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Plithogenic Superhypersoft Set and Plithogenic Forest SuperHypersoft Set

Takaaki Fujita[1*] and **Florentin Smarandache**[2]

[1] Independent Researcher, Shinjuku, Shinjuku-ku, Tokyo, Japan; t171d603@gunma-u.ac.jp.,

[2] Department of Mathematics & Sciences, University of New Mexico, Gallup, NM 87301, USA; smarand@unm.edu.

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Abstract

The Plithogenic Set is renowned for generalizing concepts such as Fuzzy Sets and Neutrosophic Sets. The Extended Plithogenic Set represents an advanced concept of the Plithogenic Set, as recently defined in [62]. A hypersoft set is a mathematical structure that maps distinct attributes with non-overlapping values to subsets of a universal set, facilitating multi-criteria decision analysis. Recent studies have explored the combination of Plithogenic Sets and Hypersoft Sets, leading to the development of the Plithogenic Hypersoft Set. This paper further extends these concepts to introduce and examine the Plithogenic SuperHypersoft Set and the Extended Plithogenic SuperHypersoft Set.

Keywords: Superhypersoft set, Plithogenic Graph, Plithogenic Set, Hypersoft Set

1 | Short Introduction of this paper

1.1 | Plithogenic Sets

Various frameworks have been developed to address uncertainty, including Fuzzy Sets [65, 66, 69, 67, 68], Intuitionistic Fuzzy Sets [9, 8, 7], Neutrosophic Sets [47, 46, 58, 22, 23], Soft Sets [35, 31, 30], Hypersoft Sets [14, 52, 42, 48], SuperHypersoft Sets [19, 53, 34], and Rough Sets [37, 40, 39, 38].

The primary focus of this paper is on Plithogenic Sets, a highly flexible and powerful concept that generalizes various approaches, including Fuzzy Sets and Neutrosophic Sets [51, 50, 25, 60, 12, 18]. Recent research has introduced concepts such as the Plithogenic Hypersoft Set[11], which extends Plithogenic Sets by incorporating elements of Soft Sets and Hypersoft Sets. This development represents a significant advancement in the study of uncertainty and decision-making frameworks.

Corresponding Author: t171d603@gunma-u.ac.jp

1.2 | Our Contribution in This Paper

In this paper, we propose and study the Plithogenic SuperHypersoft Set and the Extended Plithogenic SuperHypersoft Set. Additionally, we explore the concepts of the Plithogenic Forest SuperHypersoft Set and the SuperHyperPlithogenic SuperHypersoft Set.

2 | Preliminaries and Definitions

This section provides an introduction to the foundational concepts and definitions required for the discussions in this paper.

2.1 | Plithogenic Set

A Plithogenic Set is a mathematical framework that incorporates multi-valued degrees of appurtenance and contradictions, making it suitable for complex decision-making processes. Various studies have been conducted on Plithogenic Sets [20, 1, 64, 2, 57, 43, 15, 44]. The definition is presented below.

Definition 1. [51, 50] Let S be a universal set, and $P \subseteq S$. A *Plithogenic Set* PS is defined as:

$$PS = (P, v, Pv, pdf, pCF)$$

where:

- v is an attribute.
- Pv is the range of possible values for the attribute v.
- $pdf : P \times Pv \to [0,1]^s$ is the *Degree of Appurtenance Function (DAF)* [1]
- $pCF : Pv \times Pv \to [0,1]^t$ is the *Degree of Contradiction Function (DCF)*.

These functions satisfy the following axioms for all $a, b \in Pv$:

(1) *Reflexivity of Contradiction Function*:

$$pCF(a,a) = 0$$

(2) *Symmetry of Contradiction Function*:

$$pCF(a,b) = pCF(b,a)$$

2.2 | Extended Plithogenic Set

The Extended Plithogenic Set is an extended concept of the Plithogenic Set, which was recently defined[62]. The definition is provided below.

Definition 2 (Extended Plithogenic Set)**.** [62] Let P be a non-empty set, and let a be an attribute with a range of possible values V. An *Extended Plithogenic Set (ExPlS)* is defined as a 7-tuple:

$$\text{ExPlS} = (P,\ a,\ V,\ d_D,\ c_D,\ d_R,\ c_R),$$

where:

- $d_D : P \times V \to [0,1]^s$ is the *Degree of Appurtenance Function (DAF)* with respect to *dominant* attribute value(s), where s indicates the dimensionality (e.g., $s = 1$ for fuzzy, $s = 3$ for neutrosophic). [2]

[1] It is important to note that the definition of the Degree of Appurtenance Function varies across different papers. Some studies define this concept using the power set, while others simplify it by avoiding the use of the power set [62]. The author has consistently defined the Classical Plithogenic Set without employing the power set.

[2] It is important to note that the definition of the Degree of Appurtenance Function varies across different papers. Some studies define this concept using the power set, while others simplify it by avoiding the use of the power set [62]. The author has consistently defined the Classical Plithogenic Set without employing the power set.

- $c_D : V \times V \to [0,1]^t$ is the *Degree of Contradiction Function (DCF)* associated with dominant attribute value(s), where t is the dimensionality of contradiction. It satisfies:

$$c_D(v,v) = 0, \quad c_D(v_1,v_2) = c_D(v_2,v_1) \quad \forall\, v, v_1, v_2 \in V.$$

- $d_R : P \times V \to [0,1]^s$ is the DAF with respect to *recessive* attribute value(s), defined similarly to d_D.
- $c_R : V \times V \to [0,1]^t$ is the DCF associated with recessive attribute value(s), satisfying:

$$c_R(v,v) = 0, \quad c_R(v_1,v_2) = c_R(v_2,v_1) \quad \forall\, v, v_1, v_2 \in V.$$

2.3 | Hypersoft Set and SuperHypersoft Set

In this subsection, we provide a brief introduction to the concepts of Soft Set, Hypersoft Set, and SuperHypersoft Set. A Soft Set maps attributes (parameters) to subsets of a universal set, offering a simplified framework for parameterized decision modeling [31, 35, 27]. A Hypersoft Set extends this concept by mapping combinations of multiple attributes to subsets of a universal set, thereby facilitating enhanced multi-attribute decision analysis [48, 5, 28, 36, 6, 41, 24, 14]. A SuperHypersoft Set further generalizes this approach by mapping power set combinations of multiple attribute values to subsets of a universal set, enabling higher-dimensional decision-making frameworks [18, 29, 53, 61, 26, 59, 70, 19]. The definitions of these sets are presented as follows.

Definition 3 (Soft Set)**.** [31, 35] Let U be a universal set and A be a set of attributes. A soft set over U is a pair $(\mathcal{F}, S)$, where $S \subseteq A$ and $\mathcal{F} : S \to \mathcal{P}(U)$. Here, $\mathcal{P}(U)$ denotes the power set of U. Mathematically, a soft set is represented as:

$$(\mathcal{F}, S) = \{(\alpha, \mathcal{F}(\alpha)) \mid \alpha \in S, \mathcal{F}(\alpha) \in \mathcal{P}(U)\}.$$

Each $\alpha \in S$ is called a parameter, and $\mathcal{F}(\alpha)$ is the set of elements in U associated with α.

Definition 4 (Hypersoft Set)**.** [48] Let U be a universal set, and let $\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_m$ be attribute domains. Define $\mathcal{C} = \mathcal{A}_1 \times \mathcal{A}_2 \times \cdots \times \mathcal{A}_m$, the Cartesian product of these domains. A hypersoft set over U is a pair $(G, \mathcal{C})$, where $G : \mathcal{C} \to \mathcal{P}(U)$. The hypersoft set is expressed as:

$$(G, \mathcal{C}) = \{(\gamma, G(\gamma)) \mid \gamma \in \mathcal{C}, G(\gamma) \in \mathcal{P}(U)\}.$$

For an m-tuple $\gamma = (\gamma_1, \gamma_2, \dots, \gamma_m) \in \mathcal{C}$, where $\gamma_i \in \mathcal{A}_i$ for $i = 1, 2, \dots, m$, $G(\gamma)$ represents the subset of U corresponding to the combination of attribute values $\gamma_1, \gamma_2, \dots, \gamma_m$.

Definition 5 (SuperHyperSoft Set)**.** [53] Let U be a universal set, and let $\mathcal{P}(U)$ denote the power set of U. Consider n distinct attributes $a_1, a_2, \dots, a_n$, where $n \geq 1$. Each attribute a_i is associated with a set of attribute values A_i, satisfying the property $A_i \cap A_j = \emptyset$ for all $i \neq j$.

Define $\mathcal{P}(A_i)$ as the power set of A_i for each $i = 1, 2, \dots, n$. Then, the Cartesian product of the power sets of attribute values is given by:

$$\mathcal{C} = \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \cdots \times \mathcal{P}(A_n).$$

A SuperHyperSoft Set over U is a pair $(F, \mathcal{C})$, where:

$$F : \mathcal{C} \to \mathcal{P}(U),$$

and F maps each element $(\alpha_1, \alpha_2, \dots, \alpha_n) \in \mathcal{C}$ (with $\alpha_i \in \mathcal{P}(A_i)$) to a subset $F(\alpha_1, \alpha_2, \dots, \alpha_n) \subseteq U$. Mathematically, the SuperHyperSoft Set is represented as:

$$(F, \mathcal{C}) = \{(\gamma, F(\gamma)) \mid \gamma \in \mathcal{C}, F(\gamma) \in \mathcal{P}(U)\}.$$

Here, $\gamma = (\alpha_1, \alpha_2, \dots, \alpha_n) \in \mathcal{C}$, where $\alpha_i \in \mathcal{P}(A_i)$ for $i = 1, 2, \dots, n$, and $F(\gamma)$ corresponds to the subset of U defined by the combined attribute values $\alpha_1, \alpha_2, \dots, \alpha_n$.

2.4 | Plithogenic Hypersoft Set

A Plithogenic Hypersoft Set combines attributes, sub-criteria, appurtenance degrees, and contradictions for nuanced multi-criteria decision analysis [32, 33, 10, 49, 4, 3, 11, 63].

Definition 6 (Plithogenic Hypersoft Set)**.** [49] Let U be a universal set, $Y \subseteq U$, and $\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_m$ be attribute domains. Define $\mathcal{C} = \mathcal{A}_1 \times \mathcal{A}_2 \times \cdots \times \mathcal{A}_m$. A plithogenic hypersoft set is a tuple $(Y, \mathcal{A}, \mathcal{C}, d, c)$, where:

1. The Attribute Value Appurtenance Degree Function d is defined as:

$$d : Y \times \mathcal{C} \to \mathcal{P}([0,1]^j), \quad \forall y \in Y,$$

where $j = 1$ represents a fuzzy degree, $j = 2$ represents an intuitionistic fuzzy degree, and $j = 3$ represents a neutrosophic degree of appurtenance.

2. The Contradiction Degree Function c is defined as:

$$c : \mathcal{A}_i \times \mathcal{A}_i \to \mathcal{P}([0,1]^j), \quad \forall i = 1, 2, \ldots, m,$$

where $c(\gamma_1, \gamma_2)$ is the contradiction degree between attribute values γ_1 and γ_2, satisfying:

$$c(\gamma_1, \gamma_1) = 0, \quad c(\gamma_1, \gamma_2) = c(\gamma_2, \gamma_1).$$

The plithogenic hypersoft set is represented as:

$$f(\gamma) = \{y \mid d_y(\gamma_1), d_y(\gamma_2), \ldots, d_y(\gamma_m)\}, \quad \forall y \in Y,$$

where $\gamma = (\gamma_1, \gamma_2, \ldots, \gamma_m) \in \mathcal{C}$ and $\gamma_i \in \mathcal{A}_i$ for $i = 1, 2, \ldots, m$.

2.5 | Forest Hypersoft set and Plithogenic Forest Hypersoft Set

A Forest Hypersoft Set models multi-level attributes as trees, mapping their combinations to subsets of a universal set for decision-making. The Plithogenic Forest Hypersoft Set extends the Forest Hypersoft Set by incorporating the principles of the Plithogenic Set. The formal definition is provided below[42].

Definition 7 (Forest of Attributes)**.** Let U be a universal set, and $H \subseteq U$ be a non-empty subset relevant to our decision or classification context. Let $\mathcal{A} = \{A_1, A_2, \ldots, A_n\}$ be a finite set of root attributes ($n \geq 1$). Each attribute A_i can be expanded into a *tree* of sub-attributes (of level 1), sub-sub-attributes (of level 2), and so forth, up to potentially multiple levels. Concretely, each root attribute A_i may generate a hierarchy such as:

$$A_i \quad \to \quad \begin{cases} A_{i1},\, A_{i2},\, \ldots, & \text{(level 1)} \\ A_{i1j},\, A_{i2k},\, \ldots, & \text{(level 2)} \\ \ldots \end{cases}$$

When we collect all such trees stemming from $A_1, A_2, \ldots, A_n$, we obtain a *forest* of attributes.

Notation 8. We use $\text{Tree}(A_i)$ to denote the entire tree of sub-attributes rooted at A_i. Then the *forest* formed by all root attributes is:

$$\text{Forest}(\mathcal{A}) \; = \; \{\text{Tree}(A_1),\, \text{Tree}(A_2),\, \ldots,\, \text{Tree}(A_n)\}.$$

Each $\text{Tree}(A_i)$ has nodes that represent sub-attributes or deeper-level sub-sub-attributes, etc.

Definition 9 (Forest Hypersoft Set)**.** Let U be a universal set, and let $H \subseteq U$ be a non-empty subset relevant to the decision or classification context. Suppose we have a finite set of root attributes $\mathcal{A} = \{A_1, A_2, \ldots, A_n\}$, with $n \geq 1$. Each attribute A_i can be expanded into a *tree* of sub-attributes:

$$A_i \;\to\; \begin{cases} A_{i1}, A_{i2}, \ldots & \text{(level 1)} \\ A_{i1k}, A_{i2\ell}, \ldots & \text{(level 2)} \\ \ldots \end{cases}$$

Collecting all such trees for $A_1, A_2, \ldots, A_n$ produces a *forest* of attributes, denoted

$$\text{Forest}(\mathcal{A}) \; = \; \{\text{Tree}(A_1),\, \text{Tree}(A_2), \ldots, \text{Tree}(A_n)\}.$$

Leaf-Level Sub-Attributes. For each tree $\mathrm{Tree}(A_i)$, let $\Gamma(\mathrm{Tree}(A_i))$ be the set of all possible *leaf-level sub-attributes* stemming from A_i. A single leaf-level sub-attribute might represent a path in the tree from A_i down to one of its final sub-sub-attributes. Then define

$$\Gamma(\mathrm{Forest}(\mathcal{A})) \;=\; \bigcup_{i=1}^{n} \Gamma(\mathrm{Tree}(A_i)),$$

so that any element $\alpha \in \Gamma(\mathrm{Forest}(\mathcal{A}))$ is a final-level attribute value from one of the trees in the forest.
Forest Combinations. Consider the family of all possible subsets of $\Gamma(\mathrm{Forest}(\mathcal{A}))$:

$$\mathcal{C}_{\text{forest}} \;\subseteq\; \mathcal{P}(\Gamma(\mathrm{Forest}(\mathcal{A}))).$$

Each $\gamma \in \mathcal{C}_{\text{forest}}$ is understood as a *forest-based combination* of leaf-level sub-attributes (potentially from different root attributes).
Mapping to the Universe. A *Forest Hypersoft Set* over (U, H) is given by a pair

$$(G,\, \mathcal{C}_{\text{forest}}),$$

where $G : \mathcal{C}_{\text{forest}} \to \mathcal{P}(H)$ satisfies the condition that for each $\gamma \in \mathcal{C}_{\text{forest}}$, $G(\gamma) \subseteq H$. Concretely,

$$(G,\, \mathcal{C}_{\text{forest}}) \;=\; \Big\{ (\gamma,\, G(\gamma)) \;\Big|\; \gamma \in \mathcal{C}_{\text{forest}},\, G(\gamma) \subseteq H \Big\}.$$

In words, $G(\gamma)$ is the subset of H that corresponds to the collective influence or membership of all final-level sub-attributes in γ.

Definition 10 (Plithogenic Forest Hypersoft Set (PFHS))**.** [42] Let U be a universal set, $H \subseteq U$ a non-empty subset, and $\mathcal{A} = \{A_1, \dots, A_n\}$ be n root attributes. Suppose each A_i expands into a tree of sub-attributes up to various levels, collectively forming a forest $\mathrm{Forest}(\mathcal{A})$. We define:

- $\Gamma(\mathrm{Tree}(A_i))$ to be the set of *all possible leaf-level* sub-attribute combinations in the tree of A_i. - For instance, if A_i has a branch A_{i1} leading to further sub-attributes A_{i11}, A_{i12}, etc., each path to a leaf represents one final sub-attribute value.
- Let
$$\Gamma(\mathrm{Forest}(\mathcal{A})) \;=\; \Gamma(\mathrm{Tree}(A_1)) \;\cup\; \Gamma(\mathrm{Tree}(A_2)) \;\cup\; \dots \;\cup\; \Gamma(\mathrm{Tree}(A_n)).$$
Each element in $\Gamma(\mathrm{Forest}(\mathcal{A}))$ is effectively a *final-level attribute value* from one of the trees.

A **Plithogenic Forest Hypersoft Set (PFHS)** over (U, H) is specified by a tuple

$$\mathrm{PFHS} \;=\; \Big(U,\, H,\, \mathcal{A},\, d,\, c,\, G\Big),$$

where:

(1) $d : H \times \Gamma(\mathrm{Forest}(\mathcal{A})) \;\longrightarrow\; [0,1]^j$ is the *attribute value appurtenance degree function*, possibly multi-dimensional ($j \in \{1,2,3\}$). For each $y \in H$ and final sub-attribute $\alpha \in \Gamma(\mathrm{Forest}(\mathcal{A}))$, $d_y(\alpha) \in [0,1]^j$ indicates the membership degree(s) of y with respect to α. (E.g., fuzzy, intuitionistic, or neutrosophic membership.)

(2) $c : \Gamma(\mathrm{Forest}(\mathcal{A})) \times \Gamma(\mathrm{Forest}(\mathcal{A})) \;\longrightarrow\; [0,1]^t$ is the *contradiction degree function*, satisfying

$$c(\alpha,\alpha) = 0, \quad c(\alpha,\beta) = c(\beta,\alpha) \quad \text{for all } \alpha, \beta \in \Gamma(\mathrm{Forest}(\mathcal{A})).$$

Here, $t \in \{1,2,3\}$ denotes the dimensionality of contradiction (e.g., fuzzy or neutrosophic contradiction measures).

(3) G is a mapping that assigns each *forest-based combination* of sub-attributes to a subset of H. In practice, a *forest-based combination* is a selection of $\{\alpha_1, \alpha_2, \dots, \alpha_m\} \subseteq \Gamma(\mathrm{Forest}(\mathcal{A}))$ (one or more leaf-level values possibly from different root attributes). Formally, we can consider

$$\mathcal{C}_{\text{forest}} \;\subseteq\; \mathcal{P}(\Gamma(\mathrm{Forest}(\mathcal{A})))$$

to be the set of all feasible combinations of final-level sub-attributes across the forest. Then,

$$G : \mathcal{C}_{\text{forest}} \;\longrightarrow\; \mathcal{P}(H),$$

and for each $\gamma \in \mathcal{C}_{\text{forest}}$, $G(\gamma) \subseteq H$. The set $G(\gamma)$ is determined by:

$$G(\gamma) \;=\; \{\, y \in H \;\mid\; d_y(\alpha_k) \,\text{and}\, c(\alpha_i,\alpha_j) \;\text{(for } \alpha_i,\alpha_j \in \gamma\text{) satisfy the plithogenic composition rules}\}.$$

In essence, a PFHS extends *Hypersoft Set* ideas to a forest of multi-level attributes and further incorporates *plithogenic* elements (contradiction degrees and multi-valued membership).

2.6 | HyperPlithogenic Set and SuperHyperPlithogenic Set

Next, the definitions of the HyperPlithogenic Set and the SuperHyperPlithogenic Set are presented below [20, 13, 18]. The HyperPlithogenic Set is a concept defined using hyperstructures, while the SuperHyperPlithogenic Set is defined using superhyperstructures [21, 12, 55, 55, 56, 56, 54].

Definition 11 (n-th Powerset)**.** (cf.[56, 17, 45])

The n-th powerset of a set H, denoted $P_n(H)$, is defined recursively. Starting with the standard powerset, the construction proceeds as:

$$P_1(H) = P(H), \quad P_{n+1}(H) = P(P_n(H)), \quad \text{for } n \geq 1.$$

The n-th non-empty powerset, denoted $P_n^*(H)$, excludes the empty set:

$$P_1^*(H) = P^*(H), \quad P_{n+1}^*(H) = P^*(P_n^*(H)).$$

Here, $P^*(H)$ is the powerset of H excluding the empty set.

Definition 12 (HyperPlithogenic Set)**.** [20, 13, 18] Let X be a non-empty set, and let A be a set of attributes. For each attribute $v \in A$, let Pv be the set of possible values of v. A *HyperPlithogenic Set* HPS over X is defined as:

$$HPS = (P, \{v_i\}_{i=1}^n, \{Pv_i\}_{i=1}^n, \{\tilde{pdf}_i\}_{i=1}^n, pCF)$$

where:

- $P \subseteq X$ is a subset of the universe.
- For each attribute v_i, Pv_i is the set of possible values.
- For each attribute v_i, $\tilde{pdf}_i : P \times Pv_i \to \tilde{P}([0,1]^s)$ is the *Hyper Degree of Appurtenance Function (HDAF)*, assigning to each element $x \in P$ and attribute value $a_i \in Pv_i$ a set of membership degrees.
- $pCF : \left(\bigcup_{i=1}^n Pv_i\right) \times \left(\bigcup_{i=1}^n Pv_i\right) \to [0,1]^t$ is the *Degree of Contradiction Function (DCF)*.

Definition 13 (n-SuperHyperPlithogenic Set)**.** [20, 13, 18] Let X be a non-empty set, and let $V = \{v_1, v_2, \ldots, v_n\}$ be a set of attributes, each associated with a set of possible values P_{v_i}. An n-SuperHyperPlithogenic Set ($SHPS_n$) is defined recursively as:

$$SHPS_n = (P_n, V, \{P_{v_i}\}_{i=1}^n, \{\tilde{pdf}_i^{(n)}\}_{i=1}^n, pCF^{(n)}),$$

where:

- $P_1 \subseteq X$, and for $k \geq 2$,
  $$P_k = \tilde{\mathcal{P}}(P_{k-1}),$$
  represents the k-th nested family of non-empty subsets of P_1.
- For each attribute $v_i \in V$, P_{v_i} is the set of possible values of the attribute v_i.
- For each k-th level subset P_k, $\tilde{pdf}_i^{(n)} : P_n \times P_{v_i} \to \tilde{\mathcal{P}}([0,1]^s)$ is the *Hyper Degree of Appurtenance Function (HDAF)*, assigning to each element $x \in P_n$ and attribute value $a_i \in P_{v_i}$ a subset of $[0,1]^s$.
- $pCF^{(n)} : \bigcup_{i=1}^n P_{v_i} \times \bigcup_{i=1}^n P_{v_i} \to [0,1]^t$ is the *Degree of Contradiction Function (DCF)*, satisfying:
  (1) Reflexivity: $pCF^{(n)}(a,a) = 0$ for all $a \in \bigcup_{i=1}^n P_{v_i}$,
  (2) Symmetry: $pCF^{(n)}(a,b) = pCF^{(n)}(b,a)$ for all $a,b \in \bigcup_{i=1}^n P_{v_i}$.

- s and t are positive integers representing the dimensions of the membership degrees and contradiction degrees, respectively.

3 | Results of This Paper

This section presents the results obtained in this paper.

3.1 | Plithogenic SuperHypersoft Set

The Plithogenic SuperHypersoft Set is a concept that integrates the principles of the Plithogenic Set and the SuperHypersoft Set. Definitions and related details are provided below.

Definition 14 (Plithogenic SuperHypersoft Set)**.** Let:

- U be a universal set, and $\mathcal{P}(U)$ its power set;
- $\{a_1, a_2, \dots, a_n\}$ be n distinct attributes, with $n \geq 1$;
- A_i be the (disjoint) set of possible values for attribute a_i, i.e., $A_i \cap A_j = \emptyset$ for $i \neq j$, $1 \leq i, j \leq n$;
- $\mathcal{P}(A_i)$ be the power set of A_i, for each $i = 1, 2, \dots, n$;
- $\mathcal{C} = \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \cdots \times \mathcal{P}(A_n)$ be the Cartesian product of these power sets;
- $Y \subseteq U$ be a (potentially proper) subset of U.

We define two primary functions:
1. Attribute Value Appurtenance Degree Function.

$$d : Y \times \mathcal{C} \longrightarrow \mathcal{P}([0,1]^j),$$

where $j \in \{1, 2, 3\}$ typically represents the dimensionality of the membership structure:

- $j = 1$: Fuzzy membership degree;
- $j = 2$: Intuitionistic fuzzy membership (membership and non-membership);
- $j = 3$: Neutrosophic membership (truth, indeterminacy, falsity).

For each $y \in Y$ and $\gamma \in \mathcal{C}$, $d_y(\gamma)$ belongs to $[0,1]^j$, indicating the degree(s) of appurtenance of y with respect to the combination of attribute-values in γ.
2. Contradiction Degree Function. For each attribute a_i, define

$$c : \mathcal{P}(A_i) \times \mathcal{P}(A_i) \longrightarrow \mathcal{P}([0,1]^j),$$

which assigns a contradiction degree $c(\alpha_1, \alpha_2)$ to any two subsets $\alpha_1, \alpha_2 \subseteq A_i$. This function satisfies:

$$c(\alpha, \alpha) = 0, \quad c(\alpha_1, \alpha_2) = c(\alpha_2, \alpha_1),$$

for all $\alpha, \alpha_1, \alpha_2 \subseteq A_i$.

A **Plithogenic SuperHypersoft Set (PSHSS)** over U is then defined as a 7-tuple

$$\text{PSHSS} = (U, Y, \{a_i\}, \{A_i\}, \mathcal{C}, d, c),$$

together with a mapping

$$F : \mathcal{C} \longrightarrow \mathcal{P}(U),$$

where each $\gamma \in \mathcal{C}$ is of the form $\gamma = (\alpha_1, \alpha_2, \dots, \alpha_n)$ with $\alpha_i \subseteq A_i$. The image $F(\gamma) \subseteq U$ is determined by the degrees of appurtenance and contradictions between these subsets, typically expressed as:

$$F(\gamma) = \left\{ y \in Y \;\middle|\; d_y(\alpha_1, \dots, \alpha_n) \text{ and } c(\alpha_i, \alpha_i) \text{ or } c(\alpha_i, \alpha_j) \text{ (for } i \neq j\text{) satisfy the plithogenic composition rules} \right\}.$$

Intuitively, the function d encodes how strongly each element $y \in Y$ belongs to the combination of attribute-subsets $\alpha_1, \dots, \alpha_n$, while c quantifies contradictions among those subsets within the same attribute. The final set $F(\gamma)$ is thus shaped by both appurtenance and contradiction considerations in a plithogenic manner.

Theorem 15 (PSHSS Generalizes SHS and PHS)**.** *Let*

$$\text{PSHSS} = (U, Y, \{A_i\}, \mathcal{C}, d, c, F)$$

be a Plithogenic SuperHypersoft Set as in Definition 14. Then:

(1) ***Reduction to SuperHypersoft Set (SHS):*** *By setting the contradiction degree* $c(\alpha, \beta) = 0$ *for all* $\alpha, \beta \subseteq A_i$ *and by treating the multi-dimensional membership degrees in* d *as classical (crisp) memberships (i.e., restricting* $d_y(\gamma) \in \{0, 1\}$ *for all* $\gamma \in \mathcal{C}$*), the PSHSS reduces to the conventional SuperHypersoft Set.*

(2) ***Reduction to Plithogenic Hypersoft Set (PHS):*** *By restricting each* $\alpha_i \subseteq A_i$ *in the power-set product* $\mathcal{C} = \mathcal{P}(A_1) \times \cdots \times \mathcal{P}(A_n)$ *to* only singletons $\{\omega_i\} \subseteq A_i$*, the PSHSS becomes a Plithogenic Hypersoft Set over the direct product* $\mathcal{A}_1 \times \cdots \times \mathcal{A}_n$*. Concretely, if* α_i *is always a singleton (or empty set), then* γ *reduces to an* m*-tuple* $\gamma = (\omega_1, \ldots, \omega_m)$*, and the plithogenic conditions on* d *and* c *precisely match those in a PHS definition.*

Proof: We prove each reduction separately.
(1) PSHSS to SuperHypersoft Set. Recall that a SuperHypersoft Set (SHS) is defined by

$$(F, \mathcal{C}) \quad \text{where} \quad \mathcal{C} = \mathcal{P}(A_1) \times \cdots \times \mathcal{P}(A_n),$$

and

$$F : \mathcal{C} \longrightarrow \mathcal{P}(U),$$

without any explicit contradiction or multi-valued membership dimension. In a Plithogenic SuperHypersoft Set, we have two additional features:

- The *Contradiction Degree Function* $c(\alpha_1, \alpha_2)$ for subsets of A_i.
- The *Appurtenance Degree Function* $d_y(\gamma)$ (often multi-dimensional: fuzzy, intuitionistic, neutrosophic, etc.).

To recover an SHS, we simply:

- *Nullify the contradiction*: set $c(\alpha_1, \alpha_2) = 0$ for all $\alpha_1, \alpha_2 \subseteq A_i$. This means no contradiction penalty applies when combining subsets of an attribute's domain.
- *Restrict to crisp membership*: enforce $d_y(\gamma) \in \{0, 1\} \subseteq [0, 1]^j$ for every $y \in U$ and $\gamma \in \mathcal{C}$. In other words, an element either belongs or does not belong, without any fractional membership.

Under these two restrictions, the plithogenic composition reduces to a simple mapping $F(\gamma) \subseteq U$ that depends only on whether $d_y(\gamma) = 1$. Since there is no contradiction penalty, the set membership precisely matches the SuperHypersoft definition. Hence, PSHSS becomes an SHS.
(2) PSHSS to Plithogenic Hypersoft Set. A Plithogenic Hypersoft Set (PHS) typically works on the Cartesian product of base attribute domains $\mathcal{A}_1 \times \cdots \times \mathcal{A}_m$, rather than the power sets $\mathcal{P}(A_1) \times \cdots \times \mathcal{P}(A_n)$. In Definition 14, each γ is an n-tuple

$$\gamma = (\alpha_1, \alpha_2, \ldots, \alpha_n), \quad \alpha_i \subseteq A_i.$$

If we *restrict each* α_i to be either a *singleton* or possibly the empty set (though typically the empty set is excluded if one wants a pure domain approach), we effectively revert to a direct product of singletons in each A_i. Concretely:

$$\alpha_i = \{\omega_i\} \quad (\text{or } \emptyset),$$

so $\gamma = (\{\omega_1\}, \ldots, \{\omega_n\})$. Identifying each $\{\omega_i\}$ with $\omega_i \in A_i$ yields $\gamma \in A_1 \times \cdots \times A_n$, matching the standard domain-based product in PHS.

Meanwhile, the *contradiction* and *appurtenance* structure in PSHSS remains:

- The contradiction function $c(\{\omega_i\}, \{\omega_i'\})$ for each attribute domain A_i collapses to $c(\omega_i, \omega_i')$ (the same form used in Plithogenic Hypersoft Sets).
- The membership degrees $d_y(\{\omega_1\}, \ldots, \{\omega_n\})$ match those in PHS (often fuzzy, intuitionistic, or neutrosophic).

Thus, by restricting the power-set dimension to singletons, the PSHSS precisely mirrors the structure of a Plithogenic Hypersoft Set. This establishes the second claim. □

Theorem 16. *Let* $\mathrm{PSHSS}_1 = (U, Y_1, \{a_i\}, \{A_i\}, \mathcal{C}, d^{(1)}, c^{(1)}, F_1)$ *and* $\mathrm{PSHSS}_2 = (U, Y_2, \{a_i\}, \{A_i\}, \mathcal{C}, d^{(2)}, c^{(2)}, F_2)$ *be two Plithogenic SuperHypersoft Sets over the same universe* U *and same attribute domains* $\{A_i\}$*. Suppose that*

$$d_y^{(1)}(\gamma) \ \le\ d_y^{(2)}(\gamma) \quad \text{and} \quad c^{(1)}(\alpha,\beta) \ \le\ c^{(2)}(\alpha,\beta)$$

for all $y \in Y_1 \cap Y_2$*,* $\gamma \in \mathcal{C}$*, and* $\alpha, \beta \in \mathcal{P}(A_i)$*. If* $Y_1 \subseteq Y_2$*, then*

$$F_1(\gamma) \ \subseteq\ F_2(\gamma) \quad \text{for all } \gamma \in \mathcal{C}.$$

Proof: Fix any $\gamma = (\alpha_1, \dots, \alpha_n) \in \mathcal{C}$. Take an arbitrary $y \in F_1(\gamma)$. By Definition 14, $y \in Y_1 \subseteq Y_2$ and satisfies the appurtenance and contradiction thresholds for F_1. Concretely,

$$d_y^{(1)}(\gamma) \ \text{ and } \ c^{(1)}(\alpha_i, \alpha_j) \text{ (for } 1 \le i < j \le n)$$

yield membership of y in $F_1(\gamma)$.

Since $d_y^{(1)}(\gamma) \le d_y^{(2)}(\gamma)$ componentwise in $[0,1]^j$, and $c^{(1)}(\alpha_i, \alpha_j) \le c^{(2)}(\alpha_i, \alpha_j)$, the conditions for y to belong to $F_2(\gamma)$ are at least as permissive as those in $F_1(\gamma)$. Hence $y \in F_2(\gamma)$. Because y was chosen arbitrarily in $F_1(\gamma)$, we conclude

$$F_1(\gamma) \ \subseteq\ F_2(\gamma) \quad \text{for all } \gamma \in \mathcal{C}.$$

Thus the theorem holds. □

Theorem 17. ***(Union of Two PSHSSs)*** *Let* $\mathrm{PSHSS}_1 = (U, Y, \{a_i\}, \{A_i\}, \mathcal{C}, d^{(1)}, c^{(1)}, F_1)$ *and* $\mathrm{PSHSS}_2 = (U, Y, \{a_i\}, \{A_i\}, \mathcal{C}, d^{(2)}, c^{(2)}, F_2)$ *be two Plithogenic SuperHypersoft Sets defined over the same* $U, Y, \mathcal{C}$*. Define a new mapping*

$$F_\cup(\gamma) \ =\ F_1(\gamma) \ \cup\ F_2(\gamma), \quad \gamma \in \mathcal{C}.$$

Then

$$\mathrm{PSHSS}_\cup \ =\ (U,\, Y,\, \{a_i\},\, \{A_i\},\, \mathcal{C},\, d^{(\cup)},\, c^{(\cup)},\, F_\cup)$$

is also a Plithogenic SuperHypersoft Set, where

$$d_y^{(\cup)}(\gamma) \ =\ \max\Big\{\, d_y^{(1)}(\gamma),\, d_y^{(2)}(\gamma)\Big\}, \quad c^{(\cup)}(\alpha,\beta) \ =\ \max\Big\{\, c^{(1)}(\alpha,\beta),\, c^{(2)}(\alpha,\beta)\Big\}.$$

Proof: We need to verify that $d^{(\cup)}$ and $c^{(\cup)}$ satisfy the plithogenic properties (non-negativity, reflexivity, symmetry, etc.) and that $F_\cup$ indeed follows from these definitions.

Step 1: Properties of $d^{(\cup)}$*.* For each $y \in Y$ and $\gamma \in \mathcal{C}$,

$$d_y^{(\cup)}(\gamma) \ =\ \max\Big\{\, d_y^{(1)}(\gamma),\, d_y^{(2)}(\gamma)\Big\}.$$

Since $d_y^{(1)}(\gamma)$ and $d_y^{(2)}(\gamma)$ both lie in $[0,1]^j$, their componentwise maximum also lies in $[0,1]^j$. Thus $d_y^{(\cup)}(\gamma)$ is a valid appurtenance degree.

Step 2: Properties of $c^{(\cup)}$*.* For each attribute a_i, let $\alpha, \beta \subseteq A_i$. Then

$$c^{(\cup)}(\alpha,\beta) \ =\ \max\Big\{c^{(1)}(\alpha,\beta),\, c^{(2)}(\alpha,\beta)\Big\}.$$

Because both $c^{(1)}$ and $c^{(2)}$ are non-negative and symmetric, their maximum is also non-negative and symmetric. Reflexivity follows from

$$c^{(\cup)}(\alpha,\alpha) \ =\ \max\Big\{c^{(1)}(\alpha,\alpha),\, c^{(2)}(\alpha,\alpha)\Big\} \ =\ \max\{\,0,0\} \ =\ 0.$$

Step 3: Construction of $F_\cup$*.* By definition,

$$F_\cup(\gamma) \ =\ F_1(\gamma) \ \cup\ F_2(\gamma),$$

where $F_i(\gamma)$ are determined by $d^{(i)}$ and $c^{(i)}$, respectively. The effect of taking the maximum of d-values and c-values in each coordinate naturally relaxes the membership conditions, thus covering the union of the original plithogenic superhypersoft membership sets. Consequently,

$$F_{\cup}(\gamma) \;=\; \{y \in Y \mid d_y^{(\cup)}(\gamma) \text{ and } c^{(\cup)}(\alpha,\beta) \text{ satisfy the plithogenic conditions}\},$$

which aligns with the standard definition of membership in a PSHSS using $d^{(\cup)}$ and $c^{(\cup)}$. Therefore, $\text{PSHSS}_{\cup}$ is indeed a Plithogenic SuperHypersoft Set. □

3.2 | Extended Plithogenic SuperHypersoft Set

As previously mentioned, the *Extended Plithogenic Set* framework introduces dominant and recessive viewpoints for attributes, each with its own degree of appurtenance and contradiction functions. Here, we extend these ideas to the SuperHypersoft environment.

Definition 18 (Extended Plithogenic SuperHypersoft Set)**.** Using the notation from Definition 14, let:

- V be the overall range of potential attribute values (in the classical sense), where each $A_i \subseteq V$ and $A_i \cap A_j = \emptyset$ for $i \neq j$;
- $\mathcal{P}(A_i)$ be the power set of A_i, with $\mathcal{C} = \mathcal{P}(A_1) \times \cdots \times \mathcal{P}(A_n)$;
- $Y \subseteq U$ be a subset of the universe.

An *Extended Plithogenic SuperHypersoft Set (ExPSHSS)* is an 11-tuple

$$\text{ExPSHSS} \;=\; (U,\, Y,\, \{A_i\},\, \mathcal{C},\, d_D,\, c_D,\, d_R,\, c_R,\, F,\, j_D,\, j_R),$$

where:

(1) $d_D : Y \times \mathcal{C} \to \mathcal{P}([0,1]^{j_D})$ is the *dominant* degree of appurtenance function, with dimensionality $j_D \in \{1,2,3\}$.

(2) $c_D : \mathcal{P}(A_i) \times \mathcal{P}(A_i) \to \mathcal{P}([0,1]^{j_D})$ is the *dominant* contradiction degree function, satisfying

$$c_D(\alpha,\alpha) = 0, \quad c_D(\alpha,\beta) = c_D(\beta,\alpha).$$

(3) $d_R : Y \times \mathcal{C} \to \mathcal{P}([0,1]^{j_R})$ is the *recessive* degree of appurtenance function, with dimensionality $j_R \in \{1,2,3\}$.

(4) $c_R : \mathcal{P}(A_i) \times \mathcal{P}(A_i) \to \mathcal{P}([0,1]^{j_R})$ is the *recessive* contradiction degree function, satisfying

$$c_R(\alpha,\alpha) = 0, \quad c_R(\alpha,\beta) = c_R(\beta,\alpha).$$

(5) $F : \mathcal{C} \longrightarrow \mathcal{P}(U)$ assigns each $\gamma \in \mathcal{C}$ a subset of U, typically determined by combining d_D, d_R, c_D, c_R under plithogenic aggregation rules. Concretely,

$$F(\gamma) \;=\; \{\, y \in Y \mid \; \Theta(d_D, y, \gamma) \;\wedge\; \Theta(d_R, y, \gamma) \;\wedge\; \Omega(c_D, c_R, \gamma)\},$$

for some plithogenic composition operators Θ and Ω.

This extended structure allows each attribute subset $\alpha_i \subseteq A_i$ to be interpreted in two ways: one where it is *dominant*, another where it is *recessive*. The overall membership of $y \in Y$ in $F(\gamma)$ then depends on how these dual perspectives balance or contradict each other.

Theorem 19 (ExPSHSS Generalizes PSHSS)**.** *Let*

$$\text{ExPSHSS} \;=\; (U,\, Y,\, \{A_i\},\, \mathcal{C},\, d_D,\, c_D,\, d_R,\, c_R,\, F,\, j_D,\, j_R)$$

be an Extended Plithogenic SuperHypersoft Set *(ExPSHSS) as in Definition 18. By enforcing that the recessive viewpoint coincides with the dominant viewpoint—i.e., setting*

$$d_R \;=\; d_D, \quad c_R \;=\; c_D, \quad j_R \;=\; j_D,$$

the ExPSHSS reduces exactly to a Plithogenic SuperHypersoft Set (PSHSS).

Proof: In the Extended Plithogenic SuperHypersoft Set, each attribute domain and subset is evaluated by both a *dominant* and a *recessive* membership/contradiction scheme:

$$d_D,\ c_D \quad \text{and} \quad d_R,\ c_R.$$

Additionally, each has its own dimensionality j_D and j_R. To recover a standard PSHSS (cf. Definition 14), we remove the distinction between dominant and recessive by identifying them:

$$d_R(\gamma, y) \ = \ d_D(\gamma, y), \quad c_R(\alpha, \beta) \ = \ c_D(\alpha, \beta), \quad j_R \ = \ j_D.$$

Under these conditions, each attribute-value combination $\gamma \in \mathcal{C}$ and element $y \in Y$ has a *single* appurtenance function $d(\gamma, y)$ (no separate d_D or d_R), and a *single* contradiction function $c(\alpha, \beta)$. The mapping $F(\gamma) \subseteq U$ then depends on these unified values exactly as in the Plithogenic SuperHypersoft Set definition. Hence, the Extended Plithogenic SuperHypersoft Set collapses to a Plithogenic SuperHypersoft Set when dominant and recessive notions coincide. □

3.3 | Forest SuperHypersoft Set and Plithogenic Forest SuperHypersoft Set

The Plithogenic Forest SuperHypersoft Set is a concept that combines the principles of the Plithogenic Set and the Plithogenic Forest Hypersoft Set. The definition is presented below.

Definition 20 (Forest SuperHypersoft Set (FSHS))**.** Let:

- U be a universal set, and let $H \subseteq U$ be a non-empty subset.
- $\mathcal{A} = \{A_1, A_2, \ldots, A_n\}$ be $n \geq 1$ root attributes, each expanding into a tree of sub-attributes, collectively forming a forest $\text{Forest}(\mathcal{A})$.
- $\Gamma(\text{Forest}(\mathcal{A}))$ denote the set of all final-level sub-attributes (the leaves) across all attribute trees.
- Define the *forest super-domain* as the power set of leaf-level sub-attributes:

$$\widetilde{\mathcal{C}}_{\text{forest}} \ = \ \mathcal{P}(\Gamma(\text{Forest}(\mathcal{A}))).$$

  An element $\beta \in \widetilde{\mathcal{C}}_{\text{forest}}$ is thus a subset of final-level sub-attributes (possibly from different root attributes).

A **Forest SuperHypersoft Set** (FSHS) over (U, H) is a pair

$$(G,\ \widetilde{\mathcal{C}}_{\text{forest}}),$$

where

$$G : \widetilde{\mathcal{C}}_{\text{forest}} \ \longrightarrow \ \mathcal{P}(H)$$

maps each set of leaf-level sub-attributes $\beta \subseteq \Gamma(\text{Forest}(\mathcal{A}))$ to a subset $G(\beta) \subseteq H$. In other words,

$$(G,\ \widetilde{\mathcal{C}}_{\text{forest}}) \ = \ \Big\{(\beta,\ G(\beta)) \ \Big| \ \beta \in \widetilde{\mathcal{C}}_{\text{forest}},\ G(\beta) \subseteq H\Big\}.$$

Remark 21 (Forest Superhypersoft set)**.** *In contrast to a standard* Forest Hypersoft Set*, which considers single final-level attributes in each combination, a Forest SuperHypersoft Set allows each node in the forest to be selected as a* power-set combination *of its leaf-level sub-attributes. Thus, the function G assigns to each such collection of leaf-level values (potentially from multiple root attributes) a corresponding subset of H. This framework accommodates higher-dimensional or more flexible attribute modeling than the simpler forest hypersoft paradigm.*

Theorem 22. ***(Forest SuperHypersoft Set Generalizes Forest Hypersoft Set)*** *Let*

$$\text{FSHS} \ = \ (G,\ \widetilde{\mathcal{C}}_{\text{forest}})$$

be a Forest SuperHypersoft Set over (U, H), as in Definition 20. By restricting each subset $\beta \subseteq \Gamma(\text{Forest}(\mathcal{A}))$ to be a singleton*, we recover exactly a* Forest Hypersoft Set.

Proof: Recall that in a *Forest Hypersoft Set*, we consider only single leaf-level attributes in each combination. Concretely, if we denote

$$\mathcal{C}_{\text{forest}} \subseteq \mathcal{P}(\Gamma(\text{Forest}(\mathcal{A})))$$

to be those subsets containing exactly one element, then each $\gamma \in \mathcal{C}_{\text{forest}}$ can be written as $\gamma = \{\alpha\}$ for some $\alpha \in \Gamma(\text{Forest}(\mathcal{A}))$. Hence,

$$\mathcal{C}_{\text{forest}} = \{ \{\alpha\} \mid \alpha \in \Gamma(\text{Forest}(\mathcal{A}))\}.$$

If we take the given FSHS mapping

$$G : \widetilde{\mathcal{C}}_{\text{forest}} \longrightarrow \mathcal{P}(H),$$

and restrict its domain to singletons $\{\alpha\}$, we obtain

$$G(\{\alpha\}) \subseteq H.$$

Renaming the mapping G to a simpler notation (such as F), we have a pair

$$(F,\, \mathcal{C}_{\text{forest}}) = \Big\{(\{\alpha\},\, F(\{\alpha\})) \;\Big|\; \{\alpha\} \in \mathcal{C}_{\text{forest}}\Big\},$$

which is precisely the definition of a *Forest Hypersoft Set* (where one final-level attribute α is chosen per combination). Thus, allowing only singleton subsets in the domain recovers the Forest Hypersoft Set scenario. This completes the proof. □

Definition 23 (Plithogenic Forest SuperHypersoft Set (PFSHS))**.** Using the same notation as in Definition 10, let

$$\Gamma(\text{Forest}(\mathcal{A})) = \bigcup_{i=1}^{n} \Gamma(\text{Tree}(A_i))$$

be the set of final-level sub-attributes. Instead of taking singletons $\alpha \in \Gamma(\text{Forest}(\mathcal{A}))$, we now consider *power sets* of these final-level values. Define

$$\widetilde{\mathcal{C}}_{\text{forest}} = \mathcal{P}(\Gamma(\text{Forest}(\mathcal{A}))),$$

the power set of all possible leaf-level attributes in the forest. A **Plithogenic Forest SuperHypersoft Set (PFSHS)** over (U, H) is a tuple

$$\text{PFSHS} = \Big(U,\, H,\, \mathcal{A},\, d,\, c,\, F\Big),$$

where:

(1) $d : H \times \widetilde{\mathcal{C}}_{\text{forest}} \longrightarrow [0,1]^j$ is the multi-valued *appurtenance degree function*, analogous to PFHS but now each argument is a *set of leaf-level sub-attributes* $\beta \subseteq \Gamma(\text{Forest}(\mathcal{A}))$.

(2) $c : \widetilde{\mathcal{C}}_{\text{forest}} \times \widetilde{\mathcal{C}}_{\text{forest}} \longrightarrow [0,1]^t$ is the *contradiction degree function*, measuring contradiction between any two subsets $\beta_1, \beta_2 \subseteq \Gamma(\text{Forest}(\mathcal{A}))$. It satisfies:

$$c(\beta, \beta) = 0, \quad c(\beta_1, \beta_2) = c(\beta_2, \beta_1).$$

(3) $F : \widetilde{\mathcal{C}}_{\text{forest}} \longrightarrow \mathcal{P}(H)$ assigns each subset $\beta \in \widetilde{\mathcal{C}}_{\text{forest}}$ a subset $F(\beta) \subseteq H$, determined by plithogenic aggregation of the multi-valued appurtenances and contradictions among the chosen subsets. Formally,

$$F(\beta) = \{ y \in H \mid d_y(\beta) \text{ and } c(\beta_1, \beta_2) \subseteq [0,1]^t \text{ (for } \beta_1, \beta_2 \subseteq \beta) \text{ fulfill plithogenic membership rules}\}.$$

Hence, PFSHS allows each node in the forest to be taken as a *power-set combination of final-level sub-attributes*, capturing a higher-dimensional structure than the standard PFHS.

Theorem 24. *Let*

$$\text{PFSHS} = (U,\, H,\, \mathcal{A},\, d,\, c,\, F)$$

be a Plithogenic Forest SuperHypersoft Set as in Definition 23. Then:

(1) *By restricting each selected subset* $\beta \subseteq \Gamma(\text{Forest}(\mathcal{A}))$ *to a* singleton *(or to a single path from the forest), we recover a* Plithogenic Forest Hypersoft Set (PFHS)*.*

(2) *By restricting the forest to* single-level attributes *(or a single root attribute per domain) and simultaneously allowing multi-subattribute expansions only in a single dimension, we recover a* Plithogenic SuperHypersoft Set (PSHSS) (without the multi-level forest).

Proof: We prove each statement separately.
(1) Reducing PFSHS to PFHS. In PFSHS, each element in $\widetilde{\mathcal{C}}_{\text{forest}} = \mathcal{P}(\Gamma(\text{Forest}(\mathcal{A})))$ is a *power set* of leaf-level sub-attributes. To recover PFHS:

- Constrain each subset $\beta \subseteq \Gamma(\text{Forest}(\mathcal{A}))$ to contain *exactly one* leaf-level attribute (i.e., $\beta = \{\alpha\}$).
- Equivalently, for any multi-element subset β, disregard it or treat it as a single path-based combination, so that β is effectively a *singleton* representing a final sub-attribute.
- The contradiction function $c(\beta_1, \beta_2)$ then collapses to $c(\{\alpha_1\}, \{\alpha_2\})$, which is the standard plithogenic contradiction among final-level sub-attributes α_1, α_2.
- Likewise, $d_y(\beta)$ becomes $d_y(\{\alpha\})$, i.e., a single sub-attribute membership measure exactly matching the PFHS setup.

Under these restrictions, the tuple $(U, H, \mathcal{A}, d, c, G)$ (where G is the restricted mapping) is precisely a Plithogenic Forest Hypersoft Set (Definition 10), because each argument in the mapping is now $\alpha \in \Gamma(\text{Forest}(\mathcal{A}))$, not a *power set* of them. Hence, PFSHS generalizes PFHS.
(2) Reducing PFSHS to Plithogenic SuperHypersoft Set (PSHSS). Recall a Plithogenic SuperHypersoft Set PSHSS typically has the structure:

$$\text{PSHSS} = (U, Y, \{A_i\}, \mathcal{C}, d, c, F),$$

where $\{A_i\}$ are distinct attributes, $\mathcal{C}$ might be $\mathcal{P}(A_1) \times \cdots \times \mathcal{P}(A_n)$, and there is no notion of multiple hierarchical levels (no forest). To embed a PSHSS in PFSHS, we:

- Restrict each $\text{Tree}(A_i)$ to a single level (no deeper branches). Thus $\Gamma(\text{Forest}(\mathcal{A}))$ is effectively $\{A_1, \ldots, A_n\}$ (or the relevant expansions if each A_i itself is a set).
- Let each $\beta \subseteq \Gamma(\text{Forest}(\mathcal{A}))$ correspond to subsets from $\mathcal{P}(A_1) \times \cdots \times \mathcal{P}(A_n)$ in the classical sense.
- The contradiction function c and the membership function d now replicate exactly the plithogenic superhypersoft definitions. The hierarchical structure collapses, so the "forest" is a trivial forest with each root having no deeper sub-attributes.

Hence, the PFSHS framework (with additional multi-level capabilities) indeed specializes to the existing Plithogenic SuperHypersoft Set under these limitations. This completes the proof that PFSHS generalizes both PFHS and PSHSS. □

3.4 | m-SuperHyperPlithogenic n-SuperHypersoft Set

The m-SuperHyperPlithogenic n-SuperHypersoft Set is defined as follows. It is a concept that combines the principles of the SuperHypersoft Set and the SuperHyperPlithogenic Set.

Definition 25 (m-SuperHyperPlithogenic n-SuperHypersoft Set)**.** Let:

(1) U be a universal set, and let $Y \subseteq U$ be a (possibly proper) subset of U.

(2) $\{a_1, a_2, \ldots, a_n\}$ be n distinct attributes ($n \geq 1$).

(3) For each attribute a_i, let A_i be a non-empty set of possible values, satisfying $A_i \cap A_j = \emptyset$ for $i \neq j$. Let $\mathcal{P}(A_i)$ denote its power set.

(4) Define

$$\mathcal{C}_n = \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \cdots \times \mathcal{P}(A_n),$$

called the *n-SuperHypersoft domain*. An element $\gamma \in \mathcal{C}_n$ is an n-tuple $(\alpha_1, \ldots, \alpha_n)$ with each $\alpha_i \subseteq A_i$.

(5) Let X be a (potentially larger) universal space containing U. We construct a sequence $P_1, P_2, \dots, P_m$ with $m \geq 1$ as follows:

$$P_1 \subseteq X, \quad P_{k+1} = \tilde{\mathcal{P}}(P_k), \quad k = 1, 2, \dots, m-1.$$

Here, $\tilde{\mathcal{P}}(\cdot)$ denotes a suitable family of non-empty subsets or hyper-subsets of P_k, so that P_m is an *m-level nested family* derived from P_1.

(6) A *Hyper Degree of Appurtenance Function (HDAF)*:

$$\tilde{d} : P_m \times \mathcal{C}_n \longrightarrow \tilde{\mathcal{P}}([0,1]^s),$$

where $s \geq 1$ is the dimension of the membership degrees. For each $x \in P_m$ and $\gamma \in \mathcal{C}_n$, $\tilde{d}_x(\gamma) \subseteq [0,1]^s$ encodes a *hyper* (possibly set-valued or multi-level) membership degree regarding how x appends to or belongs with the attribute combination γ.

(7) A *Degree of Contradiction Function (DCF)*:

$$pCF^{(m,n)} : \Big(\bigcup_{i=1}^{n} A_i\Big) \times \Big(\bigcup_{i=1}^{n} A_i\Big) \longrightarrow [0,1]^t,$$

where $t \geq 1$ is the dimension of contradiction. It satisfies:

$$pCF^{(m,n)}(a,a) = 0, \quad pCF^{(m,n)}(a,b) = pCF^{(m,n)}(b,a),$$

for all $a, b \in \bigcup_{i=1}^{n} A_i$.

(8) A mapping

$$F : \mathcal{C}_n \longrightarrow \mathcal{P}(U),$$

where each $\gamma \in \mathcal{C}_n$ is mapped to a subset $F(\gamma) \subseteq U$. Concretely, $F(\gamma)$ is determined by plithogenic rules that combine:

- The hyper degree(s) $\tilde{d}_x(\gamma)$ for $x \in P_m$,
- The contradiction degrees $pCF^{(m,n)}(\alpha_i, \alpha_j)$ within each attribute domain (for $\alpha_i, \alpha_j \subseteq A_i$ when $i = j$, or extended forms for cross-attribute contradictions, if applicable),
- The chosen dimension(s) of membership s and contradiction t.

A structure

$$\text{m-SHPS n-SHS} = \Big(U,\ Y,\ \{a_i\},\ \{A_i\},\ \mathcal{C}_n,\ P_m,\ \tilde{d},\ pCF^{(m,n)},\ F\Big)$$

satisfying the above properties is called an m**-SuperHyperPlithogenic** n**-SuperHypersoft Set**.

Theorem 26. ***(m-SuperHyperPlithogenic n-SuperHypersoft Set Generalizes PSHSS and SHPS$_n$)***

Let

$$\text{m-SHPS n-SHS} = \Big(U,\ Y,\ \{a_i\},\ \{A_i\},\ \mathcal{C}_n,\ P_m,\ \tilde{d},\ pCF^{(m,n)},\ F\Big)$$

be an m-SuperHyperPlithogenic n-SuperHypersoft Set as per Definition 25. Then:

(1) ***Reduction to Plithogenic SuperHypersoft Set (PSHSS).***
If we fix $m = 1$ (so that $P_m = P_1 \subseteq X$ has no higher-level nesting) and *let $\tilde{d}_x(\gamma)$ be singleton (or crisp) membership degrees in $[0,1]^s$ rather than a hyper-set for each $x \in P_1$, the resulting structure precisely coincides with the standard Plithogenic SuperHypersoft Set.*

(2) ***Reduction to SuperHyperPlithogenic Set (SHPS$_n$).***
If we fix $n = 1$ (i.e., we have only one attribute domain A_1, so $\mathcal{C}_1 = \mathcal{P}(A_1)$) and keep the hyper-level recursion for $m \geq 1$, the construction reverts to an m-level SuperHyperPlithogenic *Set in the sense of* [20, 13, 18]*, where the single attribute domain A_1 (and its possible values) combine with hyper membership degrees $\tilde{d}$ and the contradiction function $pCF^{(m,n)}$ (now simplified to $pCF^{(m,1)}$).*

Proof: We prove each statement separately.

(1) From m-SHPS n-SHS to Plithogenic SuperHypersoft Set. By setting $m = 1$, we have no nested expansions: $P_1 \subseteq X$, and

$$P_2, P_3, \ldots, P_m \quad \text{are not defined.}$$

Hence $\tilde{d} : P_1 \times \mathcal{C}_n \to \tilde{\mathcal{P}}([0,1]^s)$ becomes a simple membership degree function from P_1 to $[0,1]^s$. If we further assume it takes on *single* values (e.g., $\tilde{d}_x(\gamma) \in [0,1]^s$ in a pointwise manner, rather than subsets of $[0,1]^s$), then $\tilde{d}$ effectively matches the attribute value appurtenance function d in the Plithogenic SuperHypersoft Set definition [16, 50]. Likewise, the contradiction function $pCF^{(1,n)}$ merges seamlessly with the standard contradiction function c. Finally, the mapping

$$F(\gamma) \;\subseteq\; U$$

is determined by these membership degrees and contradiction values just as in Definition of PSHSS. Thus, under these restrictions, we recover the conventional PSHSS.
(2) From m-SHPS n-SHS to SuperHyperPlithogenic Set. If $n = 1$, then the domain

$$\mathcal{C}_1 \;=\; \mathcal{P}(A_1),$$

so each γ is effectively a subset of A_1. This coincides with the single-attribute scenario in the *SuperHyperPlithogenic Set* [20, 13, 18], which considers a (possibly nested) HDAF over a single family of attribute values. The recursion on m remains valid for building P_m from P_1 via $\tilde{\mathcal{P}}$. The contradiction function $pCF^{(m,1)}$ becomes the usual contradiction measure on $\bigcup_{i=1}^{1} A_i = A_1$. The mapping

$$F : \mathcal{P}(A_1) \;\longrightarrow\; \mathcal{P}(U)$$

then behaves as in the hyperplithogenic approach. Therefore, under $n = 1$, we retrieve exactly an m-level SuperHyperPlithogenic Set structure.

In conclusion, each specialized case ($m = 1$ or $n = 1$) yields one of the known structures, proving that an m-SuperHyperPlithogenic n-SuperHypersoft Set generalizes both the Plithogenic SuperHypersoft Set and the SuperHyperPlithogenic Set. □

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Data Availability

This paper does not involve any data analysis.

Ethical Approval

This article does not involve any research with human participants or animals.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Disclaimer

This study primarily focuses on theoretical aspects, and its application to practical scenarios has not yet been validated. Future research may involve empirical testing and refinement of the proposed methods. The authors have made every effort to ensure that all references cited in this paper are accurate and appropriately attributed. However, unintentional errors or omissions may occur. The authors bear no legal responsibility for inaccuracies in external sources, and readers are encouraged to verify the information provided in the references independently. Furthermore, the interpretations and opinions expressed in this paper are solely those of the authors and do not necessarily reflect the views of any affiliated institutions.

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