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Editorial

Symmetry in Geometric Theory of Analytic Functions

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1. Introduction

This Special Issue, titled "Symmetry in the Geometric Theory of Analytic Functions", is addressed to researchers in the complex analysis domain. This Special Issue covered all aspects of this topic, starting with special classes of univalent functions, operator-related results studied using the theory of differential subordination and superordination, or any other techniques that can be applied in the field of complex analysis and its applications, thereby valuing the symmetric properties of the studied object.

The aim of the present Special Issue is to exchange ideas globally among eminent mathematicians as a tribute to the geometric function theory. This Special Issue boosted cooperation among mathematicians working on a broad variety of pure and applied mathematical areas.

In this Special Issue, comprised of ideas and mathematical methods, we included a wide area of applications in which the geometric function theory plays an important role, resulting in having an extreme influence on everyday life, as the development of new tools means revolutionary research results have been obtained, bringing scientists even closer to exact science and encouraging the emergence of new approaches, techniques, and perspectives in the field of complex analysis.

2. Brief Overview of the Contributions

In [1], Esra Damar examined ruled surfaces where the direction vectors are unit vectors derived from Smarandache curves, and the base curve is taken as an adjoint curve constructed using the integral curve of a Smarandache-type curve generated from the first and second Bishop normal vectors. The newly generated ruled surfaces will be referred to as Bishop adjoint ruled surfaces. Explicit expressions for the Gaussian and mean curvatures of these surfaces have been obtained, and their fundamental geometric properties have been analyzed in detail. Additionally, the conditions for developability, minimality, and singularities have been investigated. The asymptotic and geodesic behaviors of parametric curves have been examined, and the necessary and sufficient conditions for their characterization have been derived. Furthermore, the geometric properties of the surface generated by the Bishop adjoint curve and its relationship with the choice of the original curve have been established. The constructed ruled surfaces exhibit a notable degree of geometric regularity and symmetry, which naturally arise from the structural behavior of the associated adjoint curves and direction fields. This underlying symmetry plays a central role in their formulation and classification within the broader context of differential geometry. Finally, the obtained surfaces are illustrated with figures.

In [2], Loriana Andrei and Vasile-Aurel Căuș introduce a new class of harmonic functions defined through a generalized symmetric q-differential that acts on both the

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analytic and co-analytic parts of the function. By combining concepts from symmetric q-calculus and geometric function theory, they develop a framework that extends several well-known operators as special cases. The main contributions of this study include new criteria for harmonic univalence, sharp coefficient bounds, distortion theorems, and covering results. The operator offers increased flexibility in modeling symmetric structures, with potential applications in complex analysis, fractional calculus, and mathematical physics. To support these theoretical developments, we provide concrete examples and highlight potential directions for future research, including extensions to higher-dimensional settings.

In [3], Baskaran Sudharsanan, Saravanan Gunasekar, and Teodor Bulboacă introduce and explore the properties of the m-leaf function $\mathcal{Q}m$ defined in the open-unit disk $\mathbb{D}$, where $m \in \mathbb{N}$; this function is defined by the relation $\mathcal{Q}m(z) := 1 + (m+1)/(m+2)z + 1(m+2)zm + 1$. They determine the main geometrical characterization of this function, focusing on its univalency and its sharp bounds for the real and imaginary moduli. Also, the authors find the radius of convexity and give a subordination and inclusion result. Using this function, they introduce a new subclass of analytic functions normalized as usual in $\mathbb{D}$, denoted by $\mathcal{A}r$, and an investigation of this class reveals some interesting properties.

In [4], İbrahim Aktaş and Luminiţa-Ioana Cotîrlă study certain geometric properties such as the starlikeness of order ζ and the convexity of order ζ of the generalized k-Bessel function. In addition, they establish several requirements for the parameters so that the generalized k-Bessel function belongs to some subclasses of analytic functions. Furthermore, as an application of the geometric properties, the authors establish certain results concerning the Hardy spaces of the generalized k-Bessel function. On the other hand, some corollaries concerning the classical Bessel function $J\rho$ and the modified Bessel function $I\rho$ are presented. To support the geometric results, the authors present some specific examples of functions that map the open-unit disk onto the symmetric domains with respect to the real axis.

The significant characteristics of associate Laguerre polynomials (ALPs) have noteworthy applications in the fields of complex analysis and mathematical physics. The article [5] mainly focuses on the inclusion relationships of ALPs and various analytic domains. Starting with the investigation of admissibility conditions of the analytic functions belonging to these domains, Sa'ud Al-Sa'di, Ayesha Siddiqa, Bushra Kanwal, Mohammed Ali Alamri, Saqib Hussain, and Saima Noor obtained the conditions on the parameters of ALPs under which an ALP maps an open-unit disk inside such analytical domains. The graphical demonstration enhances the outcomes and also proves the validity of the obtained results.

In [6], the objective of the authors is to define and study a new subclass of analytic functions associated with the q-analog of the sine function, operating in conjunction with a convolution operator. By manipulating the parameter q, Adeel Ahmad, Hanen Louati, Akhter Rasheed, Asad Ali, Saqib Hussain, Shreefa O. Hilali, and Afrah Y. Al-Rezami observe that the image of the unit disk under the q-sine function exhibits a visually appealing resemblance to a figure-eight shape that is symmetric about the real axis. Additionally, they investigate some important geometrical problems like necessary and sufficient conditions, coefficient bounds, Fekete-Szegö inequality, and partial sum results for the functions belonging to this newly defined subclass.

A new class $\mathcal{B}\lambda\Sigma(\gamma,\kappa)$ of bi-starlike λ-pseudo functions related to the second Einstein function is presented in [7]. $c2$ and $c3$ indicate the initial Taylor coefficients of $\phi \in \mathcal{B}\lambda\Sigma(\gamma,\kappa)$, and the bounds for $|c2|$ and $|c3|$ are obtained. Additionally, for $\phi \in \mathcal{B}\lambda\Sigma(\gamma,\kappa)$, Alaa H. El-Qadeem, Gangadharan Murugusundaramoorthy, Borhen Halouani, Ibrahim S. Elshazly, Kaliappan Vijaya, and Mohamed A. Mamon calculate the Fekete–Szegö functional.

The paper [8] examines subordination conclusions for a specific subclass of p-valent meromorphic functions on the punctured unit disk of the complex plane where the function

has a pole of order p. A new linear operator is used to define the subclass that is being studied. Furthermore, Rabha M. El-Ashwah, Alaa Hassan El-Qadeem, Gangadharan Murugusundaramoorthy, Ibrahim S. Elshazly, and Borhen Halouani present several corollaries with intriguing specific situations of the results.

In [9], Sushil Kumar, Daniel Breaz, Luminita-Ioana Cotîrlă, and Asena Çetinkaya consider a subclass of normalized analytic functions associated with the hyperbolic secant function. They compute the sharp bounds on third- and fourth-order Hermitian–Toeplitz determinants for functions in this class. Moreover, the authors determine the bounds on second- and third-order Hankel determinants, as well as on the generalized Zalcman conjecture. Here, a Briot–Bouquet-type differential subordination involving the Bernardi integral operator is examined. Finally, they obtain a univalent solution to the Briot–Bouquet differential equation and discuss the majorization property for such function classes.

In [10], Adeel Ahmad, Jianhua Gong, Akhter Rasheed, Saqib Hussain, Asad Ali, and Zeinebou Cheikh apply differential subordination and quantum calculus to introduce and investigate a new class of analytic functions associated with the q-differential operator and the symmetric balloon-shaped domain. They obtain sharp results concerning the Maclaurin coefficients, the second- and third-order Hankel determinants, the Zalcman conjecture, and its generalized conjecture for this newly defined class of q-starlike functions with respect to symmetric points.

Recently, Sergey K. Sekatskii established and used the generalized Littlewood theorem concerning contour integrals of the logarithm of analytical functions to obtain new criteria equivalent to the Riemann hypothesis. The same theorem was subsequently applied to calculate certain infinite sums and study the properties of zeroes of a few analytical functions. In [11], the author discusses what are, in a sense, inverse applications of this theorem. Lemma proves that if two meromorphic functions on the whole complex plane, $f(z)$ and $g(z)$, have the same zeroes and poles, taking into account their orders, and have appropriate asymptotics for large $|z|$, then for some integer n, $d\ln(f(z))dzn = dn\ln(g(z))dzn$. The use of this Lemma enables proof of many identities between elliptic functions, their transformation, and n-tuple product rules. In particular, the author shows how exactly for any complex number a, $\wp(z)$-a, where $\wp(z)$ is the Weierstrass $\wp$ function, can be presented as a product and ratio of three elliptic $\theta 1$ functions of certain arguments. In addition, n-tuple rules are established for a few elliptic theta functions.

In [12], Norah Saud Almutairi, Awatef Shahen, Adriana Cătaş, and Hanan Darwish propose new generalized classes of (p,q)-starlike and (p,q)-convex functions. These classes are introduced by making use of a (p,q)-derivative operator. The Fekete–Szegö estimates $|a3 - \mu a22|$ for functions belonging to the newly introduced subclasses are established. Certain subclasses of analytic univalent functions associated with quasi-subordination are defined.

The paper [13] is dedicated to the examination of maximum and minimum results based on Green's functions via delta fractional differences for a class of fractional boundary problems. For such a purpose, Pshtiwan Othman Mohammed, Carlos Lizama, Alina Alb Lupas, Eman Al-Sarairah, and Mohamed Abdelwahed built the corresponding Green's functions based on the falling factorial functions. In addition, using the constructed Green's function, the positivity of the function and its corresponding delta function are presented. The occurrence of two distinct functions with the same Green's function is also verified. The maximality and minimality of the Green's function show a good qualitative agreement. Finally, the authors considered some special examples to explain the obtained results.

In [14], taking into account the intriguing recent results of Rabotnov functions, Poisson functions, Bessel functions, and Wright functions, Tariq Al-Hawary, Basem Frasin, and Ibtisam Aldawish consider a new comprehensive subclass $\mathcal{O}\mu(\Delta 1, \Delta 2, \Delta 3, \Delta 4)$ of univalent

functions defined in the unit disk $\Lambda = \{\tau \in \mathbb{C}: |\tau| < 1\}$. More specifically, they investigate some sufficient conditions for Rabotnov functions, Poisson functions, Bessel functions, and Wright functions to be in this subclass. Some corollaries of the main results are given. The novelty and the advantage of this research could inspire researchers of further studies to find new sufficient conditions to be in the subclass ($\Delta 1$, $\Delta 2$, $\Delta 3$, and $\Delta 4$) not only for the aforementioned special functions but for different types of special functions, especially for hypergeometric functions, Dini functions, Struve functions, and others.

In [15], by using the Mittag–Leffler function associated with functions of bounded boundary rotation, Ibtisam Aldawish, Prathviraj Sharma, Sheza M. El-Deeb, Mariam R. Almutiri, and Srikandan Sivasubramanian introduce a few new subclasses of bi-univalent functions involving the Mittag–Leffler function with bounded boundary rotation in the open-unit disk $\mathbb{D}$. For these new classes, the authors establish initial coefficient bounds of $|a2|$ and $|a3|$. Furthermore, the famous Fekete–Szegö coefficient inequality is also obtained for these new classes of functions.

In [16], Prathviraj Sharma, Asma Alharbi, Srikandan Sivasubramanian, and Sheza M. El-Deeb introduce a new subclass of univalent functions $\mathcal{F}(u,\lambda)$ and a subclass of bi-univalent functions $\mathcal{F}o,\Sigma(u,\lambda)$ with bounded boundary and bounded radius rotation. Some examples of the functions belonging to the classes $\mathcal{F}(u,\lambda)$ are also derived. For these new classes, the authors derive many interesting relations between these classes and the existing familiar subclasses in the literature. Furthermore, the authors establish new coefficient estimates for these classes. Apart from the above, the first two initial coefficient bounds for the class $\mathcal{F}o,\Sigma(u,\lambda)$ are established.

Various researchers have considered different forms of bi-univalent functions in recent times, and this has continued to gain more attention in geometric function theory (GFT), but not much study has been conducted in the area of application of the certain probability concept in geometric functions. In [17], the motivation of Sunday Olufemi Olatunji, Matthew Olanrewaju Oluwayemi, Saurabh Porwal, and Alina Alb Lupas is the application of analytic and bi-univalent functions. In particular, the researchers examine bi-univalency of a generalized distribution series related to Bell numbers as a family of Caratheodory functions. A few coefficients of the class of the function are obtained. The results are new as far as work on bi-univalency is concerned.

Picard iteration is on the basis of a great number of numerical methods and applications of mathematics. However, it has been known since the 1950s that this method of fixed-point approximation may not converge in the case of nonexpansive mappings. In [18], an extension of the concept of nonexpansiveness is presented in the first place. Unlike the classical case, the new maps may be discontinuous, adding an element of generality to the model. Some properties of the set of fixed points of the new maps are studied. Afterwards, two iterative methods of fixed-point approximation are analyzed in the frameworks of b-metric and Hilbert spaces. In the latter case, it is proved that the symmetrically averaged iterative procedures perform well in the sense of convergence with the least number of operations at each step. As an application, the second part of the article is devoted to the study of fractal mappings on Hilbert spaces defined by means of nonexpansive operators. The paper considers fractal mappings coming from φ-contractions as well. In particular, the new operators are useful for the definition of an extension of the concept of α-fractal function, enlarging its scope to more abstract spaces and procedures. The fractal maps studied here by María A. Navascués have quasi-symmetry, in the sense that their graphs are composed of transformed copies of themselves.

In [19], Ebrahim Amini, Wael Salameh, Shrideh Al-Omari, and Hamzeh Zureigat present a new class of linear fractional differential operators that are based on classical Gaussian hypergeometric functions. Then, the authors utilize the new operators and the concept

of differential subordination to construct a convex set of analytic functions. Moreover, through an examination of a certain operator, they establish several notable results related to differential subordination. In addition, inclusion relation results were derived by employing Briot–Bouquet differential subordinations. The authors also introduced a perspective study for developing subordination results using Gaussian hypergeometric functions and providing certain properties for further research in complex dynamical systems.

In [20], Xinguang Zhang, Peng Chen, Lishuang Li, and Yonghong Wu focus on the existence of positive solutions for a singular tempered sub-diffusion fractional model involving a quasi-homogeneous nonlinear operator. By using the spectrum theory and computing the fixed point index, some new sufficient conditions for the existence of positive solutions are derived. It is worth pointing out that the nonlinearity of the equation contains a tempered fractional sub-diffusion term and is allowed to possess strong singularities in time and space variables. In particular, the quasi-homogeneous operator is a nonlinear and nonsymmetrical operator.

Starlike and convex functions have gained increased prominence in both academic literature and practical applications over the past decade. Concurrently, logarithmic coefficients play a pivotal role in estimating diverse properties within the realm of analytic functions, whether they are univalent or nonunivalent. In [21], Pishtiwan Othman Sabir rigorously derives bounds for specific Toeplitz determinants involving logarithmic coefficients pertaining to classes of convex and starlike functions concerning symmetric points. Furthermore, he presents illustrative examples showcasing the sharpness of these established bounds. The findings represent a substantial contribution to the advancement of the understanding of logarithmic coefficients and their profound implications across diverse mathematical contexts.

The aim of [22] is to define the linear operator based on the generalized Mittag–Leffler function and the Lambert series. By using this operator, Jamal Salah introduces a new subclass of β-uniformly starlike functions $T(\alpha i)$. Further, the author obtains coefficient estimates, convex linear combinations, and radii of close-to-convexity, starlikeness, and convexity for functions $f \in T(\alpha i)$. In addition, he investigates the inclusion conditions of the Hadamard product and the integral transform. Finally, the second Hankel inequality for functions belonging to this subclass was determined.

Recently, nonlinear fractional models have become increasingly important for describing phenomena occurring in science and engineering fields, especially those including symmetric kernels. In [23], Pshtiwan Othman Mohammed, Ravi P. Agarwal, Iver Brevik, Mohamed Abdelwahed, Artion Kashuri, and Majeed A. Yousif examine two reliable methods for solving fractional coupled nonlinear Schrödinger models. These methods are known as the Sardar-subequation technique (SSET) and the improved generalized tanh-function technique (IGTHFT). Numerous novel soliton solutions are computed using different formats, such as periodic, bell-shaped, dark, and combination single bright, along with kink, periodic, and single soliton solutions. Additionally, single solitary wave, multi-wave, and periodic kink combined solutions are evaluated. The behavioral traits of the retrieved solutions are illustrated by certain distinctive two-dimensional, three-dimensional, and contour graphs. The results are encouraging, since they show that the suggested methods are trustworthy, consistent, and efficient in finding accurate solutions to the various challenging nonlinear problems that have recently surfaced in applied sciences, engineering, and nonlinear optics.

The paper [24] presents a novel investigation that utilizes the integral operator Inp in the field of geometric function theory, with a specific focus on sandwich theorems. Norah Saud Almutairi, Awatef Shahen, and Hanan Darwish obtained findings about the

differential subordination and superordination of a novel formula for a generalized integral operator. Additionally, certain sandwich theorems were discovered.

In [25], Roman Dmytryshyn and Vitaliy Goran consider the representation and extension of the analytic functions of three variables by special families of functions, namely branched continued fractions. In particular, the authors establish new symmetric domains of the analytical continuation of Lauricella–Saran's hypergeometric function *FK* with certain conditions on real and complex parameters using their branched continued fraction representations. They use a technique that extends the convergence, which is already known for a small domain, to a larger domain to obtain domains of convergence of branched continued fractions and the PC method to prove that they are also domains of analytical continuation. In addition, the authors discuss some applicable special cases and vital remarks.

In [26], Daniel Breaz, Trailokya Panigrahi, Sheza M. El-Deeb, Eureka Pattnayak, and Srikandan Sivasubramanian defined new subclasses of analytic functions, namely $Rlim(l,\nu)$ and $Clim(l,\nu)$, defined by subordination linked with a limacon-shaped domain. The authors found a few initial coefficient bounds and Fekete–Szegő inequalities for the functions in the above-stated new classes. The corresponding results have been derived for the function $h - 1$. Additionally, they discuss Poisson distribution as an application of our consequences.

In [27], the geometric nature of solutions to two second-order differential equations, $z\mathrm{y}''(z) + a(z)\mathrm{y}'(z) + b(z)\mathrm{y}(z) = 0$ and $z2\mathrm{y}''(z) + a(z)\mathrm{y}'(z) + b(z)\mathrm{y}(z) = d(z)$, is studied. Here, (z), (z), and $d(z)$ are analytic functions defined on the unit disk. Using differential subordination, Reem Alzahrani and Saiful R. Mondal established that the normalized solution $F(z)$ (with $F(0) = 1$) of the above differential equations maps the unit disk to the domain bounded by the lemniscate curve $\sqrt{1 + z}$ The authors construct several examples by the judicious choice of $a(z)$, $b(z)$, and $d(z)$. The examples include Bessel functions, Struve functions, the Bessel–Struve kernel, confluent hypergeometric functions, and many other special functions. They also established a connection with the nephroid domain. Directly using subordination, the authors construct functions that are subordinated by a nephroid function. Two open problems are also suggested in the conclusion.

The purpose of [28] is to introduce and investigate new subclasses of analytic function classes of bi-univalent functions defined in open-unit disks connected with a linear q-convolution operator, which are associated with quasi-subordination. We find coefficient estimates of $|h2|$ and $|h3|$ for functions in these subclasses. Several known and new consequences of these results are also pointed out. There is symmetry between the results of the subclass $\mathscr{f}\mu q,\sum(\zeta,n,\rho,\sigma,\vartheta,\gamma,\delta,\varphi)$ and the results of the subclass $\aleph q,\delta\sum(\lambda,\zeta,n,\rho,\sigma,\vartheta,\varphi)$.

Over the past ten years, analytical functions' reputation in the literature and their application have grown. In [29], Ibtisam Aldawish, Sheza M. El-Deeb, and Gangadharan Murugusundaramoorthy study some practical issues pertaining to multivalent functions with bounded boundary rotation that associate with the combination of confluent hypergeometric functions and binomial series. A novel subset of multivalent functions is established through the use of convolution products, and specific inclusion properties are examined through the application of second-order differential inequalities in the complex plane. Furthermore, for multivalent functions, the authors examined inclusion findings using Bernardi integral operators. Moreover, they demonstrate how the class proposed in the study, in conjunction with the acquired results, generalizes other well-known (or recently discovered) works that are called out as exceptions in the literature.

The aim of [30] is to discuss some conditions for Touchard polynomials to be in the classes $\mathfrak{T}(\rho,\sigma)$ and $\mathfrak{T}\Re b(\rho,\sigma)$. Also, Ekram E. Ali, Waffa Y. Kota, Rabha M. El-Ashwah, Abeer M. Albalahi, Fatma E. Mansour, and R. A. Tahira obtain some connection between $\Re(D,E)$ and $\mathfrak{T}\Re b(\rho,\sigma)$. Also, the authors investigate several mapping properties involving these

subclasses. Further, they discuss the geometric properties of an integral operator related to the Touchard polynomial. Additionally, briefly mentioned are specific instances of our primary results. Also, several particular examples are presented.

It is well known that both concepts of symmetry and convexity are directly connected. Similarly, in fuzzy theory, both ideas behave alike. It is important to note that real and interval-valued mappings are exceptional cases of fuzzy number-valued mappings (FNVMs) because fuzzy theory depends upon the unit interval that makes a significant contribution to overcoming the issues that arise in the theory of interval analysis and fuzzy number theory. In [31], the new class of p-convexity over up and down (UD) fuzzy relation has been introduced, which is known as UD-p-convex fuzzy number-valued mappings (UD-p-convex FNVMs). Nasser Aedh Alreshidi, Muhammad Bilal Khan, Daniel Breaz, and Luminita-Ioana Cotirla offer a thorough analysis of Hermite–Hadamard-type inequalities for FNVMs that are UD-p-convex using the fuzzy Aumann integral. Some previous results from the literature are expanded upon and broadly applied in our study. Additionally, the authors offer precise justifications for the key theorems that Kunt and İşcan first deduced in their article titled "Hermite–Hadamard–Fejer-type inequalities for p-convex functions". Some new and classical exceptional cases are also discussed. Finally, they illustrate our findings with well-defined examples.

In [32], Nataliia Baziv and Andriy Zagorodnyuk consider algebras of polynomials and analytic functions that are invariant with respect to semidirect products of groups of bounded operators on Banach spaces with symmetric bases. In particular, the authors consider algebras of so-called block-symmetric and double-symmetric analytic functions on Banach spaces $\ell(\mathbb{C}n)$ and the homomorphisms of these algebras. In addition, they describe an algebraic basis in the algebra of double-symmetric polynomials and discuss a structure of the spectrum of the algebra of double-symmetric analytic functions on $\ell(\mathbb{C}n)$.

The paper [33] begins with a discussion of the quantum calculus operator theory and proceeds to develop and investigate a new family of close-to-convex functions in an open-unit disk. Considering the quantum difference operator, Suha B. Al-Shaikh, Mohammad Faisal Khan, Mustafa Kamal, and Naeem Ahmad define and study a new subclass of close-to-convex functions connected with generalized Janowski functions. The authors prove the necessary and sufficient conditions for functions that belong to newly defined classes, including the inclusion relations and estimations of the coefficients. The Fekete–Szegő problem for a more general class is also discussed. The results of this investigation expand upon those of the previous study.

The paper [34] introduces a novel operator termed q-convolution, strategically integrated with foundational principles of q-calculus. Leveraging this innovative operator alongside q-Bernoulli polynomials, a distinctive class of functions emerges, characterized by both analyticity and bi-univalence. The determination of initial coefficients within the Taylor-Maclaurin series for this function class is accomplished, showcasing precise bounds. Additionally, explicit computation of the second Hankel determinant is provided by Timilehin Gideon Shaba, Serkan Araci, Babatunde Olufemi Adebesin, and Ayhan Esi. These pivotal findings, accompanied by their corollaries and implications, not only enrich but also extend previously published results.

Zoriana Novosad and Andriy Zagorodnyuk consider the backward shift operator on a sequence Banach space in the context of two infinite-dimensional phenomena: the existence of topologically transitive operators and the existence of entire analytic functions of the unbounded type. It is well known that the weighted backward shift (for an appropriated weight) is topologically transitive on $1 \leq p < \infty$ and on $c0$. The authors construct in [35] some generalizations of the weighted backward shift for non-separable Banach spaces, which remain topologically transitive. Also, they show that the backward shift, in some

sense, generates analytic functions of the unbounded type. The authors introduce the notion of a generator of analytic functions of the unbounded type on a Banach space and investigate its properties. In addition, they show that, using this operator, one can obtain a quasi-extension operator of analytic functions in a germ of zero for entire analytic functions. The results are supported by examples.

The basic aspect of the research on coefficient problems for numerous families of univalent functions is to describe the coefficients of functions in a specific family by the coefficients of the Carathéodory functions. Thus, in utilizing the inequalities that are known for the class of Carathéodory functions, coefficient functionals may be examined by Huo Tang, Muhammad Arif, Muhammad Abbas, Ferdous M. O. Tawfiq, and Sarfraz Nawaz Malik. Several coefficient problems will be addressed in [36] by utilizing the methodology for the abovementioned functions' family. The family of starlike functions with respect to symmetric points connected to a three-leaf-shaped image domain is the topic of our investigation.

In [37], Tariq Al-Hawary, Ala Amourah, Hasan Almutairi, and Basem Frasin present a novel family of holomorphic and bi-univalent functions, denoted as $E\mathbf{\Omega}(\eta,\varepsilon;F)$. The authors establish the coefficient bounds for this family by utilizing the generalized telephone numbers. Additionally, they solve the Fekete–Szegö functional for functions that belong to this family within the open-unit disk. Moreover, the results have several consequences.

In view of the subclass $SL*(\beta)$, which for $\beta = 0$ reduces to the class $SL*$, two more subclasses, $\mathscr{C}L(\beta)$ and $\mathscr{G}L(\beta)$, are introduced in [38]. For all these three subclasses, Nehad Ali Shah, Naseer Bin Turki, Sang-Ro Lee, Seonhui Kang, and Jae Dong Chung investigate upper bounds of second Hankel and second inverse Hankel determinants. In most cases, the results are sharp.

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