



m -Variable n -Refined Neutrosophic AH-Isometry

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ABSTRACT. We extend new for the first time, the (one-variable) n -refined neutrosophic AH-isometry to the m -variable n -refined neutrosophic AH-isometry. The reason is similar: if we knew the determinate and non-indeterminate parts of the m arguments:

$$\alpha_1 = a_0^1 + a_1^1 I_1 + a_2^1 I_2 + \cdots + a_n^1 I_n,$$

$$\alpha_2 = a_0^2 + a_1^2 I_1 + a_2^2 I_2 + \cdots + a_n^2 I_n,$$

\vdots

$$\alpha_m = a_0^m + a_1^m I_1 + a_2^m I_2 + \cdots + a_n^m I_n$$

how the we similarly find the determinate and non-indeterminate parts of a function (of operation) of these m arguments $f(\alpha_1, \alpha_2, \dots, \alpha_m)$? This paper demonstrate that.

Keywords: Indeterminacy, Neutrosophic, n -refined, AH-isometry.

AMS Mathematics Subject Classification [2020]: ?????????????

1. Introduction

The literal indeterminacy (I) was for the first time refined/split into literal sub-indeterminacies (I_1, I_2, \dots, I_n) by Smarandache [5] in 2015, who defined a multiplication law of these literal sub-indeterminacies to be able to build the Refined I -Neutrosophic Algebraic Structures. The AH-isometry [2] was firstly introduced by Mohammad Abobala and Ahmad Hatip in 2021, where AH stands for Abobala-Hatip. The 2-refined AH-isometry [3] was introduced by Celik and Hatip in 2022. Many papers (e.g. see [1]) were published about this fundamental AH-isometry in the rings of neutrosophic literal numbers. The n -refined AH-isometry [4] was introduced by Smarandache and Abobala in 2024. Now we extend it for the first time, extend one variable to many variable, called m -variable n -refined AH-isometry.

2. Main results

One-variable AH-isometry

$$f(x + yI) = f(x) + I.[f(x + y) - f(x)]$$

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Two-variable AH-isometry

$$f(x_1 + y_1 I, x_2 + y_2 I) = f(x_1, x_2) + I[f(x_1 + y_1, x_2 + y_2) - f(x_1, x_2)]$$

Three-variable AH-isometry

$$f(x_1 + y_1 I, x_2 + y_2 I, x_3 + y_3 I) = f(x_1, x_2, x_3) + I[f(x_1 + y_1, x_2 + y_2, x_3 + y_3) - f(x_1, x_2, x_3)]$$

m-variable AH-isometry, for integer $m \geq 1$

$$f(x_1 + y_1 I, x_2 + y_2 I, \dots, x_m + y_m I) = f(x_1, x_2, \dots, x_m) + I[f(x_1 + y_1, x_2 + y_2, \dots, x_m + y_m) - f(x_1, x_2, \dots, x_m)]$$

One-variable 2-refined AH-isometry

$$f(a_0 + a_1 I_1 + a_2 I_2) = f(a_0) + [f(a_0 + a_1 + a_2) - f(a_0 + a_2)]I_1 + [f(a_0 + a_2) - f(a_0)]I_2$$

Two-variable 2-refined AH-isometry

$$f(a_0 + a_1 I_1 + a_2 I_2, b_0 + b_1 I_1 + b_2 I_2) = f(a_0, b_0) + [f(a_0 + a_1 + a_2, b_0 + b_1 + b_2) - f(a_0 + a_2, b_0 + b_2)]I_1 + [f(a_0 + a_2, b_0 + b_2) - f(a_0, b_2)]I_2$$

Three-variable 2-refined AH-isometry

$$f(a_0 + a_1 I_1 + a_2 I_2, b_0 + b_1 I_1 + b_2 I_2, c_0 + c_1 I_1 + c_2 I_2) = f(a_0, b_0, c_0) + [f(a_0 + a_1 + a_2, b_0 + b_1 + b_2, c_0 + c_1 + c_2) - f(a_0 + a_2, b_0 + b_2, c_0 + c_2)]I_1 + [f(a_0 + a_2, b_0 + b_2, c_0 + c_2) - f(a_0, b_0, c_0)]I_2$$

m-variable 2-refined AH-isometry

$$f(a_0^1 + a_1^1 + a_2^1 I_2, a_0^2 + a_1^2 I_1 + a_2^2 I_2, \dots, a_0^m + a_1^m I_1 + a_2^m I_2) = f(a_0^1, a_0^2, \dots, a_0^m) + [f(a_0^1 + a_1^1 + a_2^1, a_0^2 + a_1^2 + a_2^2, \dots, a_0^m + a_1^m + a_2^m) - f(a_0^1 + a_2^1, a_0^2 + a_2^2, \dots, a_0^m + a_2^m)]I_1 + [f(a_0^1 + a_2^1, a_0^2 + a_2^2, \dots, a_0^m + a_2^m) - f(a_0^1, a_0^2, \dots, a_0^m)]I_2$$

m-variable n-refined AH-isometry

$$\begin{aligned} f(a_0^1 + a_1^1 I_1 + a_2^1 I_2 + \dots + a_n^1 I_n, a_0^2 + a_1^2 I_1 + a_2^2 I_2 + \dots + a_n^2 I_n, \dots, a_0^m + a_1^m I_1 + a_2^m I_2 + \dots + a_n^m I_n) = f(a_0^1, a_0^2, \dots, a_0^m) + [f(\sum_{i=0}^n a_i^1, \sum_{i=0}^n a_i^2, \dots, \sum_{i=0}^n a_i^m) - f(\sum_{i \neq 1}^n a_i^1, \sum_{i \neq 1}^n a_i^2, \dots, \sum_{i \neq 1}^n a_i^m)]I_1 \\ + [f(\sum_{i \neq 1}^n a_i^1, \sum_{i \neq 1}^n a_i^2, \dots, \sum_{i \neq 1}^n a_i^m)]I_2 + [f(\sum_{i \neq 1,2}^n a_i^1, \sum_{i \neq 1,2}^n a_i^2, \dots, \sum_{i \neq 1,2}^n a_i^m) - f(\sum_{i \neq 1,2,3}^n a_i^1, \sum_{i \neq 1,2,3}^n a_i^2, \dots, \sum_{i \neq 1,2,3}^n a_i^m)]I_3 + \dots + [f(\sum_{i \neq 1,2,\dots,k-1}^n a_i^1, \sum_{i \neq 1,2,\dots,k-1}^n a_i^2, \dots, \sum_{i \neq 1,2,\dots,k-1}^n a_i^m) - f(\sum_{i \neq 1,2,\dots,k}^n a_i^1, \sum_{i \neq 1,2,\dots,k}^n a_i^2, \dots, \sum_{i \neq 1,2,\dots,k}^n a_i^m)]I_k + [f(\sum_{i \neq 1,2,\dots,n-1}^n a_i^1, \sum_{i \neq 1,2,\dots,n-1}^n a_i^2, \dots, \sum_{i \neq 1,2,\dots,n-1}^n a_i^m) - f(a_0^1, a_0^2, \dots, a_0^m)]I_n \end{aligned}$$

3. Conclusion

We have introduced, for the first time, the (one-variable) n -refined neutrosophic AH-isometry to m -variables n -refined. Neutrosophic AH-isometry, which is needed in finding the determinate and non-indeterminate parts of a function (or, equation) of m arguments.

References

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