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New Types of Topologies and Neutrosophic Topologies

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Abstract: In this paper, we recall the six new types of topologies that we introduced in the last years (2019-2022), such as: Refined Neutrosophic Topology, Refined Neutrosophic Crisp Topology, NeutroTopology, AntiTopology, SuperHyperTopology, and Neutrosophic SuperHyperTopology.

Keywords: Refined Neutrosophic Topology; Refined Neutrosophic Crisp Topology; NeutroTopology; AntiTopology; SuperHyperTopology.

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1. Refined Neutrosophic Topology

The neutrosophic set has been extended to the Refined Neutrosophic Set (Logic, Probability) [1], where there are multiple parts of the neutrosophic components, as such T was split into subcomponents $T_1, T_2, \ldots, T_p$, and I into $I_1, I_2, \ldots, I_r$, and F into $F_1, F_2, \ldots, F_s$, with $p + r + s = n \geq 2$ and integers $t, r, s \geq 0$ and at least one of them is ≥ 2. Even more: the subcomponents T_j, I_k, and/or F_l can be countable or uncountable infinite subsets of [0, 1].

This definition also includes the Refined Fuzzy Set, when $r = s = 0$ and $p \geq 2$; and the definition of the Refined Intuitionistic Fuzzy Set, when $r = 0$, and either $p \geq 2$ and $s \geq 1$, or $p \geq 1$ and $s \geq 2$.

All other fuzzy extension sets can be refined in a similar way.
The *Refined Neutrosophic Topology* is a topology defined on a Refined Neutrosophic Set.

{Similarly, the Refined Fuzzy Topology is defined on a Refined Fuzzy Set, while the Refined Intuitionistic Fuzzy Topology is defined on a Refined Intuitionistic Fuzzy Set.

And, as a generalization, on any type of fuzzy extension set [such as: Pythagorean Fuzzy Set, Spherical Fuzzy Set, Fermatean Fuzzy Set, etc.] one can define a corresponding fuzzy extension topology.}

2. Refined Neutrosophic Crisp Topology

The *Neutrosophic Crisp Set* was defined by Salama and Smarandache in 2014 and 2015.
Let X be a non-empty fixed space. And let D be a Neutrosophic Crisp Set [2], where $D = \langle A, B, C \rangle$, with A, B, C as subsets of X.
Depending on the intersections and unions between these three sets A, B, C one gets several:
Types of Neutrosophic Crisp Sets [2, 3]
The object having the form D = is called:

1. A neutrosophic crisp set of Type 1 (NCS-Type1) if it satisfies: $A \cap B = B \cap C = C \cap A = \emptyset$ (empty set).

2. A neutrosophic crisp set of Type 2 (NCS-Type2) if it satisfies: $A \cap B = B \cap C = C \cap A = \emptyset$ and $A \cup B \cup C = X$.
3. A neutrosophic crisp set of Type 3 (NCS-Type 3) if it satisfies: $A \cap B \cap C = \emptyset$ and $A \cup B \cup C = X$.

Of course, more types of Neutrosophic Crisp Sets may be defined by modifying the intersections and unions of the subsets A, B, and C.

The *Refined Neutrosophic Crisp Set* was introduced by Smarandache in 2019, by refining/splitting D (and denoting it by RD = Refined D) by refining/splitting its sets A, B, and C into sub-subsets as follows:

RD = ($A_1, \ldots, A_p; B_1, .., B_r; C_1, .., C_s$), with $p, r, s \geq 1$ be positive integers and at least one of them be ≥ 2,

and $A = \bigcup_{i=1}^{p}, B = \bigcup_{j=1}^{r}, C = \bigcup_{k=1}^{s}$, and many Types of Refined Neutrosophic Crisp.

Therefore, the Refined Neutrosophic Crisp Topology is a topology defined on the Refined Neutrosophic Crisp Set.

3. NeutroTopology

NeutroTopology [3] is a topology that has at least one topological axiom which is partially true, partially indeterminate, and partially false, or (T, I, F), where T = True, I = Indeterminacy, F = False, and no topological axiom is totally false, in other words: $(T, I, F) \notin \{(1,0,0), (0,0,1)\}$, where (1, 0, 0) represents the classical Topology, while (0, 0, 1) represents the below AntiTopology.

Therefore, NeutroTopology is a topology in between classical Topology and AntiTopology.

4. AntiTopology

AntiTopology [3] is a topology that has at least one topological axiom that is 100% false (T, I, F) = (0, 0, 1).

The NeutroTopology and AntiTopology are particular cases of NeutroAlgebra and AntiAlgebra [3] and, in general, they all are particular cases of the NeutroStructure and AntiStructure respectively, since we consider "Structure" in any field of knowledge [4].

5. SuperHyperTopology

SuperHyperTopology [5] is a topology build on the nth-PowerSet of a given non-empty set H that excludes the empty set. Therefore: $P_*(H)$ is the first powerset of the set H, without the empty set (∅); ${P_*}^2(H) = P_*(P_*(H))$, is the second powerset of H (or the powerset of the powerset of H), without the empty sets; and so on, the n-th powerset of H,

$$\mathbf{P}_*^{n}(\mathbf{H}) = \mathbf{P}_*(\mathbf{P}_*^{n-1}(\mathbf{H})) = \underbrace{\mathbf{P}_*\Big(\mathbf{P}_*\Big(\ldots\Big(\mathbf{P}_*(\mathbf{H})\Big)\ldots\Big)\Big)}_{n}$$

, where P_* is repeated n times ($n \geq 2$), and without the empty sets.

6. Neutrosophic SuperHyperTopology

Neutrosophic SuperHyperTopology [6] is, similarly, a topology build on the nth-PowerSet of a given non-empty set H, but includes the empty sets [that represent indeterminaciesy] too.

As such, in the above formulas, $P_*(H)$ that excludes the empty set, is replaced by $P(H)$ which includes the empty set. $P(H)$, is the first powerset of the set H, including the empty set (∅); $P_*^2(H) = P(P(H))$ is the second powerset of H (or the powerset of the powerset of H), which includes the empty sets; and so on, the n-th powerset of H,

$$P^n(H) = P(P^{n-1}(H)) = \underbrace{P\Big(P\big(\ldots\big(P(H)\big)\ldots\big)\Big)}_{n}$$

, where P is repeated n times (n ≥ 2), and includes the empty-sets.

7. Conclusion

These six new types of topologies were introduced by Smarandache in 2019-2022, but they have not yet been much studied and applied, except the NeutroTopology and AntiTopology which got some attention from researchers.

Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

References

1. F. Smarandache, n-valued Refined Neutrosophic Set and Logic and its Applications in Physics, Progress in Physics, 143-146, Vol. 4, 2013, https://arxiv.org/ftp/arxiv/papers/1407/1407.1041.pdf and http://fs.unm.edu/RefinedNeutrosophicSet.pdf
2. A.A. Salama, F. Smarandache, Neutrosophic Crisp Set Theory, Educational Publisher, Columbus, Ohio, USA, 2015; http://fs.unm.edu/NeutrosophicCrispSetTheory.pdf
3. Florentin Smarandache, Refined Neutrosophic Crisp Set (RNCS), in the book Nidus Idearum, pp. 114-116, Vol. VII, third edition, 2019, Editions Pons, Brussels, Belgium; Nidus Idearum book:http://fs.unm.edu/NidusIdearum7-ed3.pdf Refined Neutrosophic Crisp Set (RNCS) – chapter: http://fs.unm.edu/RefinedNeutrosophicCrispSet.pdf
4. F. Smarandache, NeutroAlgebra & AntiAlgebra are generalizations of classical Algebras, http://fs.unm.edu/NA/NeutroAlgebra.htm and http://fs.unm.edu/NeutroAlgebra-general.pdf, 2019-2022.
5. Florentin Smarandache, Structure, NeutroStructure, and AntiStructure in Science, International Journal of Neutrosophic Science (IJNS), Volume 13, Issue 1, PP: 28-33, 2020; http://fs.unm.edu/NeutroStructure.pdf
6. F. Smarandache, The SuperHyperFunction and the Neutrosophic SuperHyperFunction, Neutrosophic Sets and Systems, Vol. 49, 2022, pp. 594-600, http://fs.unm.edu/NSS/SuperHyperFunction37.pdf

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