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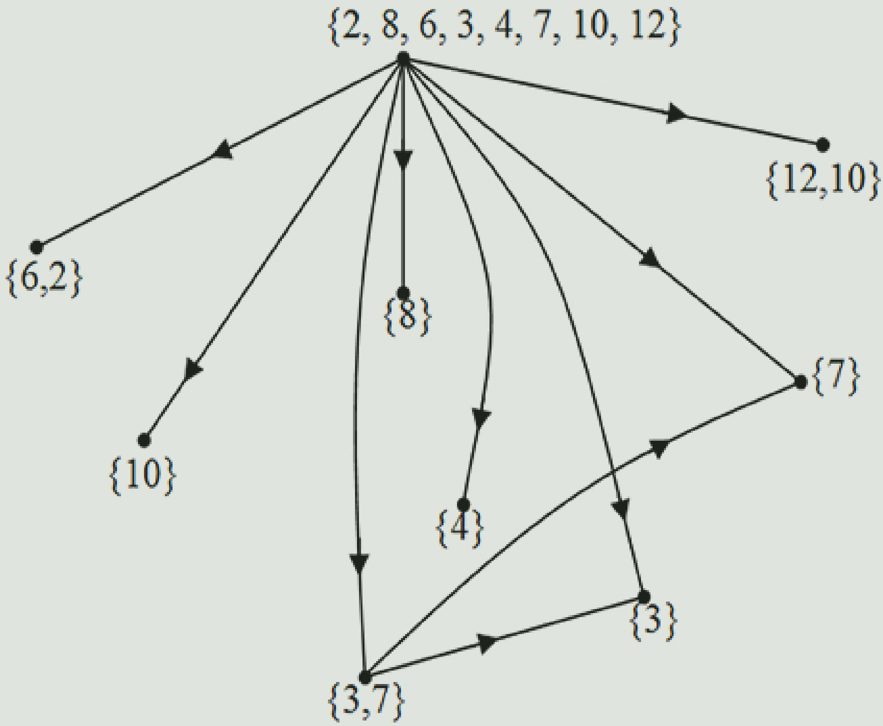

SPECIAL SUBSET VERTEX SUBGRAPHS FOR NETWORKS

Special Subset Vertex Subgraphs for Social Networks

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PREFACE

In this book authors for the first time introduce the new notion of special subset vertex subgraph of subset vertex graphs introduced recently in [46]. These subset vertex graphs takes the vertex set values from the power set P(X) of any set X. The main speciality of these subset vertex graphs is that once a set of subsets from P(X) is given, the edges of the graph are fixed in a unique way, so for a given collection of subset vertices the graph is always unique [46].

The special subset vertex subgraphs of G are the ones, which have the same number of vertices as that of the subset vertex graph G. This special property enables one to use these subgraphs as fault tolerant graphs for fault tolerant networks. In this book authors define the notion of special subset vertex hyper subgraphs, which are better suited for fault tolerant graphs.

We have defined two types of special subset vertex subgraphs and studied their special properties like local complement and so on. However we prove we are not always guaranteed of the local complement given the special subset vertex subgraph of a subset vertex graph G, conditions for its existence is also given.

These special subset vertex subgraphs can serve as fault tolerant graphs and in fact one can also define super subset vertex graphs of any given subset vertex graphs and utilize them for fault tolerant network.

We wish to acknowledge Dr. K Kandasamy for his sustained support and encouragement in the writing of this book.

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Chapter One

BASIC CONCEPTS

In this chapter we recall the basic properties of subset vertex graphs of type I. We also describe the concept of universal complements of the subset vertex graphs of type I. This concept is then localized in case of special subset vertex subgraphs which are introduced in chapter III of this book. This chapter also provides a overview of different networks in which subset vertex graphs can be utilized.

We recall that if S is a finite set (infinite set can also be considered) and P(S) be the power set of S which includes the universal set and the empty set. Let us consider a graph with vertex set from P(S), the edges from v_i to v_j exist if the intersection of v_i and v_j is non empty. That is $v_i \cap v_j \neq \phi$. Thus the edge from v_i to v_j exists if and only if $v_i \cap v_j \neq \phi$.

We will illustrate this situation by some examples. However for more about these concepts please refer [46].

Example 1.1. Let S = {1, 2, 3, 5, 4, 6} be the given set. P(S) be the power set of S.

Let G be the subset vertex graph of type I with vertex set from P(S) given by the following figure :

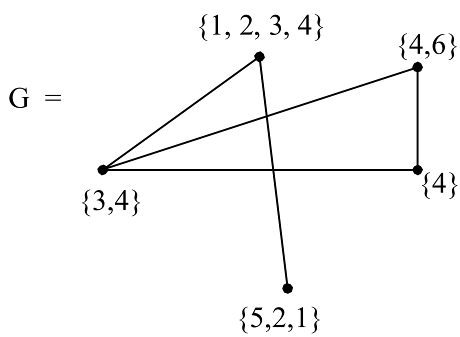


Figure 1.1

Clearly the subset vertex graph of type I is not a directed graph. Further the convention if ϕ is in the vertex set as we know $\phi \subseteq$ A for every A $\in$ P(S) so there is an edge connecting ϕ with every other edge.

Example 1.2. Let S = {1, 2, …, 18} be the given set P(S) the power set of S. Let H be the subset vertex graph of type I given by the following figure.

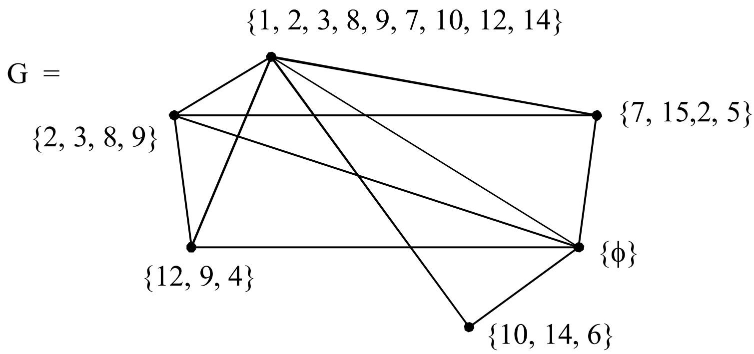


Figure 1.2

Several properties of these subset vertex graphs of type I has been dealt elaborately in [46].

We now show by examples the new notion of universal complement of subset vertex graphs of type I in the following.

Example 1.3. Let S = {1, 2, …, 12} be a set of order 12. P(S) be the power set of S. Let B be the subset vertex graph of type I given by the following figure.

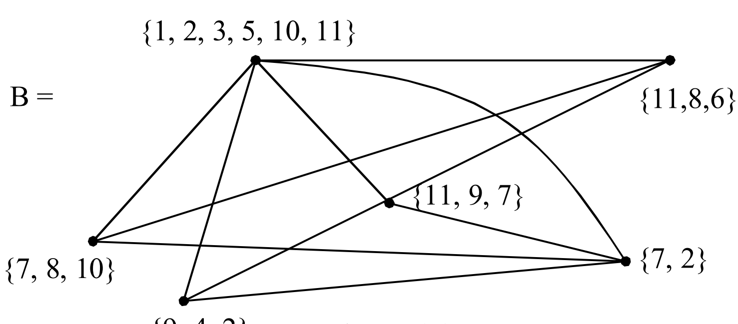


Figure 1.3

Now the universal complement of the subset vertex graph B is constructed with the complements of the vertex sets of B which is given below.

Complement of {1, 2, 3, 5, 10, 11} is {4, 6, 7, 8, 9, 12}. Complement of {7, 8, 10} is {1, 2, 3, 4, 5, 6, 9, 11, 12}. Complement of {6, 8, 11} is {1, 2, 3, 4, 5, 7, 9, 10, 12}. The complement of {9, 4, 2} is {1, 3, 5, 6, 7, 8, 10, 11, 12}. Complement of {7, 9, 11} is {1, 2, 3, 4, 5, 6, 8, 10, 12}. Complement of {7, 2} is {1, 3, 4, 5, 6, 8, 11, 9, 10, 12}. Now the universal complement subset vertex graph of B has the following vertex set V = {{4, 6, 7, 8, 9, 12}, {1, 2, 3, 4, 5, 6, 9, 11, 12}, {1, 3, 5, 6, 7, 8, 10, 11, 12}, {1, 2, 3, 4, 5, 6, 8, 10, 12}, {1, 3, 4, 5, 6, 8, 11, 9, 10, 12} {1, 2, 3, 4, 5, 7, 9, 10, 12}} given by the following figure.

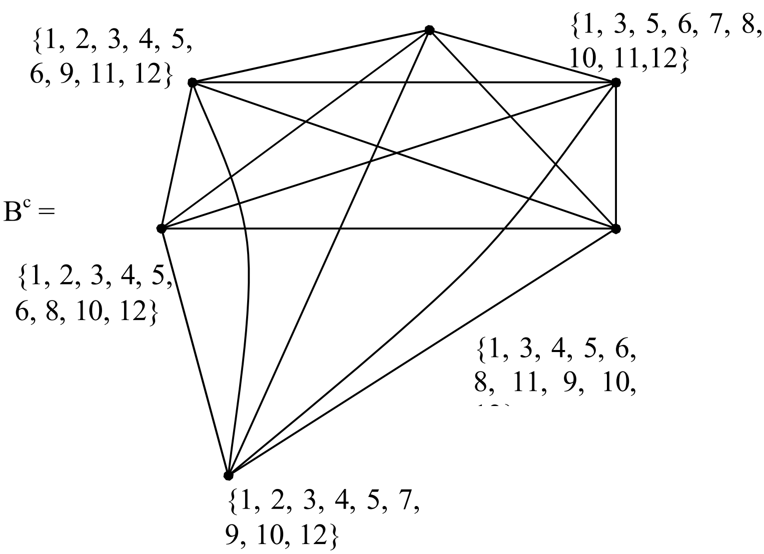


Figure 1.4

Clearly B^c is a complete subset vertex graph of type I which is defined as the universal complement subset vertex graph of B of type I.

However it is interesting to note that B is not a complete subset vertex graph of type I.

B has six vertices and has only 11 edges.

This is an extreme case. We will in due course of time characterize this case also.

Consider the following subset vertex star graph T given by the following figure.

T =

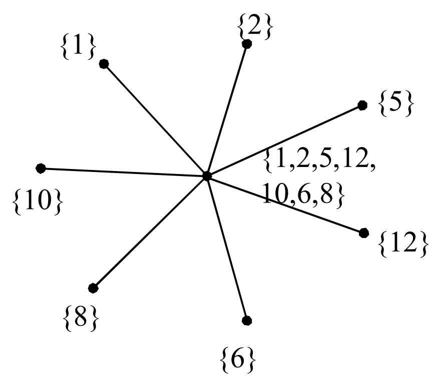


Figure 1.5

Now we find the universal complement of T in the following using the power set P(S) where S = {1, 2, …, 12}.

$\{\hat{1}, 2, 3, \ldots, 12\}$ denotes the set where one is not present in that set that it is the set {2, 3, 4, …, 11, 12}. With this convention we give the universal complement of the subset vertex graph T in the following.

v_1 is the universal complement of {1}

v_2 is the universal complement of {2}

v_3 is the universal complement of {5}

v_4 is the universal complement of {6}

v_5 is the universal complement of {8}

v_6 is the universal complement of {10}

v_7 is the universal complement of {12}

v_8 is the universal complement of {1,2,5,6,8,10,12}

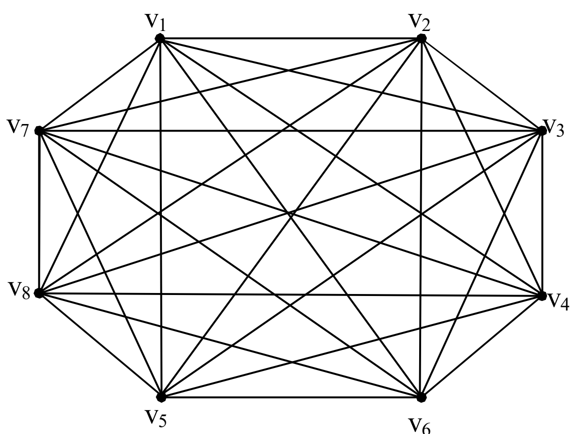


Figure 1.6

We see T^c the universal complement subset vertex graph of T is not a star graph. Such type of universal complement subset graphs are not structure preserving.

Clearly T $\cup$ T^c is just a point subset graph given by {1, 2, 3, …, 12}.

Consider the subset vertex graph K of type I given by the following figure.

K =

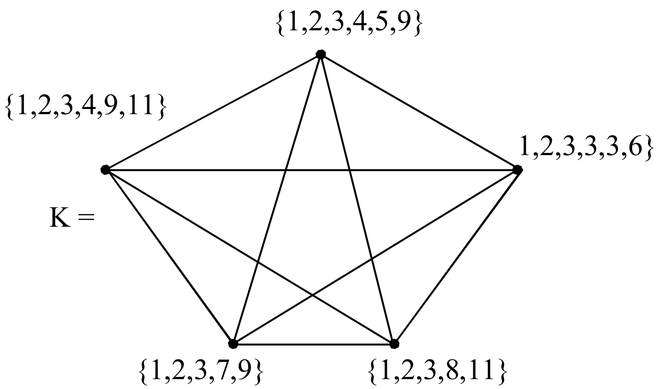


Figure 1.7

Now we find the universal complement subset vertex graph of the subset vertex graph K.

The vertex set of the universal complement K^c is {{6,7,8,10,11,12}, {4,5,7,8,10,11,12}, {4, 5, 6, 7, 9, 10, 12}, {4, 5, 6, 8, 10, 11, 12}, {5, 6, 7, 8, 10, 12}}.

The graph K^C is as follows.

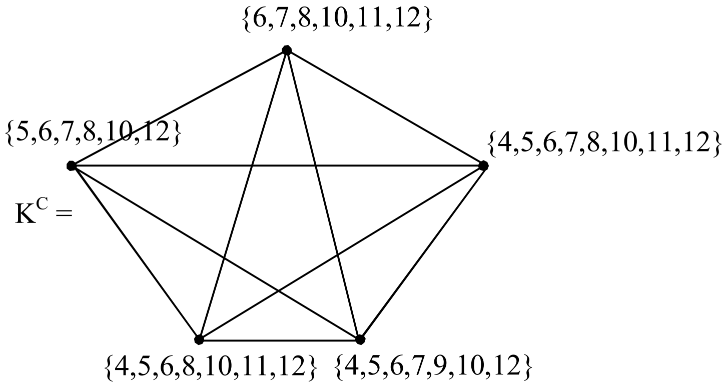


Figure 1.8

Clearly in this case both the subset vertex graph K and its universal complement K^C subset vertex graph are subset vertex complete graphs.

We now give the proper definition of the universal complement of the subset vertex graph.

Definition 1.1. *Let S be a finite set. P(S) be the power set of S. G be the subset vertex graph of type I set;* $V = \{v_1, v_2, \ldots, v_n\} \in P(S)$. *Consider the subset vertex graph H of type I with vertex set* $V^C = \{S \setminus v_1, S \setminus v_2, \ldots, S \setminus v_n\} \subseteq P(S)$.

H is defined as the universal complement subset vertex graph of the subset vertex graph G.

It is important to note that for a given subset vertex sgraph G of type I the universal complement subset vertex graph of G is unique.

Further it is interesting to note that in general the universal complement subset vertex graph need not in general be structure preserving.

We will provide some more examples of them.

Example 1.4. Let S = {1, 2, …, 10} be a set of order 10. P(S) be the power set of S. Let G be the subset vertex graph of type I given by the following figure.

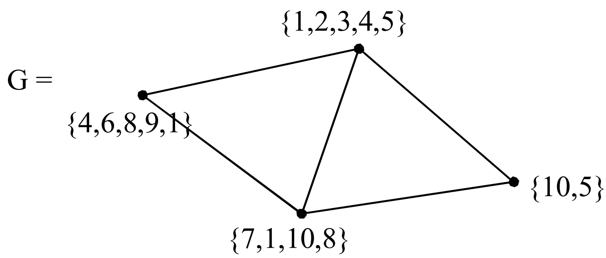


Figure 1.9

The vertex set of the universal complement of G, viz, G^C is as follows.

{{6, 7, 8, 9, 10}, {1, 2, 3, 4, 6, 7, 8, 9},

{2, 3, 4, 5, 6, 9}, {2, 3, 5, 7, 10}}

The universal complement vertex subset graph of G is as follows.

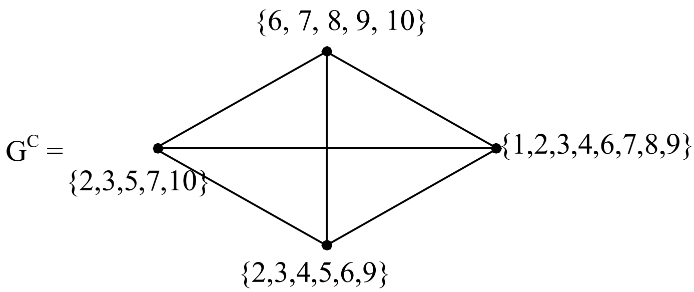


Figure 1.10

Clearly the universal complement subset vertex graph G^C of G is not structure preserving for G^C is a complete subset vertex graph of type I however G is not a complete subset vertex graph.

Thus in general universal complements do not preserve, the structure of the basic subset vertex graph for which the complement is taken.

Example 1.5. Let S = {1, 2, …, 9} be a set and P(S) the power set of S. Let B be the subset vertex graph of type I given by the following figure.

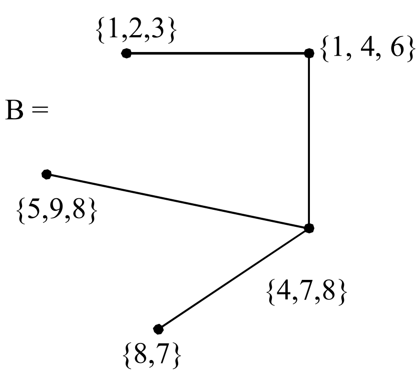


Figure 1.11

The vertex set of the universal complement of the subset vertex graph of B is W = {{4, 5, 6, 7, 9, 8}, {2, 3, 5, 9, 7, 8}, {1, 2, 3, 9, 6}, {1, 2, 3, 4, 5, 6, 9}, {1, 2, 3, 4, 6, 7}}. The universal complement of this subset vertex graph of B has W to be the vertex set the figure of which is as follows:

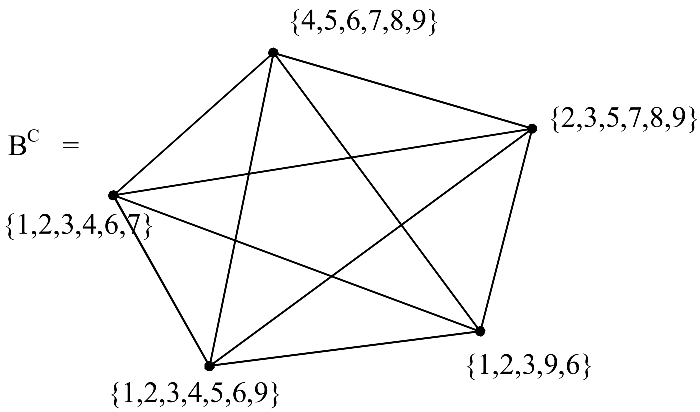


Figure 1.12

B^C is a subset vertex complete graph. Hence we see the structure of B is not preserved. However B happens to be a subset vertex graph of type I. Infact we will soon realize B will be a special subset vertex graph of type I in a different way which is very unusual for the graph structure of the subset vertex graph enjoys the substructure as a subgraph not the vertex set that is only the edges.

Now we see the B of complement B^C is a subset vertex graph of type I. Thus we can say if G^C is the universal complement subset vertex graph of G then G is the universal complement of the subset vertex graph G^C.

Thus $(G^C)^C = G$.

Now we can also define for a given subset vertex graph G of type I the subset vertex subgraph H of G and its local complement subset vertex subgraph of H relative to G. This concept will be first described by some examples.

Example 1.6. Let S = {1, 2, …, 15} be the finite set P(S) be the power set of S. Consider the subset vertex graph G of type I given by the following figure.

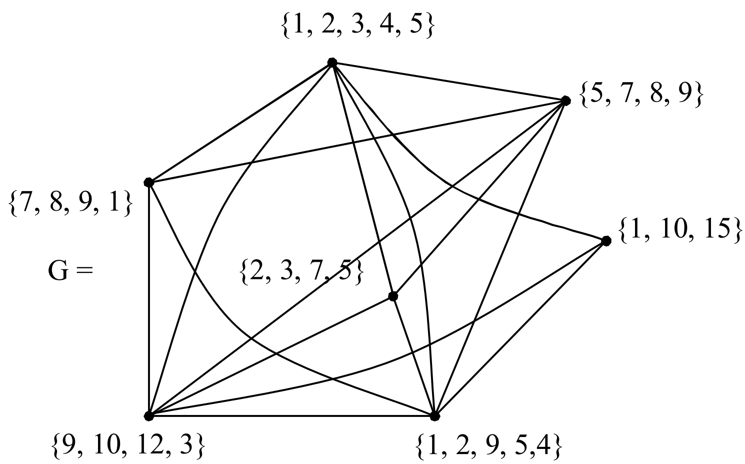


Figure 1.13

Let H be a subset vertex subgraph of G given by the following figure with vertex set, W = {{2, 3}, {5, 7, 8}, {1, 15}, {1, 2, 5, 9}, {9, 3}, {1, 9}}.

The subset vertex graph H with the above vertex set is as follows.

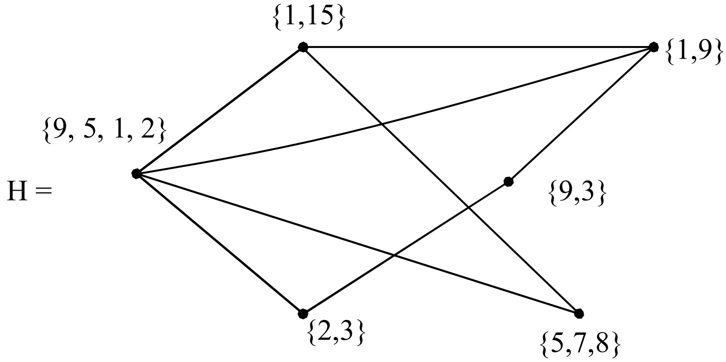


Figure 1.14

We find the complement of the vertex set H.

M = {{1, 2, 3, 4, 5} \ {2, 3}, {5, 7, 8, 9} \ {5, 7, 8}, {1, 10, 15} \ {1, 15}, {7, 8, 9, 1} \ {1, 9}, {1, 2, 9, 5, 4} \ {9, 5, 1, 2}, {9, 10, 12, 3} \ {9, 3}} = {{1, 4, 5}, {9}, {10}, {7, 8}, {4}, {10, 12}}.

The subset vertex graph N of type I associated with the vertex set M is as follows.

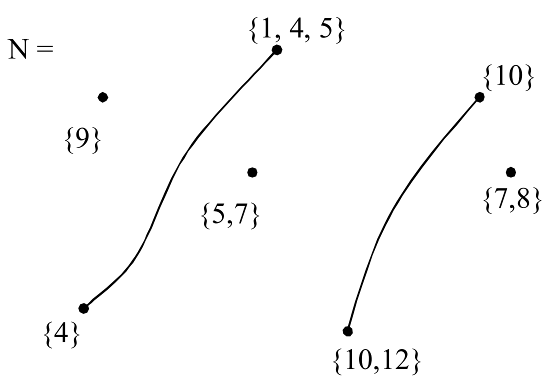


Figure 1.15

N is the local complement subset vertex subgraph of H of type I relative to G. Clearly the number of edges and vertices of the local complement subset vertex subgraph in general need not have the same number of vertices and edges as that of H or G.

We will illustrate this situation by some more examples.

Example 1.7: Let as consider the set S = {1, 2, 3, …, 10} and P(S) the power set of S.

Let G be the subset vertex graph of type I given by the following figure.

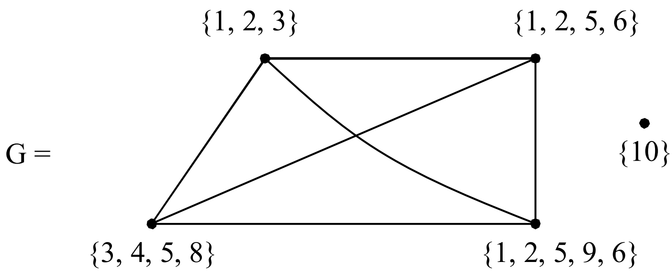


Figure 1.16

Let H be the subset vertex subgraph of G given by the following figure.

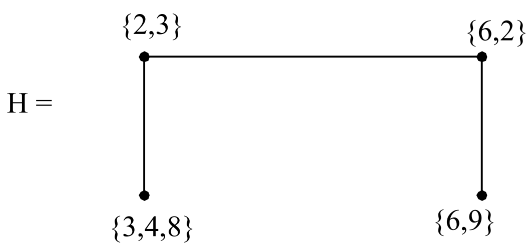


Figure 1.17

Now we find the local complement of H in G.

The vertex set of the complement subset vertex subgraph of H relative to G is {{1, 2, 3} \ {2, 3}, {1, 2, 5, 6} \ {6, 2}, {1, 2, 5, 6, 9} \ {6, 9}, {3, 4, 5, 8} \ {3, 4, 8}} = {{1}, {1, 5}, {1, 2, 5}, {5}}.

Now the subset vertex graph with the above vertex set is as follows.

H^C =

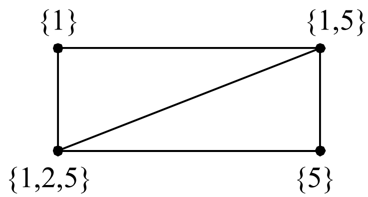


Figure 1.18

H^C is the local complement subset vertex subgraph of H relative to G.

Clearly H has only 3 edges where as H^C has 5 edges.

Consider a subset vertex subgraph K of G given by the following figure with vertex set {{1, 3}, {5, 6, 1}, {5, 4, 3} and {1, 2, 5, 6}}

K =

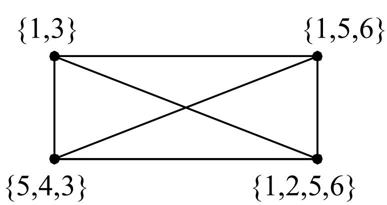


Figure 1.19

We see K is also a subset vertex subgraph which is complete, infact a subset vertex hypersubgraph of K.

Now we find the local complement K^C of K. K^C has the following vertex set {{1, 2, 3} \ {1, 3}, {1, 2, 5, 6} \ {1, 5, 6}, {3, 4, 5, 8} \ {5, 4, 3}, {1, 2, 5, 9, 6} \ {1, 2, 5, 6}} = {{2}, {2}, {8}, {9}} = {{2}, {9}, {8}}.

Thus the K^C of the subset vertex subgraph of K relative to G does not exist as two subset vertices are equal to {2}, so K^c cannot be defined as an empty subgraph with only vertices;

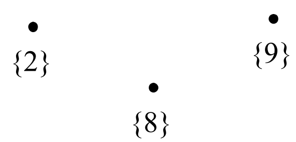


Figure 1.20

Thus the local complement of a subset vertex subgraph K^C of K does not exist whereas K is a complete subset vertex graph with four vertices.

Thus at this juncture we wish to record one cannot predict the structure of the local complement as in this case the local complement subset vertex subgraph of K does not exist.

Further it is to be noted in general that each of the subset vertex subgraph behaves in a very different way. This is clearly established by the above example.

Let P be a subset vertex subgraph of G given by the following figure with the vertex set {{1}, {1, 2, 5}, {1, 5} and {3, 5}}.

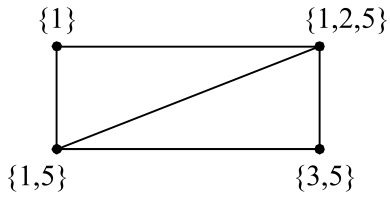
P = 

Figure 1.21

Now we find the local complement subset vertex subgraph P^C of P relative to G.

The vertex set of P^C is as follows : {{1, 2, 3, 7} \ {1}, {1, 2, 5, 6} \ {1, 2, 5}, {3, 4, 5, 8} \ {3, 5} and {1, 2, 5, 9, 6} \ {1, 5}} = {{2, 3, 7}, {6}, {4, 8}, {2, 6, 9}}. The subset vertex subgraph with the above set of vertices is given in the following.

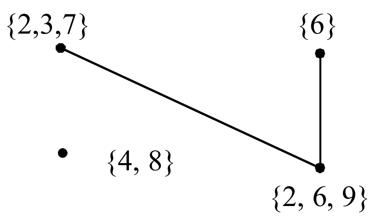
P^C = 

Figure 1.22

This local complement subset vertex subgraph P^C of P relative to G has two edges and four vertices.

Now we find the universal complement subset vertex graph G^C of G in the following. G^C has the following vertex set {{1, 2, …, 10} \ {1, 2, 3,7}, {1, 2, …, 10} \ {1, 2, 5, 6}, {1, 2, 3, …, 10} \ {3, 4, 5, 8}, {1, 2, 3, …, 10} \ {1, 2, 5, 6, 9}} = {{4, 5, 6, 8, 9, 10}, {3, 4, 7, 8, 9, 10}, {1, 2, 6, 7, 9, 10}, {3, 4, 7, 8, 10}}.

The subset vertex graph with the above set of vertices is as follows.

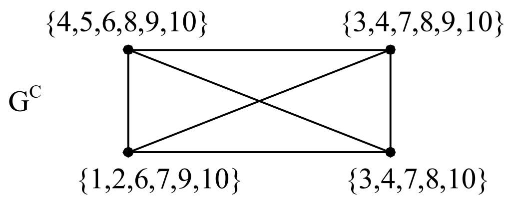


Figure 1.23

The universal complement subset vertex graph G^C of G is again a complete subset vertex graph.

In view of all these we have the following interesting theorem.

Theorem 3.1. *Let S be a finite set P(S) the power set of S. Let G be any subset vertex graph of type I with n vertices $\{v_1, v_2, \ldots, v_n\} \subseteq P(S)$. G^C the universal complement subset vertex graph of G; if $\bigcap_{i=1}^{n} (S \setminus v_i) \neq \phi$ then G^C the universal complement subset vertex graph of G is a subset vertex complete graph of type I.*

Proof. Given G is a subset vertex graph of type I with vertex set $\{v_1, v_2, \ldots, v_n\} \subseteq P(S)$.

G^C is the universal complement of G with vertex set $\{S \setminus v_1, S \setminus v_2, S \setminus v_3, \ldots, S \setminus v_n\}$.

If $\bigcap_{i=1}^{n}(S \setminus v_i) \neq \phi$ to prove G^C is a complete subset graph of type I. That is we have to prove there exists a edge between every $S \setminus v_i$ and $S \setminus v_j$; $i \neq k$, $1 \leq i, j \leq n$. Since $\bigcap_{i=1}^{n}(S \setminus v_i) \neq \phi$. There exists elements from S say $y \in S \setminus v_i$ for all $i = 1, 2, \ldots, n$. This implies $S \setminus v_i \cap S \setminus v_j \neq \phi$ for every i, j $(i \neq j)$, $1 \leq i, j \leq n$.

Hence G^C which has vertex set $\{S \setminus v_1, S \setminus v_2, \ldots, S \setminus v_n\}$ gives a subset vertex graph which is complete. Hence the result.

Here it is important to keep on record that if G^C the universal complement of a subset vertex graph G of type I is complete, then it need not necessarily imply $\bigcap_{i=1}^{n}(S \setminus v_i) \neq \phi$.

To this and we will provide an example where $\bigcap_{i=1}^{n}(S \setminus v_i) = \phi$ yet there exists a complete subset vertex graph which is the universal complement of a given subset vertex graph.

Consider the following example.

Example 1.8. Let S = {1, 2, …, 6} be a finite set. P(S) the power set of S. Consider the subset vertex graph G of type I given by the following figure.

$G =$

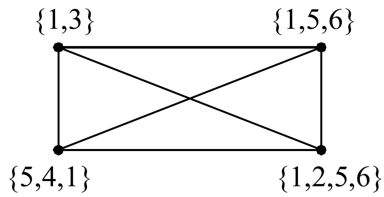


Figure 1.24

Now we find the universal complement subset vertex graph G^C of G which has the following set of vertices {{1, 2, …, 6} \ {1, 3}, {1, 2, 3, …, 6} \ {1, 5, 6}, {1, 2, …, 6} \ {1, 5, 4}, {1, 2, 3, …, 6} \ {1, 2, 5, 6}} = {{4, 2, 5, 6}, {2, 4, 3}, {2, 3, 6}, {3, 4}}.

The subset vertex graph with the above set of vertices.

$G^C =$

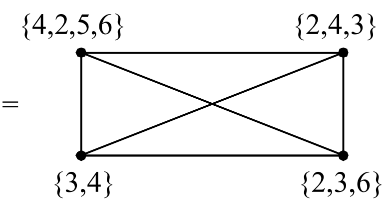


Figure 1.25

G^C is the universal complement subset vertex graph of G and we see

$\bigcap_{i=1}^{4} v_i = \phi$ where v_1 = {2, 4, 5, 6}, v_2 = {2, 4, 3}, v_3 = {3, 4} and v_4 = {2, 3, 6}.

Thus we see G^C is a subset vertex complete graph in which $\bigcap_{i=1}^{4} v_i = \phi$ where v_1, v_2, v_3 and v_4 are vertices of G^C.

However we see both G and G^C are subset vertex complete graphs.

Next we proceed onto give some more examples of subset vertex star graphs of type I and their universal complement subset vertex graphs.

Example 1.9. Let S = {1, 2, …, 12} be a set. P(S) be the power set of S. Let G be a subset vertex star graph with vertex set from P(S) given by the following figure.

G =

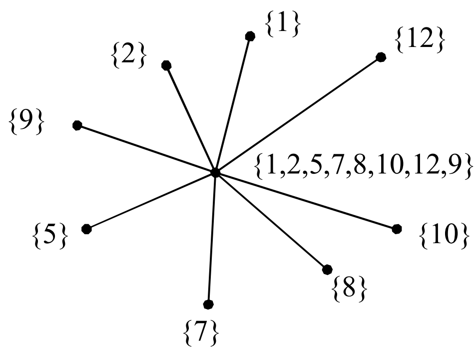


Figure 1.26

Now we find the universal complement of G. The vertex set of G the universal complement G^C is as follows. Let W = {{1, 2, 3, …, 12}\{1, 2, 5, 7, 8, 9, 10, 12}, {1, 2, …, 12} \ {1}, {1, 2, …, 12} \ {2}, {1, 2, …, 12} \ {12}, {1, 2, …, 12} \ {10}, {1, 2, …, 12} \ {8}, {1, 2, …, 12} \ {7}, {1, 2, 3, …, 12} \ {5}, {1, 2, …, 12} \ {9}} = {{3, 4, 6, 11}, {2, 3, …, 12}, {1, 3, 4, …, 12}, {1, 2, 3, …, 11}, {1, 2, 3, …, 9, 11, 12}, {1, 2, …, 7, 9, 10, 11, 12}, {1, 2, …, 6, 8, 9, …,12}, {1, 2, 3, 4, 6, 7, …, 12}, {1, 2, 3, …, 8, 10, 11, 12}.

Let the subset vertex graph G^C with vertex set W be as follows.

G^C=

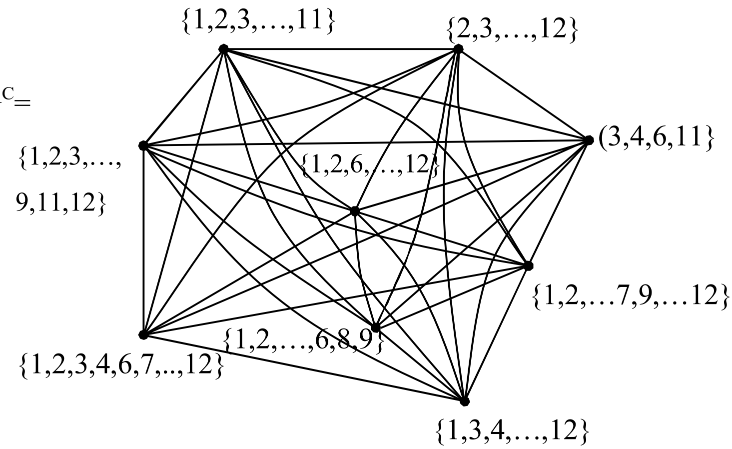


Figure 1.27

We see the universal complement of a subset vertex star graph is a complete subset vertex graph is a complete subset vertex graph. Consider the subset vertex star graph of type I given by the following figure.

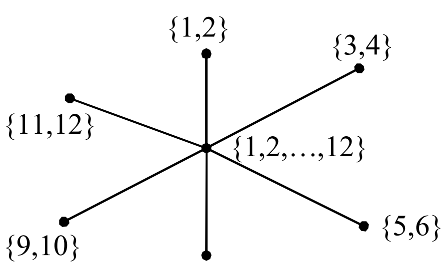


Figure 1.28

The subset vertex complement G^C of G is as follows.

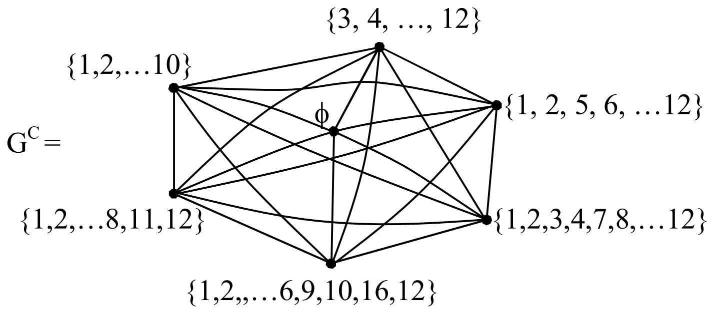


Figure 1.29

For this also G^C is a subset vertex complete graph. It is left as an exercise for the reader to find subset vertex star graphs whose universal complements are just subset vertex complete graphs.

Let us consider subset vertex graph of type I G with vertices from P(S) where S = {1, 2, …, 15}.

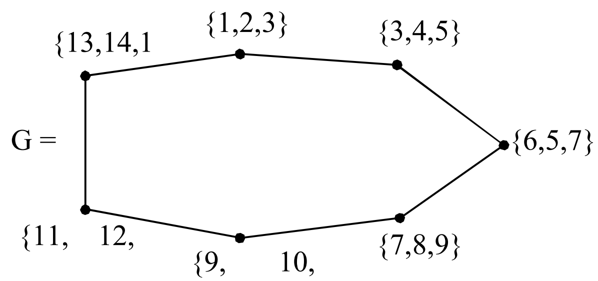


Figure 1.30

The subset vertex universal complement of graph of G is as follows.

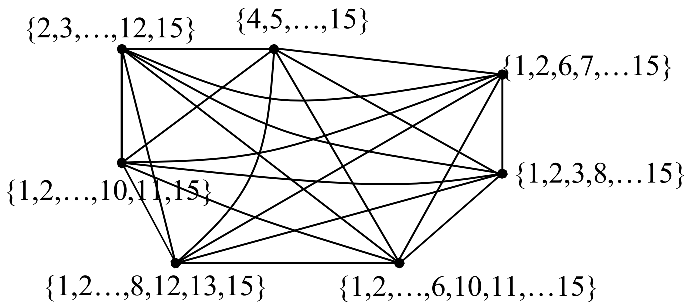


Figure 1.31

Clearly G is a subset vertex circle graph of type I. But its universal complement is a subset vertex complete graph of type I. Thus in general not only subset vertex star graphs have universal complement to be subset vertex complete graphs but also some of the subset vertex circle graphs of type I happens to have universal complements to be subset vertex complete graphs of type I.

Consider the subset vertex wheel graph given by the following figure.

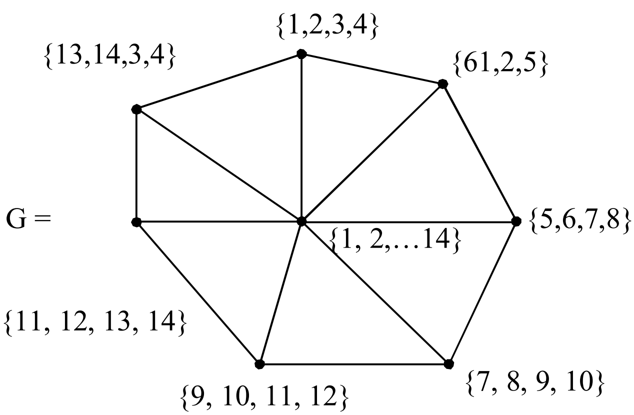


Figure 1.32

Clearly G is a subset vertex wheel graph of type I. Now we give the vertex set of the universal complement subset vertex graph of G.

W = {{5, 6, 7, …, 15}, {3, 4, 7, 8, …, 15}, {1, 2, 3, 4, 9, 10, …, 15}, {1, 2, …,6, 11, 12, 13, 14, 15}, {1, 2, …, 8, 13, 14, 15}, {1, 2, …, 10, 15}, {1, 2, 5, 6, 7, …, 12, 15}, {15}}.

Now we give the universal complement subset vertex graph G^C of the wheel G in the following.

G^C =

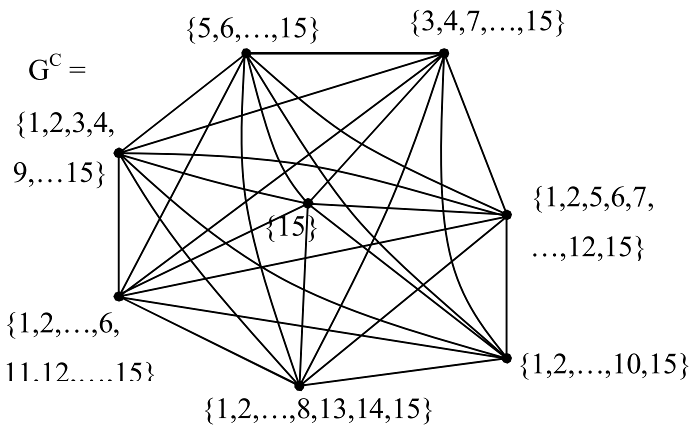


Figure 1.33

We see the universal complement subset vertex graph G^C of G is as complete subset vertex graph. Thus in this case the structure is not preserved by the universal complements.

It is left for the reader to find conditions under which both the subset vertex graph of type I and its universal complement enjoy the same structure.

Consider the subset vertex circle graph of type I given by the following figure.

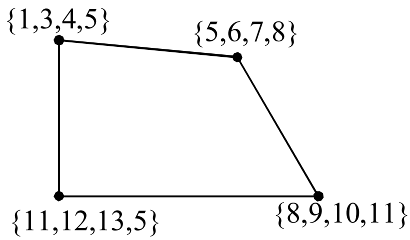


Figure 1.34

The universal complement of the subset vertex graph G is as follows.

G^C =

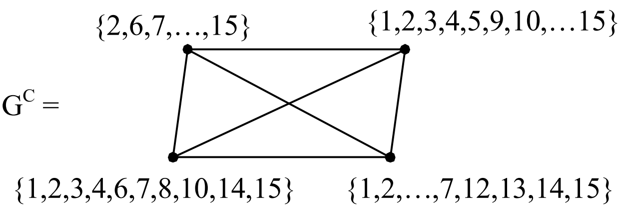


Figure 1.35

Clearly G^C of G is a subset vertex complete graph which is the universal complement of G.

We now proceed onto give more examples.

Example 1.10 Let P(S) be the power set of S; S = {1, 2, …, 9}. Let G be the subset vertex graph of type I given by the following figure.

G =

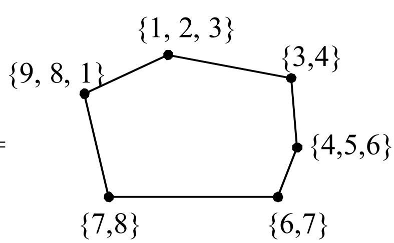


Figure 1.36

Clearly G is a subset vertex circle graph. Now we find the universal complement of G. The vertex set of G^C is as follows.

W = {{4, 5, 6, 7, 8, 9}, {1, 2, 5, 6, 7, 8, 9}, {1, 2, 3, 7, 8, 9}, {1, 2, 3, 4, 5, 8, 9}, {1, 2, 3, 4, 5, 6, 9}, {2, 3, 4, 5, 6, 7}}.

The vertex subset graph with W as vertices is as follows.

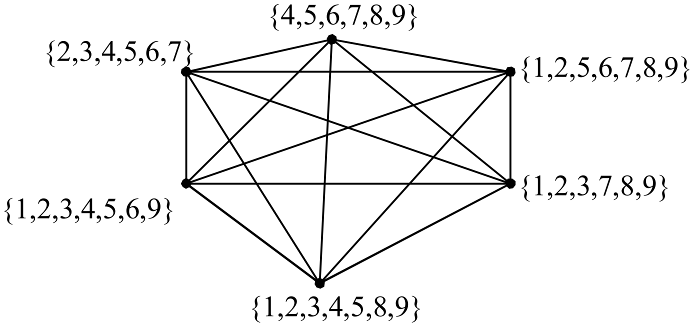


Figure 1.37

Clearly the universal complement subset vertex graph of G is a complete subset vertex subgraph.

Let G be a subset vertex graph of type I.

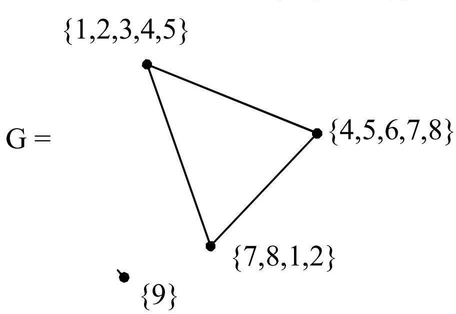


Figure 1.38

Let G^C be the universal complement of the subset vertex graph of G. The figure of G^C is given below

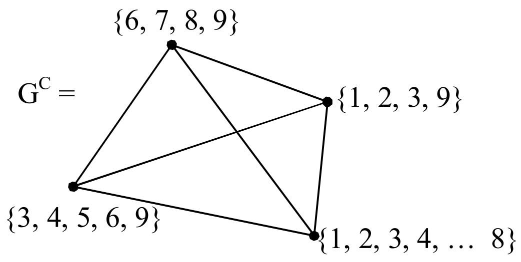


Figure 1.39

We see G^C is also a subset vertex. Consider the subset vertex subgraph H of G given by the following figure.

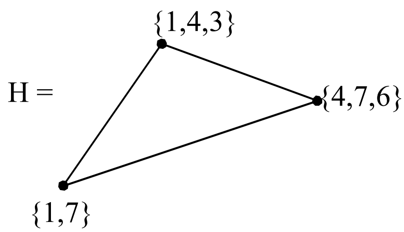


Figure 1.40

The local complement subset vertex graph H^C is as follows.

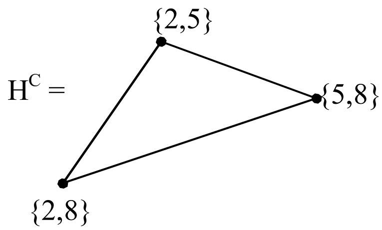


Figure 1.41

We see G, G^C, H and H^C are subset vertex complete graphs of type I.

Characterize those subset vertex complete graphs G for which G^C and (H a subset vertex complete subgraph) H^C are also complete subset vertex graphs of type I.

Subset vertex graphs can be used in any communication or social information network. Any network whose performance depends on important and basic network centrality measures can use of subset vertex graph structure to make the network more reliable and fault tolerant.

We proceed onto suggest the following problems.

Suggested problems.

1. Let S = {1, 2, …., 16} be a finite set, P(S) the power set of S. Find the universal complement of G, the subset vertex graph of type I give below.

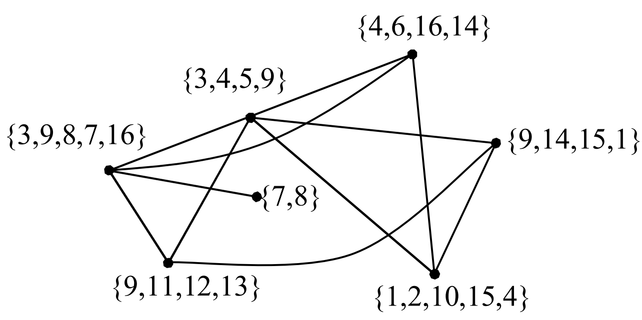


Figure 1.42

i) Find all subset vertex subgraphs of G and their local complements relative to G.

ii) How many such subset vertex subgraphs of G exist?

iii) Can G have a subset vertex hypersubgraph?

iv) If (iii) gives a nontrivial hypersubgraph then find its local complement relative to G.

2. Let S = {1, 2, …, 18} be a set of order 18. P(S) the power set of S. Let G be a subset vertex graph of type I given by the following figure.

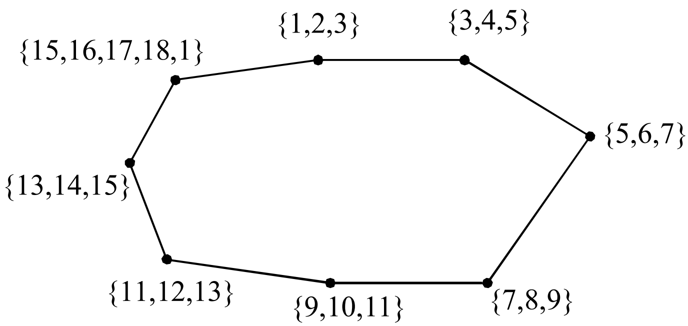


Figure 1.43

i) Study questions (i) to (iv) of problem (1) for this G.

ii) Is G^C a subset vertex complete graph?

3. Let S = {1, 2, …, 24} be a set of order 24. P(S) the power set of S.

Let G be a subset vertex graph of type I given by the following figure.

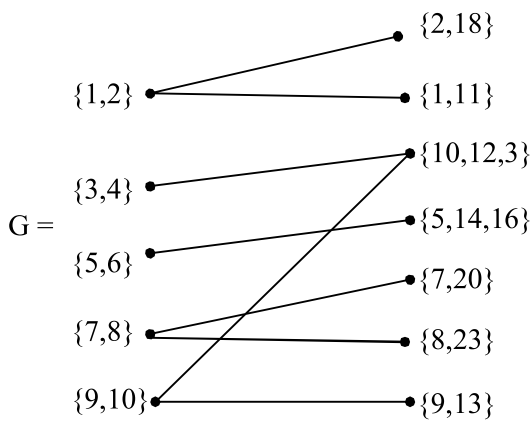


Figure 1.44

i) Find the universal complement subset vertex graph of G.

ii) Is G^C again a bipartite subset vertex graph?

iii) Study questions (i) to (iv) of problem (1) for this G.

4. Let S = {1, 2, …, 45} be a set of order 45. P(S) the power set of S.

a) Find the number of subset vertex complete graphs of type I using subsets of this P(S) as the vertex sets.

b) Find all subset vertex circle graphs of type I using this P(S) as the vertex set.

c) How many subset vertex star graphs of type I exist using this P(S) as the vertex set?

d) Enumerate all subset vertex graphs of type I which are not complete but their universal complement subset vertex graph is complete.

e) Obtain any other interesting property associated with these type of subset vertex graphs of type I and their universal complements.

5. Let S = {1, 2, …, 36} be a set of cardinality 36. P(S) be the power set of S. Let G be the subset vertex graph of type I using P(S) as the vertex set which is given in the following.

G =

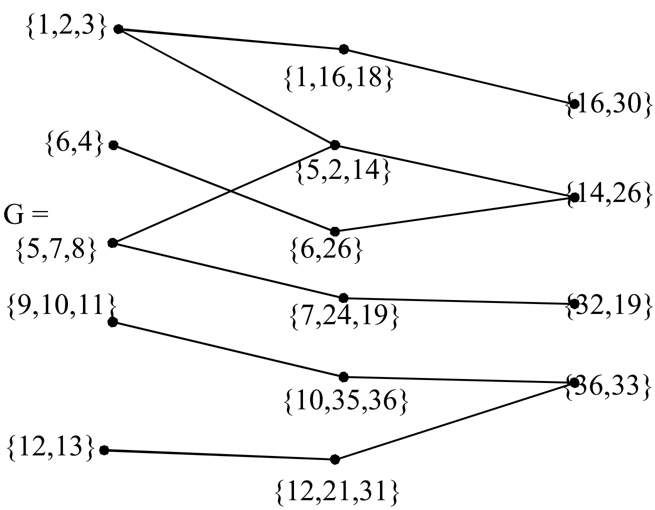


Figure 1.45

i) Will the universal complement subset vertex graph of G be a subset vertex tripartite graph?

ii) Find all subset vertex subgraphs of G and their local subset vertex subgraph complements.

iii) How many of these local complement subset vertex subgraphs of G exist?

iv) How many of these local complement subset vertex subgraphs are complete?

v) Can any of the local complement subset vertex graph be a star graph? Justify your claim.

6. Let P(S) be the power set of the set S = {1, 2, …, n}.

i) How many subset vertex complete graphs of type I can be constructed with only n vertices?

ii) How many subset vertex complete graphs of type I can be constructed using (n – 1) and (n – 2) vertices?

iii) How many subset vertex complete graphs of type I with 3 vertices can be constructed?

iv) How many subset vertex star graphs of type I with n vertices can be constructed using the power set P(S)?

v) How many subset vertex star graphs of type I with 9 vertices exists in P(S)?

vi) How many subset vertex circle graphs of type I can be constructed using P(S)?

vii) How many subset vertex circle graphs of type I with 9 vertices can be built using P(S)?

7. Let S = {1, 2, …, 54} be a set of cardinality 54. P(S) be the power set of S.

i) Study questions (i) to (vii) of problem (6) for this P(S).

8. Show such vertex subset graphs of type I will be useful in case of analysis of social networks and community networks.

9. Let S = {1, 2, …,18} be a set of cardinality 18. P(S) be the power set of S. Let G be the subset vertex graph of type I given by the following figure.

G =

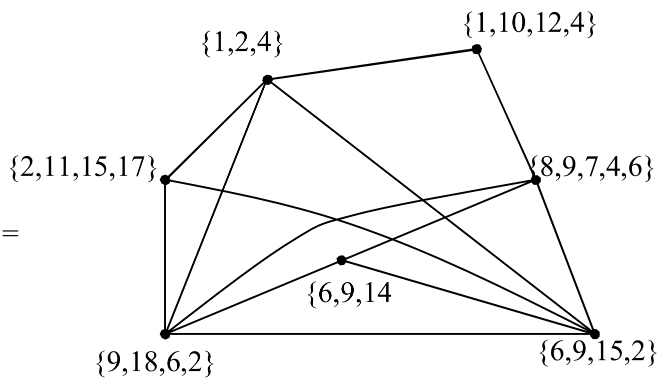


Figure 1.46

i) Find the universal complement G^C of G.

ii) Is G^C, a subset vertex complete graph?

iii) Find all subset vertex subgraphs of G and their local complement subset vertex subgraphs.

iv) How many of the local complement subset vertex subgraphs are subset vertex star graphs?

v) How many of the local complement subset vertex subgraphs are subset vertex circle subgraphs?

vi) Obtain any of the special features enjoyed by the subset vertex graphs of type I with vertex set from P(S)

10. Let S = {1, 2, 3, …, 19} be a set of cardinality 19. P(S) be the power set of S.

Let G be the subset vertex graph of type I given by the following figure.

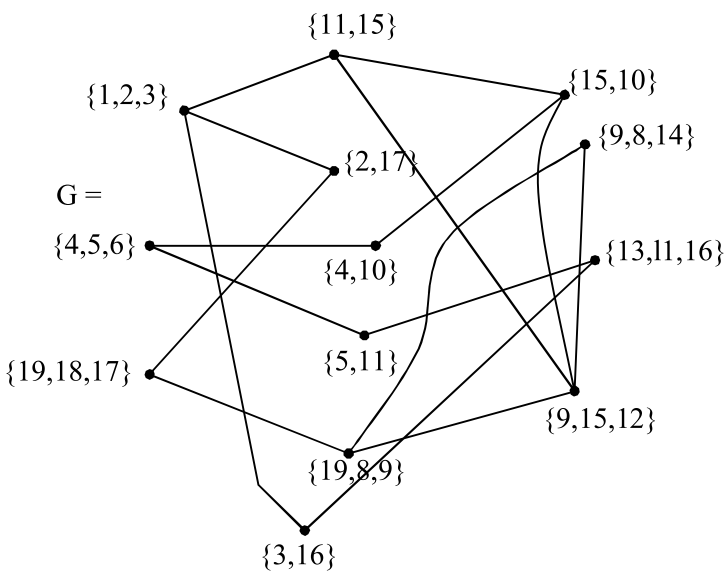


Figure 1.47

i) Find the universal complement G^C of the subset vertex graph G.

ii) Is G^C a tripartite subset vertex graph?

iii) How many subset vertex subgraphs of G be constructed?

iv) Find the local complements of those subset vertex subgraphs.

v) Will these local complements be subset vertex trigraphs of type I?

vi) Enumerate all special features enjoyed by the local complements of subset vertex subgraphs.

11. Let G be a subset vertex graph of type I with vertex set from P(S) where S = {1, 2, …, 10} given by the following figure.

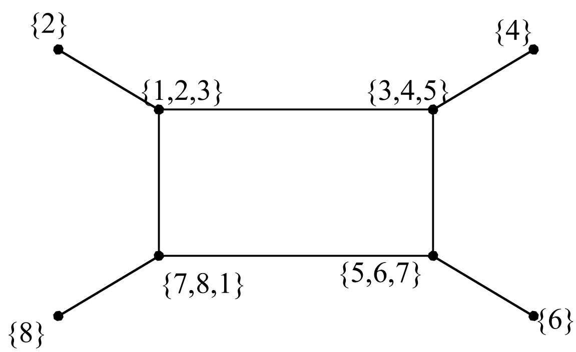
G = 

Figure 1.48

i) Find the universal complement subset vertex graph of G.

ii) How many subset vertex subgraphs of G exists?

iii) Find the local complements of each of these subset vertex subgraphs.

12. Let G_1 and G_2 be the subset vertex disconnected graph with vertex set from P(S) where S = {1, 2, …, 28} given by the following figures 1.49a and 1.49b.

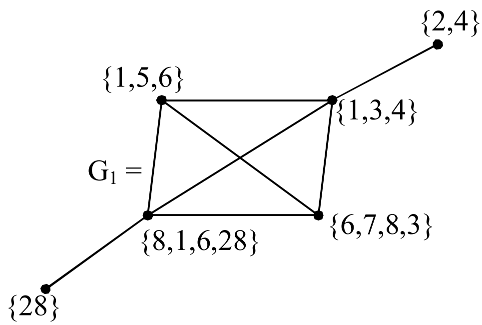


Figure 1.49(a)

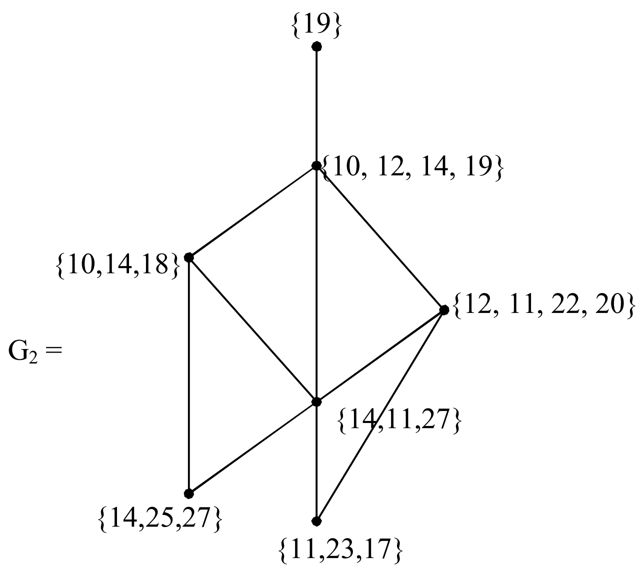


Figure 1.49(b)

i) Find the universal complement G_1^C and G_2^C of the disconnected subset vertex graph of G.

ii) Is G_1^C connected?

iii) Find all subset vertex subgraphs of G_1 and G_2.

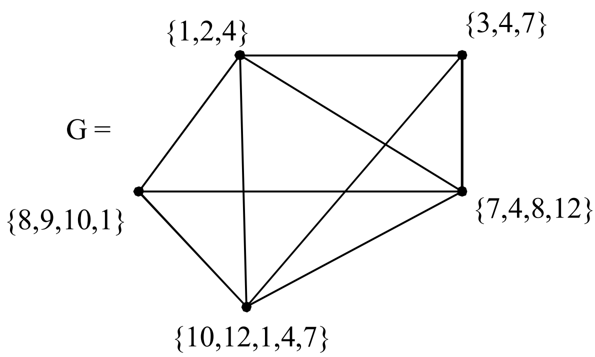
iv) How many of the local complement subset vertex subgraphs of subgraphs relative to G_1 and G_2 are connected?

v) How many of the local complement subset vertex subgraphs of subset vertex subgraphs of G relative to G_1 and G_2 are not connected.

vi) Derive any other special properties enjoyed by subset vertex disconnected graphs of type I.

13. Let S = {1, 2, …, 63} be a set of cardinality 63; P(S) be the power set of S. Let G be the set vertex graph given by the following figure.

G =



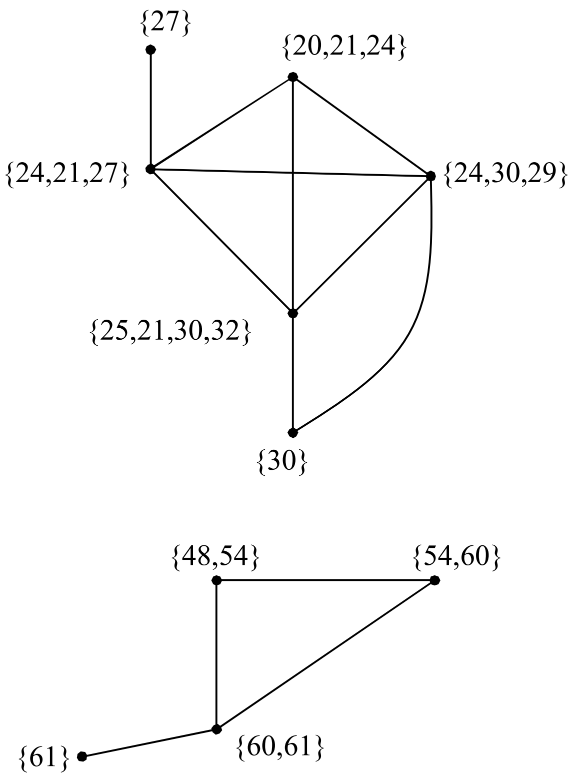


Figure 1.50

i) Study questions (i) to (vi) of problem (12) for this G given in above problem.

14. Let S = {1, 2, …, n} and P(S) be the power set of S.

i) Find all subset vertex graphs of type I which has subset vertex subgraphs which are hyper graphs.

ii) Find all subset vertex graphs G of type I which has no subset vertex subgraphs which are hyper graphs.

iii) In (i) what can one say about the global complement subset vertex graphs H of type I?

iv) In problem (ii) what can one say about the global complement of G?

15. Let S = {1, 2, …, 16} be the finite set P(S) the power set of S. Let G be the subset vertex graph of type I given by the following figure.

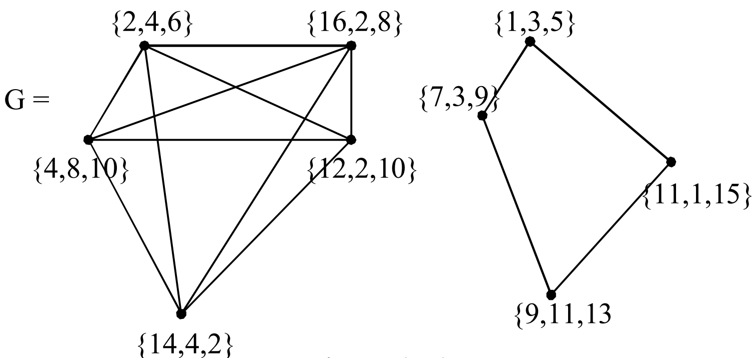


Figure 1.51

i) Find the universal complement subset vertex graph G^C of G.

ii) Find all subset vertex subgraphs of G.

iii) How many of them are disconnected?

iv) How many of the subset vertex subgraphs of G which are connected?

v) Find all local complements of the subset vertex subgraphs which are connected.

16. Explain the possible application of these subset vertex graphs in social networks and community networks.

17. Let S = {1, 2, …, 25} be the set P(S) be the power set of S. Let G be the subset vertex graph of type I given by

G =

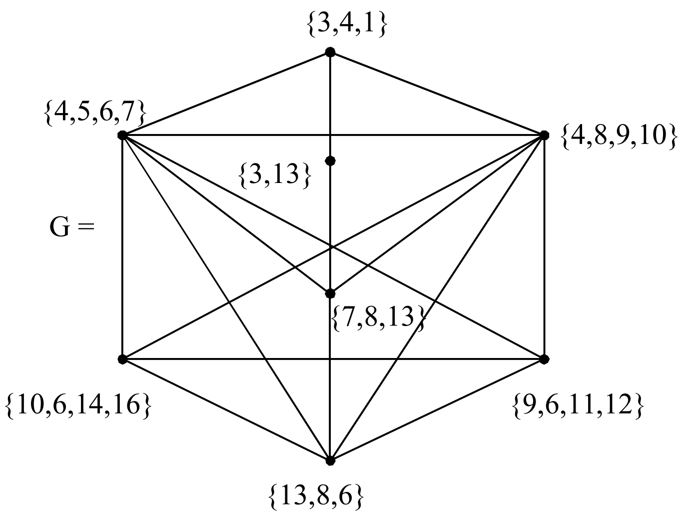


Figure 1.52

i) Find the universal complement subset vertex graph G^C of G.

ii) Can G have a subset vertex hypersubgraph?

iii) Can G have subset vertex complete subgraph?

iv) Can G have subset vertex star subgraph?

v) Find all special features associated with local complements of the subset vertex subgraphs of G.

18. Let S = {1, 2, …., n} be a set of order n(n < ∞) and P(S) be the power set of S.

    i) Find all subset vertex graphs G of type I and the universal complement subset vertex graphs G^C of G which enjoy same structure.

    ii) Find all subset vertex graphs G of type I which has at least one subset vertex subgraph H of G such that G, H and H^C have same structure where H^C is the local complement subset vertex subgraph of H.

    iii) Find all those subset vertex graphs G of type I which has all universal complements which do not enjoy same structure.

19. Prove the concept of universal complement of a subset vertex graph G of type I with vertex set from P(S) can help in the study of social network (by taking G as a social network) and its universal complement G^C.

20. Can problem (19) be studied in case of community networks?

21. Study the influence of local complements of an appropriate subset vertex subgraph of type I of the subset vertex graph G.

22. Can subset vertex hypersubgraphs of a subset vertex graph when adopted in social network or community network will be a better option than the existing ones?

23. Can subset vertex hypersubgraphs of a subset vertex graph be useful in all types of networks which wants to preserve the number of nodes and edges?

24. Let S = {1, 2, …, 18} be a set P(S) be the power set of S.

Let G be a subset vertex graph given by the following figure.

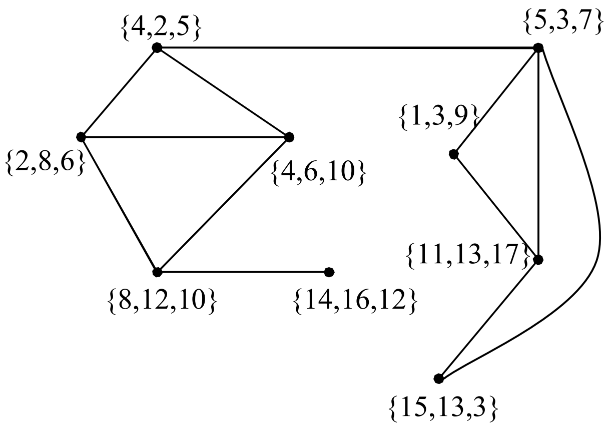


Figure 1.53

i) Find G^C the universal complement subset vertex graph of G.

ii) Adopt this graph in a social networking.

iii) Can we say if used in the community networking the community are linked weakly?

iv) Study G^C of G and test the suitability of G^C in social network / communities networks.

25. What is the role of Freeman index if subset vertex graphs of type I are appropriately used.

Chapter Two

PROJECTIVE SUBSET VERTEX GRAPHS OF TYPE II

We have in [3] introduced the notion of subset vertex graphs of type II, they can be realized as injective subset vertex graphs for if we define an edge between the two subsets {3,7} and {5, 7, 1, 3, 8, 9} by

$$\{3, 7\} \longrightarrow \{5,7, 1, 3, 8, 9\}$$

However if we have subsets {3, 7, 12} and {5, 7, 3, 9, 10, 19} then we cannot have an edge from {3, 7, 12} to {5, 7, 3, 9, 10, 19}.

We for the sake of completeness give some examples of the subset vertex graphs of type II (they are injective subset vertex graphs of type II, but we do not in general use the term injective in this book).

Example 2.1. Let S = { 1, 2, …, 9} be a set of order 9. P(S) be the power set of S. Let G be the subset vertex graph of type II (injective subset vertex graph of type II) given by the following figure.

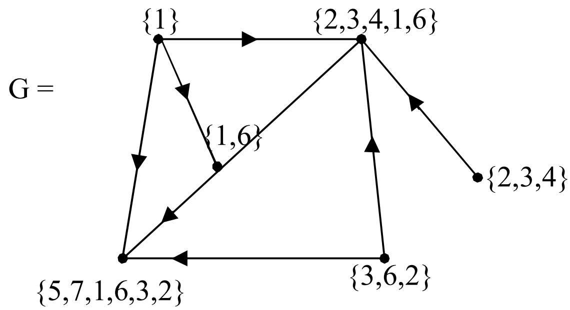


Figure 2.1

Clearly G has six vertices and 7 edges and is a directed graph.

Finding subset vertex subgraphs is a matter of routine, please refer [46].

However for the sake of completeness we provide one or two examples of them.

Let H_1 be the subset vertex subgraph of G given by the follsowing figure.

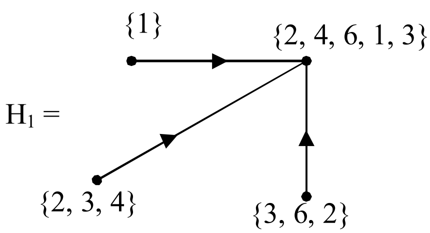


Figure 2.2

Clearly H_1 is a subset vertex subgraph which is a subset vertex star graph.

Let H_2 be the subset vertex graph given by the following figure.

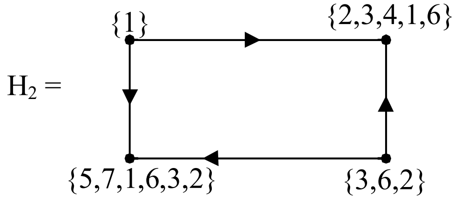


Figure 2.3

Clearly H_2 is a subset vertex subgraph of type II which is a subset vertex circle subgraph of G.

Consider H_3, a subset vertex graph given by the following figure.

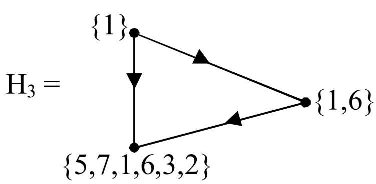


Figure 2.4

H_3 is a subset special complete subgraph of G.

Consider H_4 a subset special subgraph of G given by the following figure.

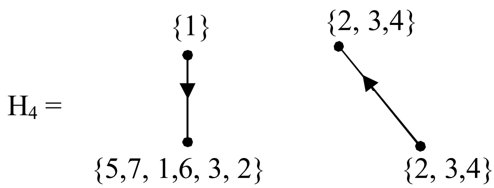


Figure 2.5

Clearly H_4 is a disconnected subset vertex subgraph of G.

Let H_5 be the subset vertex subgraph given by the following figure.

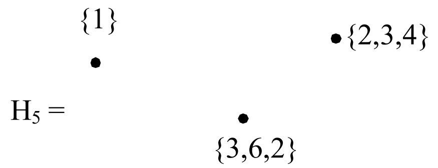


Figure 2.6

Clearly H_5 is a subset vertex empty subgraph of G.

Let H_6 be the subset vertex subgraph given by the following figure.

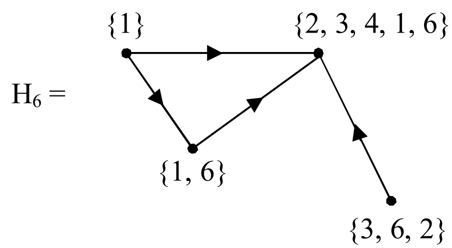


Figure 2.7

Thus G, the subset vertex graph has subset vertex subgraphs which is a star graph, complete graph, empty graph circle graph, disconnected graph and an empty graph.

Example 2.2. Let P(S) be the power set of S where S = {1, 2, …, 15} given by the following figure.

G =

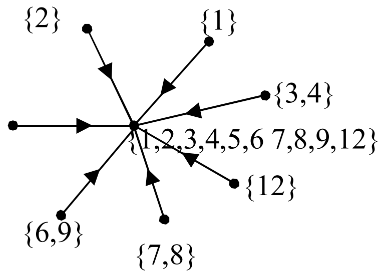


Figure 2.8

Clearly G is subset vertex star graph of type II.

We can have subset vertex subgraphs of G which are describe in the following.

H_1 =

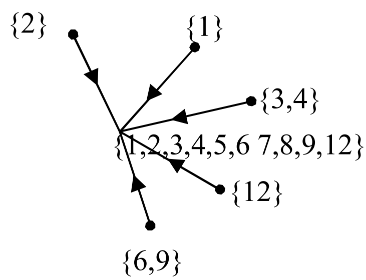


Figure 2.9

H_1 is the subset vertex subgraph of type II of G.

Clearly H_1 is also a subset vertex star subgraph of type II of G.

Let H_2 be the subset vertex subgraph of type II given by the following figure.

H_2 =

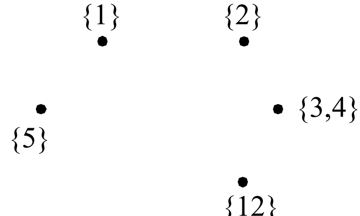


Figure 2.10

Clearly H_2 is a subset vertex empty subgraph of G.

Consider the following projective complete subset graph B of type II given by the following figure.

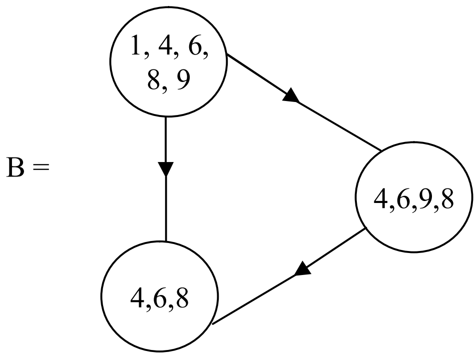


Figure 2.11

Suppose with the same set we get the injective subset graph B′ of type II.

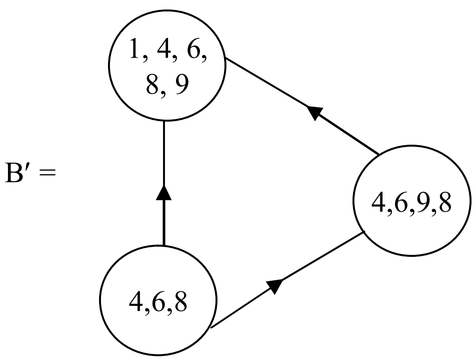


Figure 2.12

Both B and B′ are directed complete subset vertex graphs of type II.

Clearly

B″ =

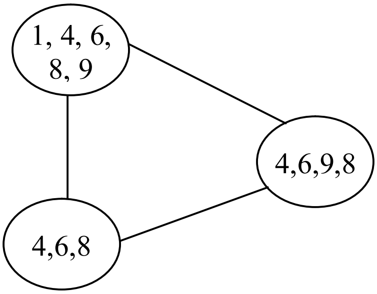


Figure 2.13

is also a subset vertex graph of type II.

Consider

C =

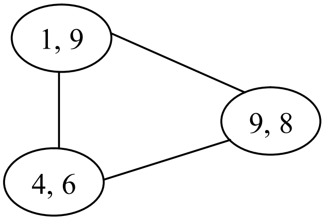


Figure 2.14

is a special subset vertex subgraph of type I.

However both projective and injective subset vertex subgraph of type II is empty given by the following figure;

D =

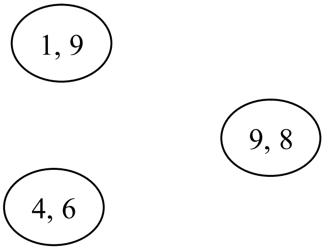


Figure 2.15

Now we find the local complement of C and D relative to B″ and B′ (and B) respectively. The local complement of C with respect to B″ is

$C^C =$

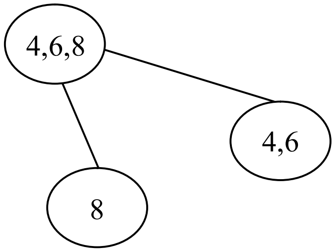


Figure 2.16

C^C is different from both C and B″

$D^C =$

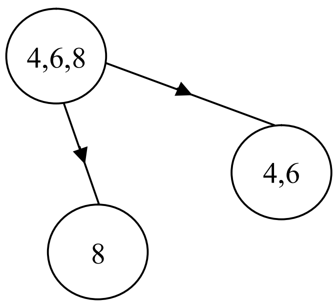


Figure 2.17

D^C is the projective local complement of the empty graph

$P^C =$

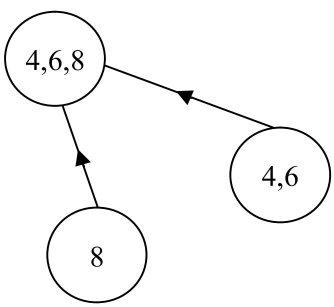


Figure 2.18

P^C is the injective local complement subset vertex sub graph B′.

None of the local complements are complete.

It is important to keep on record that all properties associated with subset vertex graphs of type II dealt in [46] which are injective hold true with appropriate modifications. In fact we have the following theorem.

Theorem 2.1: *Let G be a subset vertex graph of type II injective on the power set P(X) of X.*

Then for every graph G injective subset vertex graph of type II. We have a projective subset vertex graph of type II on the same vertex set only the directions of the edges reversed and vice versa.

Proof is direct and hence left as an exercise to the reader.

We see projective subset vertex graph of type II and injective subset graphs of type II behave in a similar way except for the direction of the arrows.

Further it is pertinent to keep on record that all results of injective subset vertex graphs of type II can be derived with no difficulty in case of projective subset vertex graphs of type II.

Further for a fixed number of vertices the number of directed edges in both projective and injective subsets graphs are the same with a simple difference the direction of each edge is different.

Subgraphs of these also enjoy similar results. Hence we only put forth a few problems for the interested reader.

Problems

1. Find all possible projective subset graphs using the power set of the set B = {I, 1 + I, 2, 0, 1}.

2. If S is a set with n elements and P(S) the power set of S.

   i) Find all projective subset vertex graphs of P(S).

   ii) Hence or otherwise find the number of projective subset vertex graphs of P(S) where the number of elements in S in 9.

3. Let S = {1 + 2I, 3I, 4I, 0, 4, 3 + I, 2, 6, 8}. P(S) be the power set of S.

   i) Find all subset projective graphs got using the power set S.

   ii) Find all subset vertex projective graphs which are complete.

   iii) Find all subset vertex projective graphs which are star graphs.

   iv) Find all subset vertex projective graphs which are circle graphs?

   v) Can we have a subset vertex projective graph which is a wheel?

4. Let $S_1 = \{x_1, \ldots, x_n\}$ be a set of order n ($2 \leq n < \infty$). P(S) the power set of S_1. Study questions (i) to (v) of problem 3 for this S_1

5. Let S = {g + I, g, 2I, 0, 4, 3, 3g + 2I + 7, I} be a set of order 8. Study questions (i) to (v) of problem (3) for this S.

6. Let M be the subset vertex projective graph of type II given by the following figure.

M =

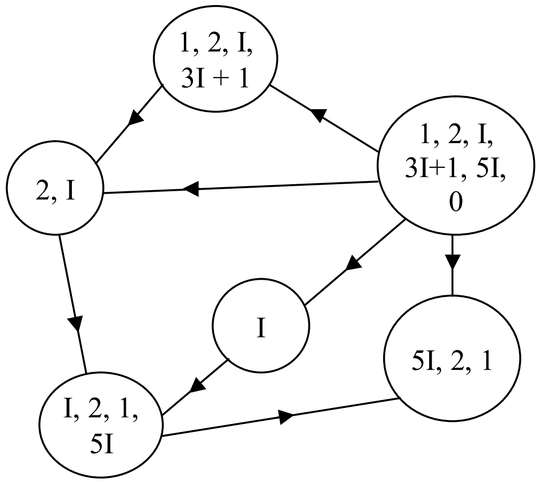


Figure 2.19

i) Can M have subset vertex projective subgraphs which are hyper subgraphs/

ii) Is it possible for M to contain subset vertex projective subgraphs which are complete?

iii) Can M have subset vertex projective subgraph which is a star graph?

iv) Enumerate all subset vertex projective subgraphs of M.

v) Find the subset vertex graph P of type I using the vertex set of M.

vi) Can P be a complete vertex subset graph of type I?

vii) Can P have subgraphs which are complete vertex subset graphs of type I?

viii) Mention all the special features enjoyed by M.

ix) Compare the subset vertex graphs P of type I with the subset vertex projective graph M (they enjoy only the same set of vertices?

x) Which of the graphs P or M has more number of edges?

xi) Can there be subgraphs P or M which have same number of vertices and edges (sans directed edges)?

7. Let W be the star graph which is a subset vertex projective graph of type II given by the following figure

W =

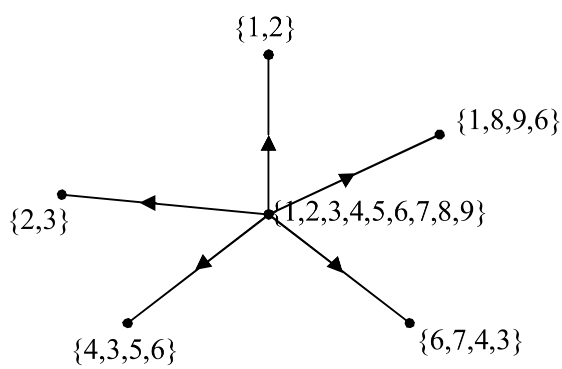


Figure 2.20

i) Using the same set of vertex set find the subset vertex graph V of type I.

ii) Is V a star subset vertex graph of type I?

iii) Is V a subset vertex wheel of type I?

iv) Can V and W have subset vertex subgraphs of type I and type II respectively which are identical except for directedness?

v) Mention the special features enjoyed by V and W.

8. Apply the notion of subset vertex projective graph of type II to some social information network problem.

9. What are the advantages / disadvantages of using subset vertex projective graphs of type II in the place of usual graphs?

10. Can we conclude the Freeman index has no role to play in case of subset vertex projective graphs of type II?

11. What subset vertex graphs type I or type II is preferable for social information networks? (Justify your claim).

12. Enumerate any other special applications of type II subset vertex projective graphs.

13. Can we say use of subset vertex projective graphs in appropriate social information networks can save time?

14. Let W be the subset projective vertex graph of type II given by the following figure:

    i) Does this W have a subset vertex projective subgraph which is complete?

    ii) Can W have a subset vertex projective subgraph which is a star graph?

iii) Obtain any other special feature enjoyed by this W.

W =

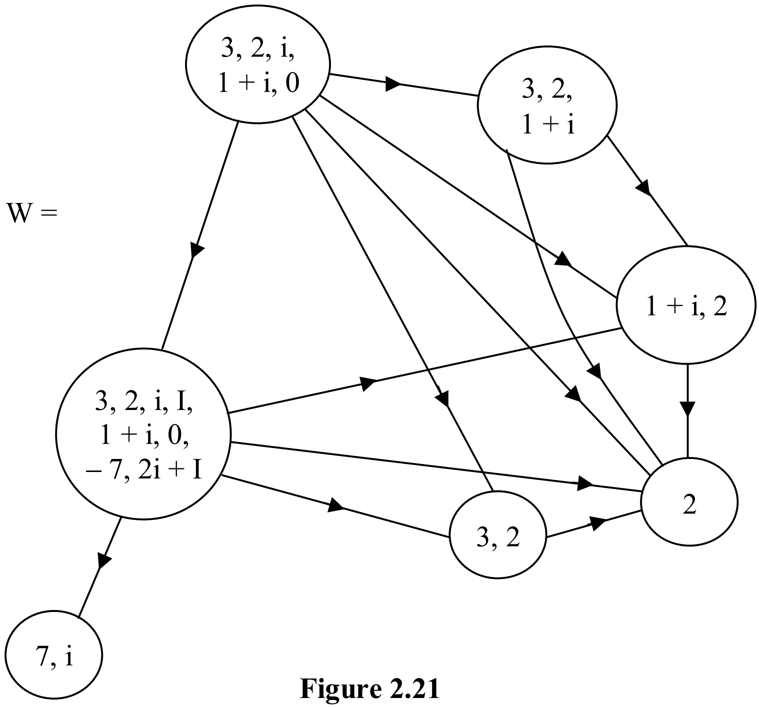


Figure 2.21

15. Can these subset vertex projective graphs of type II find application to study a class V students as a social group? Justify your claim.

16. Compare problem (15) with any other social information network model for class V students.

17. Can these subset vertex projective graphs be used in big data analysis as a social network?

    Justify or substantiate your claim!

18. Compare and contrast the results after applying subset projective vertex graphs of type II and subset vertex graphs of type I in a study of a specific social network problem.

19. Let P(S) be the power set of the set of the set S = {1, I + 3, 3I – 7, 1 – 2I, 2I, 10I, 8I, 4}.

    i) Find all subset vertex projective graphs of type I which has subset vertex project hypergraphs.

    ii) Characterize those subset vertex projective graphs which cannot contain special subset vertex projective subgraphs.

    iii) Find all subset vertex projective graphs which cannot have subset vertex hyper graphs but has special subset vertex subgraphs.

    iv) How many complete subset vertex projective graphs can be constructed using the power set P(S)?

20. V = {{1}, {2, 1}, {1, 2, 3}, {1, 3, 4, 2}, {1, 2, 3, 4, 5}} $\subseteq$ P(S) where S = {1, 2, 3, 4, 5} the vertex subset of the power set P(S).

    i) Find the projective subset vertex graph G of type II using V.

    ii) Find the subset vertex graph H of type I using this V.

    iii) Can the graphs G and H have special subset vertex subgraphs? Justify your claim.

    iv) Can G and H have subset vertex hyper graphs?

v) Obtain any other special feature enjoyed by G and H.

21. Let K be the subset vertex projective graph of type II given by the following figure.

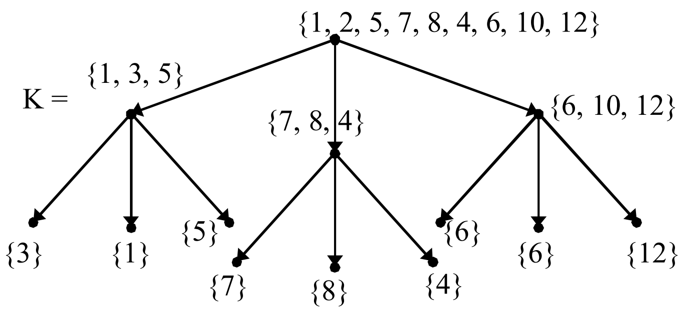


Figure 2.22

i) What type of the graph is K?

ii) What is the universal complement of K? (vertex set is from P(S), S = {1, 2, …, 12})

iii) Is the universal complement structure preserving?

iv) Find a subset vertex subgraph of K and find its local complement.

v) Can K have special subset vertex subgraph which is a hyper graph? Justify.

22. Let S = {1, 2, …, 20} be the set P(S) be the power set of S.

    i) Find all subset vertex projective binary trees.

    ii) Find all subset vertex projective trees using vertex subsets of P(S).

    iv) Find all subset vertex projective graphs which are not complete.

23. If P(S) is used as the vertex subset what is the subset vertex projective graph associated with the vertex set as P(S).

24. What is the vertex subset graph which vertex set W = {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}} $\subseteq$ P(S) where S = {1, 2, 3, …, 7}?

25. Let V_1 = {{1, 2, 3}, {2, 3}, {1}, {3}, {1, 2, 3, 4, 5}, {1, 3}, {5, 4}, {5}} $\subseteq$ P(S) be the vertex subset associated with the vertex subset projective graph G.

    i) How many edges are there in G?

    ii) Find the universal complement of G.

    iii) Find all subset vertex subgraphs of G and their local complements.

    iv) If the vertex subset of P(S) is V = {{7} $\subseteq$ {7, 6} $\subseteq$ {7, 6, 5} $\subseteq$ {7, 6, 5, 4} $\subseteq$ {7, 6, 5, 4, 3} $\subseteq$ {7, 5, 4, 6, 3, 2} $\subseteq$ {1, 2, 3, …, 7}} then what is the structure of the graph G associated with V.

26. Let S = {I, 2I, 8I – 3, 4, 2, 0, 9} be a set P(S) the power set of S.

i) Find all subset vertex projective graphs which can be obtained using subsets of P(S).

ii) Find all universal complements of graphs in (i).

iii) Show the graphs in (i) and (ii) are the same.

27. Let S = $\{a_1, \ldots, a_{18}\}$ be a set of order 18. P(S) the power set of S. Let M be the subset vertex projective graph of type II given by the following figure.

M =

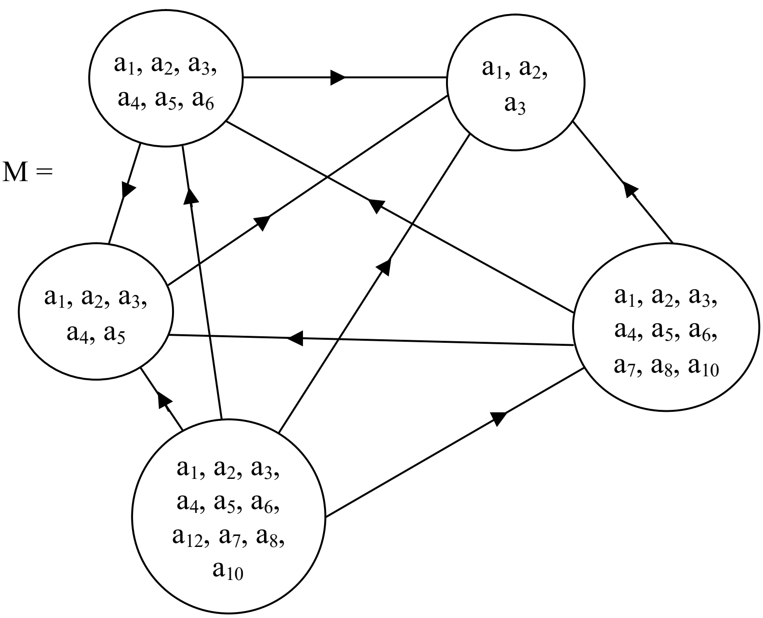


Figure 2.23

i) Find all special subset vertex projective subgraphs of M which are complete.

ii) Find the universal complement of M. Is it complete.

iii) Find all special subset vertex projective subgraphs of M and their universal complement.

If B =

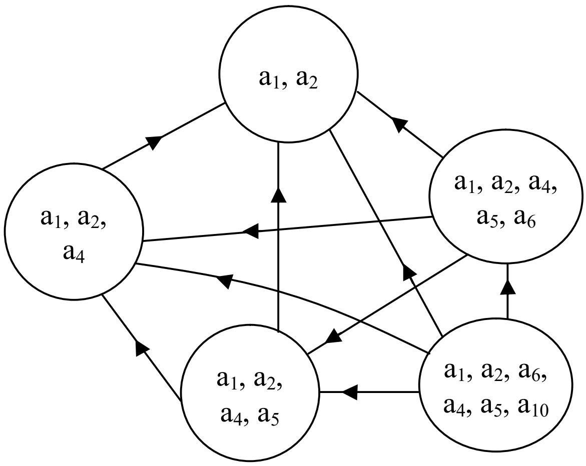


Figure 2.24

iv) Find the local complement of B relative to M.

v) Can these special subset vertex projective subgraphs which are structure preserving be used in fault tolerance networks? Justify / substantial.

vi) Obtain some special applications of these as they are special subset vertex subgraphs which preserve the structure of the initial network.

vii) Find the applications in social information network where are needs this property.

viii) Apply this concept in general networks which can function such that it is economically the best.

Chapter Three

SPECIAL SUBGRAPHS OF THE SUBSET VERTEX GRAPHS

In this chapter we describe a special type of subgraphs of subset vertex graphs which we define as special subset vertex subgraphs of the vertex subset graph. We will illustrate first this situation by some examples before we make the relevant definition and the properties associated with them. For properties of subset vertex graphs please refer [46].

Throughout this chapter P(S) will denote the power set of the finite set S (S can also be taken as infinite). In the results we have to make some appropriate modifications when $|S| = \infty$.

Example 3.1. Let S = {1, 2, 3, 4, 5} be the set of order five; P(S) = {The collection of all subsets of S including S and ϕ, the empty set} = power set of S.

Let G be the subset vertex graph given by the following figure.

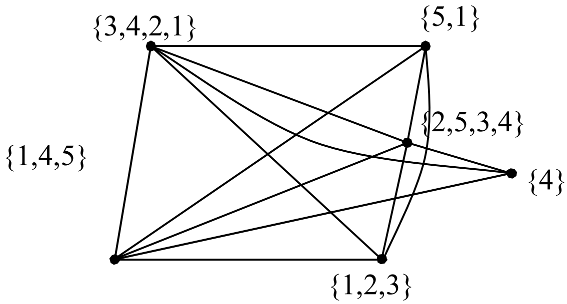


Figure 3.1

Clearly G has 6 vertices and 13 edges.

We now find all subsets of the 6 vertices. If the vertex has only one element it is taken.

Now we find all graphs with six vertices which have subsets of the six vertex but the number of edges will vary.

We call these subset vertex subgraphs the special subset vertex subgraphs of G.

Just we give the special subset vertex subgraphs of G in the following.

The proper subsets of {1, 2, 3, 4} are {{1}, {2}, {3}, {4}, {1, 2}, {1, 3}, {1, 4}, {2, 3}, {2, 4}, {4, 3}, {1, 2, 3}, {1, 2, 4}, {1, 3, 4}, {2, 3, 4}}.

Now we can have only 12 special subset vertex subgraphs where all the other vertices remain the same, we do not take the subsets {1, 2, 3} and {4} of the set {3, 4, 2, 1} as per definition we need to have six distinct vertices.

H_1 =

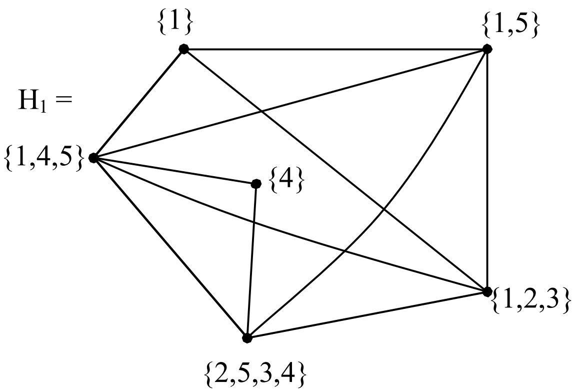


Figure 3.2

We see this special subset vertex subgraph has six vertices as that of G but only 11 edges.

H_2 =

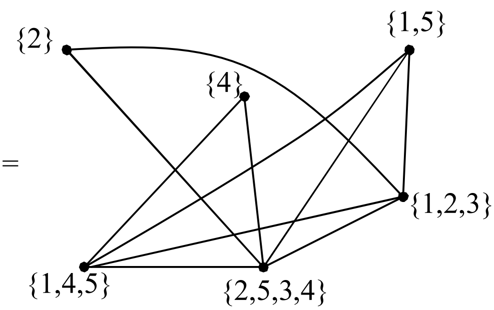


Figure 3.3

Clearly this special subset vertex subgraph H_2 has only 10 edges. We see H_1 and H_2 are different.

Now we take the vertex {3} as the subset of {1,2 3, 4}.

The special subset vertex subgraph H_3 with subset vertex {3} is as follows.

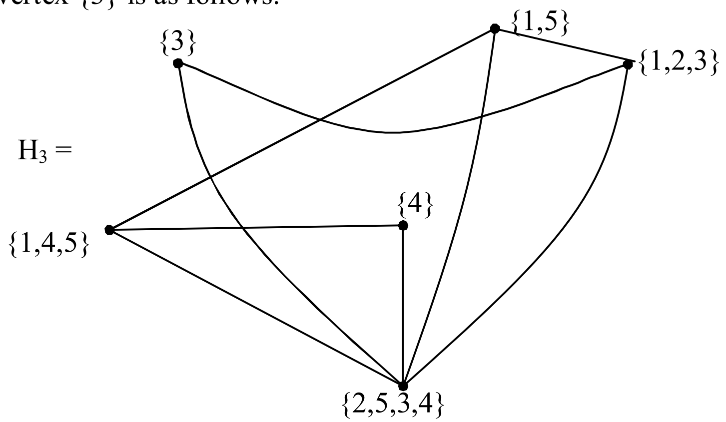


Figure 3.4

We see H_3 the subset vertex special subgraph has only 9 edges.Let H_4 be the special subset vertex subgraph with {1, 2} as the new vertex given by the following figure:

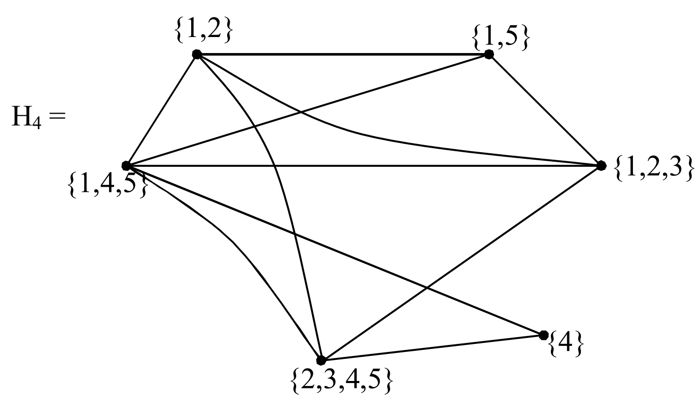


Figure 3.5

Clearly H_4 has only 11 edges and H_4 is different from H_1, H_2 and H_5.

Let H_5 be the special subset vertex subgraph of G given by the following figure.

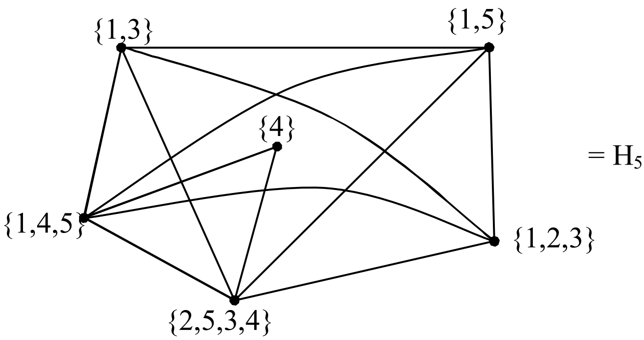


Figure 3.6

We see H_5 has 12 edges.

Let H_6 be the special subset vertex subgraph of G given by the following figure.

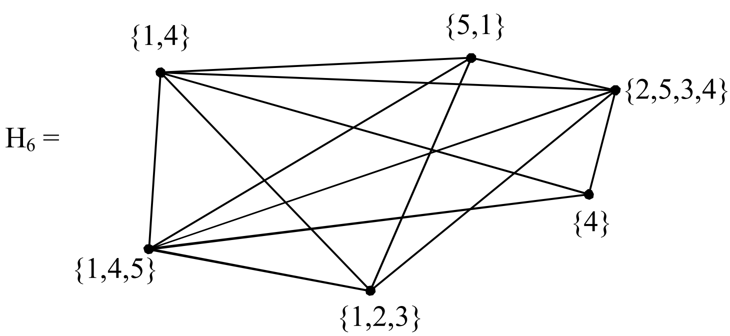


Figure 3.7

H_6 also has 13 edges and $\{1, 4\} \subseteq \{1, 2, 3, 4\}$.

H_6 has same number of edges as G we call such subgraphs as special hypersubset vertex subgraph of G when both the subset vertex graph and the special subset vertex subgraph has same number of edges.

Consider H_7 the special subset vertex subgraph with vertex {2, 3} given by the following figure.

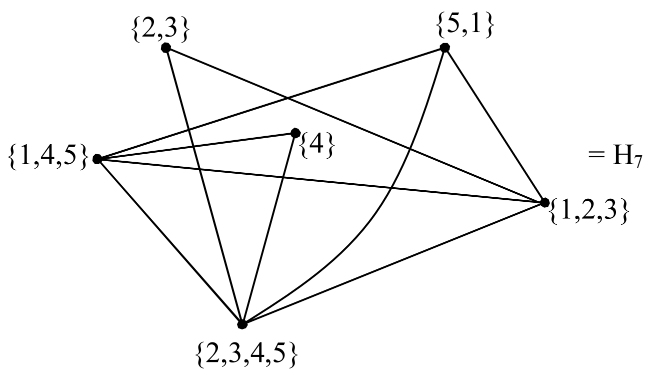


Figure 3.8

Clearly H_7 has only 10 edges, it is not a special hyper subset vertex subgraph of G.

Let H_8 be the special subset vertex subgraph of G with vertex {2,4} given by the following figure.

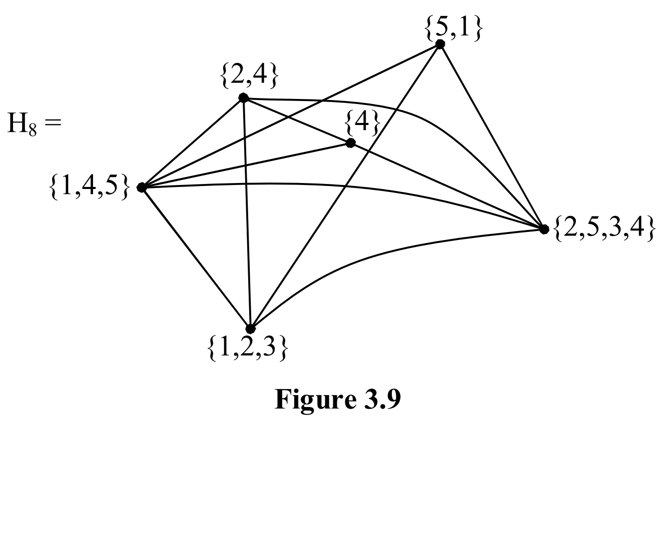


Figure 3.9

This has 12 edges and is not a special hyper subset vertex subgraph of G.

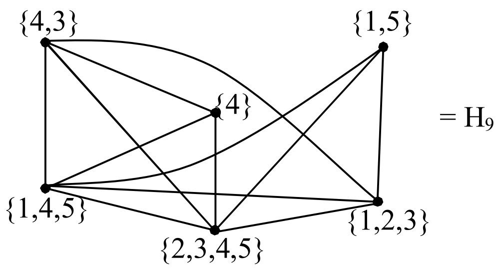


Figure 3.10

H_9 is the special subset vertex subgraph with vertex weight {4, 3}. This H_9 also has only 12 edges.

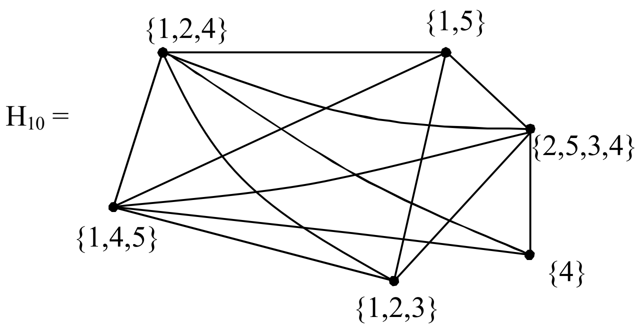


Figure 3.11

We see H_{10} is the special subset vertex subgraph with {1,2,4} as its vertex, which has 13 edges. Hence H_{10} is a special hyper subset vertex subgraph of G.

Let H_{11} be the special subset vertex subgraph with vertex {1, 3, 4}. The graph H_{11} is as follows.

$H_{11} =$

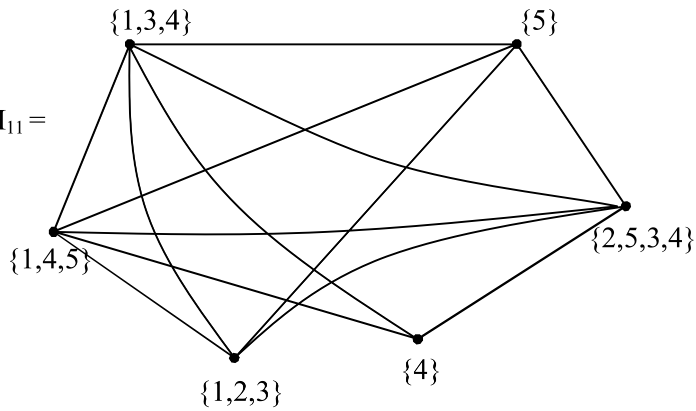


Figure 3.12

Clearly H_{11} is also a special subset vertex hyper subgraph of G.

Let H_{12} be the special subset vertex subgraph with vertex $\{2, 3, 4\} \in \{1, 2, 3, 4\}$

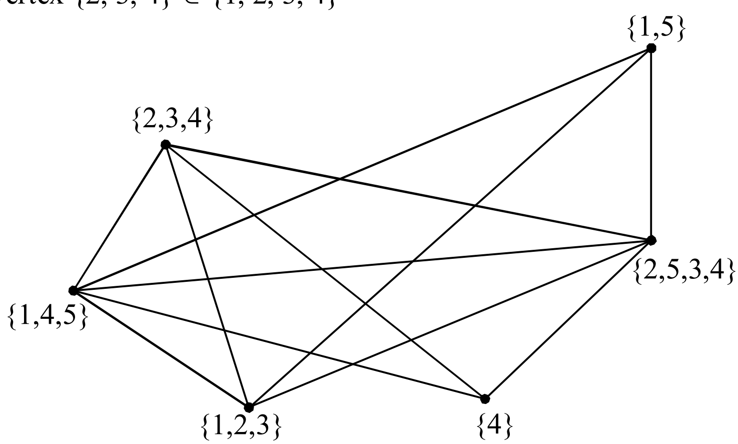


Figure 3.13

The special subset vertex subgraph H_{12} has only 12 edges so is not a hyper special subset vertex subgraph of G.

Now we get all the subsets of {1, 5}, ({1, 5}) = {{1}, {5}}, ({1, 5}) = {{1}, {5}}.

The special subset vertex subgraph with {1} as its vertex set be K_1, which is as follows.

$K_1 =$

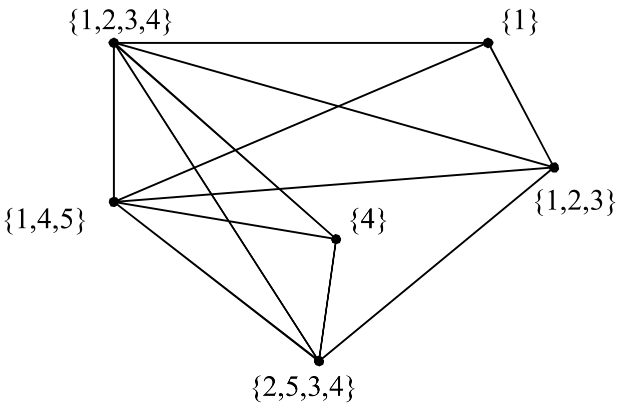


Figure 3.14

K_1 also has only 12 edges.

Let K_2 be the special subset vertex subgraph with {5} as its vertex.

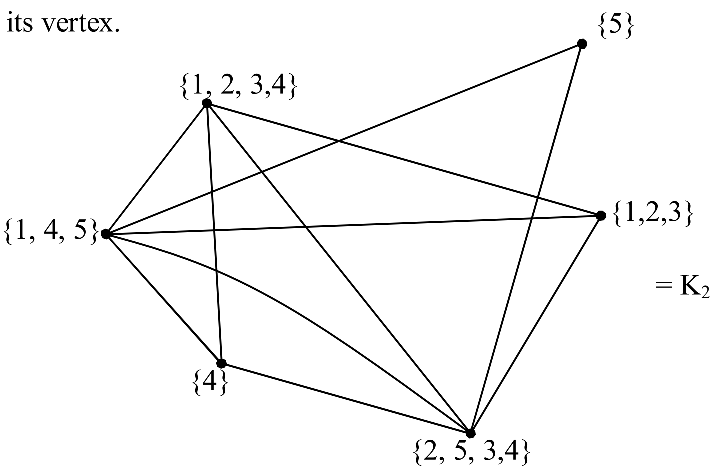


Figure 3.15

K_2 is only a special subset vertex subgraph which is not a hypergraph.

Now consider the subsets of {1, 4, 5} which is as follows: {{1}, {4}, {5}, {1,4}, {1,5}, {4, 5}}.

The only subsets which can yield nontrivial or special subset vertex subgraphs are {1}, {5}, {1, 4} and {4, 5} only.

Let P_1 be the special subset vertex subgraph with vertex {1}.

P_1 =

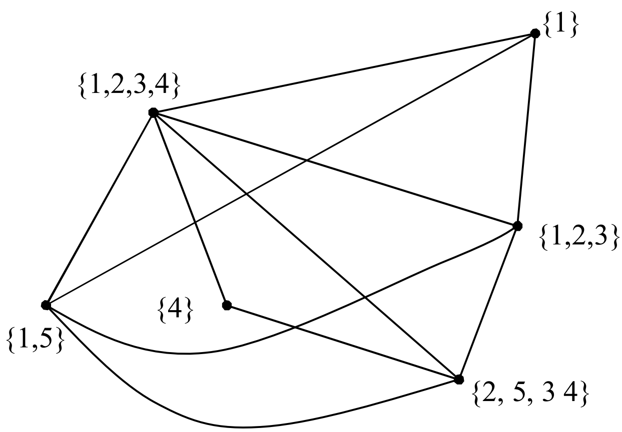


Figure 3.16

P_1 is a special subset vertex subgraph with 10 edges.

Let P_2 be the special vertex subset subgraph with vertex {5} $\subseteq$ {1, 4, 5}.

$P_2 =$

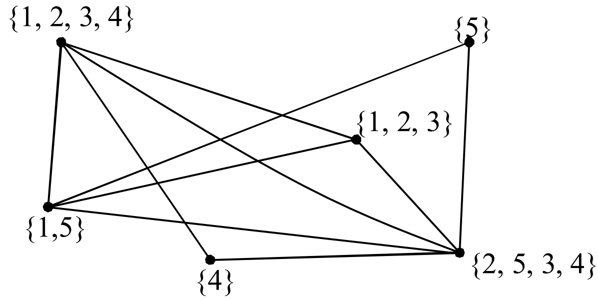


Figure 3.17

P_2 the special vertex subset subgraph has only 10 edges.

Let P_3 be special vertex subset subgraph with vertex.

$P_3 =$

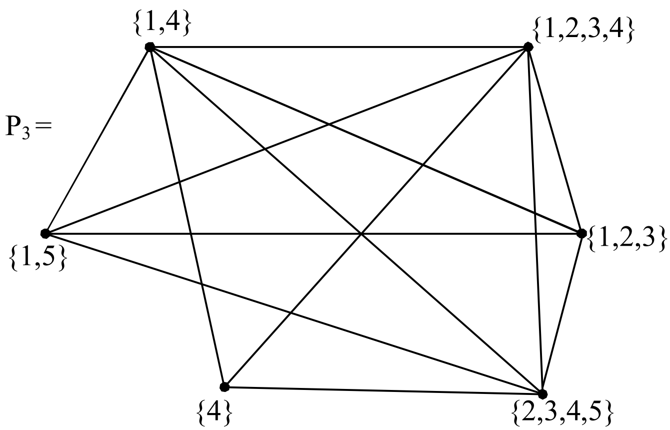


Figure 3.18

Clearly P_3 is a special subset vertex subgraph with 13 edges, hence P_3 is a special subset vertex hyper subgraph of G.

Let P_4 be the special subset vertex subgraph with vertex {4, 5}. The figure is as follows.

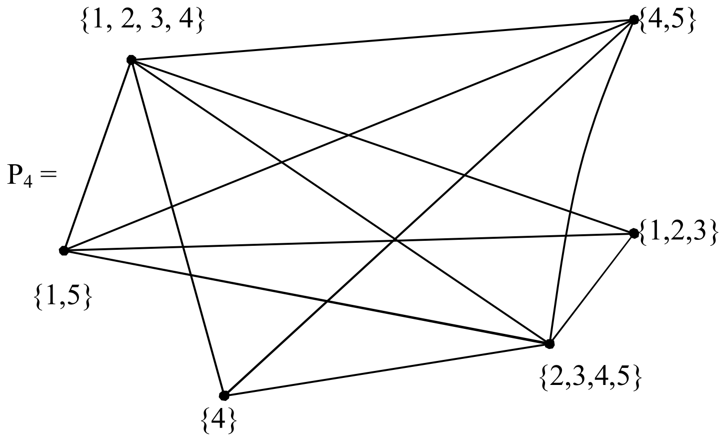


Figure 3.19

This special subset vertex subgraphs P_3 has only 12 edges.

Now the subsets of the vertex set {1, 4, 5} contributes to only one special hyper subset vertex subgraph of G.

Consider the subsets of the vertex set {1, 2, 3} is {{1}, {2}, {3}, {1, 2}, [1, 3}, {3, 2}}.

We can have six special subset vertex subgraphs using subsets of {1, 2, 3}.

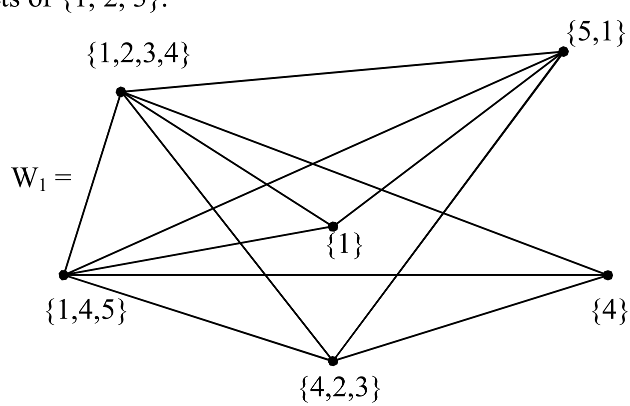


Figure 3.20

Let W_1 be the special subset vertex subgraph of G. Clearly W_1 is not a hyper subgraph.

Let W_2 be the special subset vertex subgraph with {2} $\subseteq$ {1, 2, 3} as the vertex set which is given by the following figure.

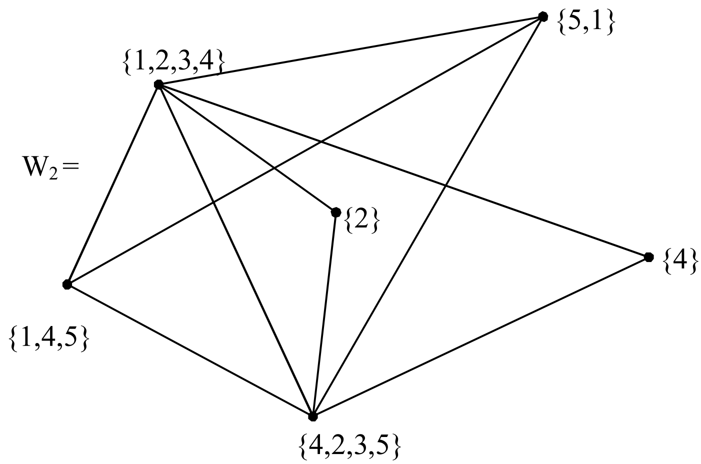


Figure 3.21

Clearly W_2 is only a special subset vertex subgraph which is not a hyper subgraph.

Let W_3 be the special subset vertex subgraph of G with vertex set {3}; given by the following figure.

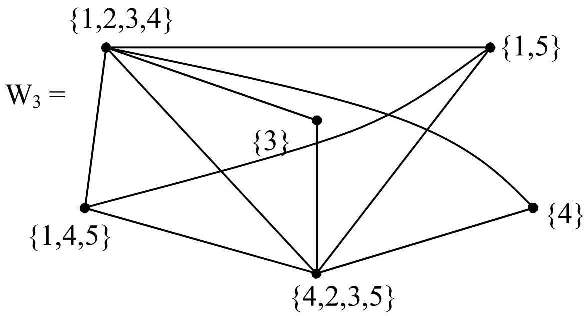


Figure 3.22

This W_3 is not a hyper special subset vertex subgraph as the edges of W_3 is only 10.

Let W_4 be the special subset vertex subgraph with vertex set $\{1,2\} \subseteq \{1, 2, 3\}$. The figure of W_4 is as follows.

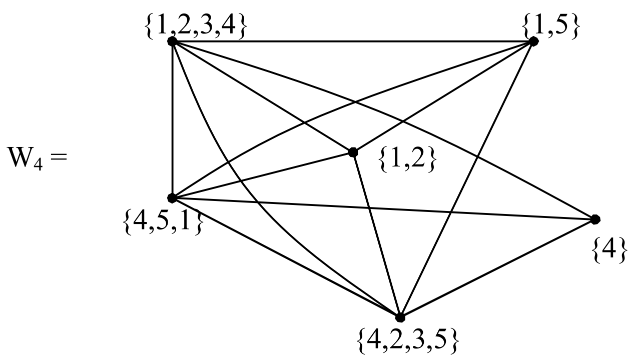


Figure 3.23

W_4 has 13 edges so W_4 is a special subset vertex hyper subgraph of G.

Let W_5 be the special subset vertex subgraph with vertex set {1, 3} given by the following figure.

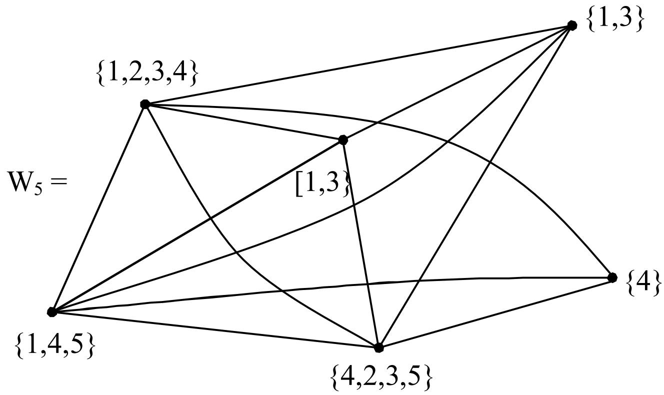


Figure 3.24

Clearly W_5 is special subset vertex subgraphs which is also a hyper special subset vertex subgraph of G.

Let W_6 be the special subset vertex subgraph of G with vertex set {2, 3}.

The graph of W_6 is as follows.

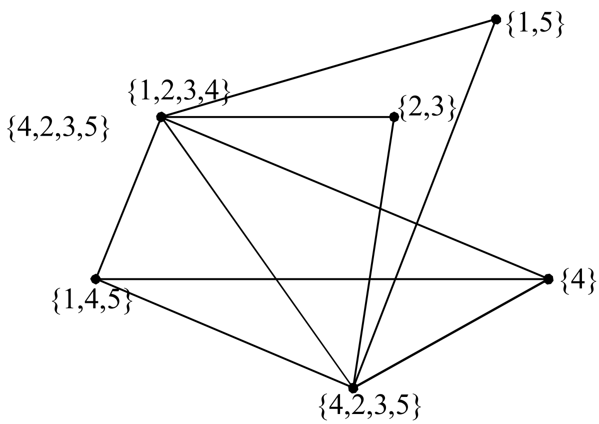


Figure 3.25

We see W_6 is only a special subset vertex subgraph of G which is not a hyper special subgraph of G. W_6 has only 10 edge.

Now consider the subsets of {4, 2, 3, 5}, {{4}, {2}, {3}, {5}, {4, 2}, {4, 3}, {4, 5}, {2, 3}, {2, 5}, {3, 5}, {4, 2, 3}, {4, 2, 5}, {4, 3, 5}, {2, 3, 5}}. Barring the subset {4} all other vertex sets are valid to contribute to special subset vertex subgraphs.

Let S_1 be the special subset vertex subgraph with {2} as its vertex. The figure of S_1 is as follows.

S_1 =

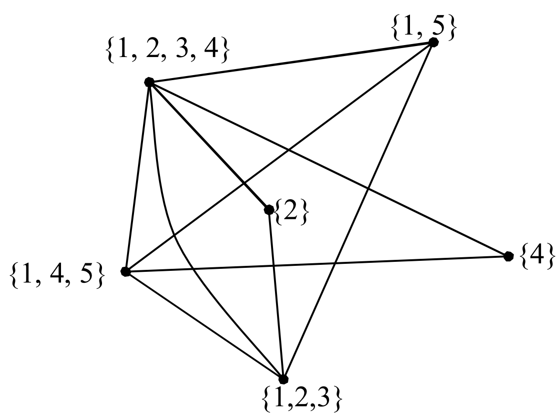


Figure 3.26

S_1 has only 10 edges so S_1 is only a special subset vertex subgraph of G.

Let S_2 be the special subset vertex subgraph with vertex set {3} given by the following figure.

S$_2$ =

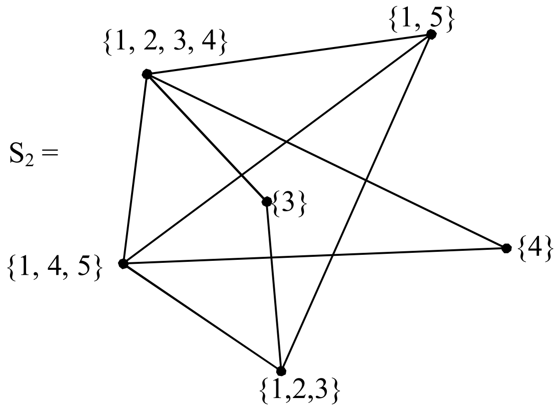


Figure 3.27

Clearly S_2 is only a special subset vertex subgraph which is not a hyper special vertex subgrpah of G.

Let S_3 be the special vertex subset subgraph with vertex set {5} is as follows.

S$_3$ =

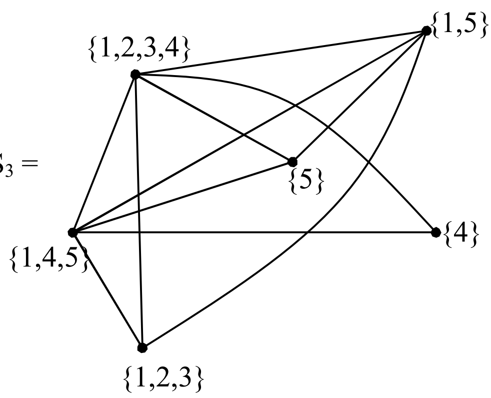


Figure 3.28

Clearly S_3 is also a special subset vertex subgraph which is not a hypersubgraph of G.

Let S_4 be the special subset vertex subgraph with vertex set {4, 2} which is given by the following figure.

$S_4 =$

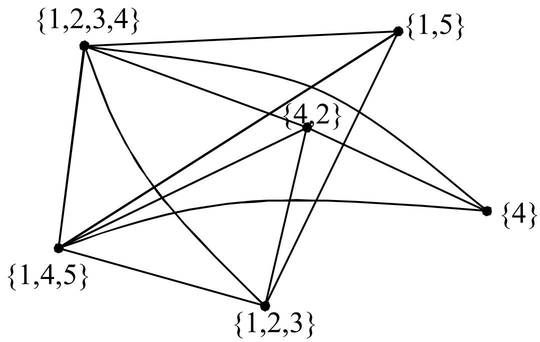


Figure 3.29

Clearly S_4 is not a hyper special subset vertex subgraph only a special subset vertex subgraph of G.

Now let S_5 be the special subset vertex subgraph with vertex set {4, 3} which is given in the following figure.

$S_5 =$

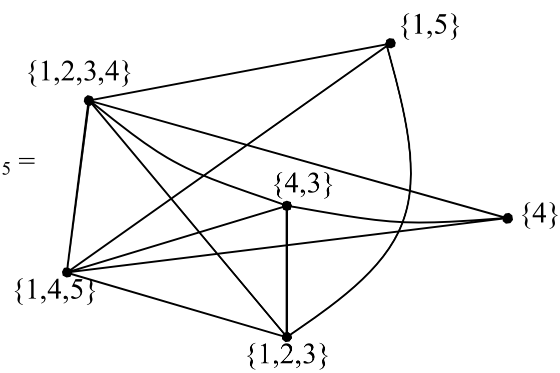


Figure 3.30

Clearly S_5 is not a hyper special vertex subset subgraph of G.

Let S_6 be the special subset vertex subgraph with vertex set {4, 5}

$S_6 =$

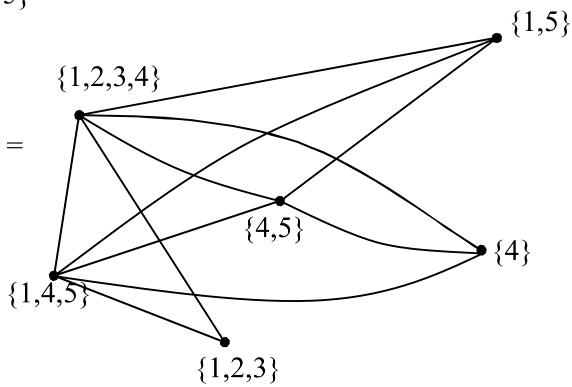


Figure 3.31

This S_6 has only 12 edges so S_6 is a hyper special subset vertex subgraph of G.

Let S_7 be the special subset vertex subgraph of G with vertex set {2, 3} which is given by the following figure.

$S_7 =$

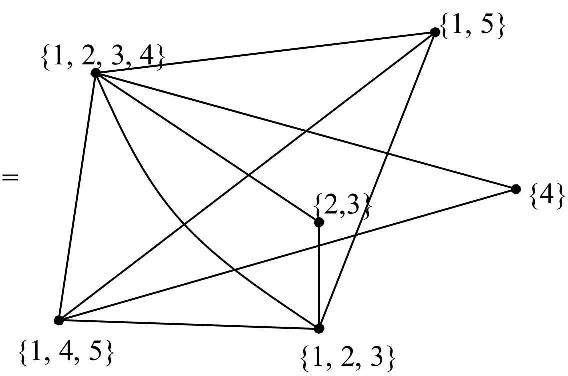


Figure 3.32

S_7 is only a special subset vertex subgraph of G which is not hyper.

Let S_8 be the special subset vertex subgraph of G with vertex set {2, 5} given by the following figure.

S_8 =

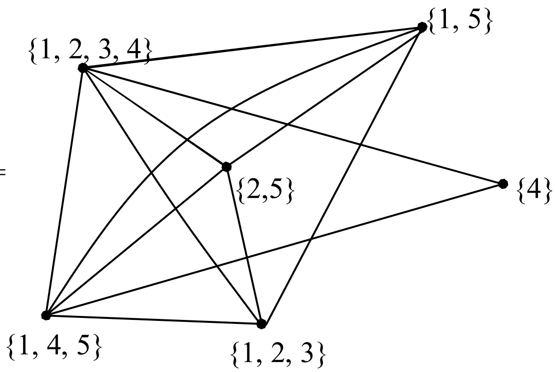


Figure 3.33

This S_8 is only a special subset vertex subgraph of G which is not hyper as it has only 12 edges.

Let S_9 be the special subset vertex graph with vertex set {3, 5} given by the following figure.

S_9 =

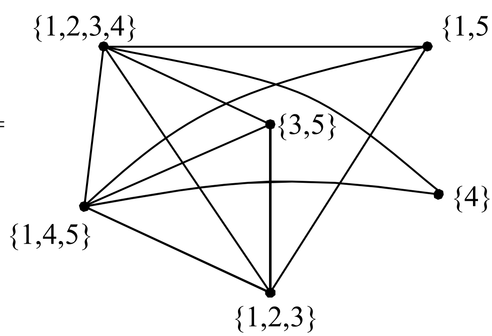


Figure 3.34

S_9 is a special subset vertex subgraph with 12 edges so is not hyper.

Let S_{10} be the special subset vertex subgraph with vertex set {4, 2, 3} which is given in the following figure.

$S_{10} =$

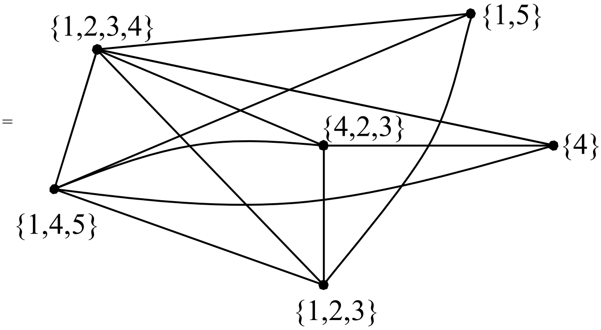


Figure 3.35

S_{10} is only a special subset vertex subgraph of G as it has only 12 edges.

Let S_{11} be the special subset vertex subgraph with vertex set {4, 2, 5} given by the following figure.

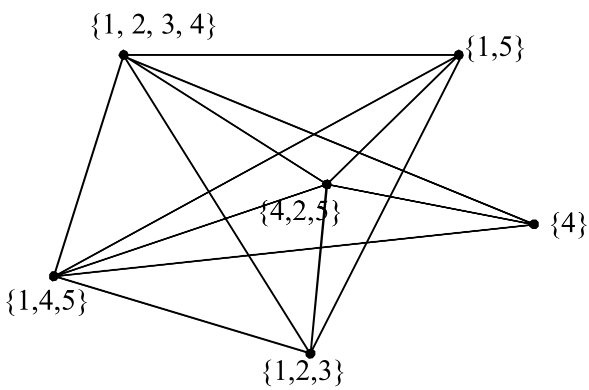


Figure 3.36

Clearly S_{11} has 13 edges so is a special subset vertex hyper subgraph of G.

Consider the vertex set {4, 3, 5}. Let S_{12} be the special subset vertex subgraph of G with this vertex.

The graph of S_{12} is as follows.

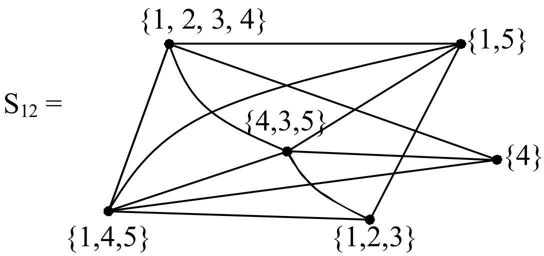


Figure 3.37

S_{12} is a special subset vertex hyper subgraph of G as S_{12} has 13 edges.

Finally let S_{13} be the special subset vertex subgraph of G which is given by the following figure

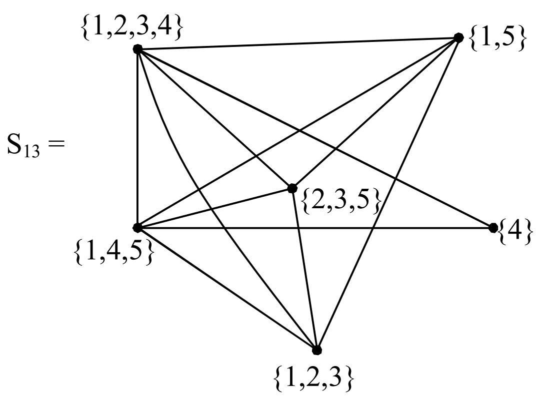


Figure 3.38

S_{13} has only 12 edges so S_{13} is a special subset vertex subgraph of G which is not a special hyper subgraph.

The reader is left with the task of finding all the (i) special subset vertex subgraphs of G (ii) All special subset vertex hypersubgraphs of G.

Thus we see finding for any given subset vertex graph G the special subset vertex subgraphs happens to be challenging one. Further finding the special subset vertex hypergraphs is an interesting one.

We will illustrate by one more example before we proceed onto define the above mentioned concepts.

Example 3.2. Let S = {1, 2, 3, 4, 5, …, 9} be the given set; P(S) be the power set of S. Let G = {V, E} be the subset vertex graph of type I with vertex set from P(S) given by the following figure.

G =

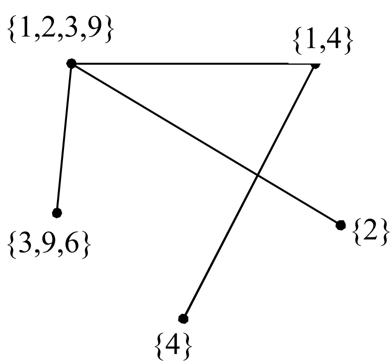


Figure 3.39

The subset vertex graph of type I; G has only 4 edges. The special subset vertex subgraphs of G are as follows.

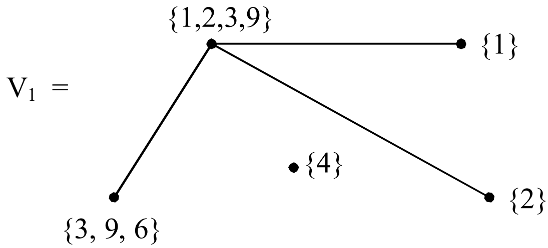


Figure 3.40

Clearly V_1 is a special subset vertex subgraph which is not hyper special subset vertex subgraphs of G as V_1 has only 3 edges.

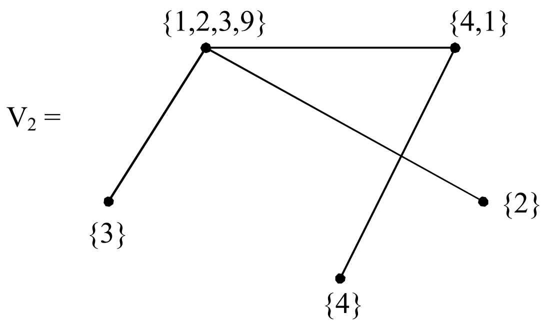


Figure 3.41

V_2 is a special subset vertex subgraph of G which has four edges, hence V_2 is a special subset vertex hyper subgraph of G.

Let V_3 be a subgraph given in the following:

V_3 =

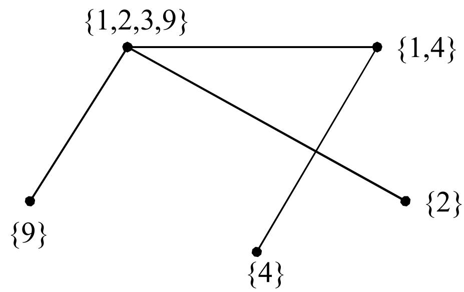


Figure 3.42

V_3 is a special subset vertex subgraph of G which is a hyper subgraph of G.

Let V_4 be the special subset vertex subgraph of G given by the following figure.

V_4 =

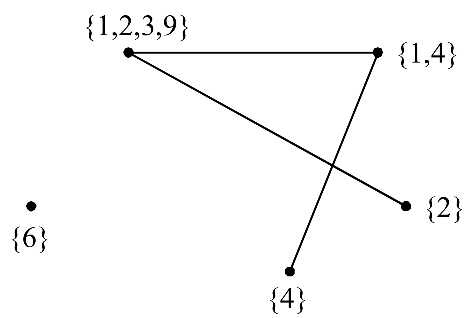


Figure 3.43

Clearly V_4 is a special subset vertex subgraph with only 3 edges so is not a hyper special subgraph of G.

Let V_5 be the special subset vertex graph with vertex set {3, 6} given by the following figure.

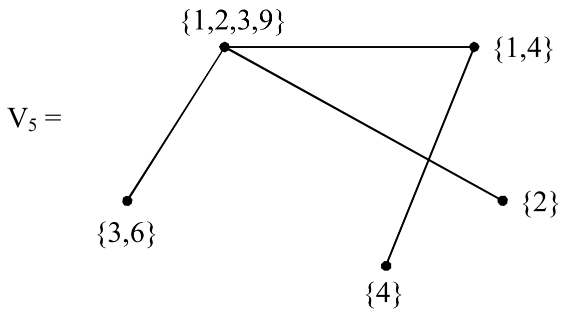


Figure 3.44

We see V_5 is a special subset vertex subgraph of G with four edges hence a special subset vertex hyper subgraph of G.

Let V_6 be the special vertex subset subgraph with vertex set {3, 9} given by the following figure.

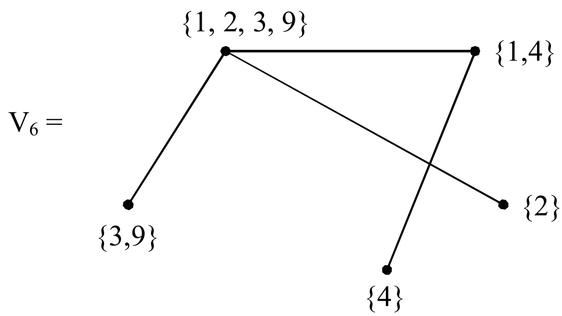


Figure 3.45

V_6 has 4 edges hence V_6 is a special vertex subset hyper subgraph of G.

Let V_7 be the special subset vertex subgraph with vertex set {9, 6} given by the following figure.

$V_7 =$

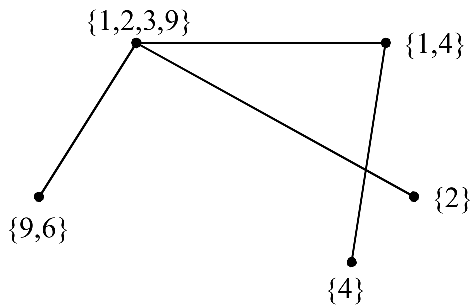


Figure 3.46

V_7 has 4 edges hence V_7 is again a special subset vertex hyper subgraph of G.

We see V_7, V_6, V_3, V_2 and V_5 the special subset vertex subgraph of G are also special subset vertex hyper subgraphs of G.

V_1 and V_4 are not hyper subgraphs of G.

Now we find the special subset vertex subgraphs of G contributed by the subsets of {1, 2, 3, 9}.

Let V_8 be the special subset vertex subgraph with vertex set {1} given by the following figure.

$V_8 =$

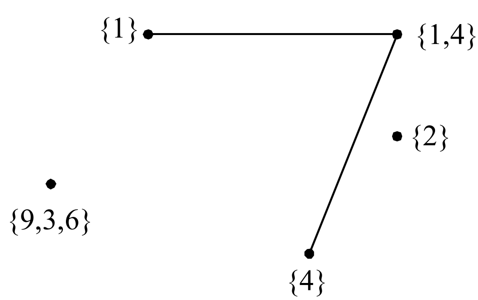


Figure 3.47

V_8 is only special subset vertex subgraph as V_8 has only 2 edges.

Let be the special vertex subset subgraph with vertex set {3} given by the following figure

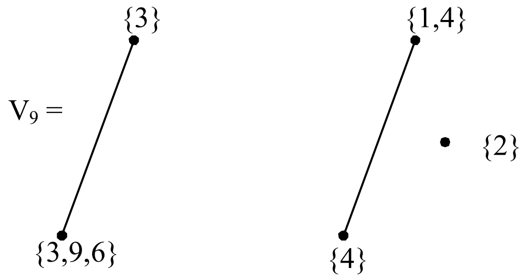


Figure 3.48

Clearly V_9 is only a special vertex subset sub graph with 2 edges.

Let V_{10} be the special subset vertex subgraph of G with vertex set {9} given by the following figure.

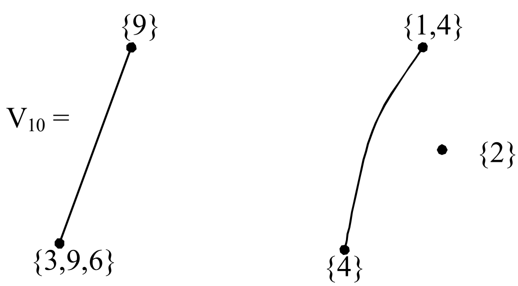


Figure 3.49

The special subset vertex subgraph V_{10} has only two edges so V_{10} is not a hyper subgraph of G. Let V_{11} be the special subset vertex subgraph with vertex set {1, 2} given by the following figure.

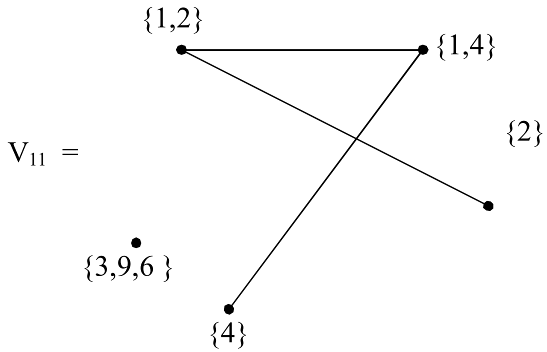


Figure 3.50

Clearly V_{11} is only a special subset vertex subgraph with three edges.

Let V_{12} be the special subset vertex subgraph with vertex set {1, 9} given by the following figure.

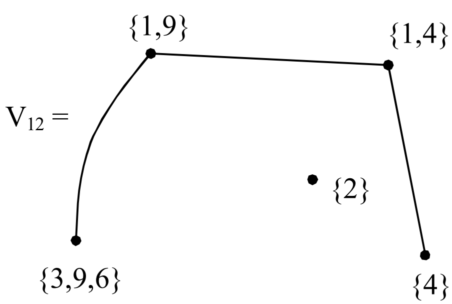


Figure 3.51

Clearly V_{12} has only 3 edges so V_{12} is not a special subset vertex hyper subgraph of G.

Let V_{13} be the special subset vertex subgraph with vertex set {1, 3} given by the following figure.

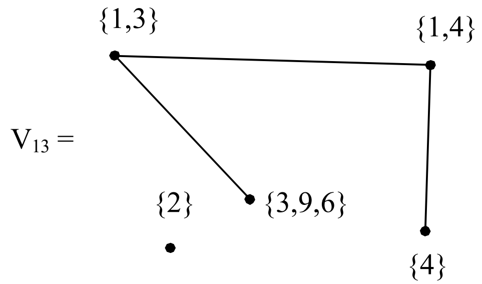


Figure 3.52

V_{13} has only 3 edges so V_{13} is a special subset vertex subgraph of G which is not a hyper subgraph of G.

Let V_{14} be the special subset vertex subgraph with vertex set {9, 3} given by the following figure.

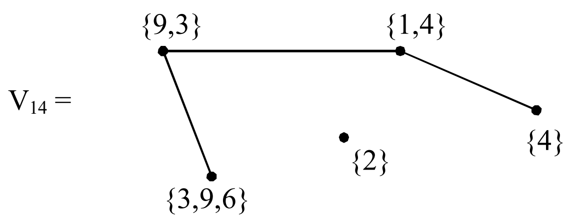


Figure 3.53

V_{14} has only 3 edges are V_{14} is not a special vertex subset hyper subgraph of G.

Let V_{15} be the special vertex subset subgraph with vertex set {1, 2, 3} is given by the following figure.

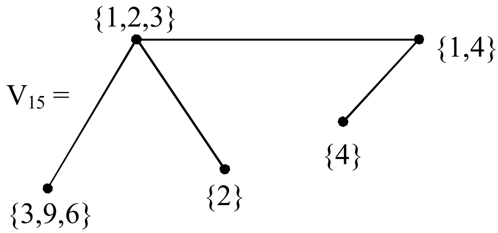


Figure 3.54

Clearly V_{15} has four edges so V_{15} is a special subset vertex subset hyper subgraph of G.

Let V_{16} be the special vertex subset subgraph with vertex set {2, 3, 9} given by the following figure.

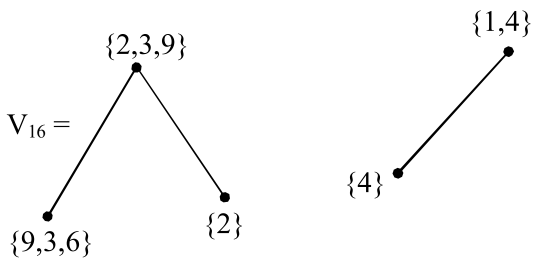


Figure 3.55

Clearly as V_{16} has only 3 edges V_6 is not a special subset vertex hyper subgraph of G.

Now having seen the special subset vertex subgraphs of G we proceed onto study the major question does there exist a subset vertex graph of type I which has no special subset vertex hyper graphs by the following example.

Example 3.3. Let G be the subset vertex graph of type I with vertex set from P(S) where S = {1, 2, 3, …, 9}

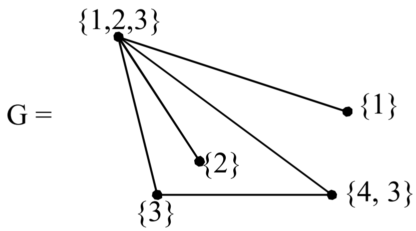


Figure 3.56

The special subset vertex subgraphs of G are

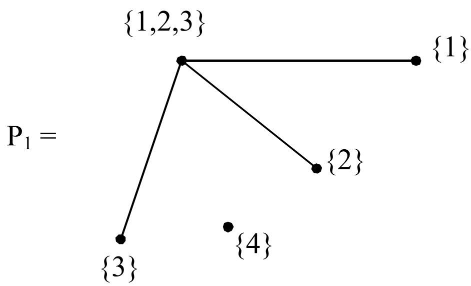


Figure 3.57

Clearly P_1 is not a hyper subgraph as P_1 has only 3 edges, so P_1 is a special subgraph vertex subgraph of G.

Let P_2 be the special subset vertex subgraph with vertex set {1, 2} given in the following figure

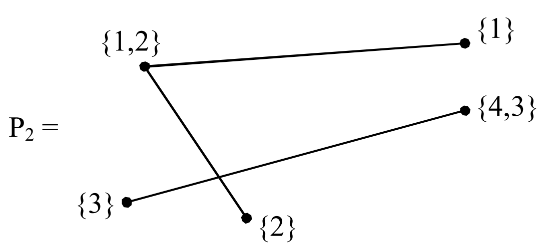


Figure 3.58

Clearly P_2 is not a special subset vertex hyper subgraph of G.

Let P_3 be the special subset vertex subgraph with vertex set {1, 3} given by the following figure.

P_3 =

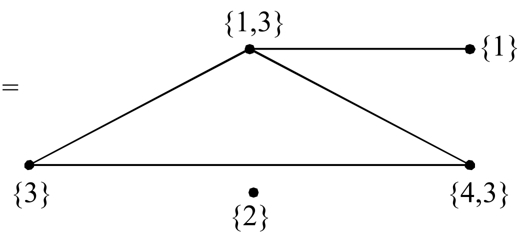


Figure 3.59

Clearly P_3 is only a special subset vertex subgraph as it has only four edges.

Let P_4 be the special subset vertex subgraph of type I with vertex {2, 3} given by the following figure.

P_4 =

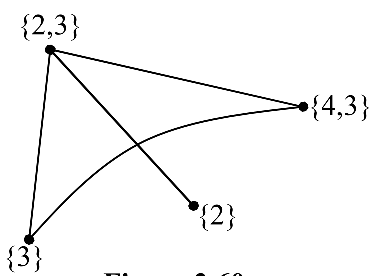


Figure 3.60

P_4 is also not a special subset vertex hyper subgraph of G as it has only four edges.

We see G behaves in a very different way as it has no special subset vertex subgraph which is hyper. So we can have vertex subset graphs of type I which has no special subset vertex subgraphs which are hyper.

Now we make the formal definitions.

Definition 3.1. *Let S be a finite set. P(S) the power set of S. Let G = {V, E} be the subset vertex graph of type I which takes the values of the vertex V from the power set P(S) and E is the collection of all edges of G. Let H be a subgraph such that the vertex sets of H are subsets of the vertex set V if two subsets in H are equal we do not define the subgraph H. So H should also have only |V| number of vertices. We define this H to be a special subset vertex subgraph of G.*

We have provided some example of this concept. However we provide one more example of it.

Example 3.4. Let G be a subset vertex graph of type I with its vertex set from P(S) where S = {1, 2, …, 12} given by the following figure.

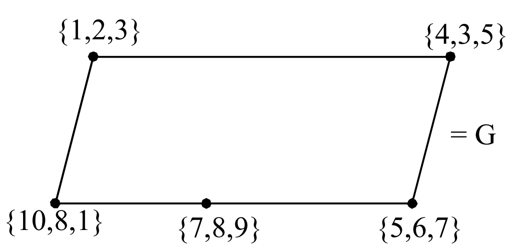


Figure 3.61

G has 5 vertices and 5 edges.

We see whether G has special subset vertex subgraphs which are hyper subgraphs.

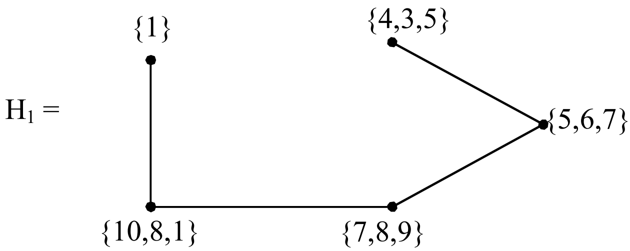


Figure 3.62

Clearly H_1 is only a special subset vertex subgraph of G and has only four edges.

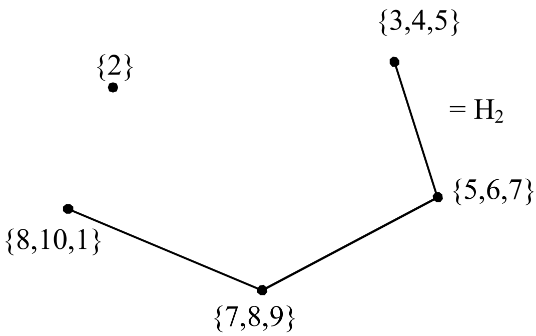


Figure 3.63

H_2 with vertex set $\{2\} \subseteq \{1, 2, 3\}$ is only a special subset vertex subgraph with three edges.

H_3 be the special subset vertex subgraph with vertex set $\{3\}$. The subgraph H_3 is as follows.

H_3 =

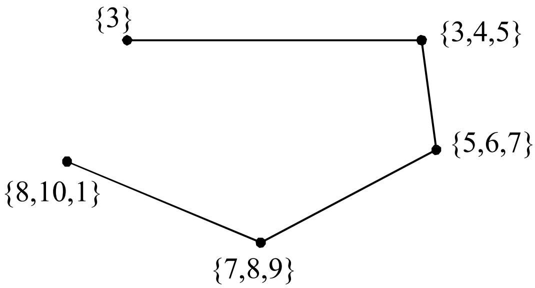


Figure 3.64

H_3 is only a special subset vertex subgraph with four edges.

Let H_4 be the special subset vertex subgraph with vertex set {1, 3}.

The subgraph H_4 is given in the following figure.

H_4 =

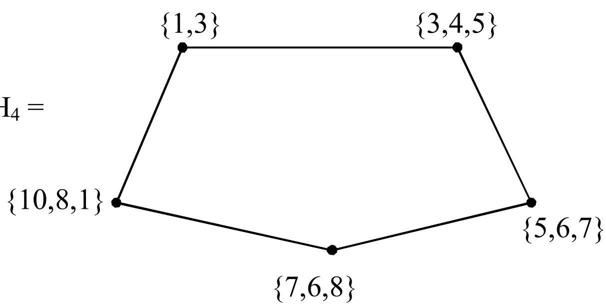


Figure 3.65

Clearly H_4 is a special subset vertex subgraph which is a hyper subgraph as H_4 has 5 edges.

None of the other subsets of {1, 2, 3} can contribute to special subset vertex subgraphs which are hyper subgraphs. Thus H_4 is the only special subset hyper subgraph of G.

Consider H_5 with vertex set $\{3\} \subseteq \{3, 4, 5\}$ the special subset vertex subgraph H_5 is given by the following figure.

H_5 =

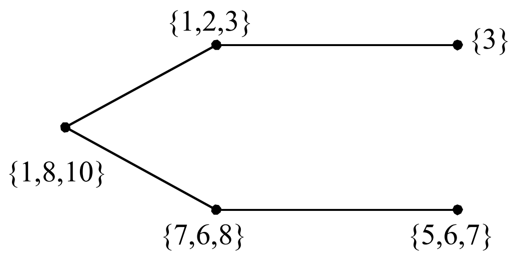


Figure 3.66

H_5 is not a special subset vertex hyper subgraph of G.

Let H_6 be the special subset vertex subgraph with vertex set $\{5\} \subseteq \{3, 4, 5\}$ given by the following figure.

H_6 =

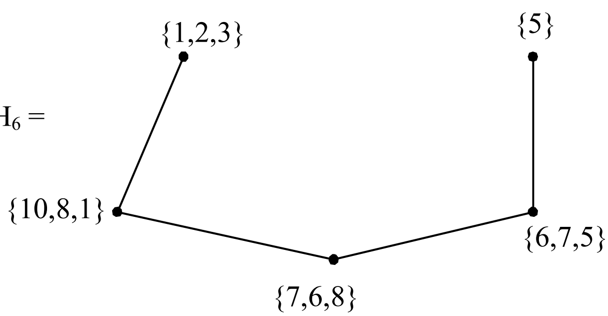


Figure 3.67

H_6 is not a special subset vertex hyper subgraph as it has only 4 edges.

Let H_7 be the special subset vertex subgraph with vertex set from $\{3, 5\} \subseteq \{3, 4, 5\}$ given by the following figure.

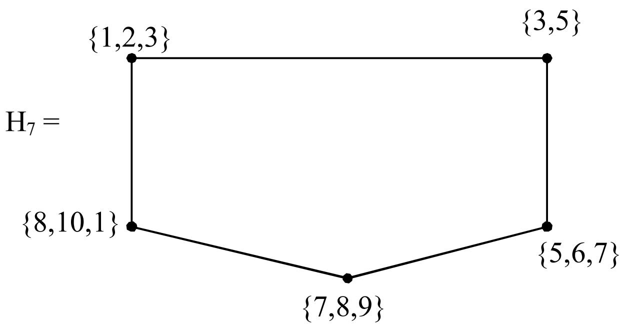


Figure 3.68

Clearly H_7 is a special subset vertex hyper subgraph of G as H_7 has five edges.

Let H_8 be the special subset vertex subgraph with vertex set $\{6\}$ from the set $\{5, 6, 7\}$ given by the following figure.

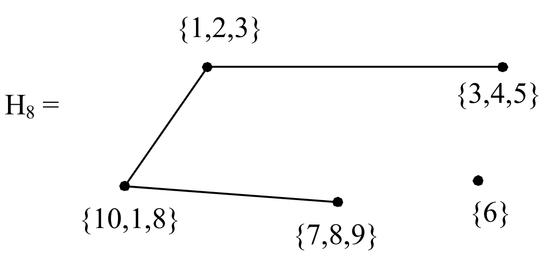


Figure 3.69

Clearly H_8 is not a special subset vertex hyper subgraph of G as it has only 3 edges.

Let H_9 be the special subset vertex subgraph with vertex set $\{5, 7\} \subseteq \{5, 6, 7\}$ given by the following figure.

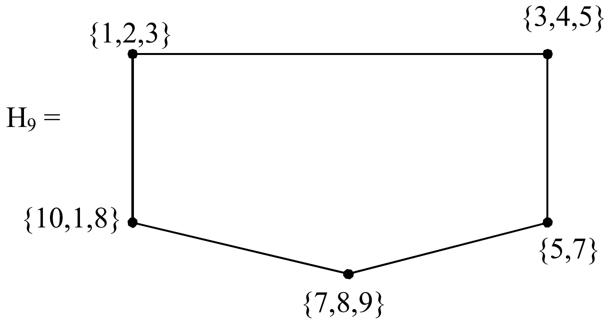


Figure 3.70

Clearly H_9 is a special subset vertex hyper subgraph of G as H_9 has 5 edges.

Consider H_{10} to be a special vertex subset subgraph with vertex set $\{\{2\}, \{4\}, \{6\}, \{9\}, \{10\}\}$ which is given by the following figure

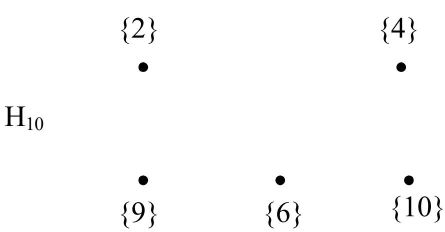


Figure 3.71

Clearly H_{10} is an empty special subset vertex subgraph of G. Several open problems are proposed for the researcher in special subset vertex subgraph in the end of this chapter also.

Conjecture 3.1. Let S be a finite set. P(S) be the power set of S, |S| = n**.**

i) Find all subset vertex graphs of type I which has atleast one special subset vertex subgraph which is empty.

ii) How many such subset vertex graphs of type I exists?

iii) Enumerate the number of subset vertex graphs of type I which cannot contain any special subset vertex subgraph which is an empty special subset vertex subgraph.

Next we proceed onto define the notion of special subset vertex hyper subgraphs of a subset vertex graph of type I.

Definition 3.2. *Let S be a finite set; P(S) power set of S. Let G = {V, E} be the subset vertex graph of type I with say t number of edges. If G has special subset vertex subgraph H with t number of edges then we define H to be a special subset vertex hyper subgraph of G.*

We have already in the earlier part of this chapter. We propose the following conjecture.

Conjecture 3.2. Let S be a finite set and P(S) the power set of S.

i) Find all subset vertex graphs of type I which has atleast one special subset vertex hyper subgraph.

ii) Find all subset vertex graphs of type I which has no special subset vertex hyper subgraphs.

Now we define the notion of special simple subset vertex graphs of type I.

Definition 3.3. *Let S be a finite set of order n. P(S) the power set of S. Let G be a subset vertex graph of type I. We define G to be a special simple subset vertex graph of type I if G has no special subset vertex subgraphs which is a special subset vertex hyper subgraph.*

We have given examples of them. Finding the number of special subset vertex subgraphs for a given subset vertex graph of type happens to be a challenging one.

Example 3.5. Let G be the subset vertex graph of type I where G takes the vertex set V from P(S) where S = {1, 2, 3, 4, 5, 6, 7} given by the following figure.

G =

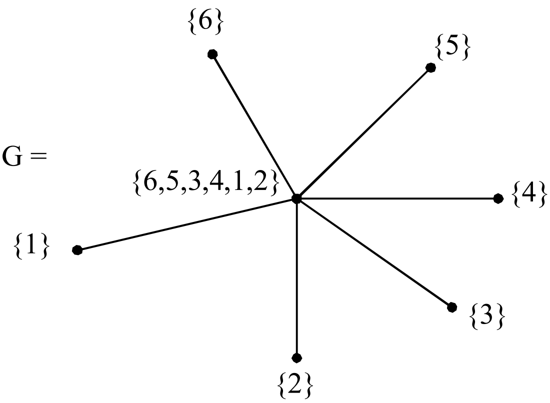


Figure 3.72

Singleton sets of {1, 2, 3, 4, 5, 6} cannot form a special subset vertex subgraphs of G.

Let P_1 be a special subset vertex subgraph with vertex set {1, 2} given by the following figure.

P_1 =

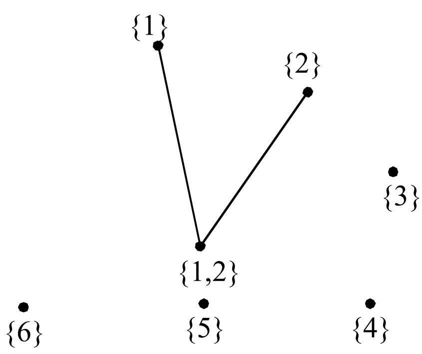


Figure 3.73

P_1 is only a special subset vertex subgraph with two edges if the cardinality of the subset vertex is two whatever be the subset of order two of {1, 2, 3, 4, 5, 6}.

Let us consider all subsets of the vertex set {1, 2, 3, 4, 5, 6} with cardinality three.

Then any special subset vertex subgraph V_2 with vertex set of cardinality 3 has only 3 edges so is only a special subset vertex subgraph which is not a hyper subgraph given in the following figure.

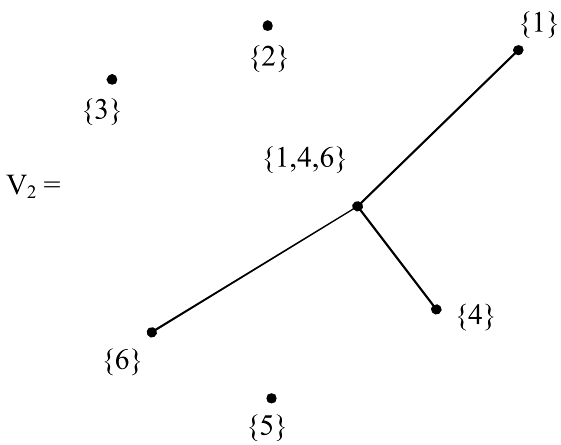


Figure 3.74

Let V_3 be the special subset vertex subgraph with vertex set of cardinality four from the set {1, 2, 3, 4, 5, 6}, say {4, 5, 6, 3} $\subseteq$ {1, 2, 3, 4, 5, 6} given by the following figure.

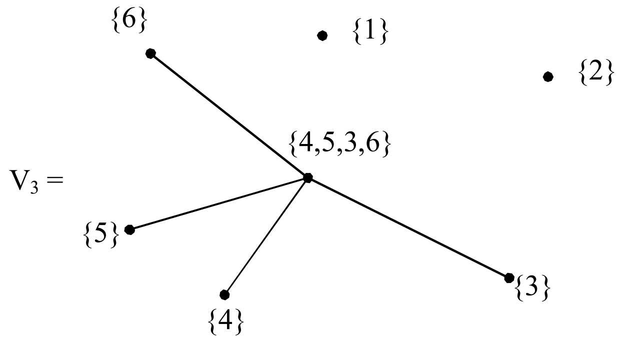


Figure 3.75

Clearly V_3 has only four edges.

Consider a subset of cardinality 5 say $\{1, 2, 3, 4, 6\} \subseteq \{1, 2, 3, 4, 5, 6\}$. Let V_4 be the special subset vertex subgraph with this vertex set. V_4 is described by the following figure.

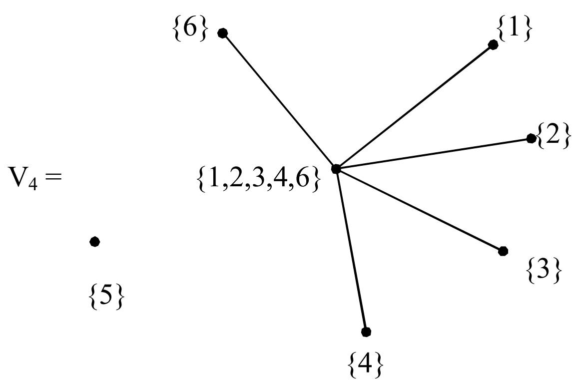


Figure 3.76

Clearly V_4 is the only special subset vertex graph with five edges. Thus this subset vertex star graph has no special subset vertex subgraph which is hyper graph or an empty graph.

Further G has only 56 special subset vertex subgraphs non of which can be a special subset vertex empty subgraph or a special subset vertex hyper subgraph.

We describe yet another example to illustrate this situation for another odd number say 15.

Example 3.6. Let S = {1, 2, 3, …, 14,15} be the set. P(S) be the power set of S.

Let G be the vertex subset graph with vertex set from {{1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}, …, {14}, {15} and {1, 2, 3, 4, …, 15}} of type I. Clearly G is a star subset vertex graph with 15 edges given by the following figure.

G =

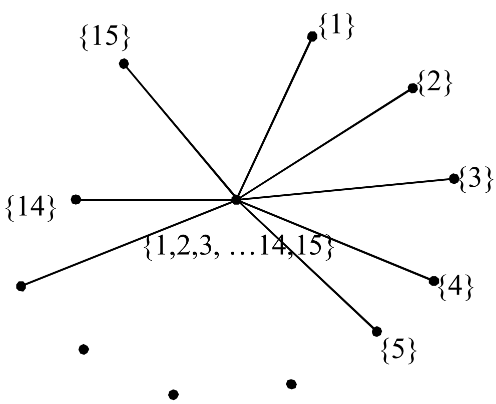


Figure 3.77

Now the subsets of {1, 2, 3, …, 15} are {{1}, {2}, {3}, …, {15}, {1, 2}, {1, 3}, …, {14, 15}, {1, 2, 3}, {1, 2, 3}, …, {1,4 15, 13}, {1, 2, 3, 4}, …., {12, 13, 14, 15}, …., {1, 2, 3, …, 14}, …, {2, 3,…, 15}}.

We see using sets {{1}, {2}, …, {15}} that is vertex sets of cardinality one we cannot get any special subset vertex subgraphs. So these subsets with cardinality one will not give any nontrivial special subset vertex subgraphs.

Hence we can get special subset vertex subgraphs of only with vertex sets of cardinality 2 or 3 or 4 or …, 12 or 13 or 14 can contribute to special subset vertex subgraphs with 2 edges or 3 edges or 4 edges or 5 edges, …, or 12 edges or 13 edges or 14 edges respectively.

Let V_1 be the special subset vertex subgraph with vertex set {5, 15} ⊆ {1, 2, …, 14,1 5} is given in the following

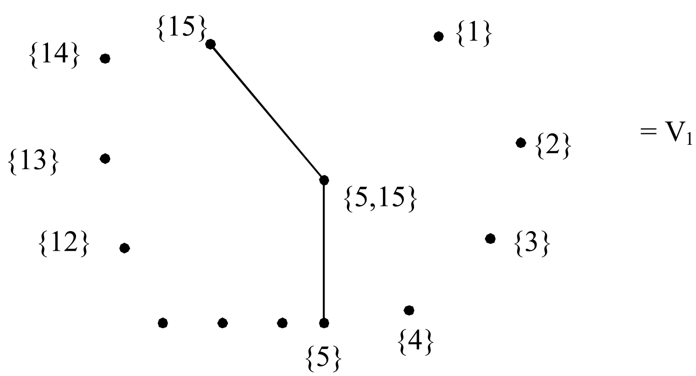


Figure 3.78

V_1 has only two edges.

Let V_2 be the special subset vertex graph with vertex set {3, 9, 10} ⊆ {1, 2, …, 15} given by the following figure.

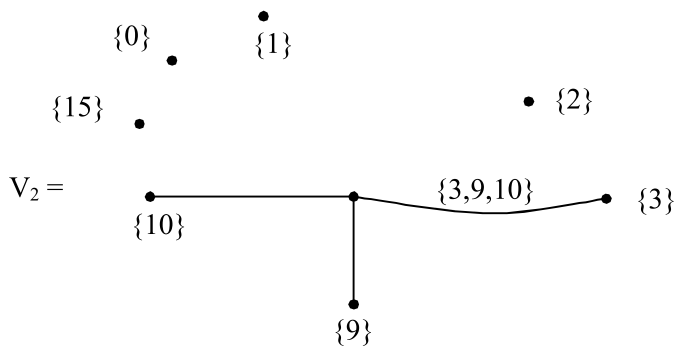


Figure 3.79

Let V_3 be the special vertex subset graph with vrtex set {1, 5, 9, 12} ⊆ {1, 2, 3, …, 10, 11,12, 13, 14, 15} given by the following figure.

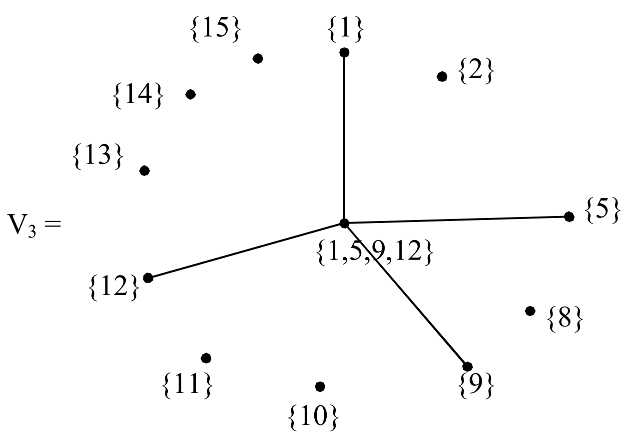


Figure 3.80

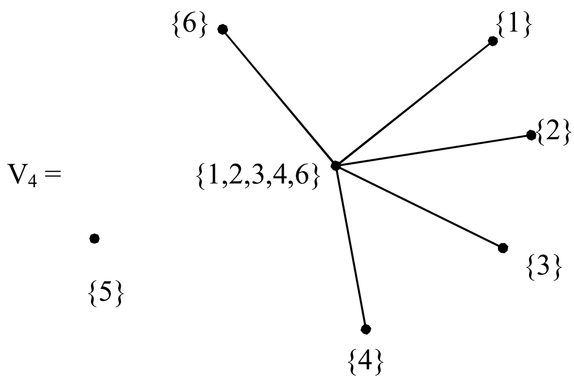


Figure 3.81

V_3 has only four edges. In this way we can proceed on with V_4, V_5, …, upto V_{14}.

We just describe V_8, V_9, V_{12} and V_{13} which will haves 9 edges, 10 edges, 13 edges and 14 edges respectively in the following.

V_8 be the special subset vertex subgraph with vertex set $\{2, 3, 4, 5, 10, 11, 15, 12,6\} \subseteq \{1, 2, 3, \ldots, 14, 15\}$.

The order of vertex set is 9. The graph of V_8 is as follows.

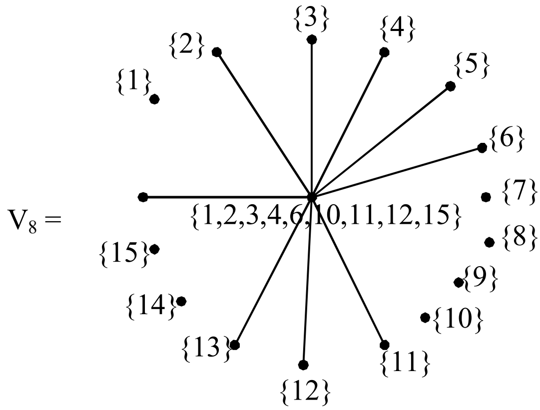


Figure 3.82

V_8 is a special subset vertex subgraph with 9 edges.

Consider V_9 the special subset vertex subgraph given by the following figure. Let {3, 4, 5, 6, 7, 8, 9, 10, 11, 15} ⊆ {1, 2, 3, 4, 5, …, 11, 12, 13, 14,15}.

The graph of V_9 is as follows.

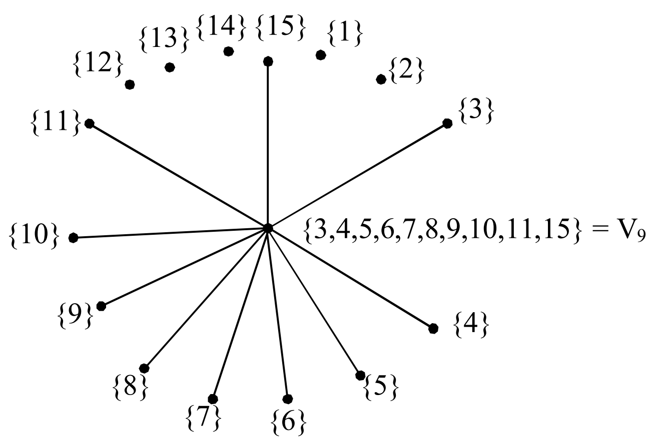


Figure 3.83

We see V_9 has only 10 edges.

Let V_{12} be the special subset vertex subgraph with vertex set {1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 15} $\subseteq$ {1, 2, 3, 4, …, 14,15}. Clearly V_{12} has 23 edges and V_{12} is not a special subset vertex hyper subgraph. The graph of V_{12} is as follows.

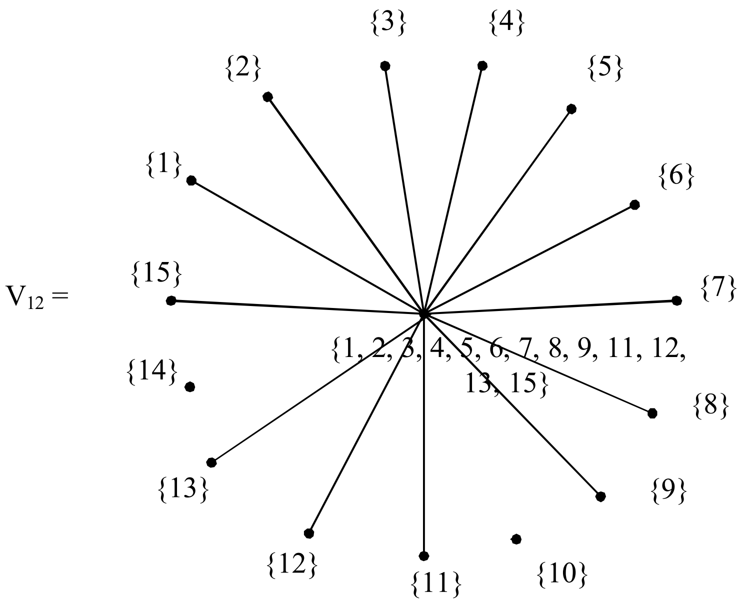


Figure 3.84

V_{12} has only 13 edges.

Let V_{13} be the special subset vertex subgraph with vertex set {1, 2, 3, 4, 5, 6, 9, 10, 11, 12, 13, 14, 15, 8} $\subseteq$ {1, 2, 3, …, 15}.

The graph of V_{13} is given by the following figure.

V_{13} =

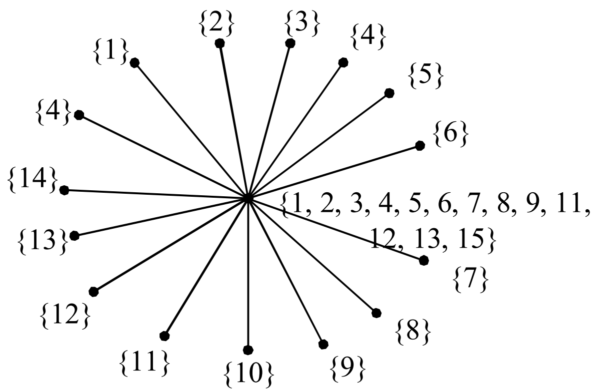


Figure 3.85

We see V_{13} has 14 edges.

The number of special subset vertex subgraphs of G is

$_{15}C_2 + {}_{15}C_3 + {}_{15}C_4 + \ldots + {}_{15}C_{14} = 2^{15} - (1 + 15 + 1)$.

In view of all these we prove the following theorem.

Theorem 3.1. *Let S = {1, 2, ..., n} be a finite set. P(S) be the power set of S.*

There exist a subset vertex star graph G with vertex set {{1}, {2}, {3}, ..., {n – 2}, {n – 1}, {n}, {1, 2, 3, ..., n – 2, n – 1, n}} such that

i) *G has no special subset vertex hyper subgraphs.*

ii) *G has no special vertex subset empty subgraph*

iii) *The number of special subset vertex subgraphs of G is* $nC_2 + nC_3 + \ldots + nC_{n-1} = 2^n - (2 + n)$.

Proof is direct and hence left as an exercise to the reader.

Next we proceed onto study the existence of star graphs which has both special vertex subset empty subgraph as well as special vertex subset hyper subgraph. For this we first give some examples.

Example 3.7. Let S = {1, 2, …, 9} be the given set P(S) the power set of S.

Let G be the star graph with the vertex set {{1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}, {1, 2, 3, …, 9}} which is a subset vertex graph of type I; given by the following figure.

G =

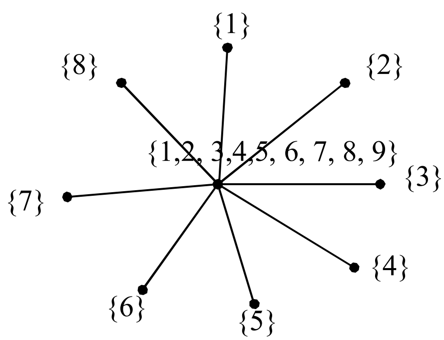


Figure 3.86

G is a subset vertex star graph of type I which has 8 edge.

Now we see if B_1 is a special subset vertex subgraph with vertex set {{1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}, {1, 2, 3, 4, 5, 6, 7,8}}.

B_1 is a special subset vertex subgraph which is given in the following figure

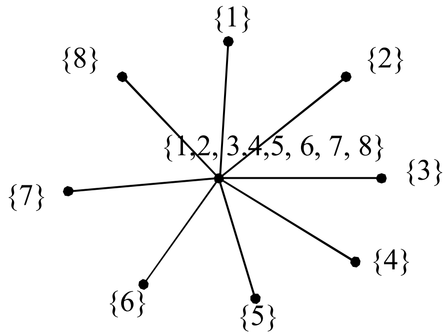


Figure 3.87

Clearly B_1 is a special subset vertex subgraph with 8 edges hence B_1 is a special subset vertex subgraph which is a special subset vertex hyper subgraph of G.

Consider the vertex set W = {{1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}, {9}}.

The special subset vertex subgraph B_2 with the associated vertex set W is given by the following figure.

B_2 =

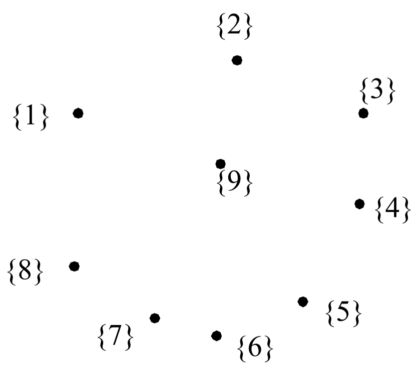


Figure 3.88

Clearly B_2 has no edges. Hence B_2 is a special subset vertex empty subgraph of G.

Thus G has both special subset vertex empty subgraph as well as special subset vertex hyper subgraph.

Infact G has several special subset vertex subgraphs with edges less than eight edges. Some of such special subset vertex subgraphs are given in the following.

Let B_3 be the special subset vertex graph with the vertex set {1, 2,3, 9, 6} given by the following figure.

Clearly B_3 is a special vertex subset subgraph with only 4 edges.

Let B_4 be the special vertex subset subgraph with vertex set {7, 3, 5, 6, 8} given by the following figure.

B_3 =

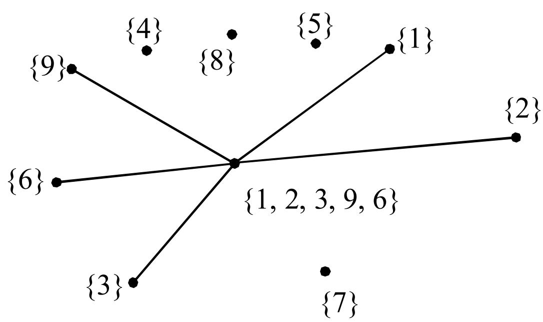


Figure 3.89

We see B_4 is a special subset vertex subgraph with only 5 edges.

B_4 =

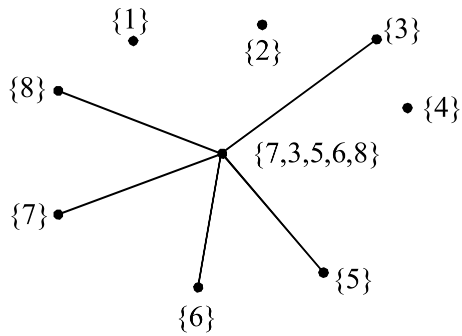


Figure 3.90

Let B_5 be the special subset vertex subgraph with vertex set {1, 9} given by the following figure.

B_5 =

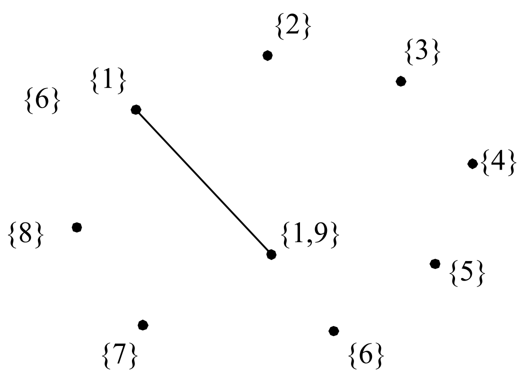


Figure 3.91

Clearly B_5 is a special vertex subset graph with only one edge.

Thus we can get several such special subset vertex subgraphs with one edge, two edge so on say upto 8 edges.

Infact there are 9 subset vertex star graphs type I with 8 edges which has only one special subset vertex hyper subgraph and a special subset vertex empty subgraph.

We give only the vertex set V_i of these 9 subset vertex star graphs of type I with 8 edges in the following: i = 1, 2,…, 9.

V_1 = {{1}, {2}, {3}, {4}, {5, {6}, {7}, {8}, {1, 2, 3, …, 9}}.

V_2 = {{2}, {3}, {4}, {5}, {6}, {7}, {8}, {9}, {1, 2, 3, 4, …, 9}}.

V_3 = {{1}, {3}, {4}, {5}, {6}, {7}, {8}, {9}, {1, 2, 3, 4, 5, 6, 7, 8, 9}}.

V_4 = {{1}, {2}, {4}, {5}, {6}, {7}, {8}, {9}, {1, 2, 3, 4, 5,6, 7, 8, 9}},

V_5 = {{1}, {2}, {5}, {3}, {6}, {7}, {8}, {9}, {1, 2, 3, 4, 5, 6, 7, 8, 9}}

V_6 = {{1}, {2}, {3}, {4}, {6}, {7}, {8}, {9}, {1, 2, 3, 4, 5, 6, 7, 8, 9}},

V_7 = {{1}, {2}, {3}, {4}, {5}, {7}, {8}, {9}, {1, 2, 3, 4, 5, 6, 7,8, 9}},

V_8= {{1}, {2}, {3}, {4}, {5}, {8}, {6}, {9}, {1, 2, 3, 4, 5, 6, 7, 8, 9}} and

V_9 = {{1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}, {1, 2, 3, 4, 5, 6, 7, 8, 9}}.

We see the subset vertex star graph of type I with maximum number of edges in this case 9 and there is only one such subgraph of type I. We call such subset vertex graph of type I as a giant subset vertex graph of type I or a subset vertex giant graph of type I. To this effect we make the following definition.

Definition 3.4. *Let S = {1, 2, 3, ..., n}, P(S) the power set of S. The subset vertex graph G of type I with this given set of vertices {{1}, {2}, {3}, {4}, {5}, {6}, ..., {n – 1}, {n}, {1, 2,3, 4, 5, 6, 7,..., n – 1, n}} is a subset vertex stargraph of type I with n edges. We define G as the subset vertex giant star graph of type I.*

Proposition 3.1. Let S = {1, 2, …, n} be set and P(S) the power set of S. There is a unique vertex set collection in P(S) which contributes to a subset vertex giant star graph with n edges.

Proof. Direct hence left as an exercise to the reader.

Proposition 3.2. Let S = {1, 2, …, n} and P(S) the power set of S.

The subset vertex giant star graph of type I has no

i) Special subset vertex hyper graph

ii) Does not contain a special subset vertex empty subgraph.

Proof follows from the theorem and is direct.

Infact it is observed there are several subset vertex star graph with less number of edges which contain both special subset vertex hyper subgraphs as well as special subset vertex empty graphs.

We provide some examples to this effect.

Example 3.8 Let S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} be a set and P(S) the power set of S.

Consider T_1 be a subset vertex star graph of type I with the given vertex set V_1; where

V_1 = {{1}, {2}, {3}, {4}, {5}, {6}, {7}, {1, 2, 3, 4, 5, 6 7, 8, 9}} given by the following figure.

T_1 =

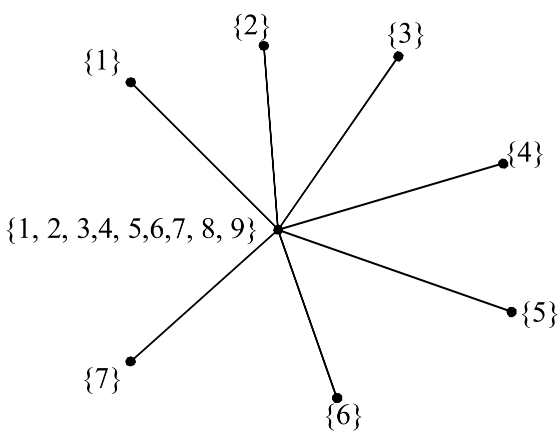


Figure 3.92

Clearly T_1 is a subset vertex star graph of type I.

Consider L_1 with vertex set $W_1 = \{\{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{6\}, \{7\}, \{8, 9\}\}$. The special subset vertex subgraph associated with the vertex set W_1 is as follows.

L_1 =

Figure 3.93

Clearly L_1 is a special vertex subset empty subgraph.

Further if the vertex set {8, 9} is replaced by {8} or by {9} once again we get a special subset vertex empty subgraphs. Thus there are four special subset vertex empty subgraphs of G.

We now find all the special subset vertex hyper subgraphs of T_1.

Consider the vertex set {{1}, {2}, {3}, {4}, {5}, {6}, {7}, {1, 2, 3, 4, 5, 6, 7, 8}}, the special vertex subset subgraph D_1 with the above vertex subset is a special vertex subset hyper subgraph of T_1 given by the following figure.

$D_1 =$

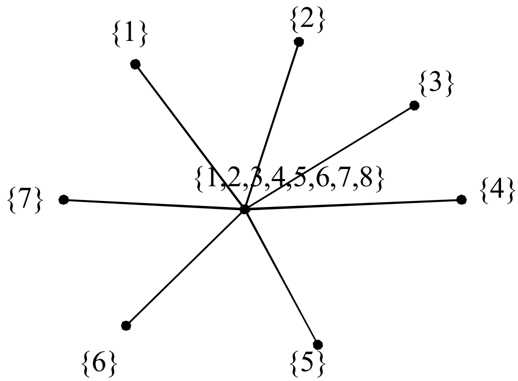


Figure 3.94

The special vertex subset subgraph D_1 also has only sevenedges. Hence D_1 is a special subset vertex hyper subgraph of T_1.

D_2 be the special subset vertex subgraph with vertex set $\{1, 2, 3, 4, 5, 6, 7\} \subseteq \{1, 2, 3, \ldots, 8, 9\}$.

The figure associated with D_2 is as follows.

$D_2 =$

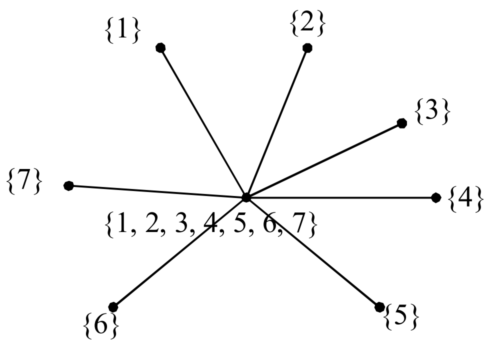


Figure 3.95

Let D_3 be the special subset vertex subgraph with vertex set $\{1, 2, 3, 4, 5, 6, 7, 9\} \subseteq \{1, 2, 3, 4, \ldots, 8, 9\}$ given by the following figure.

$D_3 =$

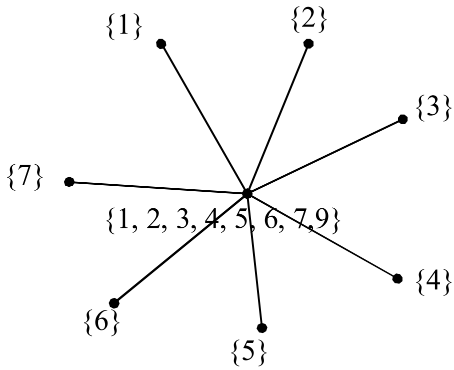


Figure 3.96

Clear D_3 has 7 edges so D_3 is a special subset vertex hyperstar subgraph of T_1.

It is important to keep on record that using P(S) one can construct several special subset star hyper subgraphs for any given subset vertex star graphs using subset of P(S) as vertex set as well as one can get several special subset vertex empty star subgraphs of the subset vertex graph.

Next we proceed onto describe the notion of subset vertex circle graphs of type I and find their special subset vertex subgraphs.

We describe this situation by some examples.

Example 3.9. Let S = {1, 2,…, 12} be the given set. P(S) be the power set of S.

Let G be the subset vertex circle graph of type I given by the following figure.

G =

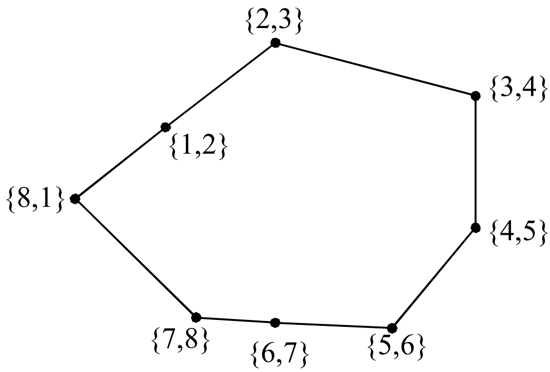


Figure 3.97

Consider the special subset vertex subgraph V_1 of G with the vertex set $\{3\} \subseteq \{3, 4\}$ given by the following figure.

V_1 =

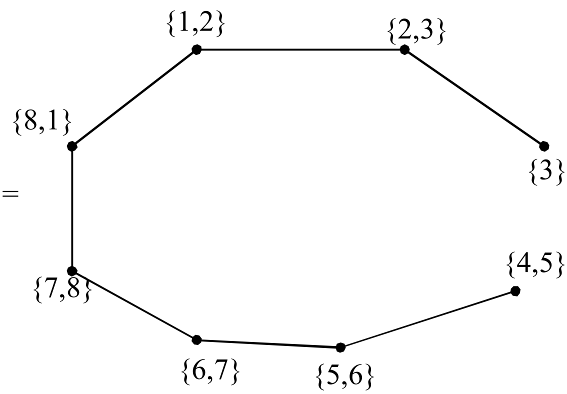


Figure 3.98

Clearly V_1 is not a special subset vertex cycle subgraph of G.

Let V_2 be the special subset vertex subgraph of G with two vertex set $\{1\} \subseteq \{1, 8\}$ and $\{5\} = \{5, 6\}$ given by the following figure.

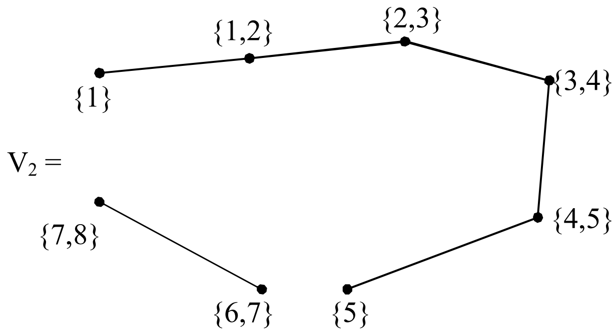


Figure 3.99

Clearly V_2 is not a special subset vertex subgraph which is a special subset vertex circle graph. We can easily prove that G has no special subset vertex circle subgraph or equivalently G has no special subset vertex hyper subgraph. That is G is special simple graph. Consider the subset vertex graph of type I, (say) H given by the following figure.

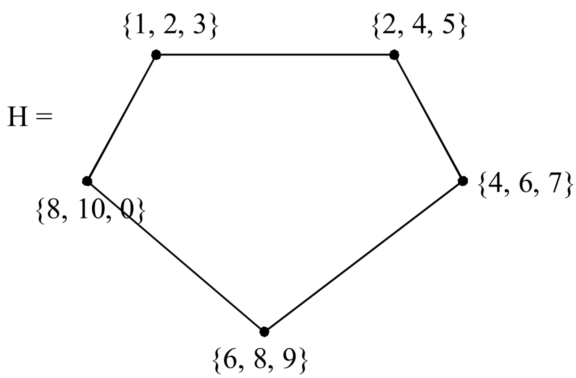


Figure 3.100

Consider the vertex set {{1,2}, {2, 4}, {4, 6}, {6, 8}, {8, 1}}. The special subset vertex subgraph K_1 of this vertex set is as follows.

K_1 =

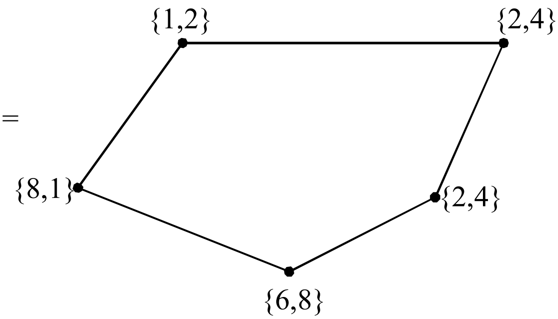


Figure 3.101

Clearly K_1 is a special subset hyper subgraph of H. K_1 also has five edges. No other subset vertex collection can yield a special subset vertex hyper graph. Further if {{1} {2}, {6}, {8} {10}} is taken as the vertex set we get the following figure.

K_2 =

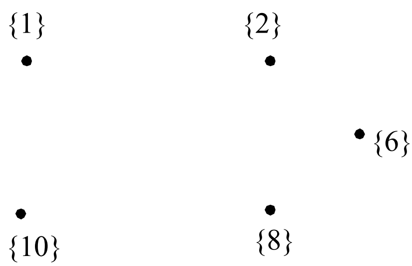


Figure 3.102

Clearly K_2 is only a special subset vertex empty subgraph of H. Thus H is not simple and H has both a special subset vertex empty subgraph and special subset vertex hyper subgraph.

K_3 =

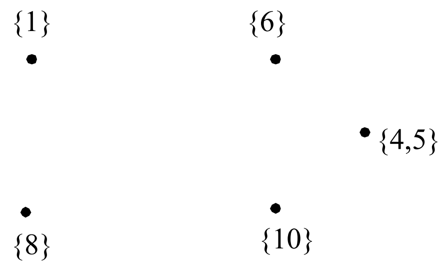


Figure 3.103

Let K_3 be the special vertex subset subgraph with vertex set {{4,5}, {1}, {6}, {8}, {10}}. K_3 is also a special subset vertex empty subgraph of H.

It is to be noted that in all cases a special subset vertex hyper subgraphs always preserves the structure of the original subset vertex graph of type I. That is if K is a subset vertex star graph then the special subset vertex hyper subgraph B of K will also be a star subgraph with same number of vertices as that of K and also same number of edges as that of K only the cardinality of all the vertex sets of K and that of the hyper subgraph B of K will not be the same.

That is special subset vertex subgraphs of the vertex subset graphs are structure preserving.

Hence in case of a subset vertex circle graph L we see the special subset vertex hyper subgraph of L we see the special subset vertex hyper subgraph of L will also be circle graph.

This property of structure preserving will certainly be a boon to researchers on social information networks and community networks.

Further these can also be useful to technologists who wants to preserve the number of nodes and edges of the network which they are analyzing in particular in case of fault tolerance. For in some cases the special hyper subgraph may yield a most economic result.

Now one of the probable problems is to find the number of special subset vertex subgraphs of a subset vertex graph of type I. Further authors at this juncture want to keep on record most of the results may not follow in case of special subset vertex subgraphs of the subset vertex graphs of type I.

Next we proceed onto define, develop and describe the concept of special subset vertex subgraphs of a subset vertex graphs of type II.

We will first illustrate these situation. All of these structures are directed ones.

Example 3.10. Let S = {1, 2, 3,…, 8} be a set of order 8. P(S) be the power set of S.

Let G be the subset vertex graph of type II given by the following figure.

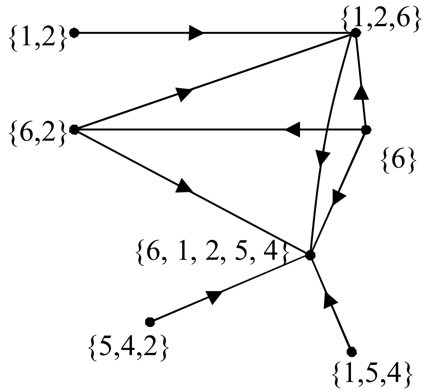
G = 

Figure 3.104

Clearly G is a directed subset vertex graph of type II. We see G has 7 vertices and 9 edges. Further G is a directed graph. Now we find the special subset vertex subgraphs of G.

Take the vertex set as {{2} {6, 1}, {6}, {1, 2, 5}, {6, 2}, {5}, {4, 1}}.

We get the following special subset vertex subgraph B_1 using these vertices given by the following figure.

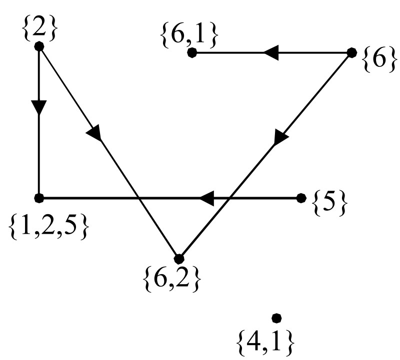
B_1 = 

Figure 3.105

B_1 is a special subset vertex subgraph of G with the 7 vertices which is mandatory and has only five edges.

Consider the special subset vertex subgraph B_2 of G with vertex set {{1, 2}, {1, 6} {5, 1}, {6}, {1, 2, 5}, {6, 2}, {1}} is given by the following figure.

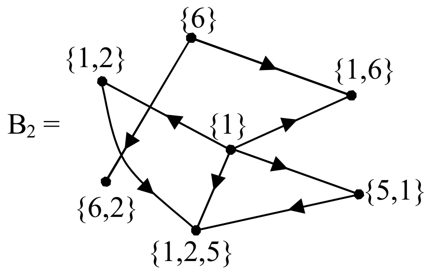


Figure 3.106

B_2 is a special subset vertex subgraph of G with 7 vertices and eight edges. If a subset vertex graph of type II G has no special subset vertex hyper subgraph then we define G to be simple special subset vertex graph of type II.

Now consider the vertex set {{6}, {5}, {1}, {2}, {4}, {5, 2}, {5, 4}}. Let B_3 be the special subset vertex subgraph of G with the above given vertex set, whose graph is given below.

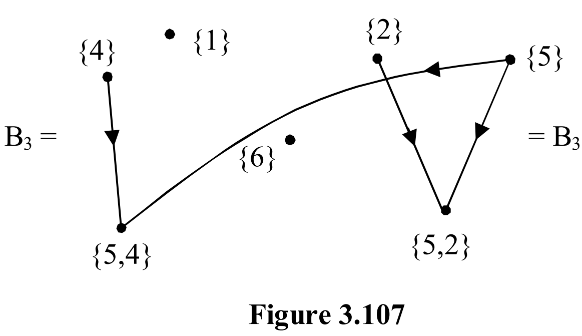


Figure 3.107

Clearly B_3 has only 4 edges, so B_3 is only a special subset vertex subgraph which is not a hyper subgraph.

Here we wish to keep on record this subset vertex graph G of type II has no special subset vertex empty subgraph. Thus we cannot in general say every subset vertex graph of type II has a special subset vertex empty subgraph.

We analyse yet another example in this direction.

Example 3.11. Let S = {1, 2,.., 15} be the set of S. Let G be the subset vertex graph of type II given by the following figure.

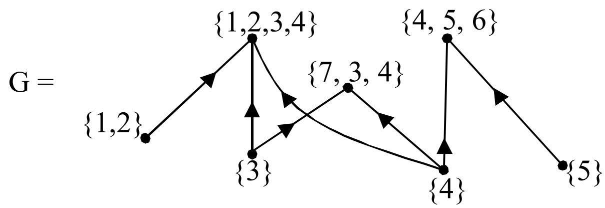


Figure 3.108

We find special subset vertex subgraphs of G in the following.

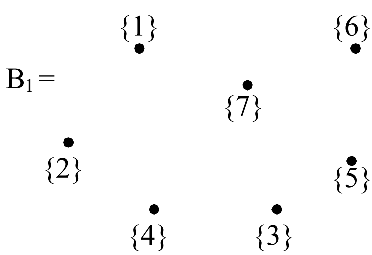


Figure 3.109

The special subset vertex subgraph B_1 must contain 7 vertices and B_1 is a special subset vertex empty subgraph of G.

Let B_2 be the special subset vertex subgraph of G given by the following figure.

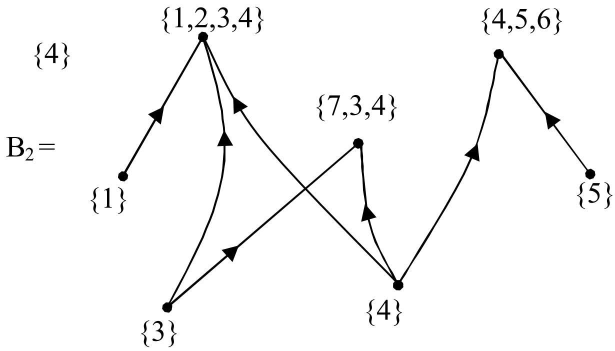


Figure 3.110

Clearly G has 7 vertices and 7 edges. Now B_2 has to contain 7 vertices. We call B_2 as the special subset vertex hyper subgraph of G.

Yet we have a special types of special subset vertex subgraphs B_3 of G with more than 7 edges that is B_3 given by the following figure has 8 edges.

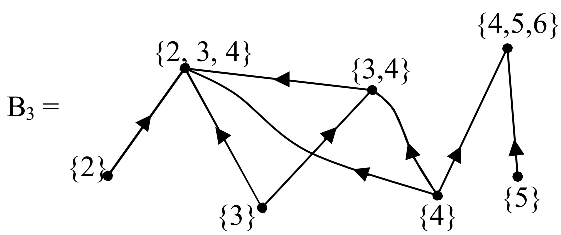


Figure 3.111

Thus these type of special type of vertex subset graphs will be known as super special subset vertex hyper subgraph or special subset vertex super hyper subgraph of G.

Consider B_4 a special subset vertex subgraph given by the following figure.

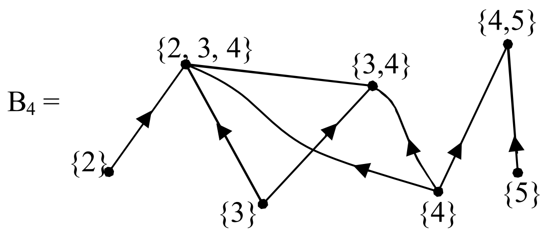


Figure 3.112

We see B_4 is a special subset vertex subgraph of G but B_4 has 8 edges and 7 vertices however G has only 7 vertices and 7 edges. Hence we call B_4 a special subset vertex super hyper subgraph of G. It is important to note that in the classical directed subgraphs one cannot face such situations.

It is important to note that certain subset vertex graphs may not have special subset vertex subgraphs. We will first illustrate this situation by some examples.

Example 3.12. Let S = {1, 2, 4, 5, 6, 7, 8, 9} be the given set and P(S) be the power set of S.

Consider the subset vertex graph G given by the following figure.

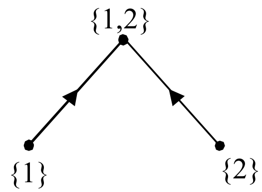
G = 

Figure 3.113

Clearly this has no special subset vertex subgraphs.

We call these subset vertex graphs as special simple graphs or special subset vertex simple graphs or simple special subset vertex graphs.

We now give the largest such simple special subset graphs of P(S) where S = {1, 2, 3} by the following figure.

Clearly P(S) = {{1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, {1, 2,3}}.

Now we see the graph with 7 vertex subset graphs which will be the largest subset vertex graph B.

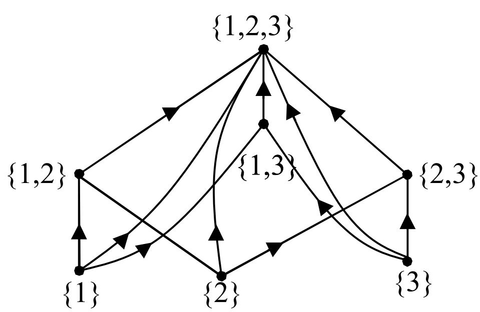
B = 

Figure 3.114

B is the biggest subset vertex graphs that can be constructed using P(S) \ ϕ. Clearly B has no special subset vertex subgraphs however it has several subset vertex subgraphs.

Further B has no special subset vertex subgraphs which is empty or hyper as in the first place B has no special subset vertex subgraphs.

Example 3.13. Let S = {1, 2, 3, 4} be finite set of order four P(S) = {{1}, {2}, {3}, {4}, {1, 2}, {1, 3}, {1, 4}, {2, 3}, {2, 4}, {4,3}, {1, 2, 3}, {1, 3, 4}, {1, 2, 4}, {3, 4, 2}, {1, 2, 3, 4}}. We give the subset vertex graph of type II using all the 15 vertices which will be the biggest subset vertex graph of type II. Let H be the subset vertex graph of type II is given by the following figure.

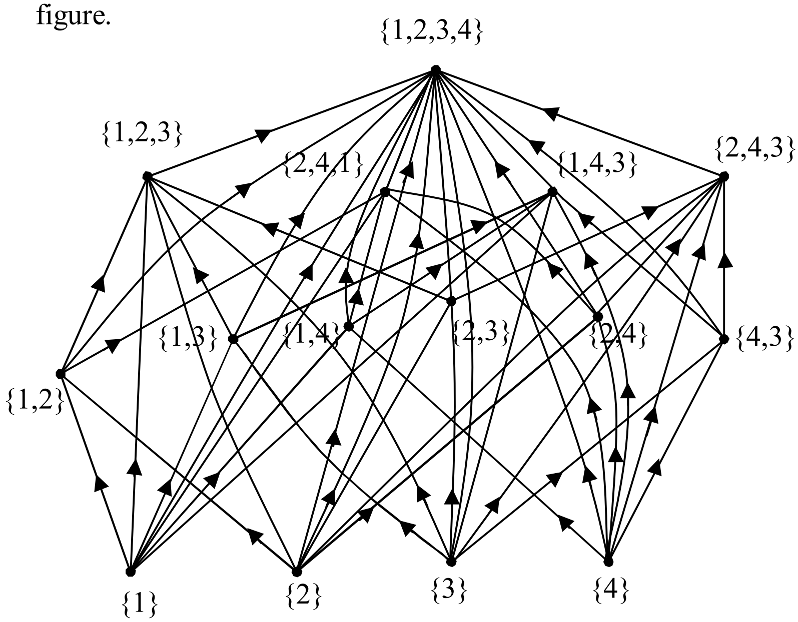


Figure 3.115

We see the top node has 14 edges incident to it and 8 nodes have 7 edges incidents to them. The middle 6 nodes has only five edges incident to them.

Now we find the properties of these graphs (subset vertex graphs of type II with $2^n - 1$ vertices, n = |S|).

Let |S| = 5, P(S) be the power set of S. We see |P(S) \ {ϕ}| = $2^n - 1$ = 31. We find the properties associated of the subset vertex graph of type II with 31 vertices.

i) All sets with same cardinality have the same number of edges incident to them.

ii) The set {1, 2, 3, 5, 4} the top set or the full set has 30 edges incident to it.

iii) The elements of P(S) are {{1}, {2}, {3}, {4}, {5}, {1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 3}, {2, 4}, {2, 5}, {3, 4}, {3, 5}, {4, 5}, {1, 2, 3}, {1, 2, 4}, {1, 2, 5}, {1, 3, 4}, {1, 3, 5}, {1, 4, 5}, {2, 3, 4}, {2, 3, 5}, {2, 4, 5}, {1, 2, 3, 4}, {1, 2, 3, 5}, {1, 2,4,5}, {1, 3, 4, 5}, {2, 3, 4, 5}, {1, 2, 3, 4, 5}}.

Clearly the nodes {1}, {2}, {3}, {4} and {5} have exactly 15 edges adjacent to them.

Similarly {1, 2, 3, 4}, {1, 2, 3, 5}, {1, 2, 4, 5}, {3, 1, 4, 5} and {2, 3, 4, 5} have exactly only 15 edges adjacent to them. We see the nodes / vertices {1, 2, 3}, {1, 2, 4}, {1, 2, 5}, …, {3, 4, 5} have only 9 edges adjacent them.

Similarly the node {1, 2}, {1, 3}, {1, 4}, {1, 5},….{3, 4}, {3, 5} also have only 9 edges adjacent to them.

So the set with highest cardinality that is 5 has 30 edges incident to them that is there are $2^5 – 2 = 30$ edges adjacent to them.

To sets with cardinality one and four has 15 edges adjacent to them, that is $2^4 – 1 = 2^{(5 – 1)} – 1 = 15$. The sets with cardinality 2 and 3 has only 9 edges adjacent to them, that is $2^3 + 1$ edges adjacent to them.

In view of all these we have the following theorem.

Theorem 3.2. *Let $S = \{1, 2, …, n\}$ be the set of order n, P(S) be the power set of S. The subset vertex graph of type II, B associated with all elements of $P(S) \setminus \{\phi\}$ that this B has $2^n – 1$ vertices and satisfies the following properties.*

i) *The vertex $\{1, 2, 3, …, n\}$ has exactly $2^n – 2$ edges incident to it and all of them are inward to the vertex.*

ii) *The sets {1}, {2}, …, {n} and their respective complements {2, 3, …, n}, {1, 3, 4, …, n}, …., {1, 2, 3, …, n – 1} as vertices of B have $2^{n–1} – 1$ edges incident to them with the vertex subsets {1}, {2}, …, {n} has all edges to be outwards and the vertex subset {2, 3, …, n}, {1, 3, 4, …, n}, …, {1, 2, 3, …, n – 1} has $2^{n–1} – 2$ of the edges to be inward and only one edge to be outward.*

iii) *The sets {1, 2}, {1, 3}, ..., {n, n – 1} and {3, 4, ..., n}, {2, 4, ..., n}, ..., {1, 2,..., n – 2} as vertex sets of B has* $2^{n-2} - 1$ *edges.* $(2^n - 1) - 2$ *of them outward and two edges inwards in case of vertex sets {1, 2}, {1, 3}, ..., {n, n – 1} and in case of {3,4, ..., n}, {2, 4, ..., n}, ..., {1, 2,..., n – 2} we see only one edge is outward and* $(2^n - 1) - 1$ *are inward.*

iv) *In case n is even we see there is a central elements of the subset vertex graph of type II with* $2^n - 1$ *vertices given by* $\{1, 2, \ldots, \frac{n}{2}\}$, $\{1, 2, \ldots, \frac{n-1}{2}, \frac{n}{2}+1\}$, ..., $\{\frac{n+1}{2}, \ldots, n\}$ *which has exactly* $2^{\frac{n}{2}} + 1$ *edges incident to it of which* $2^{\frac{n}{2}} - 1$ *of the edges are inward and the rest of the edges are outward.*

v) *In case n is odd we see the graph B has no central row infact there are two rows which are complements of each other and there are* $2^{\frac{n}{2}} + 1$ *incident to it.*

Proof: Follows from the simple working and the fact how many subsets are contained in a set in P(S).

Theorem 3.3. *Let S = {1, 2,..., n} be a finite set of order n; P(S) the power set of S.*

Let G be the subset vertex graph of type I with $2^n - 1$ vertices, G has no special subset vertex subgraphs.

Proof. Follows from the fact any subset y of x where x ∈ P(S) \ {ϕ} and y ∈ P(S) \ {ϕ} their by reducing the number of vertices of G.

Since in case of special subset vertex subgraphs H it is mandatory H also must have $2^n - 1$ number of vertices. Hence the claim.

We now define special simple subset vertex graph of type II.

Definition 3.5. *Let P(S) be the power set of S where S = {1, 2, …, n} be the set of order n. Let G be the subset vertex graph of type II with edges from P(S) \ {ϕ}. If G has no special subset vertex subgraph of type II then we define G to be a special simple vertex subset graph of type II.*

The graph mentioned in Theorem 3.3 is a simple special subset vertex graph of type II. Infact we have several such simple special subset vertex graphs of type II.

Example 3.14. Let S = {1, 2, 3, 4, 5, 6, 7, 8, 9} be the set of order 9. P(S) be the power set of S.

The graph G_1 with vertex set {{8, 9}, {8}, {9}} is given in the following.

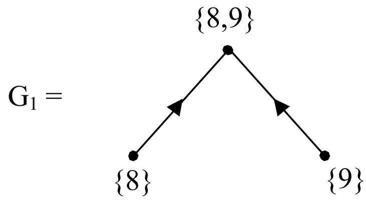


Figure 3.116

G_1 is a subset vertex graph of type II which is special simple and $o(G_1) = 3$.

Infact there are 36 such special simple subset graphs of type II of order 3.

Consider H_1 the subset vertex graph of type II with vertex set {{3}, {9}, {6}, {3, 9}, {3, 6}, {6, 9}, {3, 6, 9}} given by the following figure.

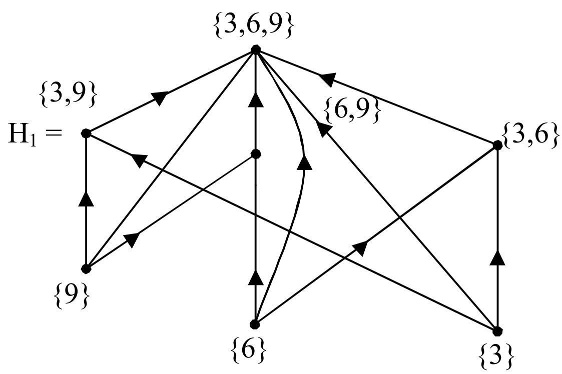


Figure 3.117

$O(H_1) = 7$. Infact fort this P(S) we have 84 such subset vertex graphs of type II which are special simple subset vertex graphs.

Consider subset vertex graph of type II given by the following figure.

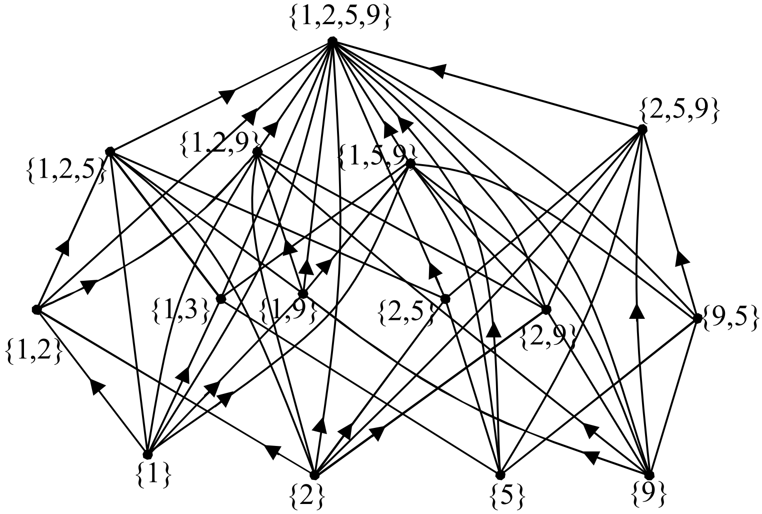


Figure 3.118

There are 126 such simple special subset vertex graph with 15 nodes.

Likewise we have 126 special simple subset vertex graphs with 31 nodes.

Further there are 84 special simple subset vertex graph of type II with 63 nodes. There are only 36 simple special subset vertex graphs of type II with 127 nodes / vertices.

There is only one special simple vertex subset graph of type I with 251 nodes / vertices.

Consider the subset vertex graph of type II, H given by

H =

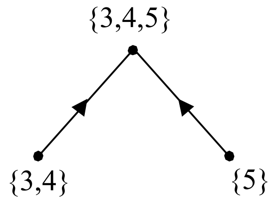


Figure 3.119

Clearly H is not a special simple subset vertex graph of type II as

P_1 =

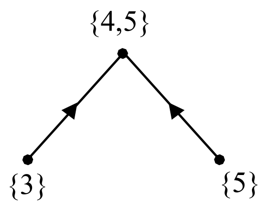


Figure 3.120

P_2 =

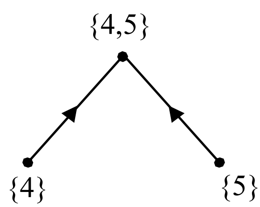


Figure 3.121

P_3 =

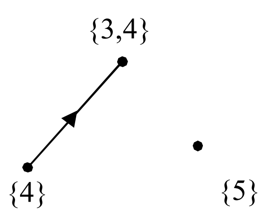


Figure 3.122

P_4=

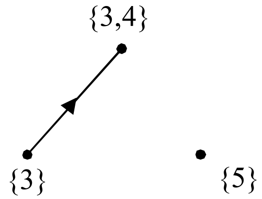


Figure 3.123

We see P_1 and P_2 are special subset vertex hyper subgraphs of H. However P_3 and P_4 are only special subset vertex subgraphs of type II of H. Consider K the subset vertex graph of type II given by the following figure.

K =

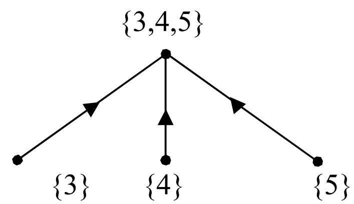


Figure 3.124

We see K is a special simple subset vertex graph of type II. However K has special subset vertex subgraphs which are not special subset vertex hyper graphs. The following figures gives the special subset vertex subgraph of type II of the subset vertex graph K.

L_1 =

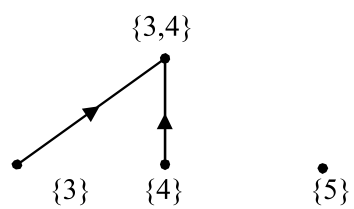


Figure 3.125

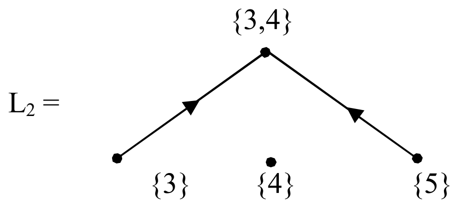


Figure 3.126

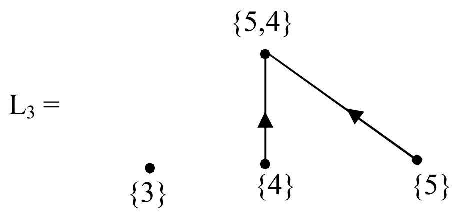


Figure 3.127

Consider the special subset vertex subgraphs of type II given by L_1, L_2 and L_3. None of them are special subset vertex hyper subgraphs of type II.

Consider the subset vertex graph M of type II given by the following figure.

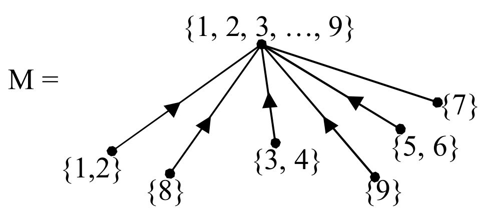


Figure 3.128

Clearly M is a tree with root {1, 2, 3, 4, …, 9}. All special subset vertex subgraph of type II are also trees. Infact M has special subset vertex subgraphs which are special subset vertex subgraphs which are trees given by the following figures.

B_1 =

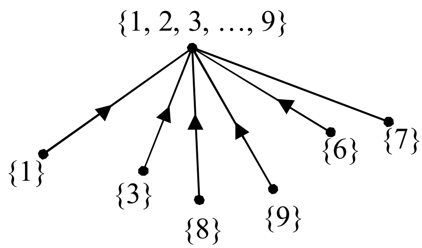


Figure 3.129

The special subset vertex subgraphs B_1 of M is a special subset vertex hyper subgraph.

Consider the subset vertex graph E of type II given by the following figure.

E =

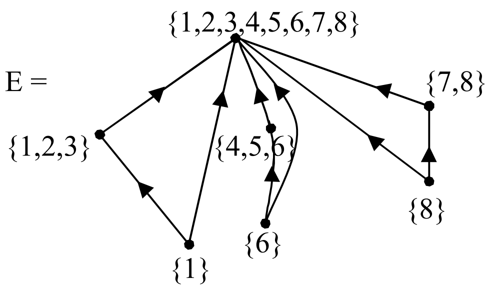


Figure 3.130

E has 7 vertices and 9 edges.

Consider the special subset vertex subgraph D_1 given by the following figure.

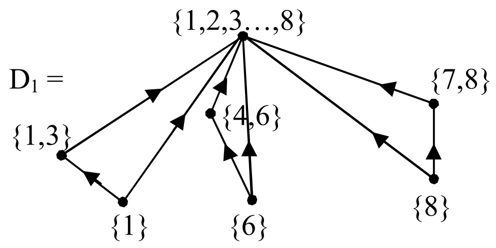


Figure 3.131

D_1 is a special subset subgraph with seven vertices and 9 edges so D_1 is a special subset hyper subgraph of E.

Consider D_2 the special subset vertex subgraph given by the following figure

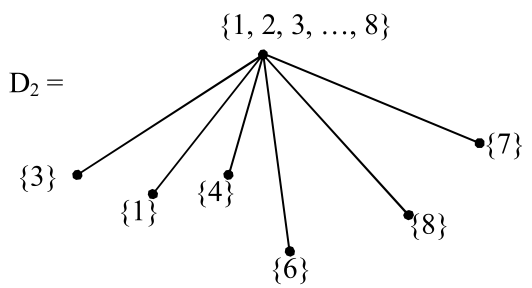


Figure 3.132

We see D_2 is only 9 special subset vertex subgraph which is not a hyper graph.

It can be easily verified E has no special subset vertex empty subgraph. Further D_2 is a tree.

Several interesting results can be obtained about the special subset vertex subgraph of type II.

It is pertinent to keep on record that in case of subset vertex graphs type II one cannot have line graphs with more than three vertices.

For if V = {{3}, {3, 6}, {5, 3, 6, 7}} be the given vertex set, we now describe the subset vertex graph of type II using these three vertices by the following figure.

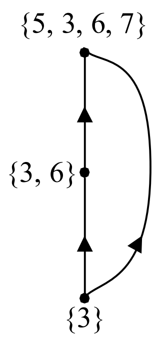


Figure 3.133a

We see for 3 vertices, one has 3 edges. Thus can we generalize in one uses n vertices where the set of n vertices forms a chain under the containment relation then can we say the subset vertex graph of type II with these n-vertices has how many edges?

Let us consider the set {{3, 7}, {3, 7, 8, 9}, {3, 7, 8, 4, 9}, {3, 7, 9, 8, 4, 11, 12}} = V the set of vertices. We have {3, 7} $\subseteq$ {3, 7, 8, 9} $\subseteq$ {3, 4, 7, 8, 9} $\subseteq$ {3, 4, 7, 8, 9, 11, 12}

The subset vertex graph of type II with V as the set of vertices is as follows

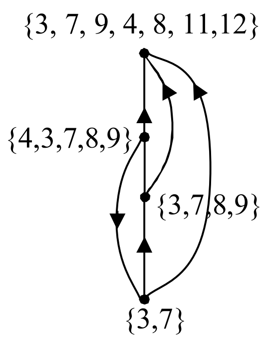


Figure 3.133b

Consider the vertex set V = {{4}, {4, 6,1}, {4, 6, 1, 5}, {4, 6, 1, 5, 8}, {1, 4, 6, 5, 8, 9}}. This set is such that {4} $\subseteq$ {4, 6, 1} $\subseteq$ {4, 6, 1, 5} $\subseteq$ {1, 4, 5, 6, 8} $\subseteq$ {1, 4,6, 8, 5, 9}.

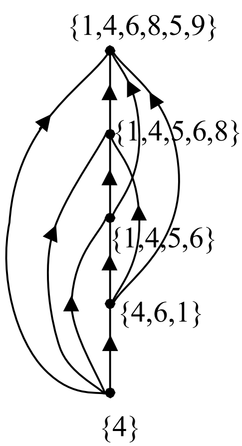


Figure 3.134

This subset vertex graph of type II using the vertex set V of cardinality 5 has 10 edges.

10 = 4 + 3 + 2 + 1

Thus if we have chain of length n then the subset vertex graph of type II with these n elements as vertices has

$(n-1) + (n-2) + \ldots + 3 + 2 + 1$ edges

that is $\frac{(n-1)\times n}{2}$ edges.

This can be proved by exploiting the set theoretic properties.

Theorem 3.4. *Let V be a collection of n subsets from P(S) where |S| = m, m finite such that the n subsets of V form a chain. The subset vertex graph of type II using these n subsets of V we get a subset vertex graph with* $\frac{n(n-1)}{2}$ *edges.*

Proof is direct.

Now we wish to prove the graph so formed is a complete directed graph.

Given $V = \{a_1, \ldots, a_n\}$ which is a vertex set with $a_i \in P(S)$; $1 \leq i \leq n$ such that $a_1 \leq a_2 \leq a_3 \leq \ldots \leq a_n$.

Now if we draw the directed graph, that is subset vertex graph of type I, then vertex a_1 has $n-1$ edges incident to it all of which are outward.

This is so because a_1 is contained in all the $(n-1)$ of the subsets $a_2, a_3, \ldots, a_n$ as the collection form a chain under inclusion relation.

Similarly a_2 has (n – 1) edges adjacent to it; of which (n – 2) of them are outward and only one is inward. This is so because a_2 is contained in all the (n – 2) subsets of V and a_1 is contained in a_2 and so on. Thus for a_{n-1} has (n – 1) edges incident to it with (n – 2) of the edges are inwards and only one edge is outward.

Thus we see a chain of n sets in P(S) gives way to a complete subset vertex graph of type II. Infact this ordered chain will only be a chain lattice.

Thus V = {{6, 3}, {6}, {6, 3, 7}} gives the following subset vertex graph of type II

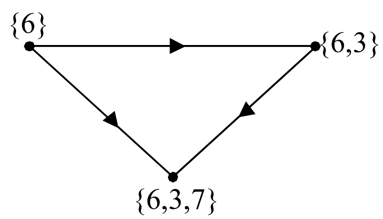


Figure 3.135

which is clearly a complete directed graph which is a subset vertex graph of type II.

{6} ⊆ {6, 3} ⊆ {6, 3, 7} is a chain or a totally ordered set.

Consider the set V = {{5}, {9,5}, {9, 5, 1, 3} and {9, 5, 3, 1, 8, 4}}. The subset vertex graph W of type II using this vertex set V is as follows.

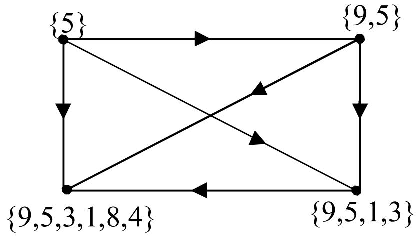
W = 

Figure 3.136

This is a subset vertex complete graph of type II. We have {5} ⊆ {9, 5} ⊆ {9, 5, 1, 3} ⊆ {9, 5, 3, 1, 8, 4} which is a totally ordered set.

If we draw the subset vertex graph with a base of chain lattice we get it as a line graph or a path or more so a directed path to be wrapped by edges. We call this configuration as a subset vertex wrapped directed path of type II.

Infact we can say every subset vertex complete directed graph of type II is a subset vertex wrapped up directed path and vice versa.

Infact they are one and the same. The subset vertex wrapped up directed path we built using the complete chain lattice as basically after all P(S) the power set of a set is a Boolean algebra of order 2^n if | S | = n.

If we add ϕ in our directed graphs then the subset vertex graph of type Ii with |P(S)| = 2^n vertices is not a Boolean algebra.

We will illustrate this situation by some examples.

Example 3.15. Let S = {1, 2, 3, 4} be a set of cardinality four. P(S) the power set of S. P(S) = {ϕ, {1}, {2}, {3}, {4}, {1, 2}, {1, 3}, {1, 4}, {2, 3}, {2, 4}, {4, 3}, {1, 2, 3}, {1, 2, 4}, {1, 3, 4}, {2, 3, 4}, {1, 2, 3, 4}}. The subset vertex graph of type II is as follows.

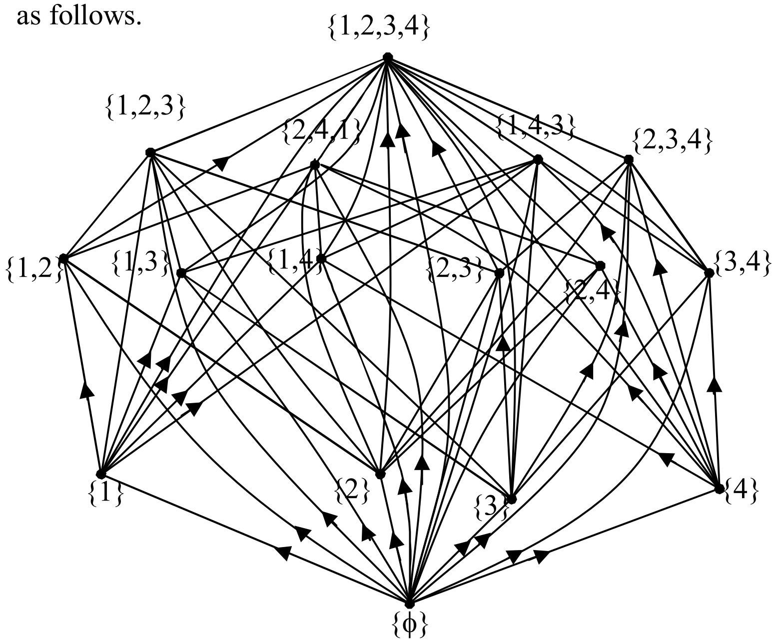


Figure 3.137

Clearly none of these subset vertex graphs of type II contain as subset vertex subgraphs of order greater than two.

We can have only the Boolean algebra of order two as subset vertex subgraph of any subset vertex graph of type II.

For this is true from the very fact that two subsets x, y $\in$ P(S) have a edge if and only if x $\not\subseteq$ y or y $\not\subseteq$ x denoted by

x $\bullet\!\!\longrightarrow\!\!\bullet$ y if x $\subseteq$ y and x $\bullet\!\!\longrightarrow\!\!\bullet$ y if y $\subseteq$ x.

Thus if we have

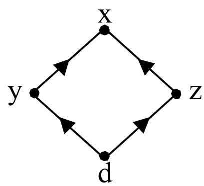


Figure 3.138

then y ⊂ x, z ⊂ x, z ⊂ x, d ⊂ z and d ⊂ y. Using these relations we have d ⊂ y ⊂ x and d ⊂ z ⊂ x hence d ⊂ x.

So the figure of the given form cannot exist as the edge $x \longrightarrow y$ is missed. So we cannot have a Boolean algebra of order two.

Now as any Boolean algebra is a direct product of order two and order four Boolean algebras we cannot have non trivial Boolean algebras as proper subset vertex subgraphs of type II for any subset vertex graph of type II.

Theorem 3.5. *Let P(S) be a power set of a finite set S. A subset vertex graph of type II cannot have a chain lattice of order greater than or equal to three.*

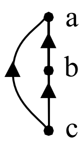
Proof is direct and hence left as an exercise to the reader. Another interesting problem about the subset vertex graph of type II is

i) Can we have lattices as subset vertex subgraph of type II or subset vertex graph of type II which are lattices? (other than chain lattice of order)

ii) If there exist a nontrivial lattices characterize them.

Only lattice is

Figure 1.139 (a)



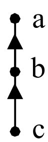


Figure 1.139 (b) **Figure 1.139(c)**

Figure 3.139(b) is a subset vertex graph of type II whereas Figure 3.139(c) is only a lattice.

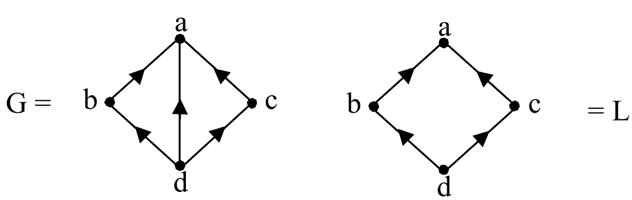


Figure 3.140

Clearly L is only a lattice and is not a subset vertex graph of type II.

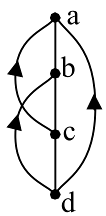


Figure 3.141

In fact a chain lattice is transformed into a subset vector complete graph of type II, hence the claim.

Authors conjecture no subset vertex graph of type II with more than two vertices is a lattice.

Next we proceed onto define the notion of complements of a special subset vertex subgraph of type II and the universal complement of a vertex subset graph.

We will illustrate this situation by examples.

Let S = {1, 2, 3, 4, 5, 6} be a set P(S) the power set of S. The subset vertex graph G of type II given by the set of vertices {1, 2, 3, 4, 6}, {4, 3} {ϕ}, {1, 2}, {6, 2}, {3}, {6, 4}, {6}} which is as follows.

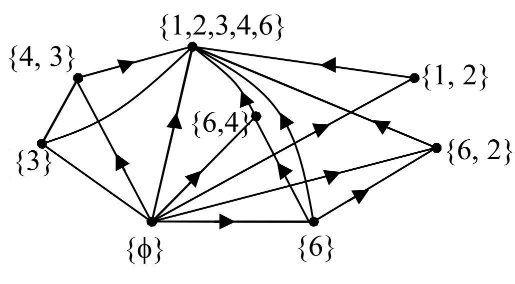


Figure 3.142

Now the universal complement subset vertex graph of type II of G is as follows which uses the following vertex subset {{5}, {1, 2, 5, 6} {1, 2, 3, 4, 5, 6}, {4, 5, 6, 3}, {1, 2, 3, 5}, {1, 3, 4, 5}, {1, 2, 4, 5, 6}, {1, 2, 3, 4, 5}}.

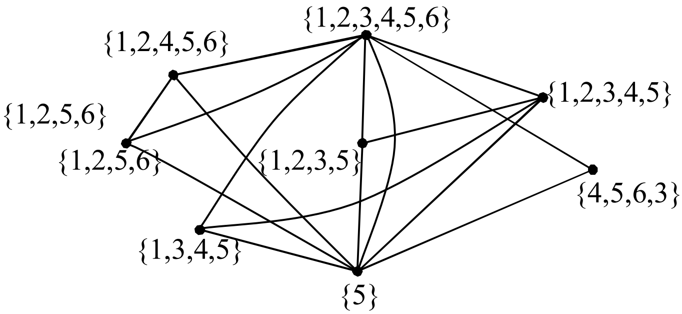


Figure 3.143

We see the set and its universal complement have same number of edges and vertices. We give another example before we proceed to define this concept abstractly.

Let {{1, 2, 3}, {1, 2, 3, 4, 5, 6}, {3, 4}, {3, 2}, {6, 5}, {4}, {3}, {2, 5, 3}, {1, 3, 6}} be the vertex set and H be the related subset vertex graph of type II given by the following figure.

H =

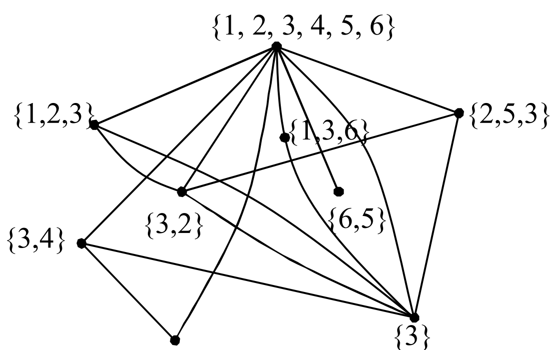


Figure 3.144

H is subset vertex graph of type II. The universal complement of H has the following set of vertices V = {ϕ, {4, 5, 6}, {3, 4, 5}, {1, 4, 6}, {1, 2, 5, 6}, {1, 4, 5, 6}, {1, 2, 3, 4}, {2, 5, 4, 6} and {1, 2, 3, 5, 6}}.

Let K be the universal complement subset vertex graph which is given by the following figure.

K =

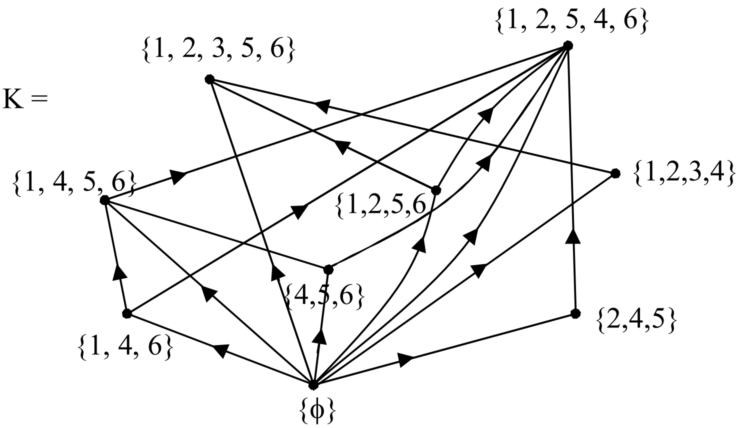


Figure 3.145

We see the degree of each of the vertex and that of its complements are equal in the subset vertex graph G and its universal complement subset vertex graph G^C.

Thus the concept of taking complements in case of subset vertex graph of type II is interesting as the universal complement subset graph of type II and the respective subset vertex graph contain same number of vertices and edges.

We now proceed onto define this concept.

Definition 3.6. *Let S be set and P(S) the power set of S. Let G be a subset vertex graph of type II with vertex set $v_1, v_2, \ldots, v_n \in P(S)$. The universal complement subset vertex graph of G has the subset vertex $\{v_1^C, v_2^C, \ldots, v_n^C\} \in P(S)$ such that $v_i \cup v_i^C = S$ for $i = 1, 2, \ldots, n$.*

We have given examples of them. We will provide some more examples of them.

Example 3.16. Let S = {1, 2, …, 12} be a set and P(S) the power set of S. Let G be the subset vertex graph of type II having the vertex set V = {{1, 2, 3, 4, 10, 6, 7, 9, 5}, {1}, {2}, {3}, {4, 10}, {6, 7}, {9, 5}} given by the following figure.

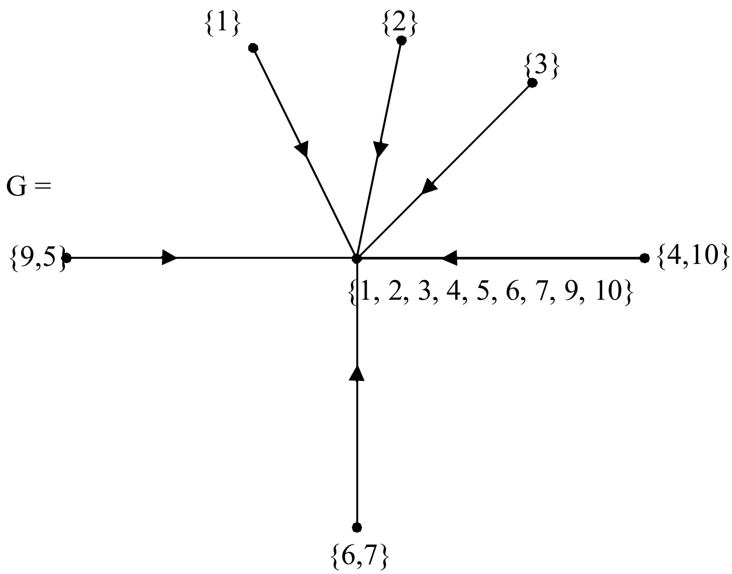


Figure 3.146

We see all edges are inward to the vertex {1, 2, 3, 4, 5, 6, 7, 9, 10} and other edges are outward and G is a subset vertex star graph of type II.

Now we find the universal complement G. Clearly the universal complement subset vertex graph of G of type II has the following set of vertices {{8, 11, 12}, {2, 3, …, 12}, {1, 3, 4, …, 12}, {1, 2, 4, 5, 6, …, 12}, {1, 2, 3, 5, 6, 7,8, 9, 11, 12}, {1, 2, 3, 4, 5, 8, 9, 10, 11, 12}, {1, 2, 3, 4, 6, 7, 8, 10, 11, 12}}. The universal complement subset vertex graph H of G is as follows.

H =

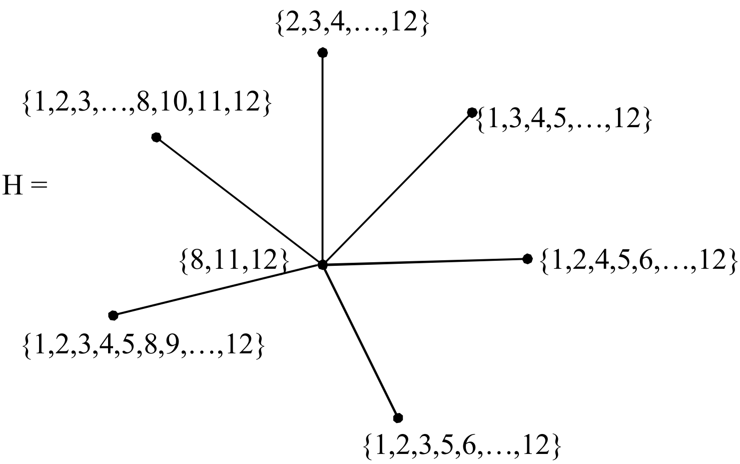


Figure 3.147

Clearly this is also a star graph in which the complement of the vertex {1, 2, 3, 4, 5, 6, 7, 8, 9} is {8, 11, 12} and for the universal complement of the star graph G = {8, 11, 12} is the central element with the major difference all edges from {8, 11, 12} are outward. We call H the universal complement of G and G the universal complement of H.

Thus these type of subset vertex subgraphs can play a vital role in social networking and the prominent node or the central node of the community network.

Now consider the subset vertex graph K with vertex set V = {{4, 5, 8, 3, 7, 1}, {1, 4, 5, 8, 3}, {4, 5, 3, 1} {1, 4, 5}, {1, 4}}. The subset vertex graph is given below.

K =

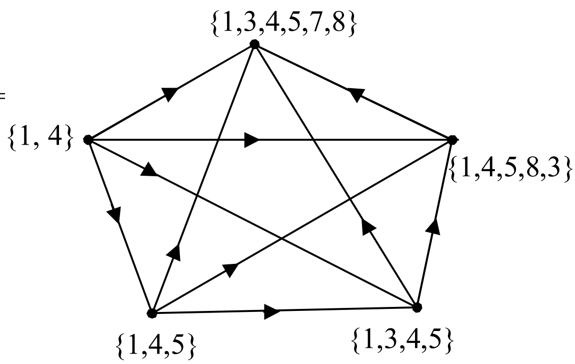


Figure 3.148

Clearly K is a subset vertex complete graph of type II.

Now we find the universal complement of K.

The vertex set of type universal complement subset graph L is as follows. V_1 = {{2, 6, 9, 10, 11, 12}, {2, 9, 6, 7, 10, 11, 12}, {2, 6, 7, 8, 9,10, 11, 12},{2, 3, 6, 7, 8, 9, 10, 11, 12}, {2, 3,5, 6, …, 12}}. The graph of L is as follows.

L =

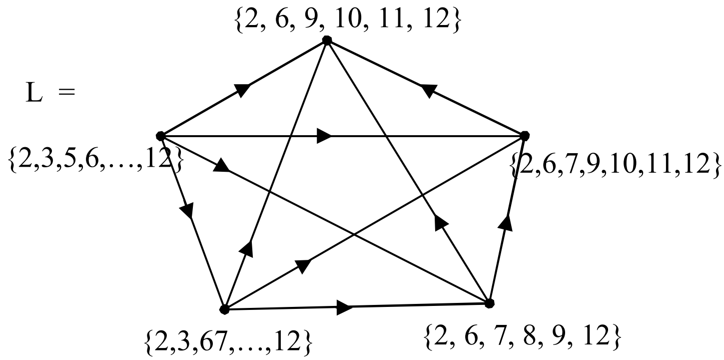


Figure 3.149

Clearly L is also a complete subset vertex graph.

In view of all these we prove the following theorem.

Theorem 3.6. *Let S be a finite set P(S) the power set of S. Let G be any subset vertex graph of type II. The universal complement of the subset vertex graph H of G preserves structure (by structure we mean if G has n vertices and m edges the universal complement H of G will also have n vertices and m edges only the direction of edges will change and the degree of the vertices / nodes does not change). If G is subset vertex complete graph where the vertex subsets forms an ordered chain then so is the universal complement of it.*

Proof follows by exploiting the set theoretic properties mainly the containment relation.

Having seen the definition and examples we proceed onto describe about the main theme of this book namely special subset vertex subgraphs of a special subset vertex graphs. Here we first give some examples of the complements of the subset vertex special subgraphs of a graph.

Let K be the subset vertex graph of type II given by the following with the vertex set V = {{1, 2, 3, 4, 7, 5, 6}, {3, 2}, {4, 6}, {5, 4, 6}, {1, 2, 3}, {2, 3, 4}, {2}, {3, 7}, {3, 4},{3}} ⊆ P(S) where S = {1, 2, 3, 4, …, 9}. The figure of the subset vertex graph K is as follows.

K =

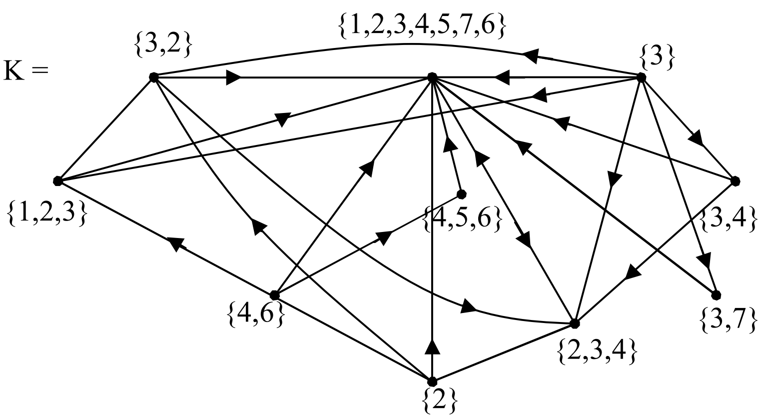


Figure 3.150

The special subset vertex subgraph W of K with vertex set A = {1, 2, 3, 4, 5, 7}, {3, 2}, {3}, {1, 2}, {4, 5}, {4}, {2}, {2, 4}, {3, 4}, {7}} is as follows.

W =

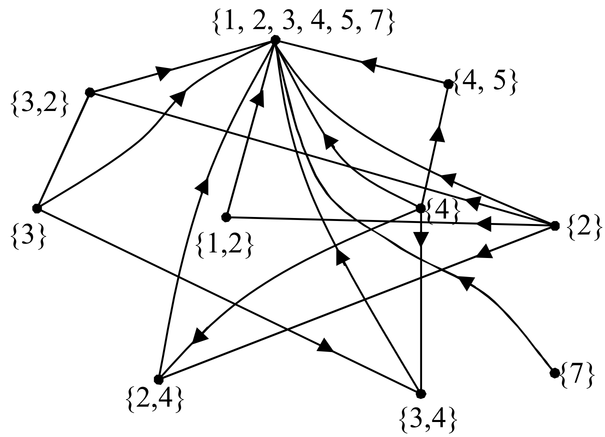


Figure 3.151

The special complement subset vertex subgraph of W relative to R is as follows.

First we give the vertex set of the complement graph of W; {{6}, {ϕ}, {3}, {6}, {6}, {ϕ}, {3}, {ϕ}, {2}} is the vertex set of local complement of the vertex set A of W.

Clearly the collection {{6}, {ϕ}, {3}, {3}} is only a empty graph and the special subset vertex empty subgraph is not defined as the vertices coincide. In this case we say the subset vertex graph W of type II has no special complement.

We have to characterize those special subset vertex graph which has speical complement subset vertex subgraphs.

Now consider the subset vertex graph G of type II given by the following figure;

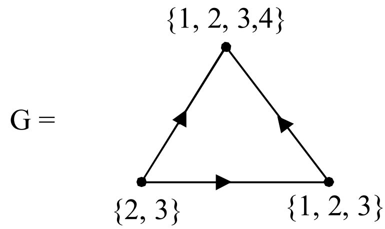


Figure 3.152

The special subset vertex subgraph H of G is given by

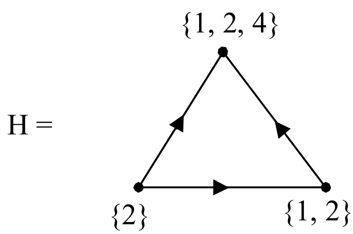


Figure 3.153

Now the complement special subset vertex subgraph of H or special complement subset vertex subgraph of H or the local complement special subset vertex subgraph of H is as follows.

First the local complement vertex subset of H is {{3}, {3}, {3}}.

Hence the local complement vertex subset of H is not a special subset vertex subgraph so local complement does not exists for this Hthat is not defined.

Consider the subset vertex graph K of type II given by the following figure.

K =

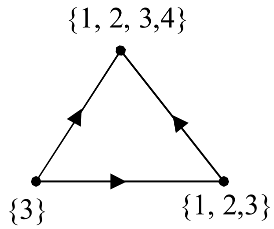


Figure 3.154

Let L be the special subset vertex subgraph of K given by the following figure.

L =

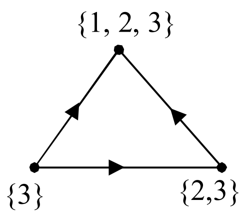


Figure 3.155

Now we find the special subset vertex subgraph L's complement relative to K that is the local complement.

Clearly the vertex set of the local special complement is {ϕ, {4}, {1}}.

Let M denote the local special complement of L which is given in the following.

M =

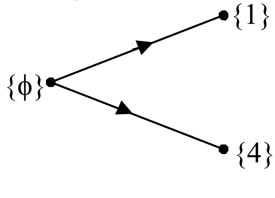


Figure 3.156

We see M is the local special complement subset vertex subgraph of L.

We see L is a complete special subset vertex subgraph in theory however M is not a complete special subset vertex subgraph. To be more specific L has 3 vertices and 3 edges but M has only 2 edges and 3 vertices which is mandatory.

Now the problem is what will one take as the complement of a special subset vertex subgraph which has the same set of vertices as that of a subset vertex graph.

The answer for this is that for such special subset vertex subgraph of G which has some of the vertices to be the same as that of the special subset vertex graph G we just state the complement does not exist for the simple reason if as in above example assume it to be ϕ then ϕ^C for the vertex set of G cannot be defined. Hence it is mandatory if we are trying to find the complement of special subset vertex subgraph of a given special subset vertex subgraph then the vertices of the given subset vertex graph and that of the special subset vertex subgraph must be proper subsets and not coincident subsets.

Consider the subset vertex graph G of type II given by the following figure.

G =

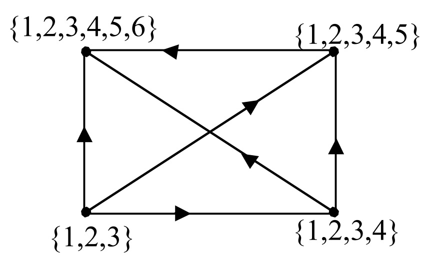


Figure 3.157

Let H be the special subset vertex subgraph with vertex set {{1, 2, 3, 4, 6}, {1, 2, 4, 5} {1, 2} and {2}} that is {1, 2, 3, 4, 6} ⊆ {1, 2, 3, 4, 5, 6}, {1, 2, 3, 4, 5} ⊇ {1, 2, 3, 4}, {1, 2} ⊆ {1, 2, 3, 4} and {2} ⊆ {1, 2, 3}. The special subset vertex subgraph H of G is as follows.

H =

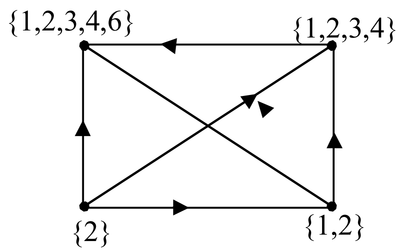


Figure 3.158

Now we try to find the complement of H that is the complement special subset vertex subgraphs of the special subset vertex subgraph.

The vertex complement subsets are {{5}, {5}, {3, 4} {1, 3}}. Clearly as the cardinality of the vertex set is only three so this vertex set does not even give a special subset vertex subgraph of type II consider K_2 the special subset vertex subgraph of G.

K_2 =

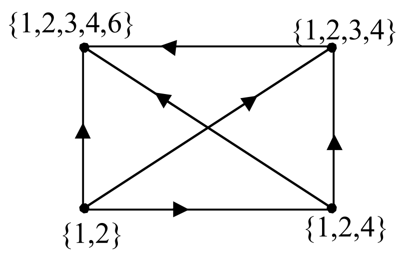


Figure 3.159

The complement of the vertices of K_2 is {5} complement of {1, 2, 3, 4, 6}

{5} is the complement of {1, 2, 3, 4} in {1, 2, 3, 4, 5}

{3} is the complement of {1, 2, 4} in {1, 2, 3, 4}

{3} is the complement of {1, 2} in {1, 2, 3}

Clearly the vertex set {{5}, {3}} does not yield a special subset vertex subgraph of type II as 4 vertices is mandatory in this case

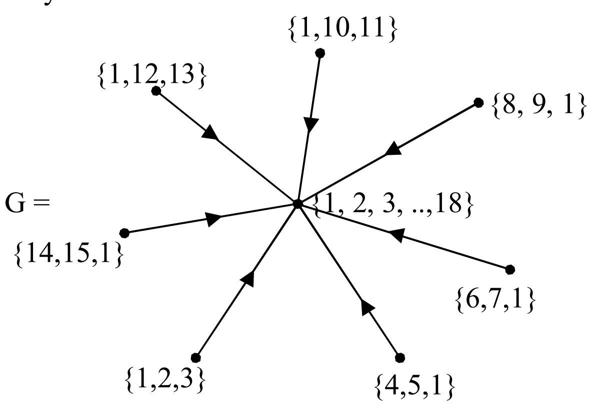


Figure 3.160

Consider the subset vertex star graph G of type II given by the following figure.

Let S be the special subset vertex subgraph of G given by the following figure.

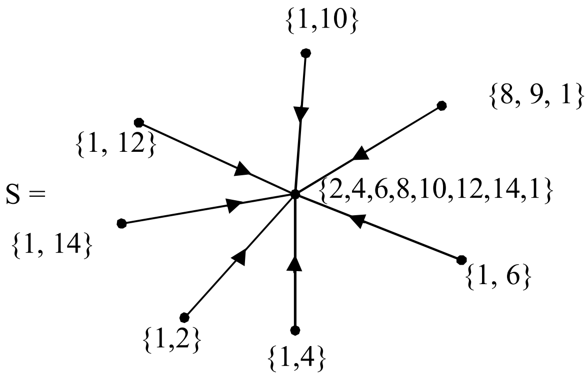


Figure 3.161

The special complement subset vertex subgraph of S is as follows.

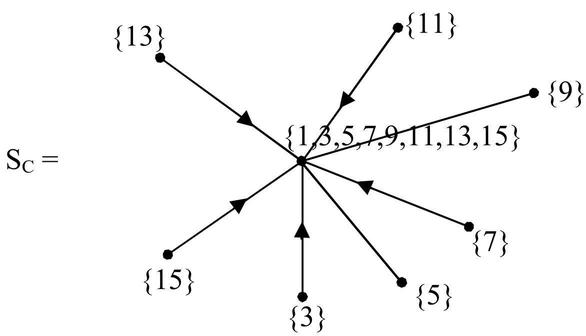


Figure 3.162

We see S_C the special complement subset vertex subgraph of S is also a star graph. Infact we can also call S_C the special complement subset vertex subgraph as special complement subset vertex hyper subgraph of S.

We see $S \cup S_C = G$.

In general the structure of the graph need not in general be inherited by the special complement subset vertex subgraph of a graph.

It is left as a open problem for the reader to characterize all those subset vertex graph of type II which has special complement subset vertex subgraphs such that they are structure preserving.

All star graphs which are subset vertex graphs need not in general be structure preserving.

Consider the star graph which is a subset vertex graph given by the following figure.

G =

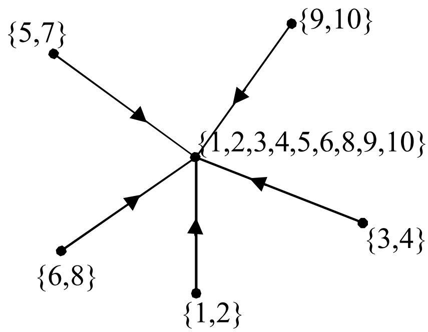


Figure 3.163

Clearly

H =

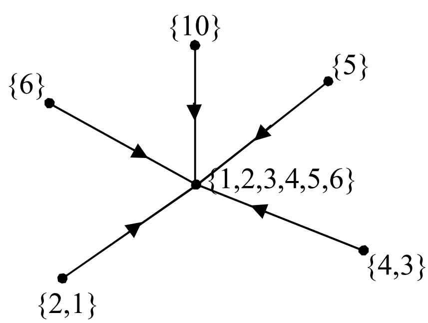


Figure 3.164

is a special subset vertex subgraph and H has no special complement subset vertex subgraph as {1, 2} and {4, 3} have no complements.

Hence the claim.

Consider the subset vertex graph G of type II given by0 the following figure.

G =

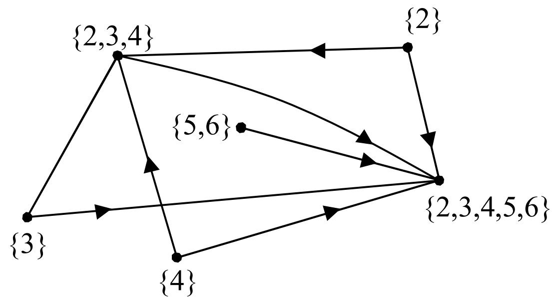


Figure 3.165

Let H =

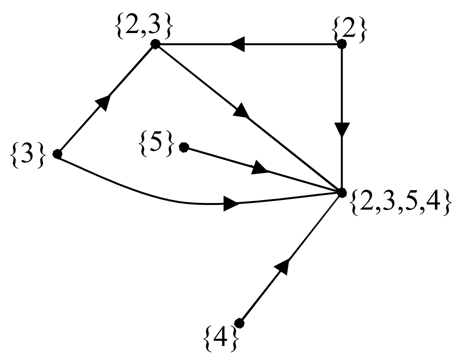


Figure 3.166

Clearly H is a special subset vertex subgraph of G which is not a special subset vertex hyper subgraph.

H has no special complement subset vertex subgraphs.

We call these special subset vertex subgraphs as devoid complement special subset vertex subgraphs. Before we give more properties and conditions for these special subset vertex subgraphs to contain special complement subset vertex subgraphs we will proceed onto make the abstract definition of special complement subset vertex subgraphs of a special subset vertex subgraph in the following.

Definition 3.7. *Let S be a finite set, P(S) the power set of S. G be the subset vertex graph of type I with vertex set $v_1, v_2, \ldots, v_n \in P(S)$. Let H be a special subset vertex subgraph of G with $\{u_1, u_2, \ldots, u_n\}$ as vertex set where $u_i \subseteq v_i$; $1 \leq i \leq n$.*

We find a vertex set W given by $\{v_i \setminus u_i; 1 \leq i \leq n\} = W$. If the special subset vertex subgraph B exists with vertex set W with n distinct vertices then we define B to be a special complement subset vertex subgraph of H relative to G.

The following observations are important.

1. If $v_i \setminus u_i = \{\phi\}$ for some i then we cannot define the special complement subset vertex subgraph for any $i = 1, 2, \ldots, n$.

2. If $v_i \setminus u_i = v_j \setminus u_j$ $i \neq j$ then also we cannot define the special complement subset vertex subgraph of H relative to G.

3. If (1) is true it implies we have to take for H also the same vertex set so only $v_i \setminus u_i = \phi$ hence we

cannot define special complement subset vertex subgraph of H relative to G.

4. It may so happen that at times the complement may be a special subset vertex empty subgraph of G.

5. Clearly the special complement subset vertex subgraph of H relative to G is also a special subset vertex graph with G = H $\cup$ B where H $\cup$ B has the vertex subset given by $\{v_i = u_i \cup v_i \setminus u_i; i = 1, 2, \ldots, n\}$.

   So the edges of H $\cup$ B is also the same as the edges of G.

6. It may so happen that at times the structure of the special complement subset vertex subgraph may be the same as that of the special subset vertex subgraph and the subset vertex graph. We call such special complement subset vertex subgraphs as structure preserving complements.

We have given an example of a structure preserving complements which were star graphs we saw all of the three graphs G, H and B happened to be star graphs.

In general we cannot always say the special complement subset vertex subgraphs are structure preserving.

We will provide some examples of them.

Example 3.17. Consider the subset vertex graph G of type II given by the following figure 3.167, the vertex set of G is taken from P(S) where S = { 1, 2, 3, …, 7}.

G =

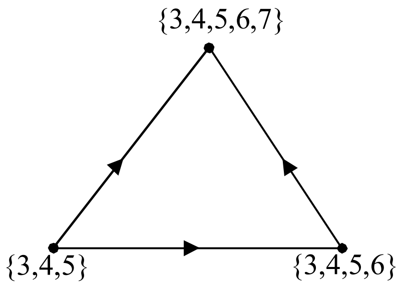


Figure 3.167

Let H be a special subset vertex subgraph of G given by the following figure;

H =

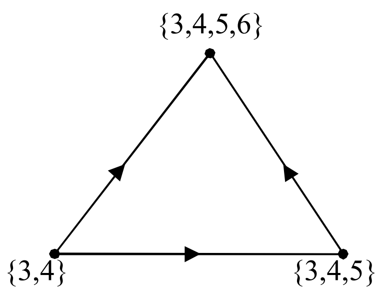


Figure 3.168

Now clearly

$\{3, 4, 5, 6\} \subseteq \{3, 4, 5, 6, 7\}$,

$\{3, 4, 5\} \subseteq \{3,4,5, 6\}$ and

$\{3, 4\} \subseteq \{3, 4,5\}$.

We find the special complement of the special subset vertex subgraph H.

Clearly both H and G are complete subset vertex graph.

Let W be the special complement subset vertex subgraph of relative to G given by the following figure.

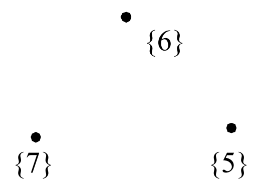


Figure 3.169

We see {3, 4, 5, 6, 7} \ {3, 4, 5, 6} = {7}; {3, 4, 5, 6} \ {3, 4, 5} = 6} and {3, 4, 5} \ {3, 4} = {5}.

Thus W has the vertex set {{5}, {6}, {7}} which only yields a special complement subset vertex subgraph which is an empty graph.

This is a very extreme case where the complement of a complete graph is only an empty graph which clearly shows that taking complements will by no means preserve the structure of the graph.

Thus we see some of the following results may list out the properties enjoyed by the special complement of the subset vertex subgraph.

Proposition 3.3. Let S = {1, 2, ..., n} be a finite set and P(S) the power set of S. If G is a subset vertex graph of type II with (v_1,

…, v_m) ∈ P(S) as vertices (distinct subsets of P(S)) then if H is a special subset vertex subgraph of G with $\{w_1, ..., w_m\}$ as vertex set then for the special complement subset vertex subgraph to exist we must have $v_i \setminus w_i \neq v_j \setminus w_j$ ($i \neq j$; $1 \leq i, j \leq n$) and $v_i \setminus w_i \neq \{\phi\}$.

Proof. Directly follows from the very definition of special complements subset vertex subgraphs of H relative to G.

Now we will examples provide to show how if $v_i - u_i = \phi$ or $v_i - u_i = v_j - u_j$; $i \neq j$; $1 \leq i, j \leq n$ and both is true we cannot have special complement subset vertex subgraphs.

To this effect we first describe how if v_i is a vertex of the graph G then if H the special subset vertex subgraph which also takes for that vertex v_i only v_i then we cannot find the complement.

For if we say ϕ it is the complement of the universal set however here complements are localized complement and we do not have the notion of universal set. So if ϕ is taken as $v_i - u_i$ then what will be its complement for it may not belong to P(S) and so on. To over come all these ambiguities we have said if $v_i = u_i$ then that vertex complement does not exist. This is mandatory for us to preserve the concept of well definedness to be preserved in the arguments and definition of special complement subset vertex subgraph.

Consider the subset vertex subgrpah G give by the following figure.

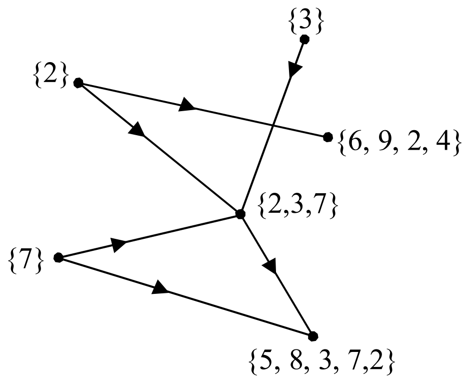


Figure 3.170

where the vertex set is taken from P(S); S = {1, 2, 3, …, 9}.

We have special subset vertex subgraph H given by the following figures.

H =

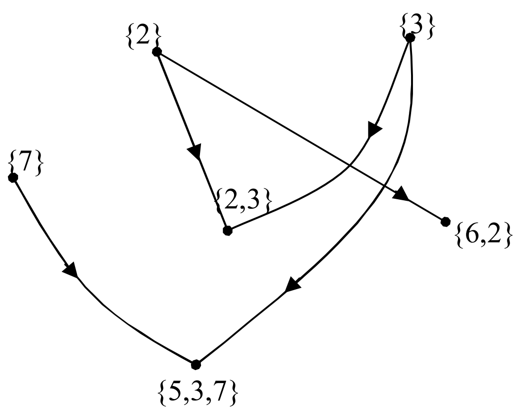


Figure 3.171

Clearly H has not special complement subset vertex subgraph as the vertices {2}, {3} and {7} cannot have proper subsets hence cannot find complements.

Thus this very subset vertex graph G has no special subset vertex subgraphs which is a special complement subset vertex subgraph.

In view of all these observations we prove the following theorem.

Theorem 3.7. *Let P(S) be the power set of a finite set S. G be a subset vertex graph of type II with vertex set* $\{v_1, v_2, \ldots, v_n\} \subseteq P(S)$*. Even if only one of the vertices* v_i*;* $1 \leq i \leq n$ *is a singleton set then that G has no special subset vertex subgraph which has special complement subset vertex subgraph.*

Proof: Follows from the fact that one of the vertices $v_i \in \{v_1, v_2, \ldots, v_n\}$ is a singleton set than by the definition of special complement subset vertex subgraph we see there can be no special subset vertex subgraph for the subset vertex graph of type II.

We characterize those subset vertex graphs of type II as special simple complement subset vertex graphs.

We proceed onto prove the interesting existing theorem.

Theorem 3.8. *Let P(S) be the power set of a set S. All subset vertex graphs with atleast one vertex set to be singleton set different from empty set is a special complement simple subset vertex graph.*

Proof. Follows from the fact that if G is a subset vertex graph of type II with one or more vertices to be singleton set then G has no special complement subset vertex subgraph proved earlier. Hence the claim.

Thus the collection of special complement simple subset vertex graph using P(S) is non empty.

Next we proceed onto describe the condition on the special subset vertex subgraphs to have special complement subset vertex subgraphs.

In the first place the vertex set of H must have well defined complements with respect to the subset vertex graph G to which H is a special subset vertex subgraph.

Secondly the vertex set of the complement subgraph must also have the same cardinality of the vertex set of H. That is the complement subgraphs must be distinct.

Let us describe this by a very simple example.

Let G =

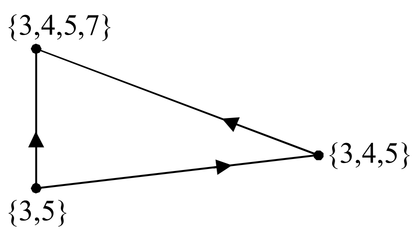


Figure 3.172

This is a complete subset vertex graph.

Let H be the special subset vertex graph.

Let H be the special subset vertex subgraph given by the following figure.

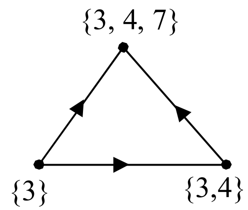


Figure 3.173

Now complement vertex set of {3, 4, 7} is {4, 5, 3, 7} \ {3, 4, 7} = {5}.

Complement set of {3, 4} is {3, 4, 5} \ {3, 4} = {5}

The complement set of the vertex 3 is {3, 5} \ {3} = {5}.

We see the set of vertices is a singleton single element {5}. So the special complement subset vertex subgraph of H does not exist infact it a point graph given by {5}. Thus it is a point graph so does not even fall under the nontrivial subset vertex graph.

So, for us to have a complement of a special subset vertex subgraph for a given special subset vertex subgraph the condition cannot be easily enumerated. Further for structure preserving special complement subset vertex graphs happens to be a very difficult one.

Now if the complement of a special subset vertex subgraph exists we can then study social networks or community network on the complement of the society or that set of which happens to be a complementing part of the network. Certainly once these graphs and their properties are analysed certainly one can develop a better understanding of the community / problem. This can also be used in case of different types of social (or otherwise) networks which has complements.

Now we give examples of universal complements of a subst vertex graph of type I.

Example 3.18 Let S = {1, 2, …,12} and P(S) be the power set of S consider the subset vertex disconnected graph G of type I given by the following figure.

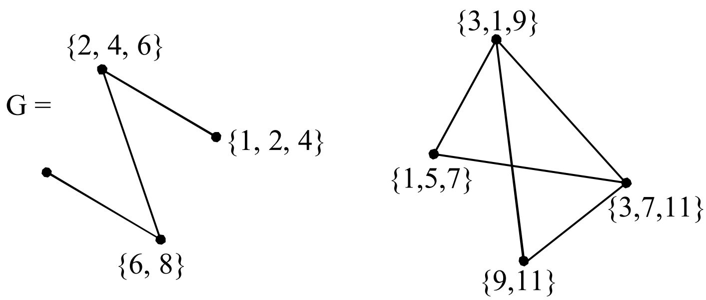


Figure 3.174

Clearly G is a subset vertex disconnected graph of type I. The universal complement G^C of G is as follows.

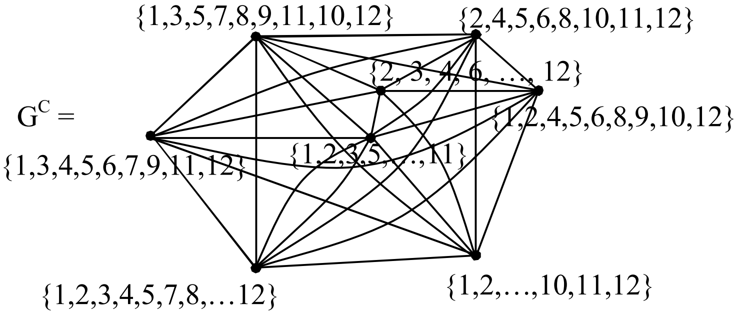


Figure 3.175

Clearly G^C the universal complement subset vertex graph of G is a subset vertex complement graph.

Now we give some examples of subset vertex disconnected graphs of type II and its universal complement in the following.

Example 3.19. Let S = {1, 2, …, 9} be a set and P(S) the power set of S. Consider the subset vertex graph of type II given by the following figure.

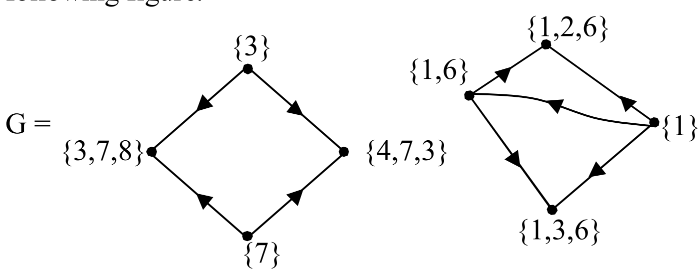


Figure 3.176

Clearly G is a subset vertex disconnected graph of type II. Now we find the universal complement G^C of which is as follows.

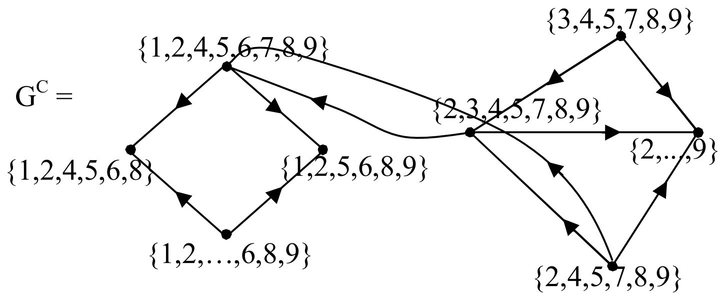


Figure 3.177

We see the universal complement subset vertex graph G^C of G is a connected subset vertex graph of type I, however G is disconnected.

We give yet another example.

Example 3.20. Let S = {1, 2, …, 9} be the set P(S) the power set of S. Let G be the subset vertex graph of type II given by the following figure.

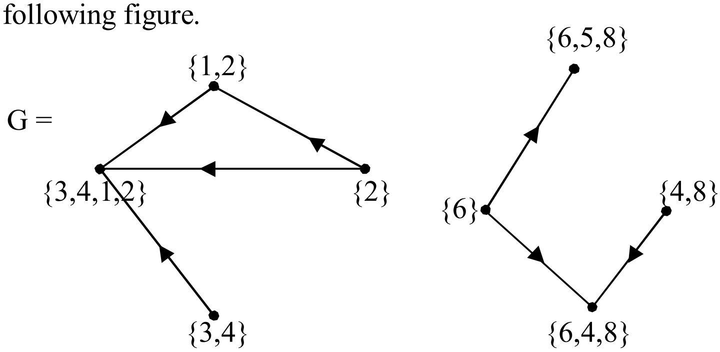


Figure 3.178

Clearly G is a subset vertex disconnected graph of type II.

Now we give the universal complement subset vertex graph G^C of G.

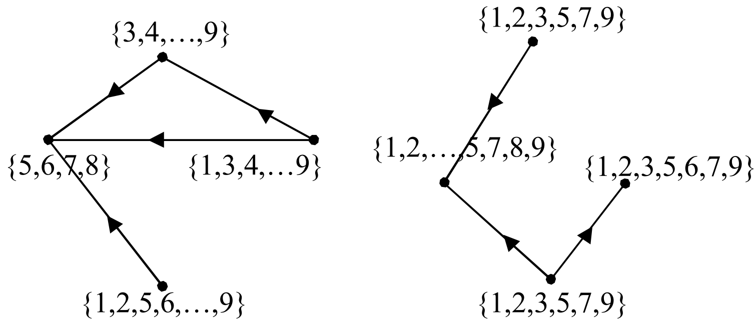


Figure 3.179

We see the G and G^C are similar except the directions in G and G^C are opposite of each other, of course the vertex set values are different.

Next we proceed onto give examples of subset vertex graphs of type II which are trees and find their universal complements.

Example 3.21 Let S = {1, 2, …, 27} be a set of order 27 and P(S) the power set of S. Let G be the subset vertex graph of type II which is a tree.

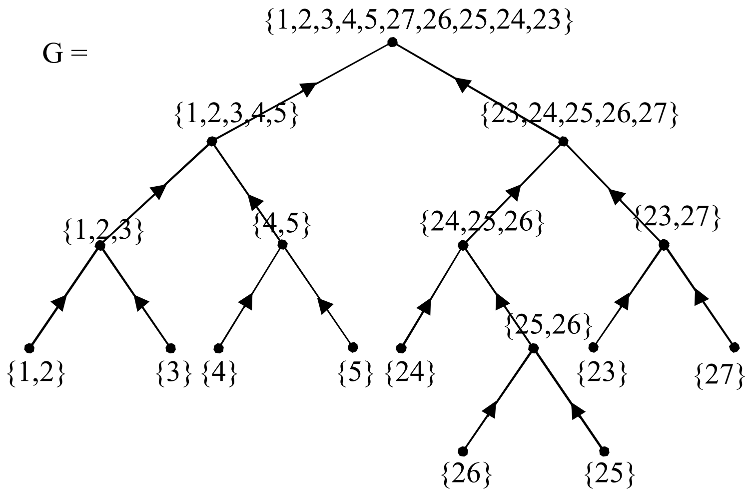


Figure 3.180

Now we find the universal complement G^C of G.

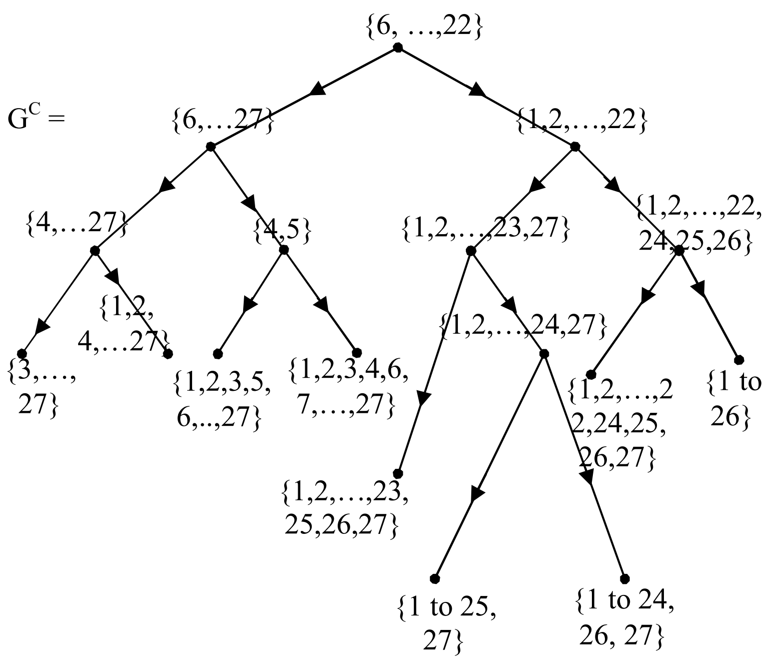


Figure 3.181

We see G^C is also a subset vertex tree of type II. Both G and G^C have same number of edges and vertices and both of them are trees however the direction of the edges in case G and G^C are reversed which is very clear from the property of subset and their complements. Study in this direction is a matter of routine so left as an exercise to the reader.

Now we proceed onto study the universal complements as well as complements of special vertex subset subgraphs and complements of just subset vertex subgraphs of a projective subset vertex graph of type II.

The concept of projective subset vertex graphs of type II was introduced in chapter II of this book. Now we proceed onto study the properties related to their complements.

Recall if {3, 5, 6} and {7, 8, 3, 5, 6, 4, 18} are two vertex subsets in case of injective subset vertex graphs we have to edge from {3, 5, 6} directed towards

{7, 8, 3, 5, 6, 4, 18} that is

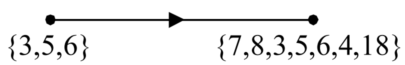


Figure 3.182

However in case of projective subset vertex graphs the edge will be from the vertex set {7, 8, 3, 5, 6, 4, 18} to {3, 5, 6} given by

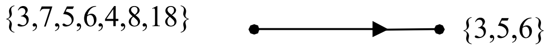


Figure 3.183

That is the large set is projected on the small set.

So these projective vertex subset graphs of type II have edges just opposite to the vertex subset graphs of type II or to be more specific injective vertex subset graphs of type II.

Now we proceed onto supply examples of special projective subset vertex subgraphs of a projective subset vertex graph of type II in the following first by examples.

Example 3.22. Let S = {1, 2, 3, …, 36} and P(S) be the power set of S. Consider the projective subset vertex graph G of type II given by the following figure:

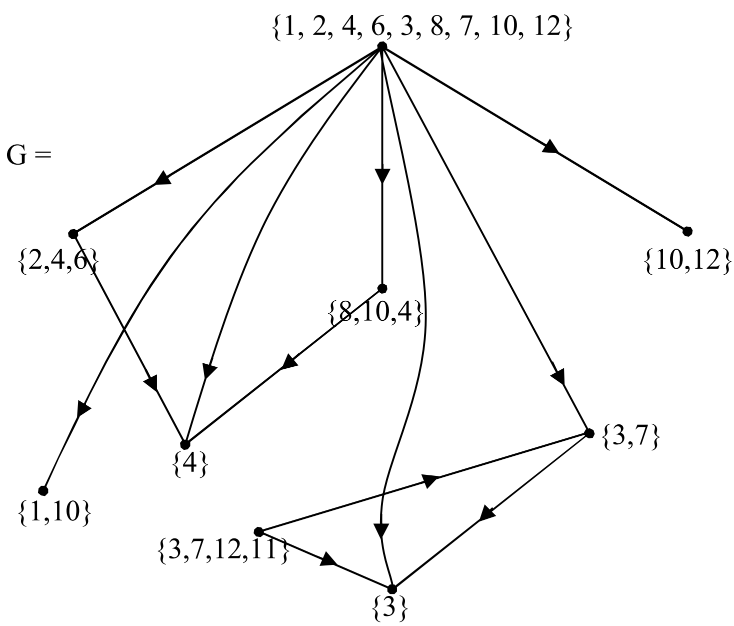


Figure 3.184

We have discussed about projective subset vertex subgraphs of type II in chapter II of this book.

Now we find special projective subset vertex subgraphs of typ II or projective special subset vertex subgraphs of type II for this G.

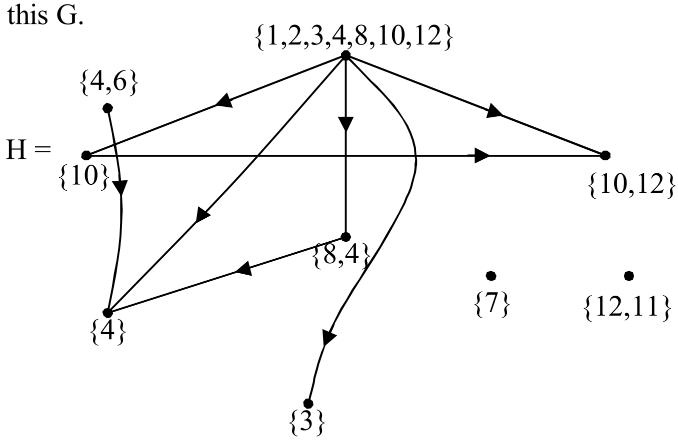


Figure 3.185

H given in the figure 3.185 is a projective special subset vertex subgraph of type II.

Clearly number of vertices of H is also 9 as that of G which is mandatory. However the number of edges in G is 12 and that of H is only 8. The graph is a disconnected projective subset vertex graph of type II.

We give yet another example of a projective special subset vertex subgraph of type II.

Let K be a projective special subset vertex subgraph of type II given by the following figure.

K =

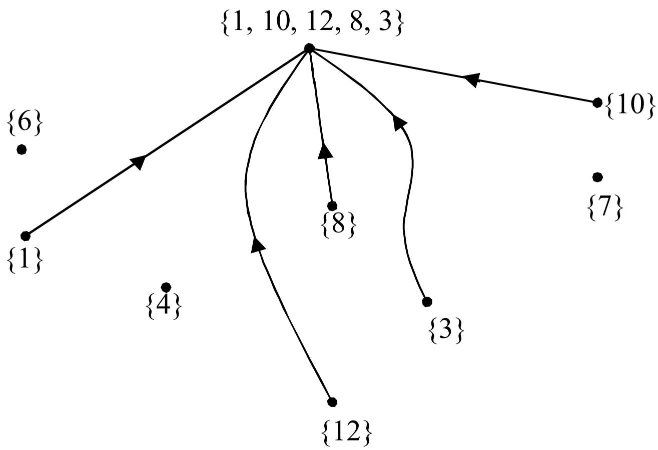


Figure 3.186

K is a special projective subset vertex subgraph of type II. Infact an incomplete star graph.

This graph is also disconnected has 9 nodes and only 5 edges.

We give yet another example of a projective special subset vertex subgraph L of G of type II in the following figure.

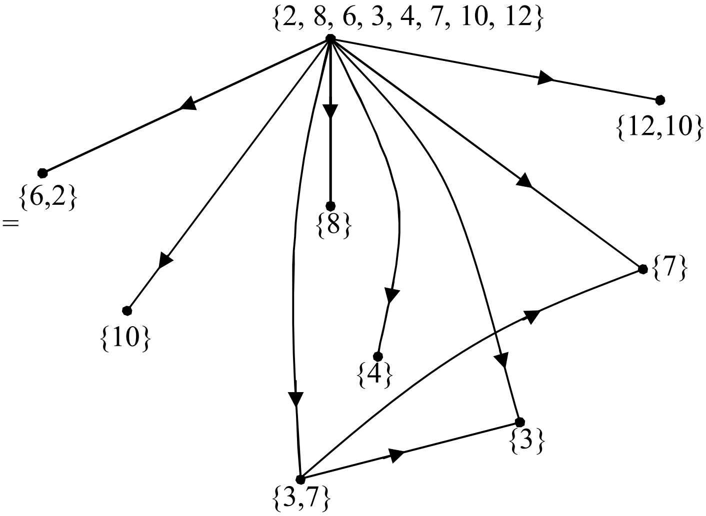
L = 

Figure 3.187

Thus we can find several special projective subset vertex subgraphs of type II.

Finding just projective subset vertex subgraphs of type II is a matter of routine. We just give one of two illustration of them in the following.

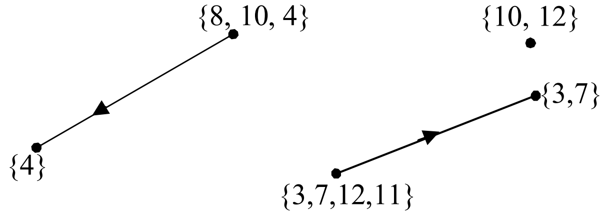
M = 

Figure 3.188

M is the projective subset vertex subgraph of type II we ee the subset of the nodes set is not taken. The nodes are taken as it is. Only the number of (nodes) is decreased from that of G.

That is the vertex subsets are just kept the same and only the number of vertex subsets from the graph. G are decreased. These will be called as ordinary subset vertex projective subgraphs of type II.

We also give examples of both ordinary projective subset vertex subgraphs as well as projective subset vertex subgraphs. We have already given examples of projective special subset vertex subgraphs of type II.

Example 3.23. Let S = {1, 2, …, 18} be a set of order 18 and P(S) the power set of S. Let G be the projective subset vertex graph of type II given by the following figure. We also use the notation of putting the vertex set in circles as shown below;

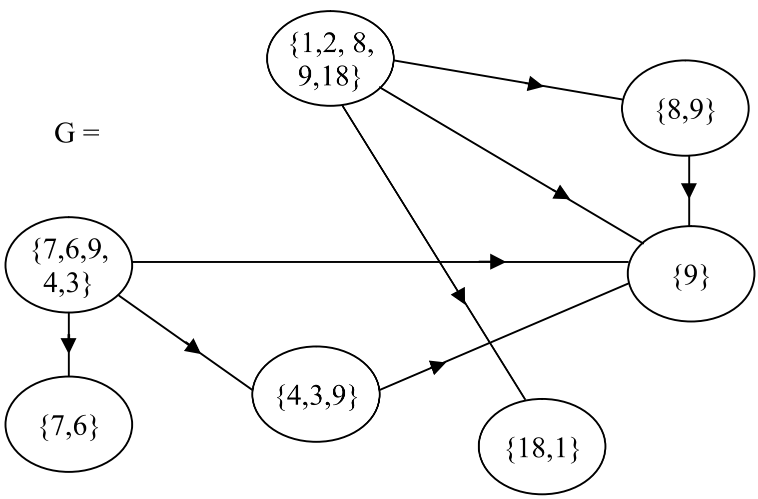


Figure 3.189

First we provide two ordinary subset vertex projective subgraphs of G. Let H_1 be the ordinary projective subset vertex subgraph of G given by the following figure

H_1 =

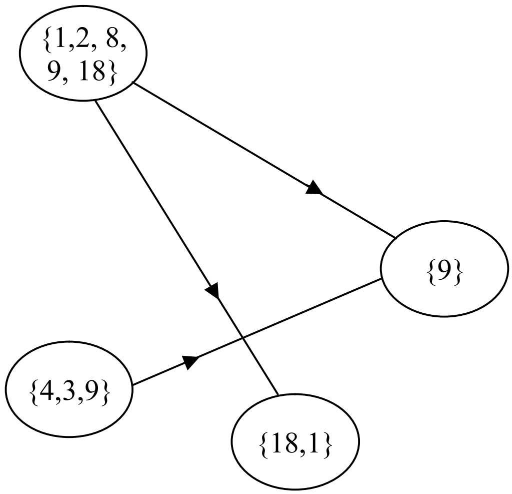


Figure 3.190

Let H_2 be the ordinary projective vertex subset subgraph of G given b y the following figure.

H_2 =

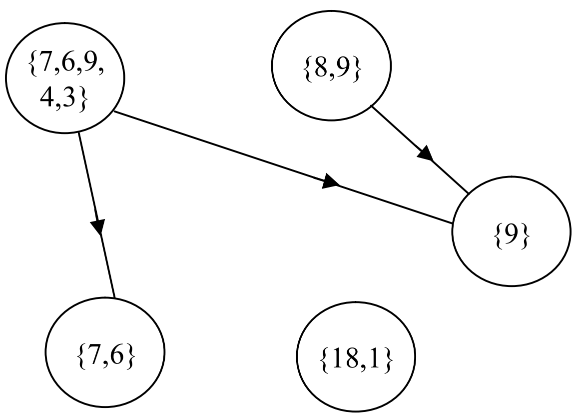


Figure 3.191

We see H_2 is a ordinary projective subset vertex subgraph of type II. It is disconnected. Consider the projective subset vertex subgraph K_1 given by the following figure.

K_1 =

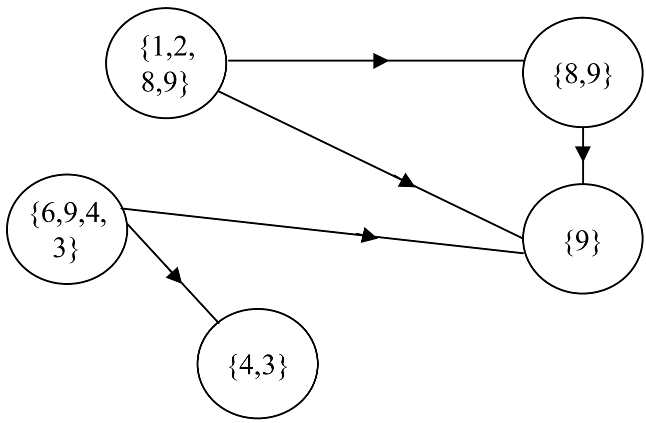


Figure 3.192

We see $\{1, 2, 8, 9\} \subseteq \{1, 2, 8, 9, 18\}$ is a subset of the vertex set $\{4,3\}$ is a subset of the vertex subset $\{4, 3, 9\}$ $\{6, 9, 4, 3\}$ is a subset of the vertex subset $\{7, 6, 9, 4, 3\}$.

Thus we see K_1 is a projective subset vertex subgraph of G.

Let K_2 be projective subset vertex subgraph of G given by the following figure.

K_2 =

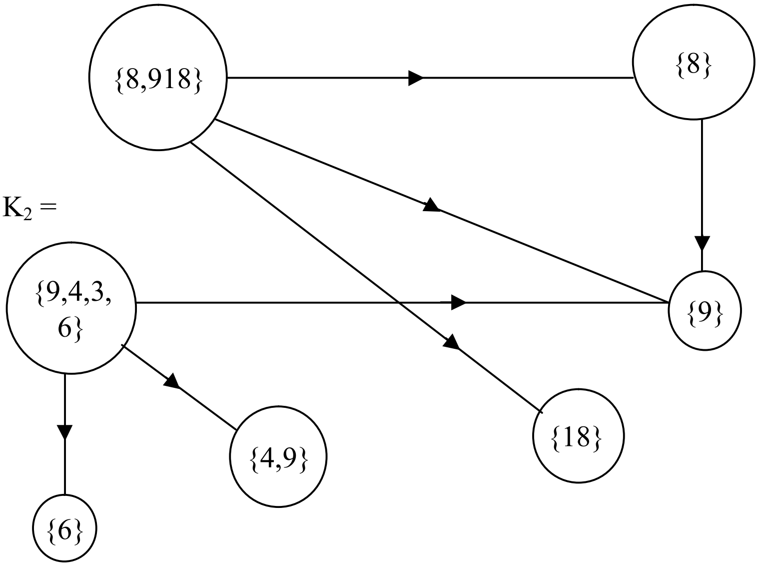


Figure 3.193

We see K_2 is a projective subset vertex subgrpah of type II. Infact K_2 has same number of edges and same number of vertices as that of G. Hence we call K_2 as the projective subset vertex hyper subgraph or K_2 is the special projective subset vertex hyper subgraph of type II of G.

Now we study the new notion of projective special subset vertex subgraphs of a projective subset vertex graphs of type II.

Consider a simple projective subset vertex graph of type II given by the following figure where S = {1, 2, …., 9} and P(S) the power set of S.

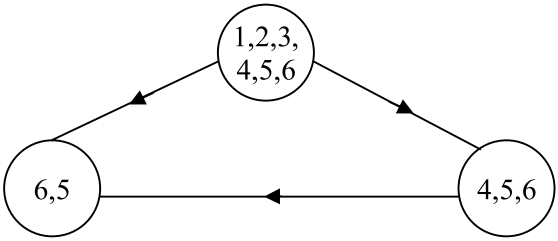


Figure 3.194

For the same set of vertices we get the following figure for the subset vertex graph (injective) of type II given by the following figure.

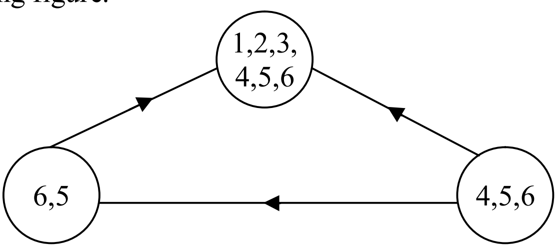


Figure 3.195

Using the same vertex set we get the subset vertex graph of type I given by the following figure.

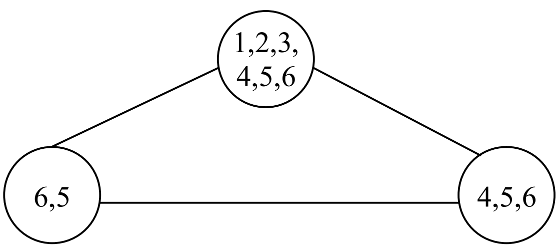


Figure 3.196

The projective subset vertex graph M of type II is as follows.

M_1 =

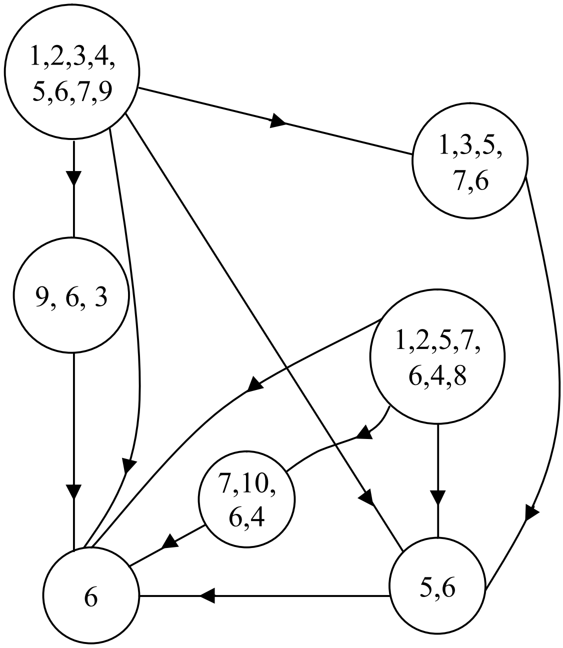


Figure 3.197

We see M_1 has 7 nodes and 10 edges.

We now give all the three types of subset vertex projective subgraphs of M.

We also give the subset vertex graph of type I using the same set of vertices and injective subset vertex graph of type II so that comparison of all the three can be made by any reader.

Let M_2 be the injective subset vertex graph of type II given by the following figure.

M_2 =

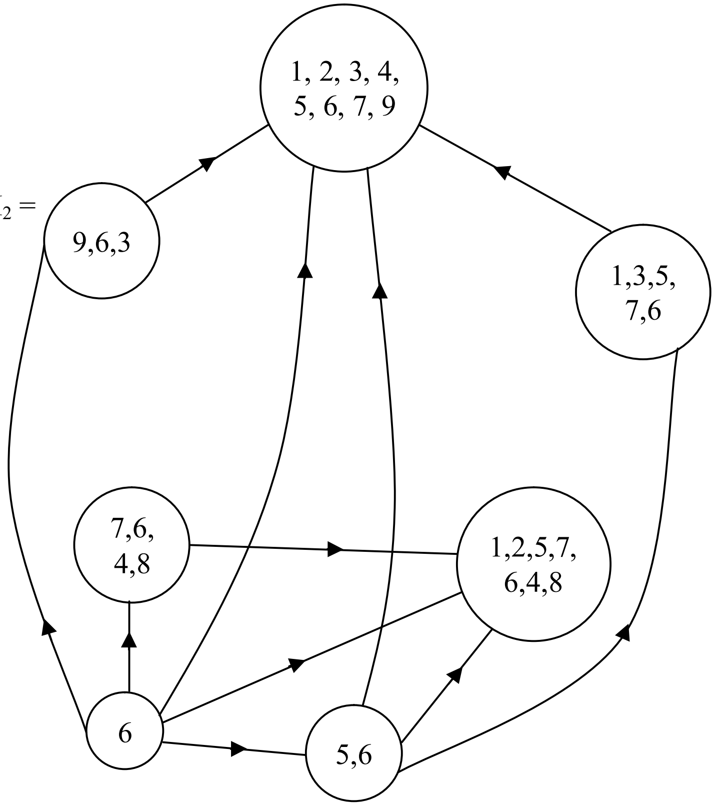


Figure 3.198

We see M_2 also has the same number of edges as that of M_1 with the only change that the direction is totally reversed.

Let M_3 be the subset vertex graph of type I using the same set of vertices as that of M_1 given by the following figure.

M_3 =

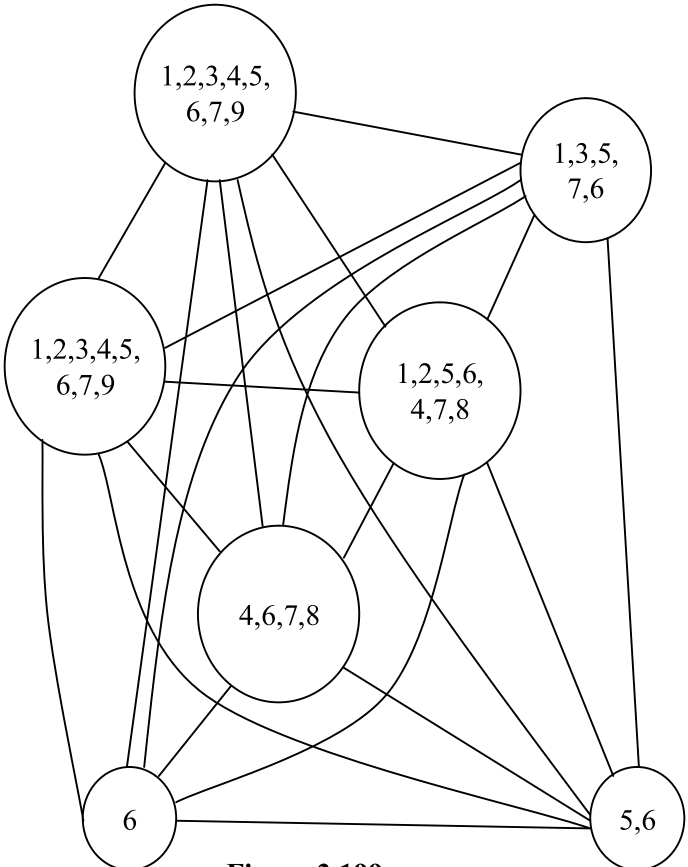


Figure 3.199

Infact M_3 is a complete subset vertex graph of type I given by the usual figure of a complete graph which is as follows.

M_4 =

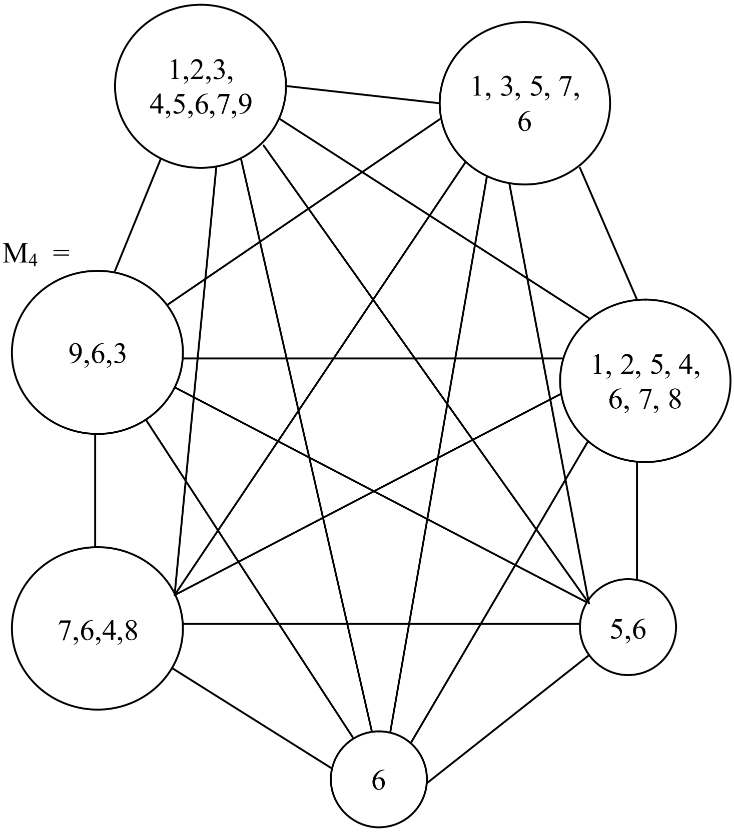


Figure 3.200

Thus we see M_3 is distinctly different from M_2 and M_1. For both M_1 and M_2 are directed graphs and are not complete graphs.

This example clearly establishes that type I subset vertex graphs are different from subset vertex graphs of type II (projective and injective).

Now we proceed onto find the universal complement of M_1.

Now we first give the vertex subset of the universal complement of M_1 complement of {1, 2, 3, 4, 5, 6, 7, 9} in {1, 2, 3, …, 9} is {8}. Complement of {1, 3, 5, 6, 7} in {1, 2, …,9} is {2, 4, 8, 9}.

Complement of {3, 6, 9} in {1, 2, …, 9} is {1, 2, 4, 5, 7, 8}.

Complement of {6} in {1, 2, …,9} is {1, 2, 3, 4, 5, 7, 8, 9}.

Complement of {6, 4, 7, 8} in {1, 2, 3, …, 9} is {1, 2, 3, 5, 9} an complement of {5, 6} in {1, 2, 3, 4, …, 9} is {1, 2, 3, 4, 7, 8, 9}, {1, 2, 5, 6, 7, 8} complement is {3, 4, 9}.

Using the vertex subset as {{3, 4, 9}, {1, 2, 3, 4, 6, 8, 9}, {1, 2, 3, 5, 9}, {1, 2, 3, 4,5, 7, 8, 9}, {1, 2, 4, 5, 7, 8}, {8}, {2, 4, 8, 9}}.

M_1^C is given by the following figure.

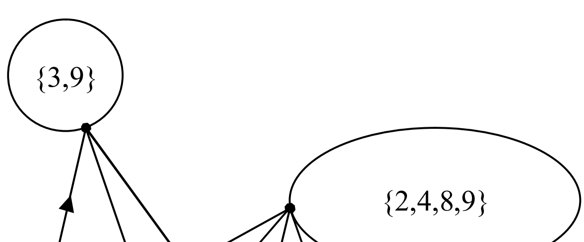


Figure 3.201

The task of finding universal complements is left as an exercise to the reader.

Further the interested reader is expected to compare the universal complements of M_1^c, M_2 and M_3.

However in the following we provide some examples.

Example 3.24. Let S = {1, 2, …, 12} be a set of order 12. P(S) the power set of S. Let G be a projective subset vertex graph of type II given by the following figure.

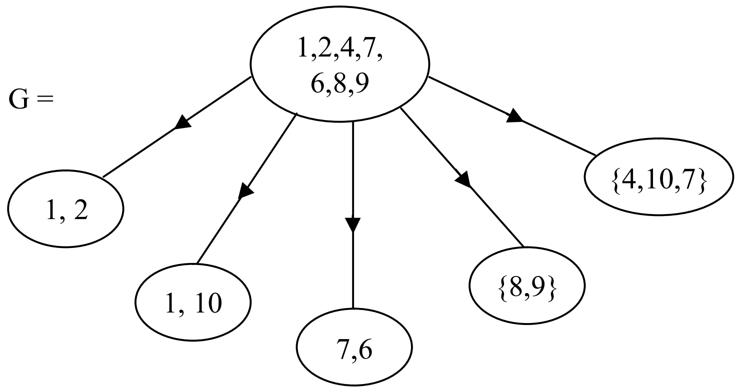


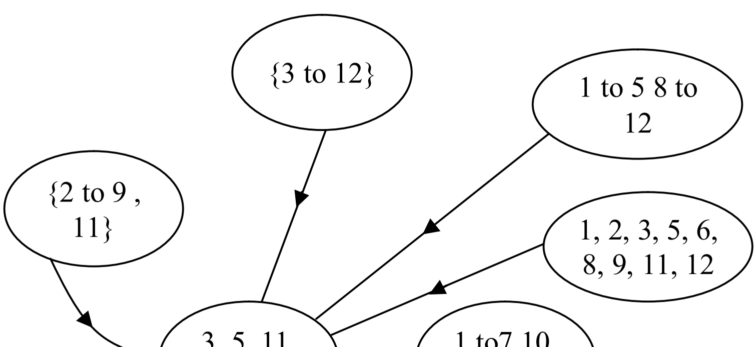
Figure 3.202

Clearly G is a projective subset vertex star graph of type II.

The universal complement vertex set of the projective subset vertex graph G^C of G is as follows.

{{2 to 9 , 11, 12}, {1 to 7, 10, 11, 12}, {1 to 5, 8 to 12}, {1, 2, 3, 5, 6, 8, 9, 11, 12}, {3, 5, 11, 12}, {3 to 12}}.

The figure of G^C, is given in the following.



$G^C =$

Figure 3.203

We see the universal complement is a projective subset vertex star graph of type II.

Thus G^C also preserves the structure of G. Consider H the projective subset vertex graph of type II.

H =

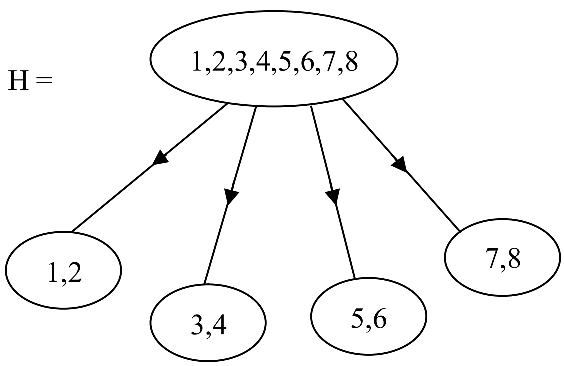
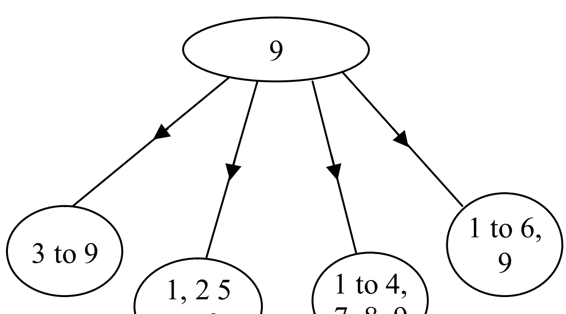


Figure 3.204

H takes its subsets from P(S) where S = {1, 2, …, 9}.

The universal complement H^C of H is as follows.



H^C =

Figure 3.205

H is a projective subset vertex graph of type II and H^C is also projective subset vertex graph of type II, both H and H^C are star graphs, however all star graphs need not in general have their universal complements to be star graphs. We will illustrate this situation by some examples.

Example 3.25 Let G be the projective subset vertex star graph of type II given by the following figure.

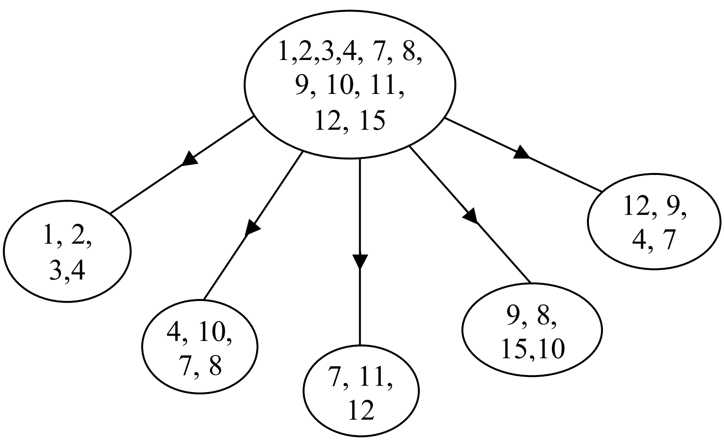


Figure 3.206

G takes its vertex set from P(S) where S = {1, 2, 3, …, 15}.

Let G^C be the universal complement of the projective subset vertex graph G of type II given by the following figure.

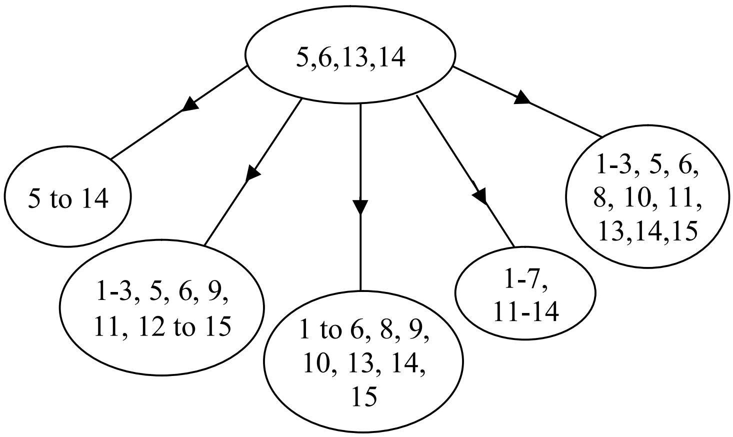


Figure 3.207

Clearly G^C is also a projective subset vertex star graph of type II. Thus both G and G^C has same structure.

Thus we propose the following problems.

Problem 3.1 Let S = {1, 2, …, n} be a set of cardinality n and P(S) the power set of S.

Let G be a projective subset vertex star graph of type II.

Will G^C the universal complement of G be also a projective subset vertex star graph of type II with direction of the edges reversed?

Problem 3.2 Can we have projective subset vertex circle graphs of type II.

Justify your claim.

Now we proceed onto give more examples of projective subset vertex graphs of type II and find the universal complement as well special projective subset vertex subgraphs and their local complement in the following.

Example 3.26. Let S = {1, 2, …, 18}, and P(S) be the power set of S.

G be the projective subset graph of type II given by the following figure.

G =

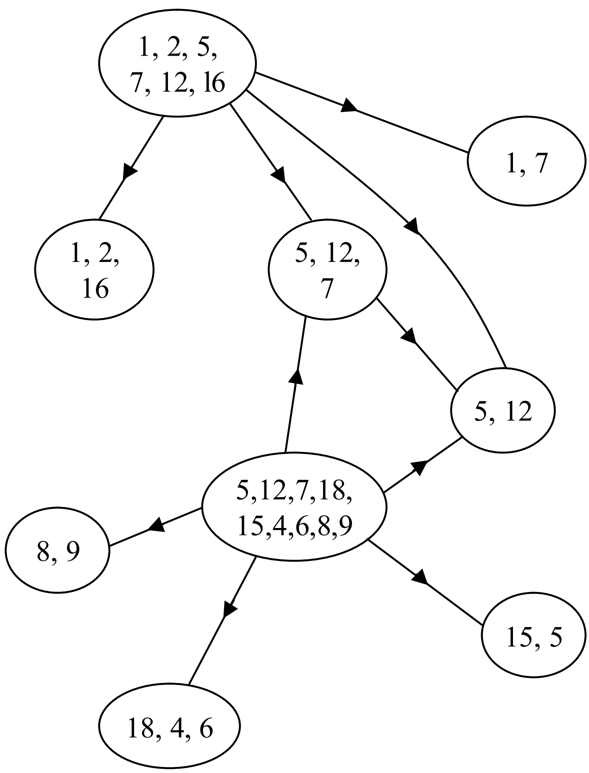


Figure 3.208

We find G^C of this G which is the universal complement.

The vertex set of G^C is as follows {{3, 4, 6, 8 to 11, 13, 14, 15, 17, 18}, {2 to 6, 8 to 18}, {3 to 15, 17, 18}, {1 to 4, 6, 8 to 11, 13 to 18}, {1 to 7, 10 to 18}, {1 to 3, 8, 10, 11, 13, 14, 16, 17}, {1 to 4, 6 to 14, 16, 17, 18}, {1 to 3, 5, 7, to 17}, {1 to 4,m 6 to 11, 13 to 18}}. Now let G^C be the projective subset vertex graph of type II which is the complement of G given by the following figure.

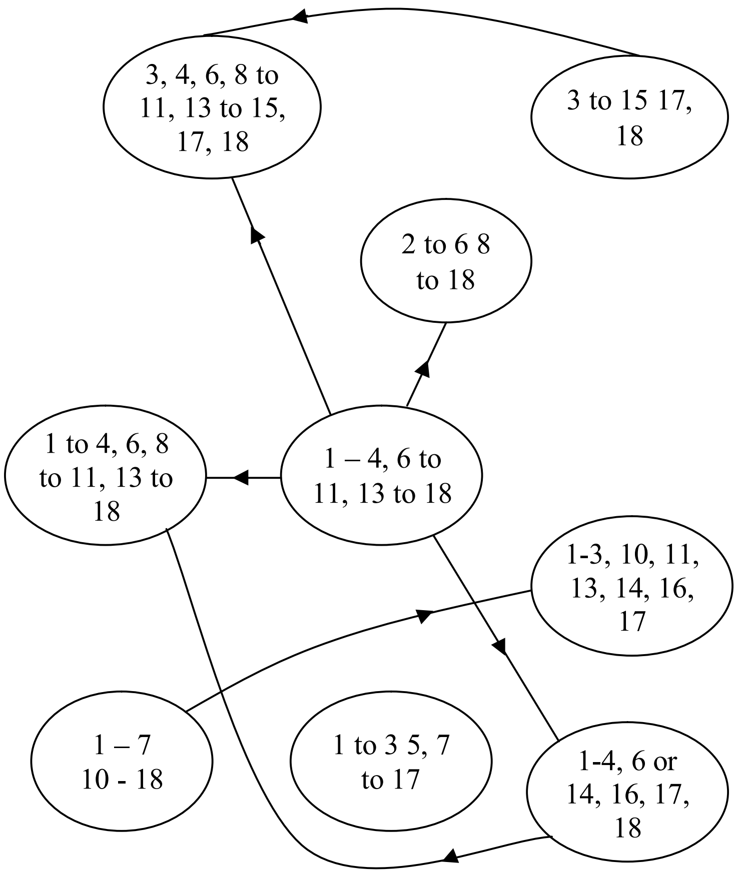


Figure 3.209

G and G^C do not enjoy the same structure.

Now we find some projective special subset vertex subgraphs of G in the following.

Consider the special subgraph projective (subset vertex subgraph H of G) given by the following figure.

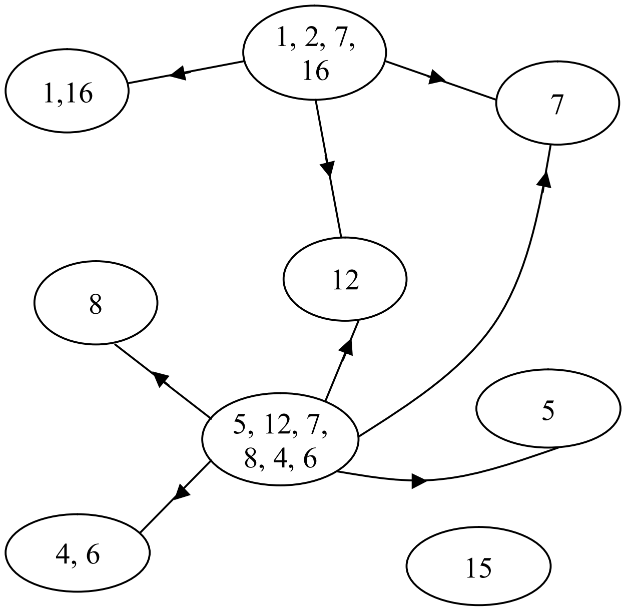


Figure 3.210

Clearly H is a special subset vertex subgraph of G.

We find the local complement of H in G given by the following figure.

Thus H is a hypergraph of G. Now we find the local complement special subset subgraph of H relative to G in the following.

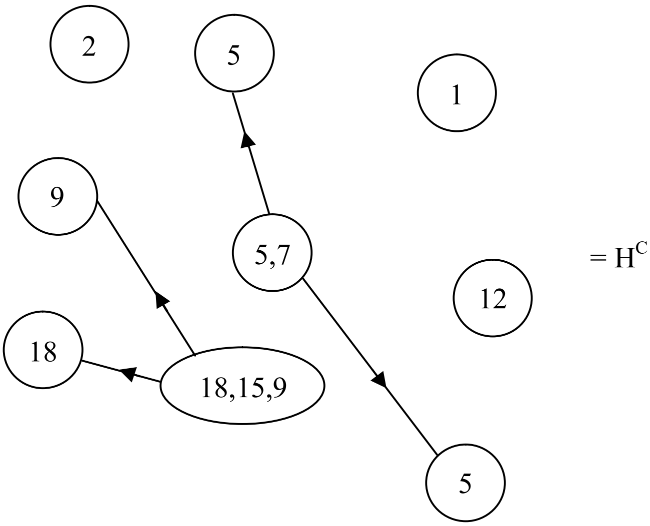


Figure 3.211

Clearly H^C does not form a special subset vertex subgraph as two of the vertices are identical with (5).

Thus it is important to record that even if H is a special subset vertex subgraph of a projective subset vertex graph G then local complement of H say H^C may or may not exist. Now we give the criteria for H^C the local complement of H to exist.

In the first place we observe even if one of the vertices of both G and its special subset vertex subgraph H of G are same then H^C cannot exist. For we do not have the concept of complement of a set in P(S) \ {S, ϕ} locally.

If one of the vertices is v = {1, 3} in G as well as in the special subset vertex subgraph H then its complement is undefined locally where S = {1, 2, 3, 4, 5} but globally the complement of v = {1, 3} is {2, 4, 5}.

However the local complement of{1, 3} with itself is not defined.

If {1, 2, 3, 4, 5} is the vertex in common with both H and G the special subset projective vertex subgraph and projective vertex subset graph G respectively then the complement of {1, 2, 3, 4, 5} is ϕ in P(S) where S = {1, 2, 3, 4, 5}.

The following theorem gives the criteria for the local complement of a special subset vertex projective subgraph H^C of a special subset vertex projective subgraph H to exist.

Theorem 3.9. *Let S = {1, 2, …, n} be a set of order n and P(S) the power set of S. Let G be a projective subset vertex graph of type II with vertices taken from P(S).*

i) *G^C the global complement of G always exists with same number of vertices as that of G and G^C is also a projective subset vertex graph of type II.*

ii) *Let H be a projective special subset vertex subgraph of G. H^C, the local complement of H relative to G exist if and only if all the vertices of H are distinct and proper subsets of each of the vertex set of G.*

Proof. Given G is a projective subset vertex graph of type II with vertex set $\{v_1, v_2, \ldots, v_n\} \subseteq P(S)$ the power set of S. Clearly $v_1 \neq v_j$ if $1 \neq j$; $1 \leq i, j \leq m$.

Now the global complement of G denoted by G^C has its vertex set as $\{S \setminus v_1, S \setminus v_2, \ldots, S \setminus v_m\}$. Clearly $S \setminus v_j = S \setminus v_j$ if and only if $v_i = v_j$, thus the m vertices are distinct hence the projective subset vertex complement graph G^C of G has also m vertices hence the claim.

Proof of (ii), H is a special projective subset vertex subgraph of G and H also has m vertices which is mandatory.

Now let the vertices of H be $t_1, t_2, \ldots, t_m$ where $t_i \neq t_j$ if $i \neq j$ and $t_i \subset v_i$; $1 \leq i \leq m$ is possible if and only if $v_i \setminus t_i \neq v_j \setminus t_j$ for $i \neq j$ and $1 \leq i, j \leq m$.

Even if one of them coincide H^C cannot exist as a special projective subset vertex subgraph of G.

Thus given H the projective special subset vertex subgraph of G H^C the local complement of H relative G may or may not exist.

To this effect we have given illustrative examples. Thus given H a special projective subset vertex subgraph of G H^C may not exist even if one of the vertices of G and H are the same. This is first illustrated by an example.

Let G be the projective subset vertex graph with vertex set from P(S) where S = {1, 2, ..., 9} given by the following figure.

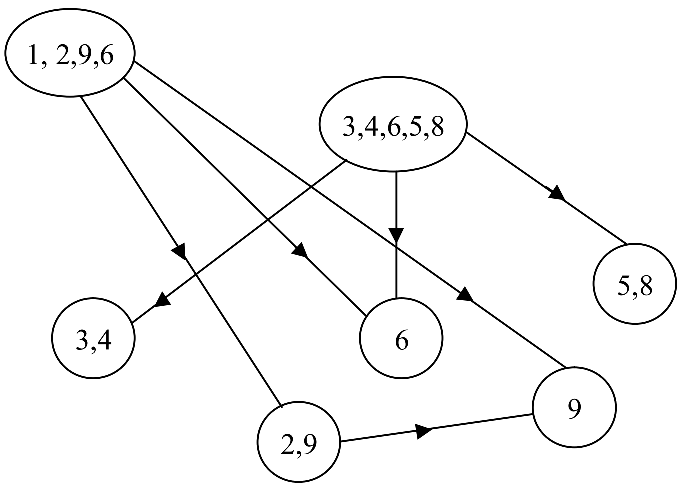


Figure 3.212

We see G has special projective vertex subset subgraphs H for which the vertex set (6) and (9) cannot be taken as proper subsets of the vertex set of G. Hence there is no special projective subset vertex subgraphs H of G which has the local complement H^C of H in G.

In view of this we have the following result.

Theorem 3.10. *Let S = {1, 2, …, n} and P(S) power set of S. Let G be a subset vertex graph of type II with vertex set projective from P(S). H a special subset vertex subgraph of G. The local complement H^C of H does not exist even if one of the vertex sets of G is a singleton set.*

Proof. Suppose G has the vertex v_1 to be singleton set {m} then the local complement of {m} where m ∈ S; with itself in undefined. Hence the claim as for H also same {m} has to be one of the vertex sets as H is a special subset vertex subgraph of G so for the local complement of {m} relative to {m} is undefined so the vertex set is undefined.

Now we proceed onto illustrate the situation in which the vertex set for the subset vertex graphs takes its values from a subset of the complex numbers C = {a + bi / a, b ∈ R; i^2 = –1}. We call these subset vertex graphs as complex subset verte3x graphs.

We give some examples of the same.

Example 3.27. Let S = {a + bi / a, b ∈ {0, 1, 2, 3, 4}; i^2 = –1} and P(S) the power set of S. Let G be the subset vertex graph of type I with vertex set from P(S) given by the following figure.

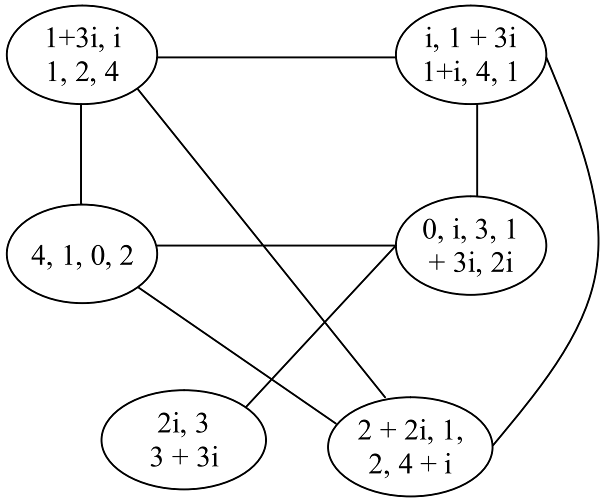


Figure 3.213

We see G is a complex subset vertex graph of type I. We can have subset vertex subgraphs which in general need not be complex.

We can also have special subset vertex graph which need not always be complex special subset vertex graphs. To this effect we find some vertex subset subgraphs of G.

Let H_1 be a vertex subset subgraph of G given by the following figure.

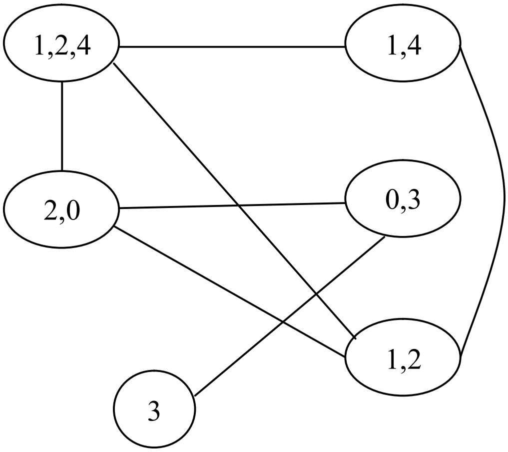


Figure 3.214

H_1 is a special subset vertex subgraph clearly H_1 is not a complex vertex subset subgraph only a real subset vertex subgraph of type I.

We find the global complement of G. The vertex set of G^C is as follows.

V = {v_1 = {0, 3, 2i, 3i, 4i, 1 + 2i,1 + 4i, 1 + i, 2 + i, 2 + 3i, 2 + 2i,2 + 4i, 3 + 2i, 3 + i, 3 + 3i, 3 + 4i, 4 + i, 4 + 2i, 4 + 3i, 4 + 4i}, v_2 = {0, 2, 3, 2i, 3i, 4i, 1 + 2i, 1 + 4i, 2 + i, 2 + 2i, 2 + 3i, 2 + 4i, 3 + i, 3 + 2i, 3 + 3i, 3 + 4i, 4 + i, 4 + 2i, 4 + 3i, 4 + 4i}, = {3, i, 2i, 3i, 4i, 1 + i, 1 + 2i, 1 + 3i, 1 + 4i, 2 + i, 2 + 2i, 2 + 3i, 2 + 4i, 3 + i, 3 + 2i, 3 + 3i, 3 + 4i, 4 + i, 4 + 2i, 4 + 3i, 4 + 4i},

v_4 = {1, 2, 4, 3i, 4i, 1 + i, 1 + 2i, 1 + 4i, 2 + i, 2 + 2i, 2 + 3i, 2 + 4i, 3 + i, 3 + 2i, 3 + 3i, 3 + 4i, 4 + i, 4 + 2i, 4 + 3i, 4 + 4i},

v_5 = {0, 1, 2, 4, i, 3i, 4i, 1 + i,1 + 2i, 1 + 3i, 1 + 4i, 2 + i, 2 + 2i, 2 + 2i, 2 + 3i, 2 + 4i, 3 + i, 3 + 2i, 3 + 4i, 4 + i, 4 + i, 4 + 2i, 4 + 3i, 4 + 4i}, v_6 = {0, 3, 4, i, 2i, 3i, 4i, 1 + i, 1 + 2i, 1 + 3i, 1 + 4i, 2 + i, 2 + 3i, 2 + 4i, 3 + i, 3 + 2i, 3 + 3i, 3 + 4i, 4 + 2i, 4 + 3i, 4 + 4i}}. Now we get the following complex subset vertex graph associated with the vertex set V = {v_1, v_2, v_3, v_4, v_5, v_6} which is of type I.

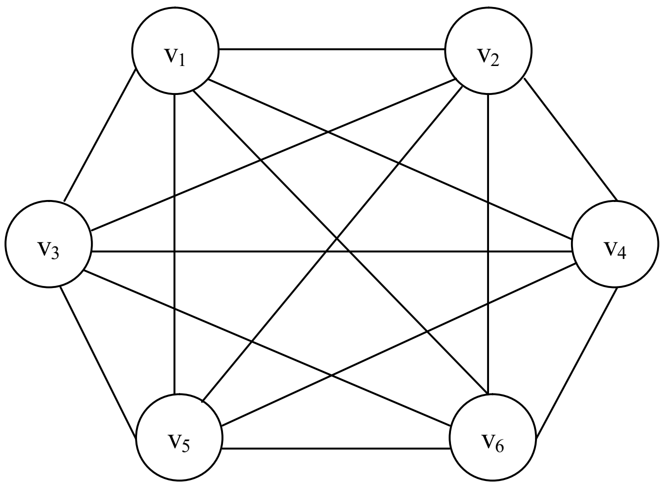


Figure 3.215

Clearly G^C is a complex subset vertex graph of type I which is complete, however G is not a complete complex subset vertex graph of type I.

Thus we see the global complements in general not preserve the structure of the basic graphs.

Next we find the local complements H^C of H the relative to the graph G given by the following figure.

$H^C =$

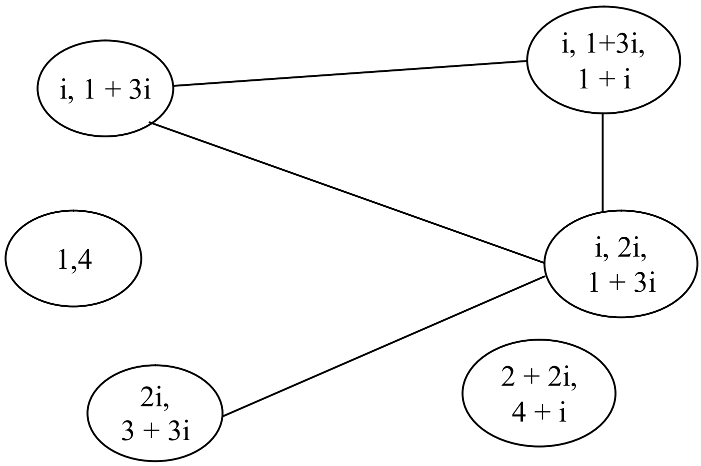


Figure 3.216

Clearly H^C exists and is also a special complex subset vertex subgraph of H which is the local complement of H.

In some cases we may not have the local complement for H the special subset vertex subgraphs to exist.

However we wish to keep the following under record in the form of theorem.

Theorem 3.11. *Let G be a complex vertex subset graph of type I with vertex set from P(S), the power set of S where S = {a + bi / a, b* $\in$ *{1, 0, 2, …, n},* $i^2 = -1$*}.*

i) *All subgraphs (special subset vertex subgraphs or subset vertex subgraphs) of G in general need not be again a complex subset vertex subgraph of type I.*

ii) *Global complements always exist for G.*

iii) *The local complement of a complex special subset vertex graphs in general may or may not exist.*

Proof is as in case of real subset vertex graphs of type I.

As basically authors feel this study is a matter of routine for any real set S can be also replaced by a finite complex set also.

However we caution that only the vertices are complex however the edges can never be complex in both type I or type II subset vertex graphs as in case of type I the edge exists if and only if $v_i \cap v_j \neq \phi$ for $i \neq j$; under the further assumption self loops do not exist. Secondly all these type I subset vertex graphs are not directed.

In case of type II subset vertex graphs we have the two concepts projective and injective so naturally type II subset vertex graphs are directed.

In case of injective subset vertex graphs of type II; if v_i and v_j are two distinct vertices for the edge to exists either $v_i \underline{\not\subset}$ v_j or $\underline{\not\subset}$ v_i then in the care $v_i \subset v_j$ we have

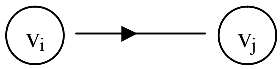


Figure 3.217

is the directed edge or if the case is $v_j \subset v_i$ then the directed edge is

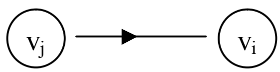


Figure 3.218

Only one such directed edge exists which is defined as injective. So we can have injective subset vertex graphs of type II usually described as subset vertex graphs of type II. However the case of projective subset vertex graphs of type II is defined in a reverse way if $v_i \underline{\not\subset}$ v_j (or $v_j \underline{\not\subset}$ v_i) then

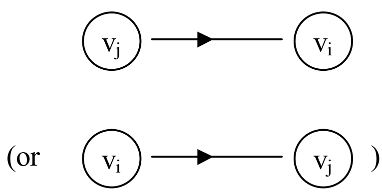


Figure 3.219

the arrows are reversed.

All results true in case of subset vertex graphs of type II using an arbitrary set S follow with appropriate modifications in case of S replaced by a complex set from C = {a + bi / a, b ∈ R; $i^2 = -1$}.

However for the sake of completeness we supply a few examples.

Example 3.28. Let S = {a + bi / a, b ∈ {1, 2, 3, 4, 5}, $i^2 = -1$} be a finite set G be the complex injective subset vertex graph of type II given by the following figure (G takes the vertex set from the power set P(S) of S given above.

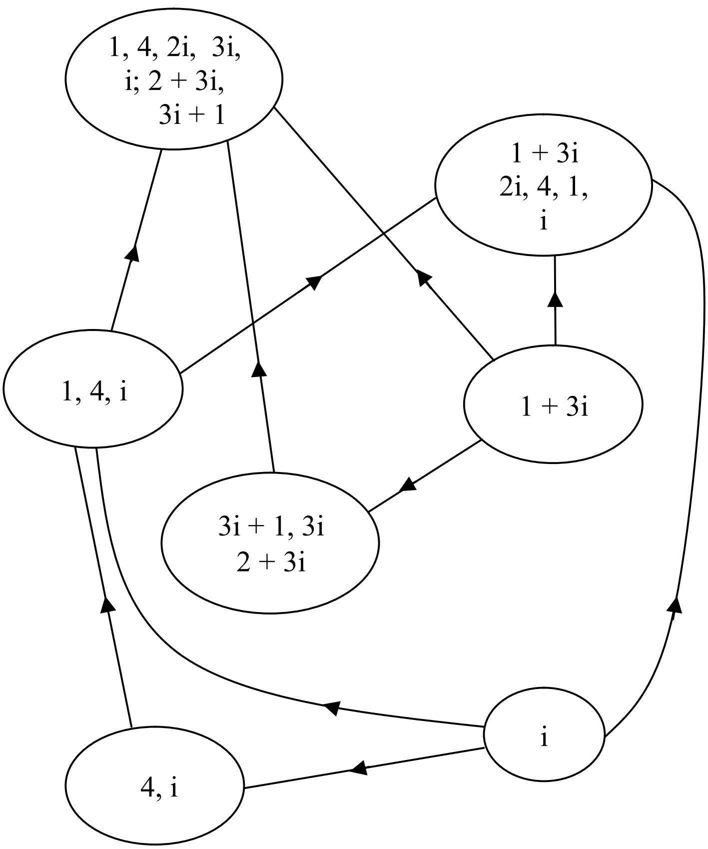


Figure 3.220

It is easily verified the global complement of G exists.

However whatever be the complex special subset vertex subgraph H of G; the local complement H^C of H does not exist. This is true from the fact that one of the vertices of G is i and the local complement of i with itself is undefined. Hence the claim.

Consider the complex (injective) subset vertex graph K given by the following figure which takes the entries of the vertex set from the complex valued power set P(S).

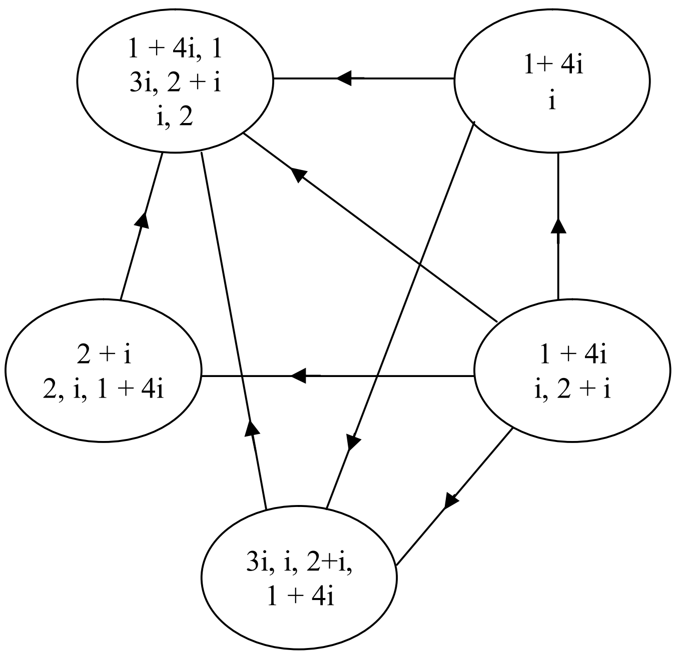


Figure 3.221

Let H be the complex special subset vertex subgraph of K given by the following figure.

H =

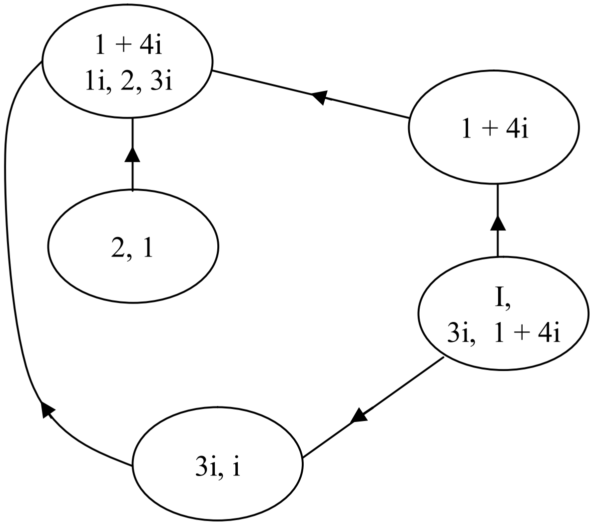


Figure 3.222

Now we find the local complement of H in G. Let H^C be the complex special vertex subset subgraph, the local complement of H in G with vertex set as the local complements forming the sets of vertices of G and H.

Does H^C exist?

Find B a complex special subset vertex subgraph of G and its local complement relative to G.

One can also find the global complement of G.

Now we give one example of a projective complex subset vertex graph of type I and find the global and local complements of special subset vertex subgraph of G.

Example 3.29. Let S = {2 + I, 4i, 2i, 0, 4 + 3i} be the complex set. P(S) be the power set of S; also will be known as complex valued power set of S.

Let G be the projective complex subset vertex graph of type II given by the following figure.

G =

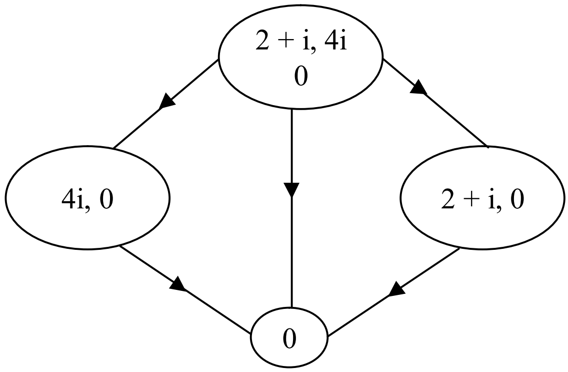


Figure 3.223

The global complement of G is given in the following

G^C =

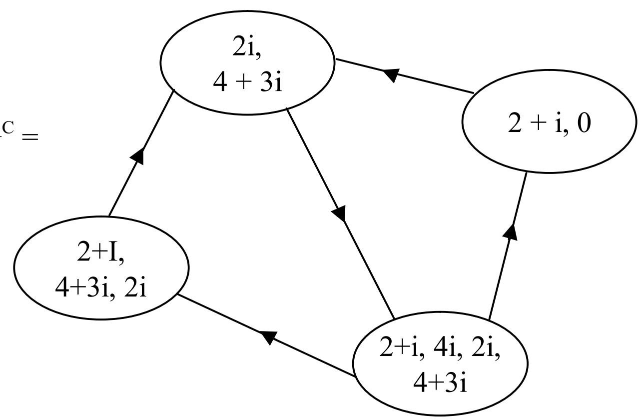


Figure 3.224

Clearly G^C the global complement of G is a complete projective complex subset vertex graph of G. However G is not a complete projective complex subset vertex graph.

Let B be the complex special vertex subset subgraph of G which is projective given by the following figure.

B =

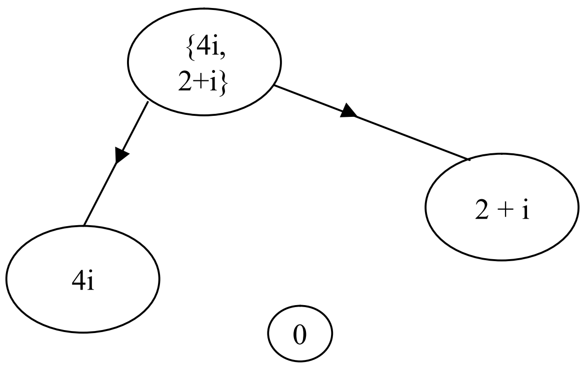


Figure 3.225

We see the local complement of B does not exist as {0}, has not local complements.

In fact none of the projective complex special subset vertex subgraphs has local complement to exist as in G the vertex {0} is a singleton set so has no local complements.

Consider R to be the projective complex subset vertex graph given by the following figure.

R =

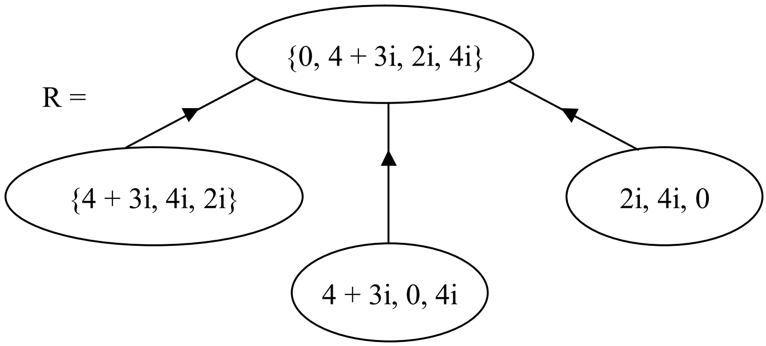


Figure 3.226

The universal or global complement of R is as follows.

$R^C =$

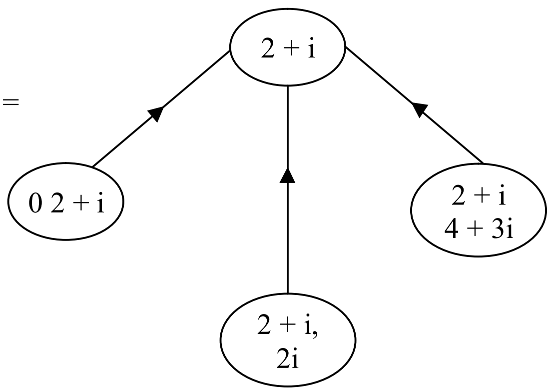


Figure 3.227

The universal (global) complement of R has also the same structure as that of R and the arrows are not reversed.

Let W be the complex projective special subset vertex subgraph of R given by the following figure.

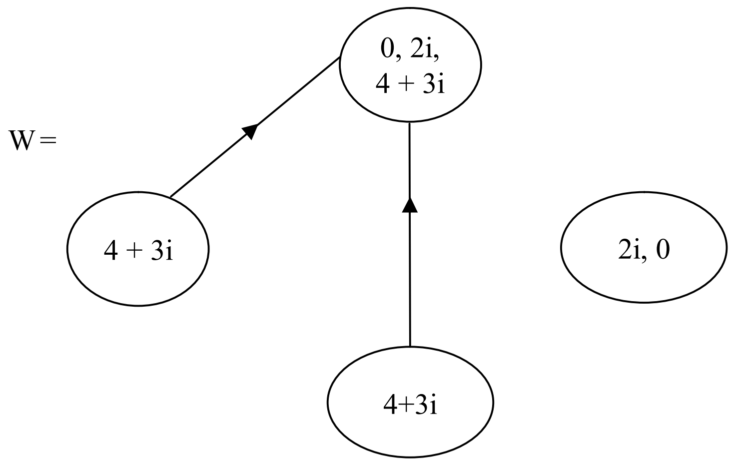


Figure 3.228

Now we try to find the local complement W^C of W relative to G.

The local complement W^C is given in the following.

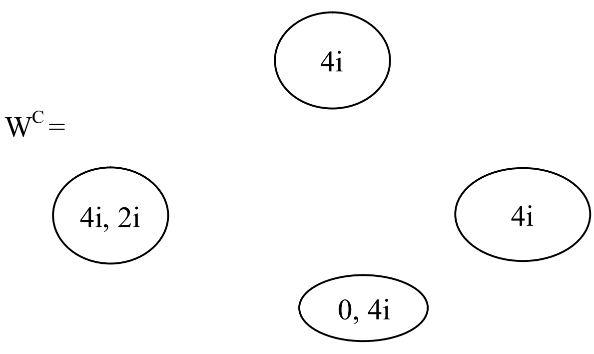


Figure 3.229

Clearly W^C is not even defined for this W.

No one can guarantee the existence of local complements for every special subset vertex subgraphs H of R.

Conditions for the non existence of local complements have been derived in the earlier part which is also applicable in this case of complex subset valued graphs of type I and type II.

Next we proceed onto describe the concept of subset vertex graphs with neutrosophic subsets vertex graphs.

Let S = {I, 1 + I, 0 5I, 8, 6 + 3I} be the neutrosophic set. We can have the power set P(S) of S.

If we construct a graph G with vertex set from P(S) we define that graph G as the neutrosophic subset vertex graphs of type I if the edge exists between v_i and v_j ($i \neq j$; $v_i, v_j \in P(S)$ then $v_i \cap v_j \neq \phi$ and vice versa.

We will illustrate this situation by some examples.

Example 3.30. Let S = {I, 3I, 0, 2, 4 + 5I, 8, 9I} be a set P(S) be the power set of S. Let G be the neutrosophic subset vertex graph of type I given by the following figure.

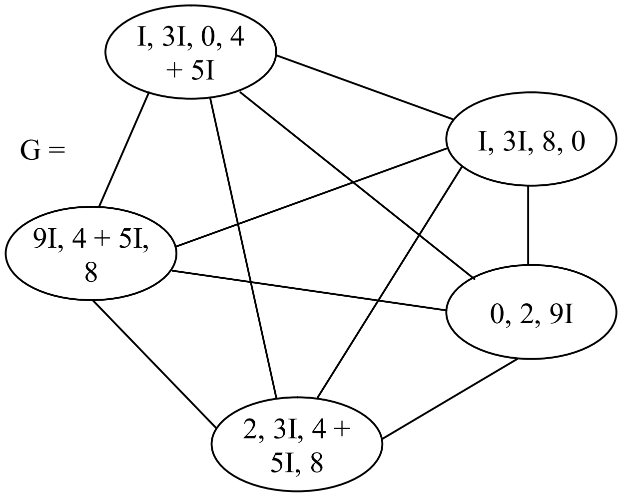


Figure 3.230

We now find the global (or universal) complement of G.

Let G^C be the global complement neutrosophic subset vertex graph given by the following figure.

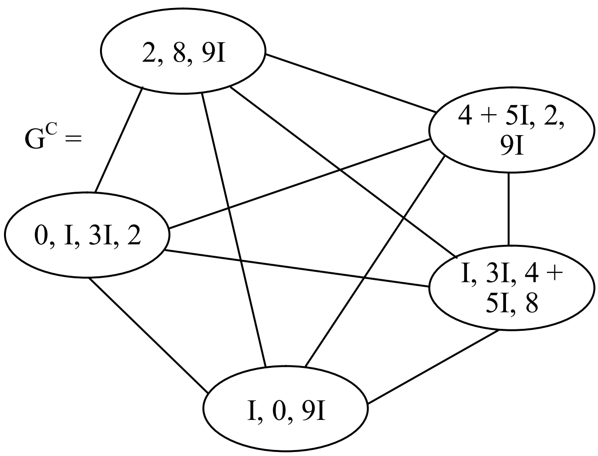


Figure 3.231

Clearly the global complement subset vertex neutrosophic graph of G is also a complete neutrosophic subset vertex graph of type I.

Next we proceed of find some special neutrosophic subset vertex subgraph of G. Let H_1 be the neutrosophic special subset vertex subgraph of G given by the following figure.

H_1 =

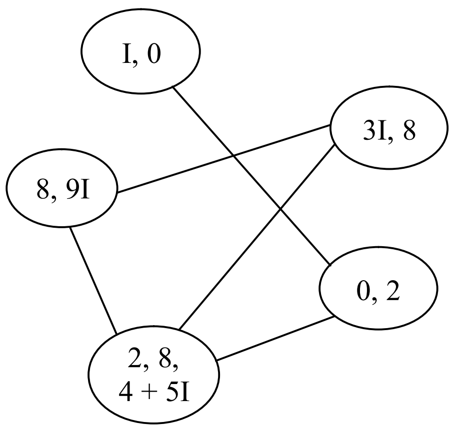


Figure 3.232

Now let H^C be the local complement neutrosophic special subset vertex subgraph of H_1 relative to G given by the following figure.

$H_1^C =$

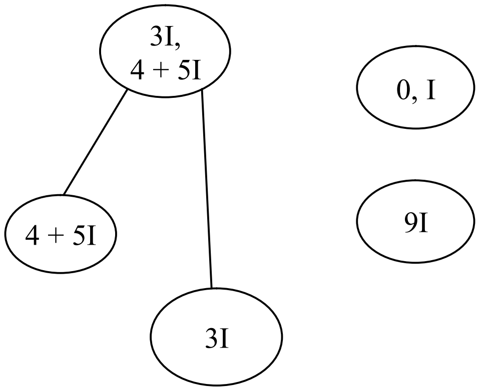


Figure 3.233

We see the local complement H_1^C of H_1 exists and they are structurally different from H_1,G and G^C. However even H_1 is not a complete subset vertex neutroosphic graph as G or G^C.

It is left as an exercise for the reader to find any other special features associated with neutrosophic vertex subset graphs of type I.

Further at this juncture it is pertinent to keep on record that though we have complex (or neutrosophic) subset vertex graphs of type I and II only the vertex sets are complex (or neutrosophic) no changes in the edges, edges remain real thus we have to make some other special definitions to make the edges complex or real. That is why white studying subset vertex graphs we have not made any special mention of this for the set S should only be finite; it can be neutrosophic, complex or both complex and neutrosophic for all types of subset vertex graphs.

Now just for the sake of better understanding of every reader we give now example of neutrosophic injective subset vertex graphs of type II and projective neutrosophic subset vertex graphs of type II.

Example 3.31. Let S = {I, 3I + 1, 0, 2I – 4, 6I + 7, 7, 8, 10I, 1} be a finite set. P(S) the power set of S. Let G be the injective neutrosophic subset vertex graphs of type II given by the following figure.

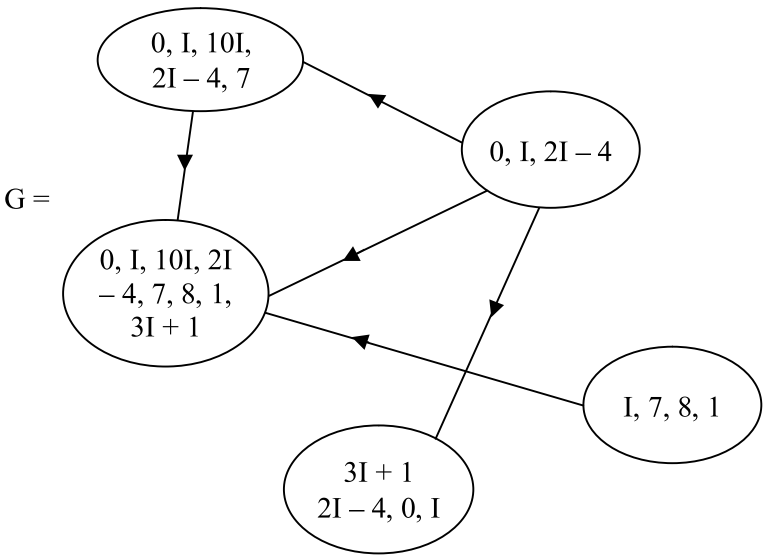


Figure 3.234

We can find the global complement of G. Let G^C be the injective neutrosophic universal complement subset vertex graph of G.

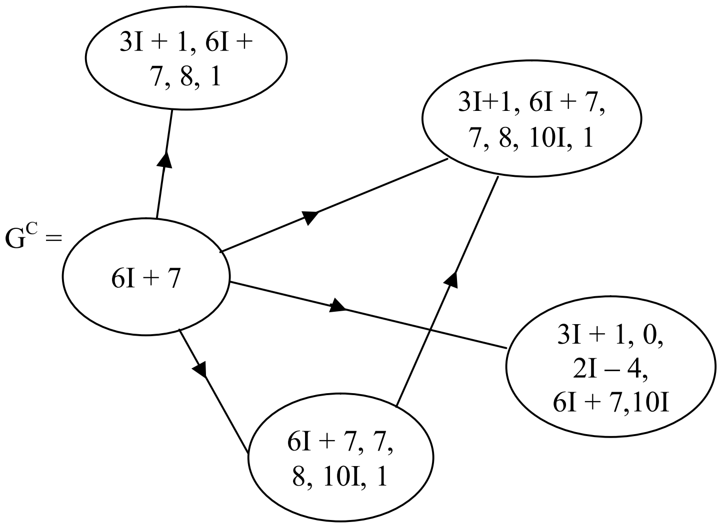


Figure 3.235

It is clear that G and G^C enjoy different structures.

Interested reader can find more such examples and obtain conditions on the vertex set of a neutrosophic subset graph G of type I so that G and its universal complement enjoy the same structure.

Let H be the neutrosophic special subset vertex subgraph of type II which is injective G^C given by the following figure

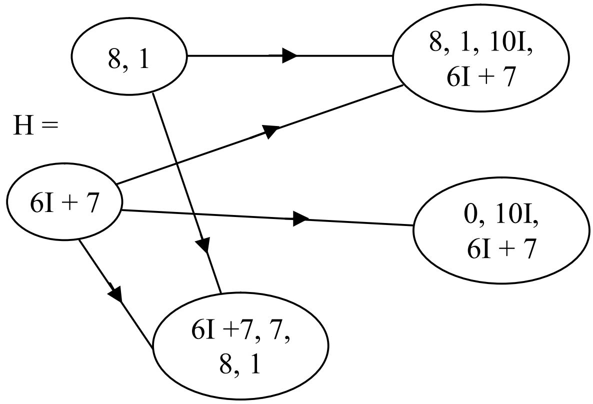


Figure 3.236

Clearly the local complement of H does not exist.

Infact G^C has no special neutrosophic subset vertex subgraph of type II (injective) that has local complement. This is due to the fact the vertex 6I + 7 is a singleton set.

Let K be the injective neutrosophic special subset vertex subgraph of G of type II given by the following figure.

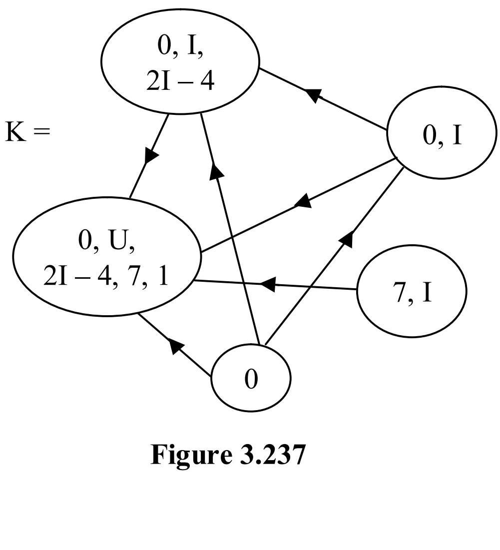


Figure 3.237

Clearly K is a neutrosophic special subset vertex subgraph of G of type II which is also injective.

Now we find the local complement K^C of K relative to G which is given by the following figure.

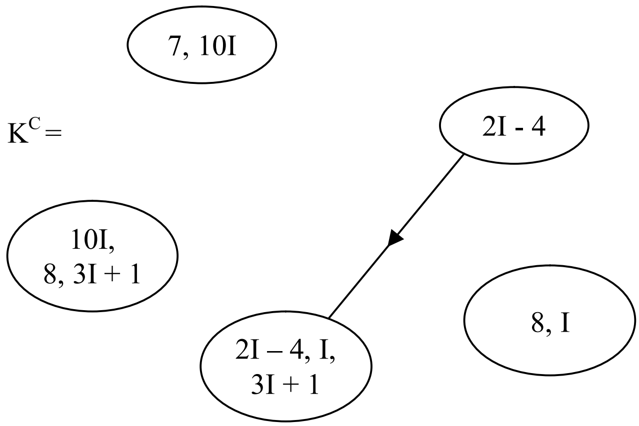


Figure 3.238

K^C of K exist with only one edge.

The reader is left with the task of finding more such special subgraphs and analylse their properties.

Next we proceed onto give an example of a neutrosophic projective subset vertex graphs of type II.

Example 3.32. Let S = {I, 2I + 3, 3I + 5, 2, 7, 9, 9 + I, 9I + 7, 0, 1, 6} be the neutrosophic set of order 11belonging to P(S) be the power set of S.

Let G be the projective neutrosophic subset vertex graph of type II given by the following figure.

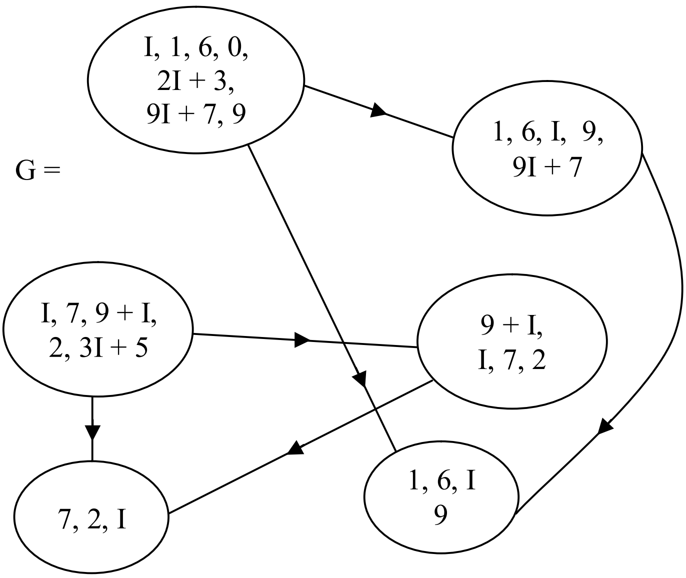


Figure 3.239

Now we find the global or universal complement of G. Let G^C be the projective neutrosophic subset vertex graph of type II the universal complement of G given by the following figure.

G^C =

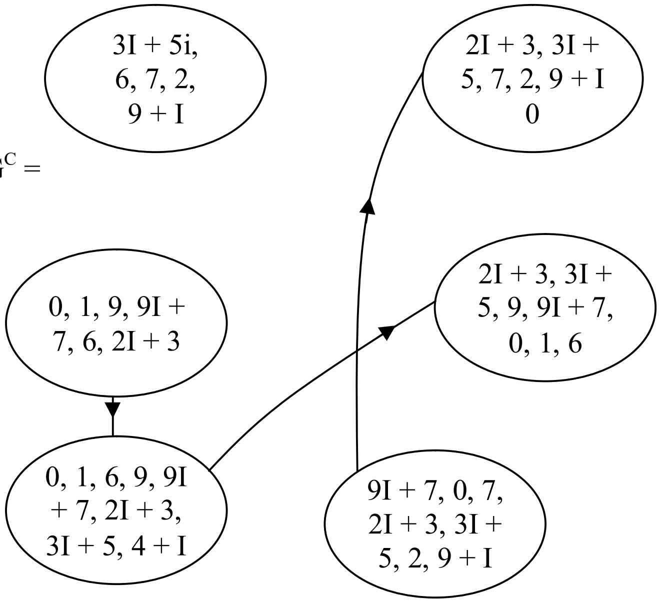


Figure 3.240

We see both G and G^C have different structures as the number of edges in them are distinct.

Now for this G we find a special projective subset vertex subgraph. Let H be the projective special subset vertex subgraph of G given by the following figure.

H =

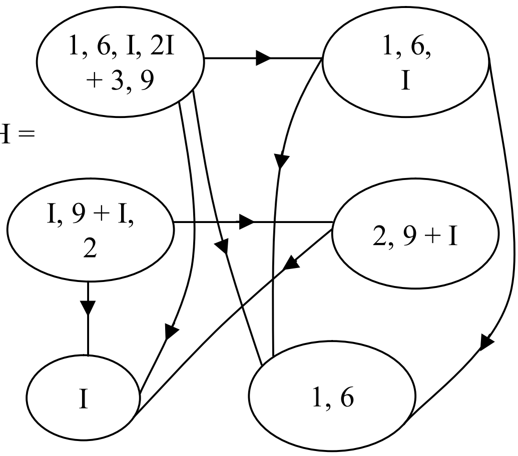


Figure 3.241

Let H^C be the local complement of H relative to G given by the following figure.

H^C =

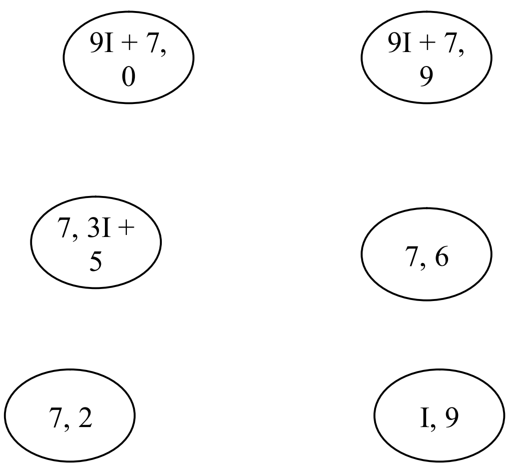


Figure 3.242

Clearly H^C is only a projective neutrosophic special subset vertex trivial subgraph. It exists but is trivial we may have situations in which H^C does not exist.

Characterize those neutrosophic projective subset vertex graphs G of type I.

i) G has only projective special subset neutrosophic vertex subgraphs which has their local complements relative to G to be trivial.

ii) Characterize all those neutrosophic projective special subset vertex graphs of type II none of whose special projective subset vertex subgraphs have local complements relative toG.

Next we proceed onto describe neutrosophic complex subset vertex graphs of type I and type II by examples.

Recall the complex - neutrosophic collection is $\langle C \cup I \rangle$ $\{a + bi + cI + diI / a, b, c, d \in R; i^2 =1, I^2 = I, (iI)^2 = -I\}$ is of finite cardinality. However we can have $C(\langle Z_n \cup I \rangle) = \{a + bi_F + cI + dIi_F / a, b, c, d \in Z_n, i_F^2 = (n-1), I^2 = I, (i_F I)^2 = (n-1) I\}$.

We can for the complex neutrosophic subset vertex graphs of type I or type I take subsets from $C(\langle Z_n \cup I \rangle$ or $\langle C \cup I \rangle$.

Example 3.33. Let S = {I, I + 3i, 7, 4 + I, 5i – 3I + 2, 4iI + 1, 2 + I, 2 + I + I, 5iI + 1} be set. P(S) be the power set of S.

Let G be the complex - neutrosophic subset vertex graph of type I given by the following figure.

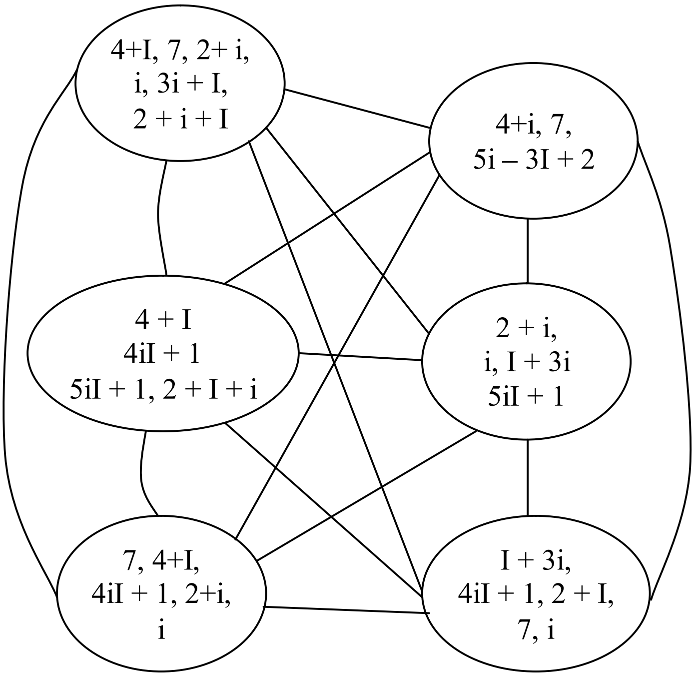


Figure 3.243

Clearly G is a complete complex – neutrosophic subset vertex graph of type I.

Reader is left with the task of finding the universal complement G^C of G. Is G^C also a complete complex neutrosophic subset vertex graph?

Further it is left as a task for the reader to find subset vertex complex neutrosophic special subgraphs H and their local complements of H relative to G.

i) Find all special complex neutrosophic subset vertex subgraphs of G which has no local complements relative to G.

ii) Find all special subset vertex complex - neutrosophic valued subgraphs which has local complements relative to G.

iii) Find all complex neutrosophic valued subset vertex subgraphs whose local complement is just the empty subgraph.

Next we provide examples of complex neutrosophic subset vertex graphs of type II and discuss some of its bsic properties in the following.

Example 3.34. Let S = {i, 1 + i + I, 2I, 7I + 4i + 1 + 3iI, 5Ii + 2, 3iI + 2I, 0, 1, 7, 8, 9i, 9iI} be the finite complex neutrosophic set.

Let P(S) be the power set of S. G be the injective complex - neutrosophic subset vertex (injective) graph of type II given by the following figure.

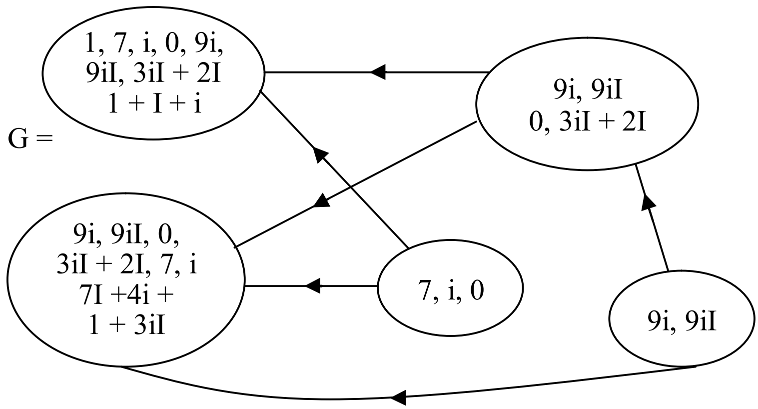


Figure 3.244

Next we find the global complement G^C of G in the following.

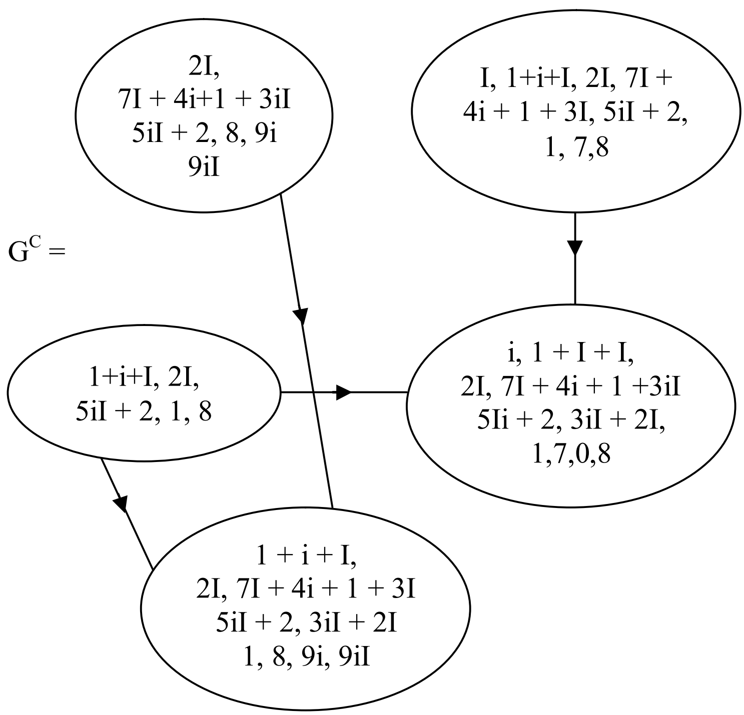


Figure 3.245

G^C and G has different structures.

Now we find a complex - neutrosophic special subset vertex subgraph H of G which is given by the following figure.

H =

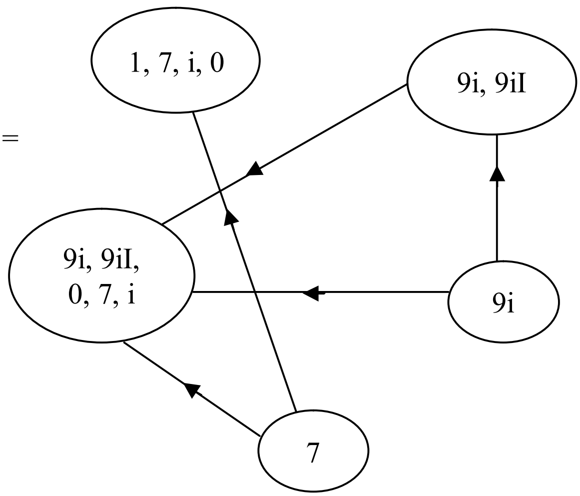


Figure 3.246

Clearly H does not preserve structure. Now we find the local complement H^C of H relative to G.

H^C =

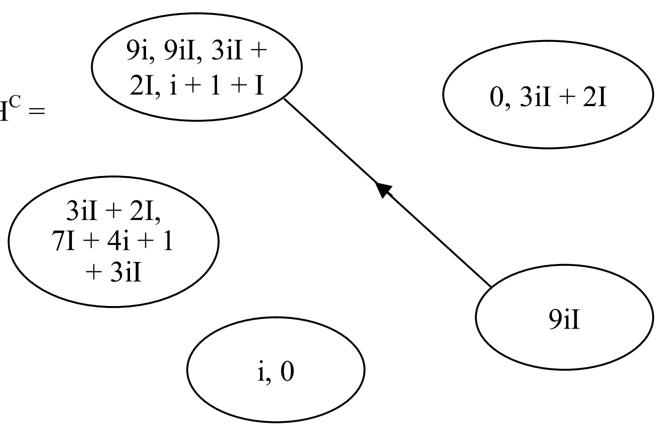


Figure 3.247

Clearly H^C does not preserve structure but H^C is a complex neutrosophic subset vertex subgraph of G.

Now we finally give an example of a complex neutrosophic subset vertex projective graph of type II.

Example 3.35. Let S = {3, i, 6i, 9i, 9I, 9iI, 9 + 9i, 9 + 9I, 9 + 9iI} be a complex - neutrosophic set. P(S) be the power set of S. Let G be the complex - neutrosophic projective subset vertex graph of type II given by the following figure.

G =

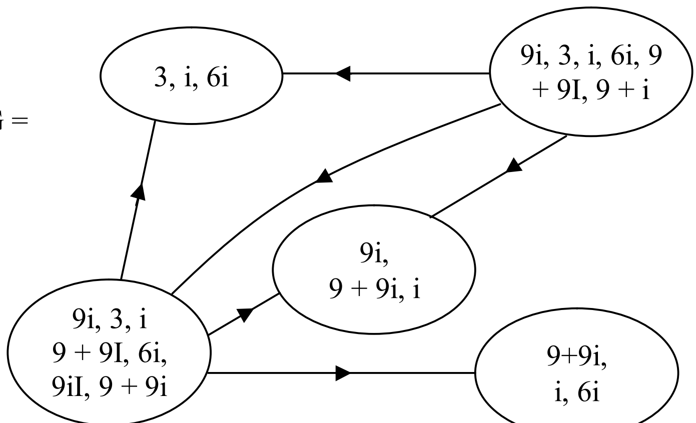


Figure 3.248

Now we find the universal complement G^C of G in the following.

$G^C =$

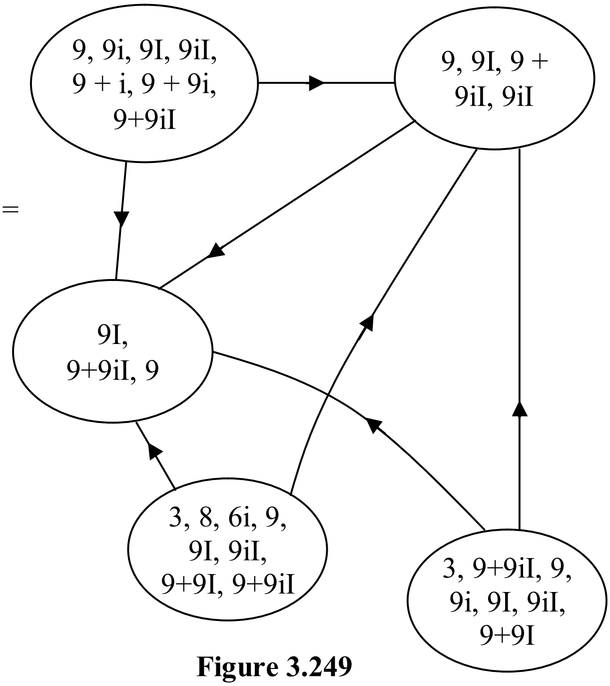


Figure 3.249

However the number of edges in G and G^C are different.

Let H be the special complex - neutrosophic subset vertex subgraph of G given by the following figure.

H =

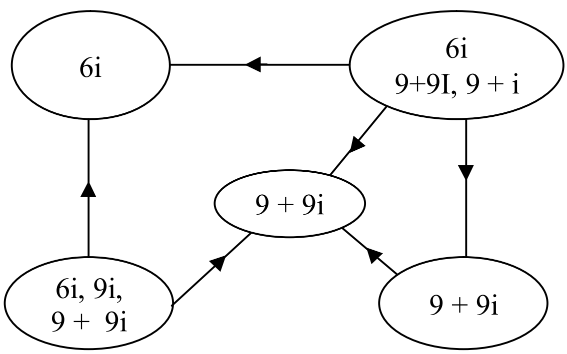


Figure 3.250

Now we find the local complement H^C of H relative to G which is given by the following figure.

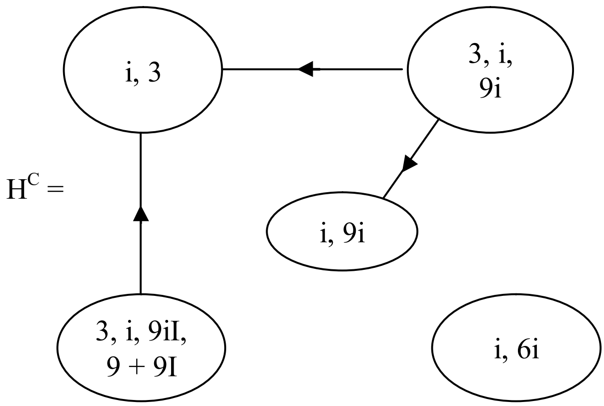


Figure 3.251

The local complement exists but it does not preserve the structure. The reader is left with the task of finding other special subset vertex complex - neutrosophic subgraphs of G and finding their complements when ever they exist.

Prove or disprove that all special complex - neutrospohic subset vertex subgraphs H of G need not have the local complement.

Let G_1 be the complex - neutrosophic subset vertex graph (projective) of type II given by the following figure.

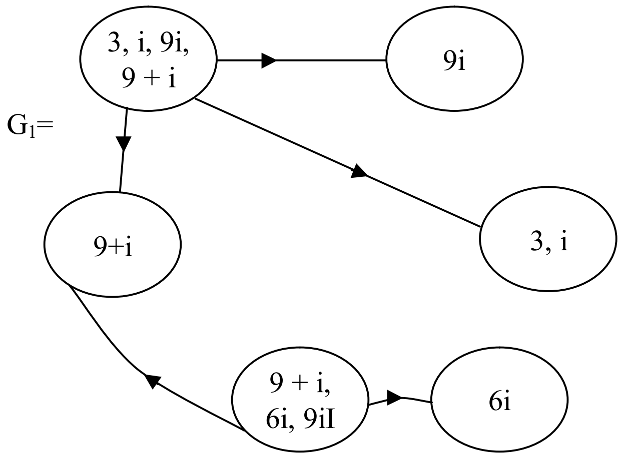


Figure 3.252

Clearly none of the special complex - neutrosophic subset vertex projective subgraphs of G_1 has local complements relative to G_1. Let H_1 be the special complex - neutrosophic subset vertex subgraph of G_1 given by the following figure.

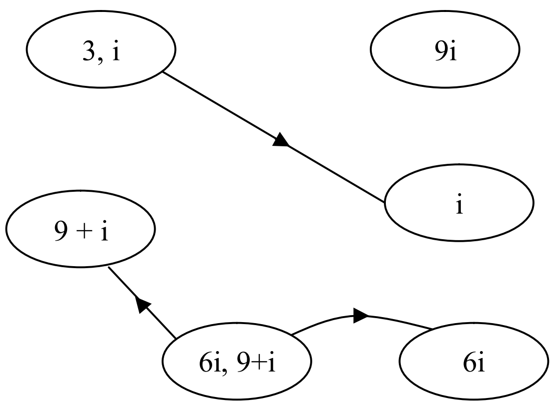


Figure 3.253

Clearly the local complement of H_1 does not exist.

Infact every complex - neutrosophic special subset vrtex subgraph of G_1 will have three vertices to be singleton sets as G_1 has three vertices to be singleton sets.

Local complements relative to G_1 can never exist as every special subgraph will have singleton vertices and for singleton sets we cannot define the notion of local complements hence the claim.

It is pertinent to keep on record at this juncture that all properties enjoyed by the usual subset vertex graphs of type I and type II can be without any difficulty extended to the case of complex subset vertex graphs of type I and type II, neutrosophic subset vertex graphs of type I and type II and complex neutrosophic subset vertex graphs of type I and type II with appropriate modifications.

Thus interested reader can prove these analogous results. Further it is important to note that in case of complex vertex subset graphs only the vertex set is complex by definition of the edges in these types of graphs the edges remain always real. Similarly in case of neutrosophic subset vertex graphs of type I and type II and also in case of complex neutrosophic subset vertex graphs of type I and type II.

So we have to make some special modifications to make the edges complex or neutrosophic or complex or neutrosophic, however this cannot be adopted for subset vertex graphs of type I and type II.

We suggest a few problems some of which are at research level and others just routine exercises.

Problems

1. Let S = {1, 2, 3, 4} and P(S) be the power set of S.

   i) Find all subset vertex graph of type I.

   ii) How many of these subset vertex graphs of type I are simple special subset vertex in graphs?

   iii) Find the number of subset vertex graphs of type I (using above S) which are not simple, that is they have atleast one special subset vertex hyper subgraph.

   iv) Let G = (V, E) be the subset vertex graph given by the following figure.

G =

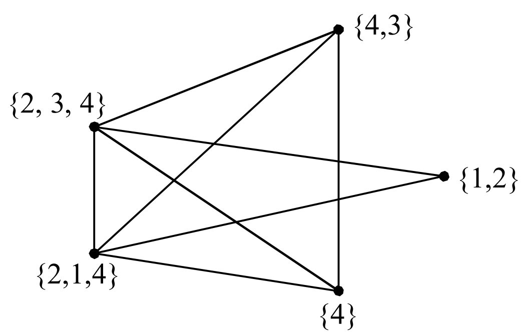


Figure 3.254

i) Find all special subset vertex subgraphs of G.

ii) Can G have special subset vertex hyper subgraphs?

iii) Prove G has no special subset vertex empty subgraph.

iv) Give one example of a simple subset vertex graph of type I using S.

2. What are the special features of special subset vertex subgraphs of a subset vertex graph of type I?

3. Distinguish between the special subset vertex subgraphs and subset vertex subgraphs of a subset vertex graph G of type I.

4. Can we find any sort of relation between subset vertex graphs of type I and type II?

5. Find any special and innovative features associated with subset vertex graphs of type I.

6. Can we say projective subset vertex graphs of type II becomes injective subset vertex graphs of type II if the arrows are reversed?

7. Using the set S = {1, 2, …, 9} and P(S) the power set of S.

   i) How many subset vertex graphs of type I can be constructed using P(S)?

ii) How many of the subset vertex graphs of type I are complete?

iii) Find for the complete subset vertex graph G of type I with 9 vertices the number of special subset vertex subgraphs which has local complements that are complete.

iv) Find the special subset vertex subgraphs of type I which are trees.

v) Can we have subset vertex graphs of type I which are trees and which has special subset vertex subgraphs which are trees?

Justify your claim.

vi) Obtain any other special feature enjoyed by these special subset vertex subgraphs of type I.

vii) Let K be the subset vertex graph of type I given by the following figure.

K =

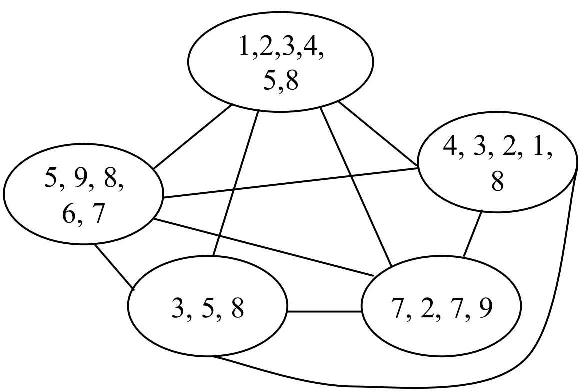


Figure 3.255

Study questions (iii) to (vi) of this problem for this K.

8. Let S = {1, 2, …, 18} be a finite set of order 18. P(S) b the power set of S.

   i) Find the number of projective subset vertex graphs of type II.

   ii) How many of these special subset vertex subgraphs mentioned in (i) has local complements?

   iii) Can we say every projective subset vertex projective graphs of type II has special subset vertex subgraphs which has no local complements?

   iv) Can we have projective special subset vertex subgraphs of type II which are trees? Justify you claim!

   v) Find all special subset vertex subgraphs of G which are complete but G is not a subset vertex complete graph of type II of G.

   vi) Enumerate all special and interesting features associated with projective special subset vertex subgraphs of type II.

9. Can we say any given projective subset-vertex graph of type II there always exists using those vertices a subset vertex graph of type I with more number of edges?

Prove or disprove the above question by an example.

10. Prove if G is a subset vertex graph of type I with vertex set from P(S) where S = {1, 2, …, n} then in general P(S); G can also be a projective (or injective) subset vertex empty graph of type II.

    a) How many such empty subset vertex graphs exist?

11. Find any interesting relation between special subset vertex subgraphs of type I and type II.

12. Write a program to find all complete subset vertex graphs of type I with vertex set from P(S); S = {1, 2,…, n}.

    i) How many complete subset vertex graphs of type I exist using P(S); S = {1, 2, …, n}?

    ii) Can we have find any complete subset vertex graph of type I be type II?

13. Compare projective and injective subset vertex graphs G of type II with type I graphs with the same set of vertices as that of G.

14. Let S = {1, 2, …, 18} be a set of order 18. P(S) be the power set of S.

    i) Draw the subset vertex graph of type I using the vertex set V = {{1, 2, 3, 4, 5, 6, 9}, {7, 8, 10, 11, 1, 12},, {14, 15, 17}, {3, 4, 5, 17}, {10, 11, 18}, {15, 1, 2, 6}, {18, 9, 10, 5}}.

ii) Using this vertex set V $\subseteq$ P(S) can we get a injective (or projective) subset vertex graph of type II which is not a empty graph? Justify your claim.

iii) How many special vertex subset subgraph of type I can built using the subset vertex graph of type I using vertex set V?

iv) Let W = {(1, 2, …, 17), (1, 2, 3,…, 16, 18), (1,2, …, 15, 17, 18), … (2, …, 18)} be the set of 18 elements from P(S). Using W find the subset vertex graph G of type I.

v) Is G a complete graph? Justify?

vi) How many special subset vertex subgraphs of G are complete?

vii) Give atleast 4 special subset vertex subgraphs of G which are not complete.

15. Let S = {1, 2, …, 16} be the set of order 16. P(S) be the power set of S.

Let G be the subset vertex graph of type I given by the following figure.

G =

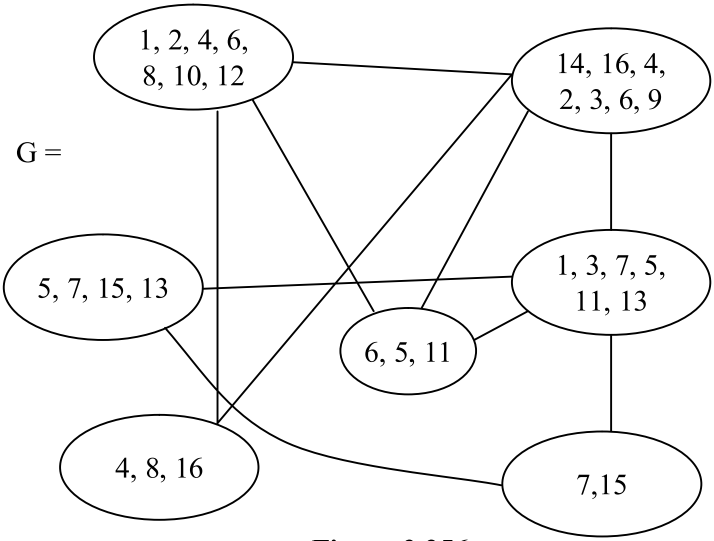


Figure 3.256

i) Find all special subset vertex subgrpahs of G.

ii) Find the global complement of G.

iii) How many of the special subset vertex subgraphs of G has local complements relative to G?

iv) Find all special vertex subgraphs of G which are empty.

v) Prove special vertex empty subgraphs of G has local complements relative to G.

vi) Find all special subset vertex subgraphs whose local complements of them relative to G is empty.

vii) Find all special subset vertex subgrpahs of G which has no local complements relative to G and characterize them.

viii) Find all special and distinct feature enjoyed by local complements of special subset vertex subgraphs.

16. Define and describe the complex subset vertex graph of type I with some examples.

17. Prove in general a special subset vertex subgraph of a complex subset vertex graph G with vertex set from P(S) where S = $C(Z_{27})$ need not be complex special subset vertex subgraphs be it of type I or projective type II or injective type II.

18. Find all special features associated with complex subset vertex graphs of type I and type II.

19. Give an example of a complex subset vertex graph whose special subset vertex subgraph and its local complements are complete.

20. Distinguish between complex subset vertex graphs of type I and type II and subset vertex graphs of type I and type II respectively.

21. Prove these complex subset vertex graphs of type I and type II can only have real edges.

22. Describe neutrosophic subset vertex graphs of type I and type II and prove the edges are always real.

23. Let S = {1 + I, 4I, 5I – 2, 3I + 7, 9I, 9, 9 + 9I, 0, 6, 9I, 2, 6I, 12} be the neutrosophic set P(S) the power set of S.

    Let V = {{1 + I, 4I, 9, 6}, {0, 6, 9I, 9 + 9I}, {12, 6, 0, 4I, 5I – 2}, {3I + 7, 9, 9I, 6I, 2}, {12, 09, 9I, 5I – 2, 1 + I, 4I}, {4I, 0, 6I, 12, 5I – 1}} $\subseteq$ P(S).

    i) Draw the subset vertex graph G of type I using the vertex set V.

    ii) Prove both subset vertex graphs of type II (projective and injective) using vertex set V are only empty graphs.

    iii) How many edges does G contain?

    iv) Is G a complete subset vertex graph of type I.

    v) Find the global complement G^C of G. Does G and G^C enjoy the same structure?

    vi) Find all neutrosophic special subset vertex subgraphs of G.

    vii) Can G have real special subset vertex subgraph?

    viii) Find the local complements for atleast 3 special subset vertex subgraphs of G.

ix) Let K be the neutrosophic subset vertex graph of type II given by the following figure.

K =

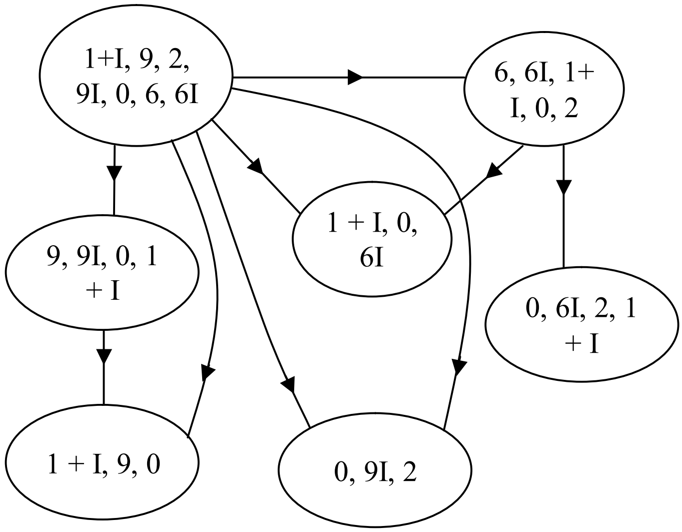


Figure 3.257

x) Find the global complement of K.

xi) Find all neutrosophic special subset vertex subgraphs of K.

xii) How many of these special subset vertex subgraphs of K are real?

xiii) Find for those neutrosophic special subset vertex subgraphs H of K their local complements H^C relative to K.

xiv) How many of the neutrosophic special subset vertex subgraphs of K are empty?

xv) Is the neutrosophic subset vertex subgraph B of K with vertex set W = {{1 + I, 9, 2}, {6I, 2}, {9I, 1+ I}, {6I, 0}, {2, 1 + I}, {9I, 2}, {0, 1 + I}} an empty special subset vertex subgraph of K?

xvi) What can be the structure enjoyed by B^C the local complement of B relative to K?

xvii) Will B^C be an empty subgraph? (Justify)

24. Let S = {i, 3i + I, 4I + 8, 9iI + 7, 10i + 12I + 4iI + 3, 0, I, ij, 7, 9, 8iI + 7i + 8I + 9, 4I, 10I, 11i, 12I + I, 15 iI + 1, 15, 2, 9iI} be the neutrosophic complex valued set and P(S) the power set of S.

Let V = {{I, 3i + I, 4I, 2}, {9iI, 15, 2, 3i+I}, {4I, 10I, 10i + 12I + 3 + 4iI, ij, I}, {I, ij, 3i + I, 4I + 8}} $\subseteq$ P(S) be the vertex set.

i) Using V find the complex - neutrosophic subset vertex graph G of type I.

ii) Prove using V we can get only a complex - valued subset vertex empty graphs of type I and type II.

iii) Find the global complement of G.

iv) Prove or disprove G cannot have real special subset vertex subgraph of type I.

v) Find all complex - neutrospohic subset vertex subgraph H of G.

vi) How many of these complex neutrosophic special subset vertex subgraphs H have local complements?

vii) Enumerate all complex neutrosophic special subset vertex subgraphs which are empty.

viii) If B = {{i}, {15}, {4I}, {ij}} ⊆ V prove using B as the vertex set one can get only a complex neutrosophic special subset vertex subgraph which is empty.

ix) Prove using C = {{2}, {ij}, {15}, {4I + 8}} ⊆ V as a vertex set we get yet another complex neutrosophic special subset vertex subgaph which is empty.

x) Let D = {{4I, 2}, {2}, {I, 4I, 10I, 2} {I}} ⊆ V. Draw the complex neutrosophic special subset vertex subgraph H using the vertex set D.

xi) Find the number of edges in H.

xii) M =

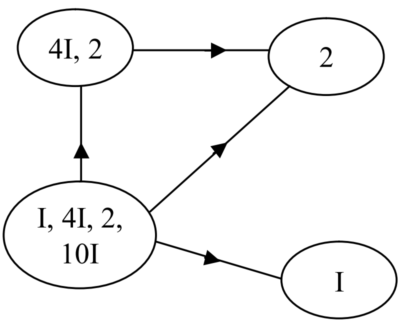


Figure 3.258

Is M given in the above figure a type II complex neutrosophic vertex subset graph of type II which is projective?

We define them as doubly special complex neutrosophic subset vertex subgraphs of type II (projective or injective).

Thus if G a complex neutrosophic (or complex or neutrosophic or real) subset vertex graph of type I and if G has a special subset vertex subgraph with a vertex st M and if one can using this vertex set M get a special subset vertex subgraph P of type II (projective or injective) then we define P to be a doubly special subset vertex subgraph of G).

xiii) Prove M is a doubly special complex neutrosophic subset vertex subgraph of G,

25, Will every subset vertex graph of type I (complex or neutorosphic or complex - neutrosophic or complex – neutrosophic or real) contain a doubly special subset vertex subgraph? Justify your claim for any general set.

26. Does there exist a subset vertex graph of type I which has no doubly special subset vertex subgraph?

27. Can we say every subset vertex graph of type I has doubly special subset vertex subgraphs of type II?

28. Now suppose we have a subset vertex graph of type II can we say it has doubly special subset vertex subgraphs of type I?

    Justify your claim.

29. Is it mandatory every type I subset vertex graph has doubly special subset vertex subgraph of type II?

30. Obtain all special features associated with doubly special subset vertex subgraphs.

31. Can we say for every subset vertex graphs of type II there exists a doubly special subset vertex subgraph of type I?

32. Let G be a subset vertex graph of type II with vertex set from P(S) where S = {1, 2, 3, 4, 5, 6, 7, 8, 9} given b y the following figure.

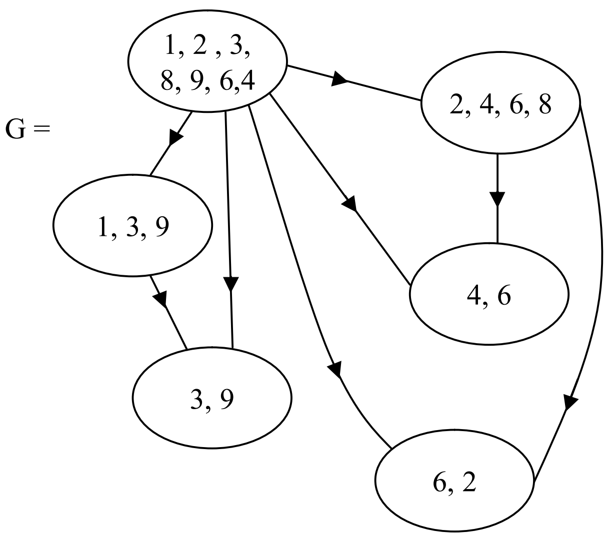


Figure 3.259

Now using the same set of vertex set from G we get the subset - vertex graph H of type I in the following.

H =

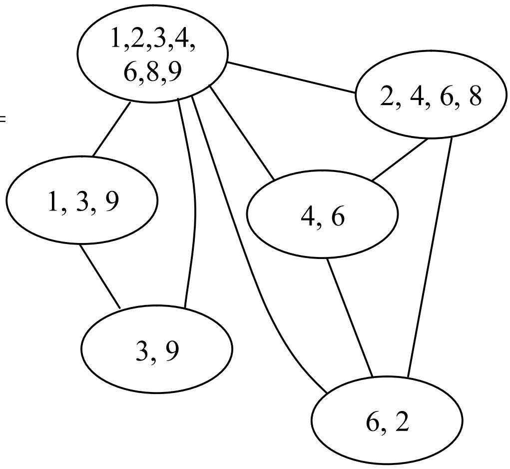


Figure 3.260

We see H is a graph with 9 edges whereas G has only 8 edges.

Thus can we conjecture that every vertex set of a subset vertex graph of type II will always yield a subset vertex graph of type I but not vice versa.

33. Let G b subset vertex graph of type I given by the following figure where the vertex set is from P(S) where S = {1, 2, 3,4,5}

G =

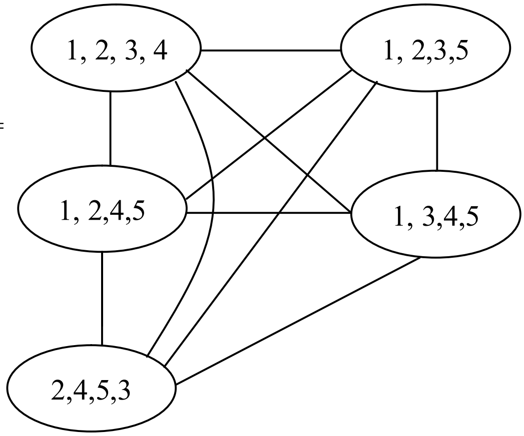


Figure 3.261

G is a complete graph. Prove using this vertex set of G subset graph of type II is a empty graph.

34. Let S = {10, 10I + 9, 9I, 9i, 9 + 9i + 9I, 12I, 0, I, i, iI, 9iI + 9i, 27i, 27iI, 9 + 18I + 27i + 36iI, 36, 1, 36I} be the neutrosophic - complex set. P(S) the power ofS.

i) How many complex - neutrosophic subset vertex graphs of type I with 9 nodes can be built?

ii) How many complex - neutrosophic subset vertex graphs of type I with 6 nodes can be built?

iii) Find for atleast one complex - neutrosophic subset vertex graph G with 9 nodes.

a) Find all complex - neutrosophic special subset vertex subgraphs of G which has local complements.

b) Those complex neutrosophic subset vertex subgraphs which has no local complements for this graph G with 9 nodes.

c) How many complex neutrosophic subset vertex subgraph which are only empty subgraphs exist for this graph G?

d) Find the global (universal) complement of this G.

e) How many complete complex neutrosophic subset vertex graphs with 9 can be complete?

f) For which number of nodes (say 5 or 6 or 7 or …) we have large number of complete complex neutrosophic subset vertex graphs of type I

iv) Find all complex - neutrosophic subset vertex graphs G of type I using the vertex set from P(S) which are such that both G and G^C are complete.

v) Characterize all those complex neutrosophic subset vertex graphs G of type I for which none of the special complex neutrosophic subset vertex subgraphs of G has no local complements.

Chapter Four

APPLICATION OF SPECIAL SUBSET VERTEX SUBGRAPHS OF TYPE I AND TYPE II

In this chapter authors briefly mention the possible applications of special subset vertex subgraphs of type I and type II. It is important to mention that these subset vertex graphs can be used in social information networks and communications networks apart from other computer networks including the state space graph of the problem solving agents in AI. However these special subset vertex subgraphs notion is used mainly in case of fault tolerant system.

We are more interested in using these graphs in social information networks. We see that certain social structures we want to preserve as it is for we realize those structures have been useful for a peaceful and understanding society, weeding of all intermittent chaos, war etc.

Thus we can by some means either by using a super subset vertex graph or using the special subset vertex subgraph retrieve the same network (they can be also used in the fault tolerant networks).

Thus these special subset vertex subgraphs can be used in technical fault tolerant system thereby using these we can save both time and money. Another practical way of using it is we will have a subset vertex graph (of type I or type II) which has special subset vertex subgraph which are hypergraphs, so to be more economic we will use the hyper subgraphs instead of the original subset vertex graph so that some units do not function at all times only they function in time of emergency or need. So one need not seek to repair the system but can be made to work in the same way, always keeping in mind the minimum number of concepts or entities or nodes have to function and repair them in times of maintenance; there by saving both time and money.

Thus these subset vertex graphs which has special subset vertex subgraphs, which are hypergraphs will be ideally best suited for fault tolerant networks. This can reduce the money and time spent on that unit or on that particular network. Thus at large they can work well in the retrievable networks.

Our basic mission of introducing these subset vertex graphs of type I and type II is to study of the social information networks. For our aim to study the social network is to know the structure of the social set up so that all sorts of disturbances or changes or dying of the network can be analysed and if possible retrieved it back to the normal one by either using the super subset vertex graph or the special subset vertex subgraph of the subset vertex graph.

This sort of study is vital for we see several nations fight with each other, it is also prevalent among the clans of the same nation etc. so that we can find the root cause of the same and try to settle the war or unrest and establish some sort of peace or understanding. A total annihilation of a clan or a tradition or art or culture or practices can be avoided.

Further certain social setups are dismantled for unknown reasons, we can try to unite them so that though not fully at least partially we can find some sort of relation to rebuilt or restructure them.

Thus these subset vertex graphs and special subset vertex subgraphs can be used to retrieve the social information networks (and in non social information networks it can be used for fault tolerant networks). These special subset vertex subgraphs of vertex subset graphs G which are hypergraphs of G are also termed or known as fault tolerant graphs.

The complements both local and global of these subset vertex graphs of type I and type II can be used in the study of dual structures of both social networks as well as fault tolerant networks. Finally these special subset vertex subgraphs can be best suited for all fault tolerant networks.

Now we give some problems for the researcher.

Problems

1. Study the various civil and social problems among nations and how they can be resolved using special subset vertex subgraphs of a subset vertex graph.

2. Study of the social setup of the terrorist groups in this world, their mission and the hidden nodes using the subset vertex graphs.

3. Study the war among different clans to separate the nation or that state. The reason for it can be studied by using social information network and adopting the subset vertex graphs.

4. Study of social information networks in class rooms, institutions, industries and political parties etc. using subset vertex graphs of type I or type II, can clearly reveal the strength or weakness or balance of these structures.

5. Study of ethnic groups using social information networks can throw light on how social structures were dismantled.

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A new class of subgraphs called special subset vertex subgraphs are defined in this book. These graphs can be used in any communication or social information network. Any network whose performance depends on important and basic network centrality measures can use subset vertex subgraph structure to make the network more reliable and fault tolerant.

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