume 33, pages 3558-3565, 2019.
- [72] Takaaki Fujita. Claw-free graph and at-free graph in fuzzy, neutrosophic, and plithogenic graphs, June 2024. License: CC BY 4.0.
- [73] Takaaki Fujita. Contact graphs in fuzzy and neutrosophic graphs. Preprint, May 2024, File available, 2024.
- [74] Takaaki Fujita. Fuzzy directed tree-width and fuzzy hypertree-width. ResearchGate(Preprint), 2024.
- [75] Takaaki Fujita. General plithogenic soft rough graphs and some related graph classes, June 2024. License: CC BY 4.0.
- [76] Takaaki Fujita. Interval graphs and proper interval graphs in fuzzy and neutrosophic graphs. 2024.
- [77] Takaaki Fujita. Permutation graphs in fuzzy and neutrosophic graphs. Preprint, July 2024. File available.
- [78] Takaaki Fujita. Pythagorean, fermatean, and complex turiyam graphs: Relations with general plithogenic graphs, June 2024. License: CC BY 4.0.
- [79] Takaaki Fujita. A review of the hierarchy of plithogenic, neutrosophic, and fuzzy graphs: Survey and applications. ResearchGate(Preprint), 2024.
- [80] Takaaki Fujita. Study for general plithogenic soft expert graphs. 2024.
- [81] Takaaki Fujita. Survey of intersection graphs, fuzzy graphs and neutrosophic graphs. ResearchGate, July 2024.
- [82] Takaaki Fujita. Survey of intersection graphs, fuzzy graphs and neutrosophic graphs, 2024.
- [83] Takaaki Fujita. Survey of planar and outerplanar graphs in fuzzy and neutrosophic graphs. ResearchGate, July 2024.
- [84] Takaaki Fujita. Survey of trees, forests, and paths in fuzzy and neutrosophic graphs. July 2024.
- [85] Takaaki Fujita. Uncertain automata and uncertain graph grammar, 2024.
- [86] Takaaki Fujita. Vague soft expert graph and complex fuzzy soft expert graph. ResearchGate, 2024.
- [87] Takaaki Fujita. Various properties of various ultrafilters, various graph width parameters, and various connectivity systems. arXiv preprint arXiv:2408.02299, 2024.
- [88] GA Ganati, VNS Rao Repalle, MA Ashebo, and M Amini. Turiyam graphs and its applications. *Information Sciences Letters*, 12(6):2423-2434, 2023.
- [89] Gamachu Adugna Ganati, VN Srinivasa Rao Repalle, and Mamo Abebe Ashebo. Social network analysis by turiyam graphs. *BMC Research Notes*, 16(1):170, 2023.
- [90] Gamachu Adugna Ganati, VN Srinivasa Rao Repalle, and Mamo Abebe Ashebo. Relations in the context of turiyam sets. *BMC Research Notes*, 16(1):49, 2023.
- [91] A Nagoor Gani and K Radha. On regular fuzzy graphs. 2008.
- [92] Harish Garg and Kamal Kumar. Linguistic interval-valued atanasov intuitionistic fuzzy sets and their applications to group decision making problems. *IEEE Transactions on Fuzzy Systems*, 27:2302-2311, 2019.
- [93] Aidong Ge. A new approach to controller design of stochastic fuzzy systems. 2013 IEEE International Conference on Information and Automation (ICIA), pages 43-47, 2013.
- [94] Jayanta Ghosh and Tapas Kumar Samanta. Hyperfuzzy sets and hyperfuzzy group. *Int. J. Adv. Sci. Technol*, 41:27-37, 2012.

- [95] Puspendu Giri, Somnath Paul, and Bijoy Krishna Debnath. A fuzzy graph theory and matrix approach (fuzzy gtma) to select the best renewable energy alternative in india. *Applied Energy*, 358:122582, 2024.
- [96] Zengtai Gong and Junhu Wang. Hesitant fuzzy graphs, hesitant fuzzy hypergraphs and fuzzy graph decisions. *Journal of Intelligent & Fuzzy Systems*, 40(1):865-875, 2021.
- [97] Georg Gottlob, Nicola Leone, and Francesco Scarcello. Hypertree decompositions and tractable queries. In *Proceedings of the eighteenth ACM SIGMOD-SIGACT-SIGART symposium on Principles of database systems*, pages 21-32, 1999.
- [98] Georg Gottlob, Nicola Leone, and Francesco Scarcello. Hypertree decompositions: A survey. In *Mathematical Foundations of Computer Science 2001: 26th International Symposium, MFCS 2001 Mariánské Lázně, Czech Republic, August 27-31, 2001 Proceedings* 26, pages 37-57. Springer, 2001.
- [99] Georg Gottlob and Reinhard Pichler. Hypergraphs in model checking: Acyclicity and hypertree-width versus cliquewidth. *SIAM Journal on Computing*, 33(2):351-378, 2004.
- [100] Jonathan L. Gross, Jay Yellen, and Mark Anderson. *Graph theory and its applications*. Chapman and Hall/CRC, 2018.
- [101] Hongjun Guan, Shuang Guan, and Aiwu Zhao. Intuitionistic fuzzy linguistic soft sets and their application in multiattribute decision-making. *J. Intell. Fuzzy Syst.*, 31:2869-2879, 2016.
- [102] Muhammad Gulistan, Naveed Yaqoob, Zunaira Rashid, Florentin Smarandache, and Hafiz Abdul Wahab. A study on neutrosophic cubic graphs with real life applications in industries. *Symmetry*, 10(6):203, 2018.
- [103] Krishna Kumar Gupta and Sanjay Kumar. Hesitant probabilistic fuzzy set based time series forecasting method. *Granular Computing*, 4:739-758, 2019.
- [104] Saira Hameed, Muhammad Akram, Noreen Mustafa, and Sovan Samanta. Extension of threshold graphs under complex fuzzy environment. *International Journal of Applied and Computational Mathematics*, 7:1-19, 2021.
- [105] Mohammad Hamidi, Florentin Smarandache, and Elham Davneshvar. Spectrum of superhypergraphs via flows. *Journal of Mathematics*, 2022(1):9158912, 2022.
- [106] Jun Han and Bao Qing Hu. Operations of fuzzy numbers via genuine set. In *Rough Sets and Knowledge Technology*, 2010.
- [107] Xin hua Wang, Fang bing Yuan, and Zong bo Xu. Multi-objective optimization qos routing based on grey fuzzy theory. 2009 International Symposium on Computer Network and Multimedia Technology, pages 1-4, 2009.
- [108] Liangsong Huang, Yu Hu, Yuxia Li, PK Kishore Kumar, Dipak Koley, and Arindam Dey. A study of regular and irregular neutrosophic graphs with real life applications. *Mathematics*, 7(6):551, 2019.
- [109] S Satham Hussain, N Durga, Muhammad Aslam, G Muhiuddin, and Ganesh Ghorai. New concepts on quadripartitioned neutrosophic competition graph with application. *International Journal of Applied and Computational Mathematics*, 10(2):57, 2024.
- [110] S Satham Hussain, R Jahir Hussain, and Ghulam Muhiuddin. Neutrosophic vague line graphs, volume 36. *Infinite Study*, 2020.
- [111] S Satham Hussain, Hossein Rashmonlou, R Jahir Hussain, Sankar Sahoo, Said Broumi, et al. Quadripartitioned neutrosophic graph structures. *Neutrosophic Sets and Systems*, 51(1):17, 2022.
- [112] Sk Rabiul Islam and Madhumangal Pal. Neighbourhood and competition graphs under fuzzy incidence graph and its application. *Computational and Applied Mathematics*, 43(7):411, 2024.
- [113] Robert E Jamison and Alan P Sprague. Multithreshold graphs. *Journal of Graph Theory*, 94(4):518-530, 2020.
- [114] Chiranjibe Jana, Tapan Senapati, Monoranjan Bhowmik, and Madhumangal Pal. On intuitionistic fuzzy g-subalgebras of g-algebras. *Fuzzy Information and Engineering*, 7(2):195-209, 2015.
- [115] Kesavan Janani and R Rakkiyappan. Complex probabilistic fuzzy set and their aggregation operators in group decision making extended to topos. *Engineering Applications of Artificial Intelligence*, 114:105010, 2022.
- [116] Pu Ji, Hong yu Zhang, and Jian qiang Wang. Selecting an outsourcing provider based on the combined mabac-electre method using single-valued neutrosophic linguistic sets. *Comput. Ind. Eng.*, 120:429-441, 2018.
- [117] Young Bae Jun, Kul Hur, and Kyoung Ja Lee. Hyperfuzzy subalgebras of bck/bci-algebras. *Annals of Fuzzy Mathematics and Informatics*, 2017.
- [118] Nor Liyana Amalini Mohd Kamal, Lazim Abdullah, Ilyani Abdullah, and Muhammad Saqlain. Multi-valued interval neutrosophic linguistic soft set theory and its application in knowledge management. *CAAI Trans. Intell. Technol.*, 5:200-208, 2020.
- [119] Vasantha Kandasamy, K Ilanthenral, and Florentin Smarandache. *Neutrosophic graphs: a new dimension to graph theory*. Infinite Study, 2015.
- [120] Bingyi Kang, Yong Deng, Kasun Hewage, and Rehan Sadiq. A method of measuring uncertainty for z-number. *IEEE Transactions on Fuzzy Systems*, 27(4):731-738, 2018.
- [121] Bingyi Kang, Yong Deng, and Rehan Sadiq. Total utility of z-number. *Applied Intelligence*, 48:703-729, 2018.
- [122] M. G. Karunambigai, R. Parvathi, and R. Buvaneswari. Arc in intuitionistic fuzzy graphs. *Notes on Intuitionistic Fuzzy Sets*, 17:37-47, 2011.
- [123] V Keerthika and M Gomathi. Neutrosophic fuzzy threshold graph. *Neutrosophic Sets and Systems*, 43(1):12, 2021.
- [124] Sang-Hyuk Lee, Sang-Min Lee, Gyo-Yong Sohn, and Jaeh-Yung Kim. Fuzzy entropy design for non convex fuzzy set and application to mutual information. *Journal of Central South University of Technology*, 18(1):184-189, 2011.
- [125] Hongxing Li and Vincent C Yen. *Fuzzy sets and fuzzy decision-making*. CRC press, 1995.
- [126] SK Li, GH Tang, L Li, and QL Kong. Right continuous fuzzy set-valued stochastic processes with left limitation. *Fuzzy sets and systems*, 110(1):123-126, 2000.

- [127] Wentao Li and Tao Zhan. Multi-granularity probabilistic rough fuzzy sets for interval-valued fuzzy decision systems. *International Journal of Fuzzy Systems*, 25:3061 - 3073, 2023.
- [128] Huchang Liao, Fan Liu, Yue Xiao, Zheng Wu, and Edmundas Kazimieras Zavadskas. A survey on z-number-based decision analysis methods and applications: What's going on and how to go further? *Information Sciences*, 663:120234, 2024.
- [129] Robert Lin. Note on fuzzy sets. *Yugoslav Journal of Operations Research*, 24:299-303, 2014.
- [130] Tsau Young Lin. Measure theory on granular fuzzy sets. 18th International Conference of the North American Fuzzy Information Processing Society - NAFIPS (Cat. No.99TH8397), pages 809-813, 1999.
- [131] W.-N. Liu, Jingtao Yao, and Yiyu Yao. Rough approximations under level fuzzy sets. In *Rough Sets and Current Trends in Computing*, 2004.
- [132] Ying-ming Liu. Some properties of convex fuzzy sets. *Journal of Mathematical Analysis and Applications*, 111(1):119-129, 1985.
- [133] Zhi Liu and Han-Xiong Li. A probabilistic fuzzy logic system for modeling and control. *IEEE Transactions on Fuzzy Systems*, 13(6):848-859, 2005.
- [134] Juan Lu, Deyu Li, Yanhui Zhai, and Hexiang Bai. Granular structure of type-2 fuzzy rough sets over two universes. *Symmetry*, 9:284, 2017.
- [135] Rupkumar Mahapatra, Sovan Samanta, Madhumangal Pal, Tofigh Allahviranloo, and Antonios Kalampakas. A study on linguistic z-graph and its application in social networks. *Mathematics*, 2024.
- [136] Pradip Kumar Maji, Ranjit Biswas, and A Ranjan Roy. Soft set theory. *Computers & mathematics with applications*, 45(4-5):555-562, 2003.
- [137] Debashish Malakar, Sulekha Gope, and Sujit Das. Correlation measure of hesitant fuzzy linguistic term soft set and its application in decision making. In *International Conference on Frontiers in Intelligent Computing: Theory and Applications*, 2015.
- [138] Rama Mallick and Surapati Pramanik. Pentapartitioned neutrosophic set and its properties. *Neutrosophic Sets and Systems*, 35:49, 2020.
- [139] Sonia Mandal. A novel concept on fermatean fuzzy threshold graph with its application on covid. *International Journal of Fuzzy Mathematical Archive*, 20(2):45-69, 2023.
- [140] Sunil Mathew, John Mordeson, and Hai-Long Yang. Incidence cuts and connectivity in fuzzy incidence graphs. *Iranian Journal of Fuzzy Systems*, 16(2):31-43, 2019.
- [141] Sunil Mathew and John N Mordeson. Connectivity concepts in fuzzy incidence graphs. *Information Sciences*, 382:326-333, 2017.
- [142] Daud Mohamad, Saidatull Akma Shaharani, and Nor Hanimah Kamis. A z-number-based decision making procedure with ranking fuzzy numbers method. In *AIP conference proceedings*, volume 1635, pages 160-166. American Institute of Physics, 2014.
- [143] Siti Nurul Fitriah Mohamad, Roslan Hasni, and Florentin Smarandache. Novel concepts on domination in neutrosophic incidence graphs with some applications. *Journal of Advanced Computational Intelligence and Intelligent Informatics*, 27(5):837-847, 2023.
- [144] Siti Nurul Fitriah Mohamad, Roslan Hasni, Binyamin Yusoff, Naeem Jan, and Muhammad Kamran. Novel concept of interval-valued neutrosophic incidence graphs with application. *Neutrosophic Sets Syst*, 43:61-81, 2021.
- [145] Alaa Fouad Momena, Shubhendu Mandal, Kamal Hossain Gazi, Bibhas Chandra Giri, and Sankar Prasad Mondal. Prediagnosis of disease based on symptoms by generalized dual hesitant hexagonal fuzzy multi-criteria decision-making techniques. *Systems*, 11(5):231, 2023.
- [146] Ignacio Montes, Nikhil Ranjan Pal, and Susana Montes. Entropy measures for atanassov intuitionistic fuzzy sets based on divergence. *Soft Computing*, 22:5051 - 5071, 2018.
- [147] John N Mordeson and Sunil Mathew. *Advanced topics in fuzzy graph theory*, volume 375. Springer, 2019.
- [148] John N Mordeson, Sunil Mathew, Davender S Malik, John N Mordeson, Sunil Mathew, and Davender S Malik. Fuzzy incidence graphs. *Fuzzy Graph Theory with Applications to Human Trafficking*, pages 87-137, 2018.
- [149] Sunil MP and J Suresh Kumar. On intuitionistic hesitancy fuzzy graphs. 2024.
- [150] Dilnoz Muhamediyeva and Baxtiyor Tagbayev. Method of converting z-number to classic fuzzy number. *Scientific Collection «InterConf+»*, (21 (109)):348-352, 2022.
- [151] Kavya Nair, MS Sunitha, et al. Operations on fuzzy incidence graphs and strong incidence domination. *arXiv preprint arXiv:2210.14092*, 2022.
- [152] Kavya R Nair and MS Sunitha. Strong incidence domination in fuzzy incidence graphs. *Journal of Intelligent & Fuzzy Systems*, 43(3):2667-2678, 2022.
- [153] Hafiza Saba Nawaz, Muhammad Akram, and José Carlos R Alcantud. An algorithm to compute the strength of competing interactions in the bering sea based on pythagorean fuzzy hypergraphs. *Neural Computing and Applications*, 34(2):1099-1121, 2022.
- [154] János Pach. Geometric graph theory. *Handbook of Discrete and Computational Geometry*, 2nd Ed., pages 219-238, 2004.
- [155] Madhumangal Pal, Sovan Samanta, and Ganesh Ghorai. Fuzzy threshold graph. In *Modern Trends in Fuzzy Graph Theory*, pages 145-152. Springer, 2020.
- [156] Madhumangal Pal, Sovan Samanta, and Ganesh Ghorai. *Modern trends in fuzzy graph theory*. Springer, 2020.

- [157] Sakshi Dev Pandey, AS Ranadive, and Sovan Samanta. Bipolar-valued hesitant fuzzy graph and its application. *Social Network Analysis and Mining*, 12(1):14, 2022.
- [158] T Pathinathan, J Jon Arockiaraj, and J Jesintha Rosline. Hesitancy fuzzy graphs. *Indian Journal of Science and Technology*, 8(35):1-5, 2015.
- [159] Witold Pedrycz. *Fuzzy sets engineering*. 1995.
- [160] Witold Pedrycz. Shadowed sets: representing and processing fuzzy sets. *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, 28(1):103-109, 1998.
- [161] Witold Pedrycz. Higher type, higher order fuzzy sets and hybrid fuzzy sets. 2020.
- [162] Sriram Pemmaraju and Steven Skiena. *Computational discrete mathematics: Combinatorics and graph theory with mathematica®*. Cambridge university press, 2003.
- [163] Soumitra Poulik and Ganesh Ghorai. Connectivity concepts in bipolar fuzzy incidence graphs. *Thai Journal of Mathematics*, 20(4):1609-1619, 2022.
- [164] Jian qiang Wang, Jia ting Wu, Jing Wang, Hong yu Zhang, and Xiao hong Chen. Interval-valued hesitant fuzzy linguistic sets and their applications in multi-criteria decision-making problems. *Inf. Sci.*, 288:55-72, 2014.
- [165] Jian qiang Wang, Yu Yang, and Lin Li. Multi-criteria decision-making method based on single-valued neutrosophic linguistic maclaurin symmetric mean operators. *Neural Computing and Applications*, 30:1529 - 1547, 2016.
- [166] Dong Qiu and Weiquan Zhang. Symmetric fuzzy numbers and additive equivalence of fuzzy numbers. *Soft Computing*, 17:1471-1477, 2013.
- [167] Shio Gai Quek, Ganeshsree Selvachandran, D Ajay, P Chellamani, David Taniar, Hamido Fujita, Phet Duong, Le Hoang Son, and Nguyen Long Giang. New concepts of pentapartitioned neutrosophic graphs and applications for determining safest paths and towns in response to covid-19. *Computational and Applied Mathematics*, 41(4):151, 2022.
- [168] Radha R. and Stanis Arul Mary. Pentapartitioned neutrosophic pythagorean set. *International Research Journal on Advanced Science Hub*, 2021.
- [169] Tadeusz Radecki. Level fuzzy sets. *Cybernetics and Systems*, 7:189-198, 1977.
- [170] M Rajeshwari, R Murugesan, and KA Venkatesh. Substantial and fragile domination in bipolar fuzzy incidence graphs. *Indian Journal of Natural Sciences*, 12(65):30605-30614, 2021.
- [171] Elisabeth Rakus-Andersson. Continuous fuzzy sets as probabilities of continuous fuzzy events. In *International Conference on Fuzzy Systems*, pages 1-7. IEEE, 2010.
- [172] Yongsheng Rao, Saeed Kosari, and Zehui Shao. Certain properties of vague graphs with a novel application. *Mathematics*, 8(10):1647, 2020.
- [173] Hossein Rashmanlou and Rajab Ali Borzooei. Vague graphs with application. *Journal of Intelligent & Fuzzy Systems*, 30(6):3291-3299, 2016.
- [174] Fahad Ur Rehman, Tabasam Rashid, and Muhammad Tanveer Hussain. Optimization in business trade by using fuzzy incidence graphs. *Journal of Computational and Cognitive Engineering*, 2(3):196-203, 2023.
- [175] David W Roberts. Analysis of forest succession with fuzzy graph theory. *Ecological Modelling*, 45(4):261-274, 1989.
- [176] Azriel Rosenfeld. Fuzzy graphs. In *Fuzzy sets and their applications to cognitive and decision processes*, pages 77-95. Elsevier, 1975.
- [177] Jonathan M. Rossiter and Toshiharu Mukai. Learning from uncertain image data using granular fuzzy sets and biomimetic applicability functions. In *EUSFLAT Conf.*, 2005.
- [178] Jean J Saade. Mapping convex and normal fuzzy sets. *Fuzzy sets and systems*, 81(2):251-256, 1996.
- [179] Seyed Hossein Sadati, Hossein Rashmanlou, and Ali Asghar Talebi. Domination in intuitionistic fuzzy incidence graph. In *2021 52nd Annual Iranian Mathematics Conference (AIMC)*, pages 88-94. IEEE, 2021.
- [180] Ridvan Şahin. An approach to neutrosophic graph theory with applications. *Soft Computing*, 23(2):569-581, 2019.
- [181] Sovan Samanta, Madhumangal Pal, Hossein Rashmanlou, and Rajab Ali Borzooei. Vague graphs and strengths. *Journal of Intelligent & Fuzzy Systems*, 30(6):3675-3680, 2016.
- [182] Qiang Shen and Zhiheng Huang. Fuzzy interpolation with generalized representative values. 2004.
- [183] Prem Kumar Singh. Four-way turiyam based characterization of non-euclidean geometry. 2023.
- [184] Prem Kumar Singh, Katy D Ahmad, Mikail Bal, and Malath Aswad. On the symbolic turiyam rings. *Journal of neutrosophic and fuzzy systems*, pages 80-88, 2022.
- [185] Prem Kumar Singh et al. Mathematical concept exploration using turiyam cognition. *Full Length Article*, 9(1):08-8, 2023.
- [186] Prem Kumar Singh et al. Non-euclidean data exploration using turiyam set and its complement. *Full Length Article*, 6(2):23-3, 2023.
- [187] Prem Kumar Singh, Naveen Surathu, Ghattamaneni Surya Prakash, et al. Turiyam based four way unknown profile characterization on social networks. *Full Length Article*, 10(2):27-7, 2024.
- [188] Prem Kumar Singh, Naveen Surathu, Ghattamaneni Surya Prakash, et al. Turiyam based four way unknown profile characterization on social networks. *Full Length Article*, 10(2):27-7, 2024.
- [189] Florentin Smarandache. Ambiguous set (as) is a particular case of the quadripartitioned neutrosophic set (qns). *nidus idearum*, page 16.
- [190] Florentin Smarandache. A unifying field in logics: Neutrosophic logic. In *Philosophy*, pages 1-141. American Research Press, 1999.
- [191] Florentin Smarandache. n-superhypergraph and plithogenic n-superhypergraph. *Nidus Idearum*, 7:107-113, 2019.

- [192] Florentin Smarandache. Extension of HyperGraph to n-SuperHyperGraph and to Plithogenic n-SuperHyperGraph, and Extension of HyperAlgebra to n-ary (Classical-/Neutro-/Anti-) HyperAlgebra. Infinite Study, 2020.
- [193] Florentin Smarandache. Introduction to the n-SuperHyperGraph-the most general form of graph today. Infinite Study, 2022.
- [194] Florentin Smarandache. Decision making based on valued fuzzy superhypergraphs. 2023.
- [195] Florentin Smarandache and Said Broumi. Neutrosophic graph theory and algorithms. IGI Global, 2019.
- [196] Florentin Smarandache, Siti Nurul Fitriah Mohamad, and Roslan Hasni. Novel concepts on domination in neutrosophic incidence graphs with some applications. 2023.
- [197] A Sudha and P Sundararajan. Robust fuzzy graph. Ratio Mathematica, 46, 2023.
- [198] Yue-Ping Sun. A gray-fuzzy evaluation method for soft foundation treatment based on genetic algorithm. In Civil Engineering and Urban Planning IV: Proceedings of the 4th International Conference on Civil Engineering and Urban Planning, Beijing, China, 25-27 July 2015, page 407. CRC Press, 2016.
- [199] AL-Hawary Talal and Bayan Hourani. On intuitionistic product fuzzy graphs. Italian Journal of Pure and Applied Mathematics, page 113.
- [200] Vakkas Uluçay and Memet Şahin. Intuitionistic fuzzy soft expert graphs with application. Uncertainty discourse and applications, 1(1):1-10, 2024.
- [201] N Venkataraman, Raman Sundareswaran, and V Swaminathan. A note on incidence graphs. Electronic Notes in Discrete Mathematics, 33:87-93, 2009.
- [202] Ju Wang, Xinghu Ai, and Li Fu. Multi-granularity neighborhood fuzzy rough set model with two universes. Journal of Intelligent Learning Systems and Applications, 2024.
- [203] Li-Xin Wang. A new look at type-2 fuzzy sets and type-2 fuzzy logic systems. IEEE Transactions on Fuzzy Systems, 25:693-706, 2017.
- [204] Guiwu Wei, Jiang Wu, Yanfeng Guo, Jie Wang, and Cun Wei. An extended copras model for multiple attribute group decision making based on single-valued neutrosophic 2-tuple linguistic environment. Technological and Economic Development of Economy, 2021.
- [205] Tong Wei, Junlin Hou, and Rui Feng. Fuzzy graph neural network for few-shot learning. In 2020 International joint conference on neural networks (IJCNN), pages 1-8. IEEE, 2020.
- [206] Douglas Brent West et al. Introduction to graph theory, volume 2. Prentice hall Upper Saddle River, 2001.
- [207] Tamunokuro Opubo William-West, Armand Florentin Donfack Kana, and Adeku Musa Ibrahim. Shadowed set approximation of fuzzy sets based on nearest quota of fuzziness. ANNALS OF FUZZY MATHEMATICS AND INFORMATICS, 2019.
- [208] Richard Willmott. Two fuzzier implication operators in the theory of fuzzy power sets. Fuzzy Sets and Systems, 4:31-36, 1980.
- [209] Shuyin Xia, Xiaoyu Lian, Guoyin Wang, Xinbo Gao, Qinghua Hu, and Yabin Shao. Granular-ball fuzzy set and its implement in svm. IEEE Transactions on Knowledge and Data Engineering, 36:6293-6304, 2024.
- [210] Zeshui Xu. Hesitant fuzzy sets theory, volume 314. Springer, 2014.
- [211] Yong Bo Xuan, Chang Qiang Huang, and Wang Xi Li. Air combat situation assessment by gray fuzzy bayesian network. Applied Mechanics and Materials, 69:114-119, 2011.
- [212] Ronald R. Yager. On z valuations using zadeh's numbers. International Journal of Intelligent Systems, 27, 2012.
- [213] Jing-Ho Yan, Jer-Jeong Chen, Gerard J Chang, et al. Quasi-threshold graphs. Discrete applied mathematics, 69(3):247-255, 1996.
- [214] Lanzhen Yang and Hua Mao. Intuitionistic fuzzy threshold graphs. Journal of Intelligent & Fuzzy Systems, 36(6):6641-6651, 2019.
- [215] Xibei Yang, Dongjun Yu, Jingyu Yang, and Chen Wu. Generalization of soft set theory: from crisp to fuzzy case. In Fuzzy Information and Engineering: Proceedings of the Second International Conference of Fuzzy Information and Engineering (ICFIE), pages 345-354. Springer, 2007.
- [216] Yiyu Yao and Jilin Yang. Granular fuzzy sets and three-way approximations of fuzzy sets. Int. J. Approx. Reason., 161:109003, 2023.
- [217] Jun Ye. An extended topsis method for multiple attribute group decision making based on single valued neutrosophic linguistic numbers. J. Intell. Fuzzy Syst., 28:247-255, 2015.
- [218] John Yen. Computing generalized belief functions for continuous fuzzy sets. International Journal of Approximate Reasoning, 6(1):1-31, 1992.
- [219] Lotfi A Zadeh. Fuzzy sets. Information and control, 8(3):338-353, 1965.
- [220] Lotfi A. Zadeh. Fuzzy algorithms. Inf. Control., 12:94-102, 1968.
- [221] Lotfi A. Zadeh. Soft computing and fuzzy logic. IEEE Software, 11:48-56, 1994.
- [222] Lotfi A. Zadeh. Fuzzy logic = computing with words. IEEE Trans. Fuzzy Syst., 4:103-111, 1996.
- [223] Lotfi A Zadeh. Fuzzy logic, neural networks, and soft computing. In Fuzzy sets, fuzzy logic, and fuzzy systems: selected papers by Lotfi A Zadeh, pages 775-782. World Scientific, 1996.
- [224] Lotfi A. Zadeh. Fuzzy sets as a basis for a theory of possibility. Fuzzy Sets and Systems, 100:9-34, 1999.
- [225] Lotfi A. Zadeh. The concept of a z -number - a new direction in uncertain computation. In IEEE International Conference on Information Reuse and Integration, 2011.
- [226] Lotfi A. Zadeh. A note on z -numbers. Inf. Sci., 181:2923-2932, 2011.

- [227] Lotfi A. Zadeh and Janusz Kacprzyk. Fuzzy logic for the management of uncertainty. 1992.
- [228] Jun-Hai Zhai, Yao Zhang, and Hongyu Zhu. Three-way decisions model based on tolerance rough fuzzy set. *International Journal of Machine Learning and Cybernetics*, 8:35 - 43, 2016.
- [229] Ping Zhang and Gary Chartrand. *Introduction to graph theory*. Tata McGraw-Hill, 2:2-1, 2006.
- [230] Haiyan Zhao, Qian Xiao, Zheng Liu, and Yanhong Wang. An approach in medical diagnosis based on z-numbers soft set. *Plos one*, 17(8):e0272203, 2022.
- [231] Hua Zhao, Zeshui Xu, Shousheng Liu, and Zhong Wang. Intuitionistic fuzzy mst clustering algorithms. *Computers & Industrial Engineering*, 62(4):1130-1140, 2012.

Disclaimer/Publisher's Note: The perspectives, opinions, and data shared in all publications are the sole responsibility of the individual authors and contributors, and do not necessarily reflect the views of Sciences Force or the editorial team. Sciences Force and the editorial team disclaim any liability for potential harm to individuals or property resulting from the ideas, methods, instructions, or products referenced in the content.