The Box and Muller Technique for Generating Neutrosophic Random Variables Follow a Normal Distribution

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Abstract

The focus of operations research is the existence of a problem that requires making an appropriate decision that helps reduce risk and achieves a good level of performance. Operations research methods depend on formulating realistic issues through mathematical models consisting of a goal function and constraints, and the optimal solution is the ideal decision, despite the multiplicity of these methods. However, we encounter many complex issues that cannot be represented mathematically, or many issues that cannot be studied directly. Here comes the importance of the simulation process in all branches of science, as it depends on applying the study to systems similar to real systems and then projecting this. The results if they fit on the real system. So simulation is the process of building, testing, and running models that simulate complex phenomena or systems using specific mathematical models. The simulation process depends on generating a series of random numbers subject to a regular probability distribution in the field [0, 1], and then converting these random numbers into random variables subject to the distribution law. Probability, according to which the system to be simulated operates, using appropriate techniques for both the probability density function and the cumulative distribution function. Classical studies have provided many techniques that are used during the simulation process, and to keep pace with the great scientific development witnessed by our contemporary world, we found that a new vision must be presented for this. Techniques A vision based on the concepts of neutrosophics, the science founded by the American mathematical philosopher Florentin Smarandache. The year 1995, in which new concepts of probabilities and probability distributions are used, as we presented in previous research some techniques from a neutrosophic perspective, and as an extension of what we presented previously, we present in this research a neutrosophic vision of the Box and Muller technique used to generate random variables that follow a normal distribution.

Keywords: Simulation; Neutrosophic simulation; Neutrosophic normal distribution; Generation of neutrosophic random variables that follow the normal distribution; Box and Muller technique

1. Introduction

In the world of rapid changes, there has emerged an urgent need to make a rational decision based on quantitative methods and methods that limit the risk ratio, especially if the decisions are crucial and the decision issues are huge and complex, which was the reason for the emergence of the science of operations research, the science that is the applied side of mathematics and helped improve the performance of many systems that used its methods in their workflow. One of the most important methods of operations research is the simulation method, which is a numerical technique used to carry out tests on a numerical computer and includes mathematical relationships that interact with each other to describe the behavior and structure of a system in the real world over a period of time without any risk or procedure. Experiments on the real world and allow the study of complex issues that are difficult to solve using other operations research methods. We know that the basis of the simulation process is generating a series of random numbers subject to a regular probability distribution in the range [0,1], and then converting these random numbers into random variables that follow the distribution. The probability according to which the system to be simulated

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works. In classical logic, there are many techniques that can be used in the conversion process, which result in random variables that follow irregular probability distributions, to keep pace with the great scientific development that our contemporary world is witnessing and after the emergence of neutrosophic logic and the accurate results presented by research and studies. presented by researchers and those interested in this logic in most fields of science, including [1-13], and given the great importance of the simulation method, we presented in previous research [14-18] a neutrosophical vision of some of the techniques used to generate random variables that follow important probability distributions, such as the inverse transformation technique. Which can be used for all probability distributions whose cumulative distribution function has an inverse function, such as the uniform distribution over the domain [a, b], the exponential distribution, the Weibull distribution, and others. We also presented a neutrosophical vision of the rejection and acceptance technique and how to use it to generate random variables that follow the beta distribution. We present in this research is a neutrosophical study of the Box and Muller technique used to generate random variables that follow complex probability distributions. The aforementioned techniques cannot be used in the process of generating random variables that follow a normal distribution.

2. Discussion:

2.1. Box and Muller technique for generating neutrosophic random variables that follow a normal distribution:

Within this technique, we use many mathematical relationships in order to be able to represent some complex probability density functions in a more simplified way so that random variables can be generated using known techniques. We know that the simulation process depends on generating a series of random numbers that follow a uniform distribution over the field [0,1], and using the appropriate transformation method we convert these random numbers into random variables that follow the probability distribution according to which the system to be simulated operates, based on which to generate random variables. Neutrosophic, we generate a series of random numbers that follow a uniform distribution over the field [0,1]. From this series we can obtain neutrosophic random variables that follow the probability distribution according to which the system to be simulated operates. Here we distinguish three cases: The first case: Neutrosophic random numbers and probability distribution in the classical form The second case: classical random numbers and the probability distribution is in the neutrosophic form. The third case: neutrosophic random numbers and probability distribution in neutrosophic form. The normal distribution is one of the most widely used probability distributions, but from the definition of its probability density function, we find that it is not possible to use the neutrosophic study presented in research [15,17] for the inverse transformation method and the rejection and acceptance method. Rather, we need a special technique for probability distributions in which the probability density function is a complex function, such as the Box-Muller technique.

2.2. neutrosophical study of the Box and Muller technique for generating random numbers following a general normal distribution:

2.2.1. This technique was presented according to classical logic in many references, including references [19,20], and its scientific basis was to derive two simple probability distributions from the general normal distribution after converting it to the standard normal distribution.

2.2.2. Neutrosophic study:

a. In Reference [21], the neutrosophic probability density function for a random variable following a normal distribution is defined by the following relation:

$$X_N \sim N_N(\mu_N, {\sigma_N}^2) = \frac{1}{\sigma_N\sqrt{2\pi}} exp^{\left(-\frac{(x-\mu_N)^2}{2{\sigma_N}^2}\right)} \quad (1)$$

b. We convert it to the standard normal distribution using the following relation:

$$z_N = \frac{(x-\mu_N)}{\sigma_N} \quad (2)$$

c. Then the neutrosophic probability density function for the standard normal distribution is written as follows:

$$f(z_N) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z_N^2}{2}} \quad (3)$$

d. Using the relationship (3), we form two independent neutrosophic variables Z_{1N} and Z_{2N} subject to the standard normal distribution as follows:

$$f(Z_{1N}) = \frac{1}{\sqrt{2\pi}} e^{-\frac{Z_{1N}^2}{2}} \qquad f(Z_{2N}) = \frac{1}{\sqrt{2\pi}} e^{-\frac{Z_{2N}^2}{2}}$$

Forming their joint distribution, we find:

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$$f(Z_{1N}, Z_{2N}) = f(Z_{1N}).f(Z_{2N}) = \frac{1}{2\pi} e^{-\frac{1}{2}(Z_{1N}^2 + Z_{2N}^2)}$$

e. We move from Cartesian coordinates to polar coordinates using transformations:

$$Z_{1N} = r_N cos\theta \qquad Z_{2N} = r_N sin\theta$$

Substituting this transformation and its extensions $|J| = r_N$, we obtain the following joint distribution in terms of the polar variables r_N, θ:

$$f(r_N, \theta) = \frac{1}{2\pi} e^{-\frac{1}{2} r_N^2} . r_N$$

From it we obtain the following two single neutrosophic distributions for r_N, θ:

$$f(\theta) = \int_0^\infty f(r_N, \theta) dr_N = \frac{1}{2\pi} \int_0^\infty r_N \, e^{-\frac{1}{2} r_N^2} dr_N = \frac{1}{2\pi} \left[-e^{\frac{r_N^2}{2}} \right]_0^{-\infty} = \frac{1}{2\pi}$$

f. The first distribution is a uniform distribution over the domain [0,2p] defined by the following relation:

$$f(\theta) = \frac{1}{2\pi} \quad (4)$$

In the same way we find:

$$f(r_N) = \int_0^{2\pi} f(r_N, \theta) d\theta = \frac{r_N e^{-\frac{r_N^2}{2}}}{2\pi} \int_0^{2\pi} d\theta = r_N e^{-\frac{r_N^2}{2}}$$

It is a Rayleigh distribution defined on the field $[0, +\infty[$. To convert this distribution to an exponential distribution, we perform the following transformation:

$$u_N = r_N^2 \Rightarrow du_N = 2 r_N dr_N \Rightarrow \frac{dr_N}{du_N} = \frac{1}{2 r_N}$$

Substituting the Rayleigh distribution, we get:

$$f(u_N) = r_N e^{-\frac{u_N}{2}} . \frac{1}{2 r_N} = \frac{1}{2} e^{-\frac{u_N}{2}}$$

g. The second distribution is the exponential distribution whose parameter is λ and is defined by the following relation:

$$f(u_N) = \frac{1}{2} e^{-\frac{u_N}{2}} \quad (5)$$

2.2.3. From the above, the Box and Müller technique is formulated in the following steps:

1) We generate two random numbers R_1 and R_2 that follow a uniform distribution over the range [0,1].
2) We convert the two random numbers into two neutrosophic random numbers, R_{1N} and R_{2N}, see [14].
3) We use the inverse transformation technique to transform the first number R_{1N} into a random variable that follows an exponential distribution whose parameter $\lambda = \frac{1}{2}$ using the following relation:

$$x_{iN} = \frac{-lnR_{iN}}{\lambda}$$

We get:

$$r_N^2 = -2lnR_{1N} \Rightarrow r_N = (-2lnR_{1N})^{\frac{1}{2}}$$

4) We use the inverse transformation technique to transform the second number R_{2N} into a random variable that follows a uniform distribution over the field $[0,2\pi]$ using the following relation:

$$x_{iN} = (b-a)R_{iN} + a$$

We get:

$$\theta_N = 2\pi R_{2N}$$

We substitute the following two relations: $x_{1N} = r_N cos\theta$, $x_{2N} = r_N sin\theta$ we get:

$$x_{1N} = (-2lnR_{1N})^{\frac{1}{2}} \cos(2\pi R_{2N}) \quad (6)$$

$$x_{2N} = (-2lnR_{1N})^{\frac{1}{2}} sin(2\pi R_{2N}) \quad (7)$$

2.2.4. The Box and Muller technique for generating random variables follows the general normal distribution $N(\mu_N, \sigma_N^2)$, defined by the following relation:

$$f_N(x) = \frac{1}{\sqrt{2\pi} . \sigma_N} e^{-\frac{1}{2}\left(\frac{x-\mu_N}{\sigma_N}\right)^2}$$

We transform random numbers subject to the standard normal distribution by the relation:

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$$x_N = \sigma_N . Z_N + \mu_N$$

Thus, we get the following generation relation:

$$x_N = \sigma_N . [-lnR_{2N}]^{\frac{1}{2}} \cos(2\pi R_{1N}) + \mu_N \quad (8)$$

Practical example:

Using the Box and Muller technique, find two random variables that follow a normal distribution with $\mu = [15,17]$ and $\sigma = [2,3]$.

Using the mean square method, we generate two regular random numbers in the range [0,1].

The mean square method is given by the following relation:

$$R_{i+1} = Mid[R_i^2] \, ; i = 0,1,2,3, \ldots$$

Where *Mid* represents the middle four levels of R_i^2, and R_i is chosen, any random fractional number consisting of four places (called a seed) that does not contain a zero in any of its four places, we get:

$$R_1 = 0.1234 \quad , R_2 = 0.5227$$

We convert these random numbers into neutrosophic random numbers and here we distinguish three states of the field [0,1] with a margin of indeterminacy, in the three states we have $\varepsilon \in [0, n]$ and $0 < n < 1$

First case: $[0 + \varepsilon, 1]$ indeterminacy at the minimum of the range. We substitute in the relation:

$$R_{1N} = \frac{R_1 - \varepsilon}{1 - \varepsilon}$$

Second case: $[0, 1 + \varepsilon]$ indeterminacy at the upper limit of the range. We substitute in the relation:

$$R_{1N} = \frac{R_1}{1 + \varepsilon}$$

Third case: $[0 + \varepsilon, 1 + \varepsilon]$ Indeterminacy in the upper and lower limits of the field. We substitute in the relation:

$$R_{1N} = R_1 - \varepsilon$$

Let us take $\varepsilon \in [0, 0.02]$ then we get:

In the first case:

$$R_{1N} = \frac{R_1 - \varepsilon}{1 - \varepsilon} = \frac{0.1234 - [0,0.02]}{1 - [0,0.02]} = \frac{[0.1034, 0.1234]}{[1, 0.98]} = [0.1034, 0.1259]$$

$$R_{2N} = \frac{R_2 - \varepsilon}{1 - \varepsilon} = \frac{0.5227 - [0,0.02]}{1 - [0,0.02]} = \frac{[0.5027, 0.5227]}{[1, 0.98]} = [0.5027, 0.5334]$$

We replace the relationship (8)

$$x_{1N} = \sigma_N . [-lnR_{2N}]^{\frac{1}{2}} cos(2\pi R_{1N}) + \mu_N$$

$$x_{1N} = [2,3]\big[-ln[0.5027, 0.5334]\big]^{\frac{1}{2}} . cos(2\pi)[0.1034, 0.1259] + [15,17]$$

$$x_{1N} = [16.6584, 19.3782]$$

We compensate with the relationship (9)

$$x_{2N} = \sigma_N . [-lnR_{2N}]^{\frac{1}{2}} sin(2\pi R_{1N}) + \mu_N$$

$$x_{2N} = [2,3]\big[-ln[0.5027, 0.5334]\big]^{\frac{1}{2}} . sin(2\pi)[0.1034, 0.1259] + [15,17]$$

$$x_{2N} = [15.0187, 17.0328]$$

Thus, we obtained two neutrosophic random variables that follow a normal distribution with $\mu = [15,17]$ and $\sigma = [2,3]$.

In the same way, we can obtain the random variables according to the second and third cases.

3. Conclusion and results:

Due to the great importance of the simulation method, and in order to obtain more accurate results that complement what we presented in previous research from the neutrosophic study of generating neutrosophic random numbers and then converting them into random variables that follow the probability distribution with which the system to be simulated operates, and since the reverse transformation method and the rejection and acceptance method do not they can be used for probability distributions defined by a complex probability density function whose cumulative distribution function is impossible to obtain. In this research, we presented a neutrosophic study of the Box and Muller technique used to treat such distributions using mathematical relationships that enable us to derive easy probability distributions in which the neutrosophic inverse transformation can be used to generate neutrosophic random variables. Then we make appropriate substitutions through which we obtain neutrosophic random variables that follow probability distributions that have a complex probability density function. To clarify the algorithm, we used the normal

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distribution, which is one of the most important and most widely used probability distributions in practical life. The result of the study was mathematical relationships that can be used directly to generate the required random variables.

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