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A multi-objective transportation model under neutrosophic environment[☆]

Rizk M. Rizk-Allah[a,1], Aboul Ella Hassanien[b,1], Mohamed Elhoseny[c,*]

[a] Faculty of Engineering, Menoufia University, Shebin El-Kom, Egypt
[b] Faculty of Computers and Information, Cairo University, Egypt
[c] Faculty of Computers and Information, Mansoura University, Egypt

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ABSTRACT

In this paper, a new compromise algorithm for multi-objective transportation problem (MO-TP) is developed, which is inspired by Zimmermann's fuzzy programming and the neutrosophic set terminology. The proposed NCPA is characterized by assigning three membership functions for each objective namely, truth membership, indeterminacy membership and falsity membership. With the membership functions for all objectives, a neutrosophic compromise programming model is constructed with the aim to find best compromise solution (BCS). This model can cover a wide spectrum of BCSs by controlling the membership functions interactively. The performance of the NCPA is validated by measuring the ranking degree using TOPSIS approach. Illustrative examples are reported and compared with exists models in the literature. Based on the provided comparisons, NCPA is superior to fuzzy and different approaches.

1. Introduction

Despite the enormous efforts to build an intelligent transportation model, the complex working environment of the recent real-life applications, i.e., Smart Cities applications [1] makes building such systems a big challenge. Thus, a Transportation Model is an urgent need to solve the Transportation Problem (TP) that aims to transport the goods from several supply points to different demand points through considering the minimization of the total transportation costs. TP model has a wide range of practical applications includes logistic systems, the supply chain management, manpower planning, inventory control, the production planning, etc. However, in reality, TP is characterized by multiple, incommensurable, and conflicting objective functions, being called the multi-objective transportation problem (MO-TP). Thus, in multi-objective transportation problem (MO-TP), the concept of optimal solution gives place to the concept of best compromise solution or the non-dominated solutions. Many studies have been investigated on MO-TPs. Aneja and Nair [2] developed a new approach to solve bi-criteria TP. Isermann [3] proposed a method to identify the set of non-dominated solutions for a MO-TP. In [4], Ringuest and Rinks were presented two interactive approaches to find the solution of the MO-TP.

In general, multi-objective transportation problems (MO-TPs) are solved with the assumptions that the cost parameters, sources, and destinations are specified in a precise way, i.e., in a crisp environment. Nevertheless, when handling real-life

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* Corresponding author.
E-mail address: mohamed_elhoseny@mans.edu.eg (M. Elhoseny).
1 Scientific Research Group in Egypt http://www.egyptscience.net.

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Nomenclature

m	number of sources
n	number of destinations
K	number of objective functions
s_i	label of the ith source
d_j	label of the ith decision variable destinations
c_{ij}^k	the penalty of transportation from s_i to d_j on the kth objective
Z^k	kth element of objective function vector
$\mathbf{Z}$	objective function vector
x_{ij}	denotes the quantity transported from s_i to d_j
Ω	the feasible set of solutions
E	the efficient set of solutions
$\mathbf{Z}_0^+$	ideal objective function vector
$\mathbf{Z}_*^-$	anti-ideal objective function vector
Y	space of points (objects)
S	neutrosophic set
T_S	truth membership
I_S	indeterminacy membership
F_S	falsity membership
t_k,s_k	predetermined real numbers in (0,1)
L_k	lower bound of the kth objective
U_k	upper bound of the kth objective
C	fuzzy constraints
G	fuzzy goal
D	fuzzy decision
α, β, γ	auxiliary parameters
FST	fuzzy set theory
FA	fuzzy approach
TP	transportation problem
BCS	best compromise solution
NCPA	neutrosophic compromise programming approach
MO-TP	multi-objective transportation problem

transportation models, the circumstances of imprecision in the description of the problem's parameters may appeared. Thus the conflict and imprecise parameters nature of the MO-TP make the mathematical formulation of the problem difficult to solve by conventional methods. To overcome this difficulty, fuzzy set theory (FST) was developed by Zadeh [5] to deal with impreciseness and uncertainties in the candidate data. A fuzzy set was defined as a set of elements where each element has some degree or grade of belongingness between 0 and 1 and such degrees determine the membership value of an element in that set. Since the new concepts by Bellman and Zadeh have been introduced [6], several inventions have been developed based on FST to solve different types of transportation problems under imprecise aspects [7–14]. In some large-scale applications, stochastic algorithms [15,16] are employed to find a near-optimal solution. In this context, some approaches based on genetic algorithm are employed to solve MO-TP [17,18].

Although FST is very useful when dealing with uncertainties, it cannot handle certain cases of uncertainties where it is hard to depict the membership degree using one specific value. To overcome the lack of knowledge of non-membership degrees, intuitionistic fuzzy set (IFS) was proposed in 1986 by Atanassov [19] as an extension of FST. In IFS, each element in a set is attached with two grades, membership grade and non-membership grade, where the sum of these two grades is restricted to less or equal to one. Thus, the grade of non-belongingness for a certain element is equal to 1 minus the grade of belongingness. The IFS has been flourished in applications of decision making [20]. Additionally, many authors have been employed IFS for solving different types of transportation problems [21,22]. Although the development of FST and IFS, dealing with all sorts of uncertainty in different areas still lack for a general framework, where the indeterminate knowledge cannot be managed. For example, let we ask the opinion of an expert regarding a particular statement, one may say that the possibility in which the statement is true is 0.6, the statement is false is 0.5 and the statement is not sure is 0.2. This issue is beyond the scope of FST and IFS and therefore dealing with a type of indeterminate situations of uncertain data undoubtedly becomes a true challenge.

Recently, a generalized form of FST and IFS is the neutrosophic set that was studied by Smarandache [23]. It provides a more general structure and very suitable form to overcome the mentioned issues. The term neutrosophy means the knowledge of neutral thought and this neutral represents the primary distinction between fuzzy and intuitionistic fuzzy logic. The neutrosophic set is established based on logic in which elements of the universe is presented by three degrees. Namely, truth degree, indeterminacy degree and falsity degree and they lie between [0, 1]. It differs from the intuitionistic fuzzy sets, where the involved uncertainty is dependent of the belongingness and non-belongingness degrees; here the uncertainty present, i.e., the indeterminacy factor, is independent of truth and falsity values. Since its inception by Smarandache [23], some attention has been developed for optimization aspects [24].

The main aim of the proposed approach is to provide a general structure for better dealing with impressions and uncertainties in available information. Furthermore, controlling the model memberships can achieve the best compromise result that does not only satisfy the decision maker's preferences, but it also presents non-dominated one. Another contribution it is represented as a realized aspect to deal with impressions by considering the truth membership, indeterminacy membership and falsity membership that are related to satisfaction, satisfaction to some extent and dissatisfaction of objectives respectively in finding the best compromise solution (BCS). Also, covering a wide spectrum of BCSs by controlling the membership functions interactively can be achieved. To the best of our knowledge, this is the first time to extend the Zimmermann's concepts to solve MO-TP.

The purpose of this study is to investigate the best compromise solution of multi-objective transportation problem (MO-TP) under neutrosophic compromise programming approach (NCPA). The proposed NCPA is developed by extending the Zimmermann concepts [25] the neutrosophic environment. The proposed NCPA presents a new insight in neutrosophic MO-TP by obtaining the best compromise solution using three memberships namely, truth membership, indeterminacy membership and falsity membership. The performance of the NCPA is tested by measuring the closeness degree using TOPSIS approach. Illustrative examples are reported and compared with exists models in the literature. Based on the provided comparisons, NCPA is superior to fuzzy approach and different approaches.

The main contributions are summarized below.

(1) To deal with uncertainties arising in data processing due to indeterminacy factors, neutrosophic compromise programming approach (NCPA) is introduced to determine the best compromise solution (BCS).
(2) The neutrosophic optimization model is constructed based on the neutrosophic decision set.
(3) A novel form of closeness to the ideal solution and the farthest from an anti-ideal solution is derived by TOPSIS.
(4) The feasibility and validity of the NCPA have been verified through two illustrative examples and comparative analysis.
(5) NCPA represents a novel form to deal with imprecise, uncertain, incomplete and inconsistent information that is very popular in scientific and engineering situations.

The paper is structured as follows: Section 2 presents some preliminaries of MO-TP and some definitions. The neutrosophic compromise programming approach (NCPA) for MO-TP is described in details in Section 3. In Section 4, two numerical examples are investigated. Section 5 discusses the performance regarding the results. Section 6 offers the conclusion and the further study.

2. Preliminaries

2.1. Problem formulation

MO-TP is concerned about transporting the product from m sources, s_1,s_2,…, s_m to n destinations d_1,d_2,…, d_n under K objectives, Z_1,Z_2,…, Z_K. Each source s_i has available supply, a_i,$i = 1, 2, \ldots, m$, and the destination d_j, ($j = 1, 2, \ldots, n$), bears a certain level of demand. Additionally, transporting a unit from source s_i to destination d_j obliges a certain penalty c_{ij}^k for the objective Z^k. The associated penalty may be shipping cost or time of delivering or delivery safety, etc. The solution of this problem is how to find an unknown quantity of goods, x_{ij} which were shipped from source s_i to destination d_j under the optimization aspect. With this description, the mathematical formulation of the MO-TP can be written as follows:

$$\begin{aligned}
&\mathrm{P}_1: \quad \operatorname{Min} Z^k(x_{ij}) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{(k)} x_{ij}, \quad k = 1, 2, \ldots, K,\\
&\quad \text{Subject to}\\
&\quad \sum_{j=1}^{n} x_{ij} = a_i, \qquad \forall i = 1, 2, \ldots, m,\\
&\quad \sum_{i=1}^{m} x_{ij} = b_j, \qquad \forall j = 1, 2, \ldots, n,\\
&\quad x_{ij} \geq 0, \qquad \forall i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n.
\end{aligned} \tag{1}$$

Where $\mathbf{Z(x)} = \{Z^1(x), Z^2(x), \ldots, Z^K(x)\}$ is a vector of objective functions. Without loss of generality, it is usual to assume that $a_i > 0\ \forall i,\ b_j > 0\ \forall j,\ c_{ij}^{(k)} \geq 0\ \forall (i, j)$ and $\sum_i a_i = \sum_j b_j$. In this respect, several methods have been developed to find the solution of the MO-TP, and they are divided into two categories based on the generated solutions [7,10]. First, some methods have been proposed to generate a set of efficient solutions. The second one is proposed based on exploring the preferred solution or the best compromise solution among the efficient set of solutions [7]. Therefore, the practical applications affirm that the existences of the many efficient sets of solutions make the decision maker fall into the hesitation, so most of the decision maker ever searches for the compromise solution, especially for complicated tasks. To deal with MO-TP, some definitions regarding optimality are defined.

2.2. Marginal evaluation

For a certain objective Z_k of the MO-TP (1) ($k=1, 2, \dots, K$), two values, U_k and L_k ($U_k > L_k$), can be identified as the upper and lower bounds of Z_k. That is U_k can be viewed as the highest acceptable level and L_k can be viewed as an aspired level of achievement for objective Z_k, respectively. With the two levels, U_k and L_k we can get the marginal evaluation of every objective Z_k. By considering a marginal evaluation for the objective Z_k, we defined a mapping $\mu_k : X \to [0, 1]$ that can tell us to what degree the decision $x \in X$ makes the objective Z_k close to its aspiration level L_k. In addition, $\mu_k(x) \in [0, 1]$ is defined as the degree of compatibility between the value $Z_k(x)$ and aspiration level L_k of the objective Z_k . Regarding to the definition of fuzzy sets, μ_k is just a fuzzy subset describing the fuzzy concept of “optimum” for objective Z_k on feasible solution space X, and $\mu_k(x) \in [0, 1]$ is the membership degree of $x \in X$ to which x is compatible with the “optimal solution” considering only the objective Z_k. By specifying the highest acceptable level U_k and the aspired level L_k, the following form can be utilized to obtain the marginal evaluation mapping $\mu_k : X \to [0, 1]$ for the objective Z_k, ($k=1, 2, \dots, K$).

$$\mu_k(X) = \begin{cases} 1 & \text{if } Z_k(x) > U_k \\ 1 - \dfrac{U_k - Z_k(x)}{U_k - L_k} & \text{if } L_k \le Z_k(x) \le U_k \\ 0 & \text{if } Z_k(x) < L_k \end{cases} \tag{2}$$

2.3. Membership function concept

The membership function is the significant part of the fuzzy approach (FA). It enables an FA to evaluate ambiguous and uncertain matters. The primary goal of the membership function is to act as a personal and subjective human perception as a member of a fuzzy set. Choosing an appropriate membership function is a critical step for building an accurate fuzzy set application. Often, a linear membership function is utilized to avoid non-linearity. Nevertheless, there are some obstacles in selecting the solution to a problem stated in a linear membership function. The fuzzy set theory is employed to solve the problem by assigning, for each kth objective, two values U_k and L_k as upper and lower bounds respectively, where L_k is the aspired level of achievement of the kth objective, U_k is the highest acceptable level of achievement of kth objective, and $d_k = U_k - L_k$ is the degradation allowance for the kth objective.

2.4. Definitions of solution concept

In this subsection, some definitions regarding the solution concept of the MO-TP are introduced.

Definition 1 (Efficient solution [9]): any feasible solution $X^* = \{x^*_{ij}\} \in \Omega$ (i.e., Ω is the feasible set of solutions) is said to be efficient (non-dominated or Pareto) solution of P_1 if there is no other feasible solution $X = \{x_{ij}\} \in \Omega$ such that

$$\begin{gathered} \sum_{i=1}^{m}\sum_{j=1}^{n} C^k_{ij} x_{ij} \le \sum_{i=1}^{m}\sum_{j=1}^{n} C^k_{ij} x^*_{ij} \qquad \forall k,\ k = 1, 2, \dots, K \\ \text{and} \\ \sum_{i=1}^{m}\sum_{j=1}^{n} C^k_{ij} x_{ij} < \sum_{i=1}^{m}\sum_{j=1}^{n} C^k_{ij} x^*_{ij} \qquad \text{for at least one } k,\ k = 1, 2, \dots, K \end{gathered} \tag{3}$$

Definition 2 (Compromise solution [9]): A feasible solution $X^* = \{x^*_{ij}\} \in \Omega$ is called a preferred or compromise solution for the problem P_1 iff $X^* \in E$ (E is the efficient set of solutions) and $Z(X^*) \le \wedge_{X \in \Omega} Z(X)$, where $\wedge$ stands for the minimum.

Definition 3 (Ideal solution). The ideal solution of the P_1 is the point $\mathbf{Z_0}^+ = (Z^1_0, Z^2_0, \dots, Z^K_0)$ such that $Z^k_0 = \mathrm{Min}_{X\in\Omega} \sum_{i=1}^{m}\sum_{j=1}^{n} c^{(k)}_{ij} x_{ij},\ k = 1, 2, \dots, K$.

Definition 4 (Anti-Ideal solution). The anti-ideal solution of the P_1 is the point $\mathbf{Z}^-_* = (Z^1_*, Z^2_*, \dots, Z^K_*)$ such that $Z^k_* = \mathrm{Max}_{X\in\Omega} \sum_{i=1}^{m}\sum_{j=1}^{n} c^{(k)}_{ij} x_{ij},\ k = 1, 2, \dots, K$.

Definition 5 (Neutrosophic set [23]): Assume a set Y be a space of points (objects) and $y \in Y$. A neutrosophic set Sin Y is characterized by three memberships namely, truth $T_S(y)$, indeterminacy $I_S(y)$ and a falsity $F_S(y)$ and it is denoted by the following form:

$$S = \{< y, T_S(y), I_S(y), F_S(y) > \ | y \in Y\} \tag{4}$$

Where $T_S(y)$, $I_S(y)$ and $F_S(y)$ are real standard or non-standard subsets belong to $]0^-,\ 1^+[$, that is, $T_S(y)$: $Y \to]0^-,\ 1^+[$, $I_S(y)$: $Y \to]0^-,\ 1^+[$, and $F_S(y)$: $Y \to]0^-,\ 1^+[$.

There is no restriction on the sum of $T_S(y)$, $I_S(y)$ and $F_S(y)$ so that $0^- \le \sup(T_S(y)) + \sup(I_S(y)) + \sup(F_S(y)) \le 3^+$.

3. Neutrosophic compromise programming for MO-TP

In this section, a new approach based on neutrosophic set to solve MOTP is presented. The proposed approach has been inspired by extending Zimmermann's concepts [25] in neutrosophic aspect. The proposed neutrosophic compromise programming approach (NCPA) introduces a new insight in handling the indeterminacy occupied into the optimization problems, with the aim to simultaneously maximizes the degrees of truth (satisfaction), falsity (dissatisfaction) and minimizes the degree of indeterminacy (satisfaction to some extent) of a neutrosophic decision. Bellman and Zadeh [6] have been introduced three concepts for the fuzzy set: fuzzy decision (D) fuzzy goal (G) and fuzzy constraints (C), and implemented these concepts in many applications of decision-making under fuzziness. The fuzzy decision is defined as follows:

$$D = G \cap C \tag{5}$$

Accordingly, the neutrosophic decision set D_N, a conjunction of neutrosophic objectives and constraints, is defined

$$D_N = \left(\bigcap_{k=1}^{K} G_k\right)\left(\bigcap_{i=1}^{m} C_i\right) = (x, T_D(x), I_D(x), F_D(x)) \tag{6}$$

where

$$T_D(x) = \min\left\{\begin{matrix} T_{G_1}(x), T_{G_2}(x), \dots, T_{G_K}(x); \\ T_{C_1}(x), T_{C_2}(x), \dots, T_{C_m}(x) \end{matrix}\right\} \text{ for all } x \in X \tag{7}$$

$$I_D(x) = \min\left\{\begin{matrix} I_{G_1}(x), I_{G_2}(x), \dots, I_{G_K}(x); \\ I_{C_1}(x), I_{C_2}(x), \dots, I_{C_m}(x) \end{matrix}\right\} \text{ for all } x \in X \tag{8}$$

$$F_D(x) = \max\left\{\begin{matrix} F_{G_1}(x), F_{G_2}(x), \dots, F_{G_K}(x); \\ F_{C_1}(x), F_{C_2}(x), \dots, F_{C_m}(x) \end{matrix}\right\} \text{ for all } x \in X \tag{9}$$

where $T_D(x)$ is the truth-membership function, $I_D(x)$ is the indeterminacy membership function, and $F_D(x)$ is falsity membership function of neutrosophic decision set D_N.

To formulate the membership functions for the MO-TP, the bounds for each objective function is determined. The lower and upper for each objective are denoted by L_k and U_k that is calculated as follows: each objective is optimized as a single objective subjected to the problem constraints. By solving K objectives individually, we obtain K solutions, $X^1, X^2, \dots, X^K$. Afterward, these solutions are substituted in each objective to explore the bounds for each objective as follows: $U_k = \max\{F_k(x)\}_{k=1}^{K}$ and $L_k = \min\{F_k(x)\}_{k=1}^{K}$.

Then the bounds for the neutrosophic environment are calculated as flows:

$$U_k^T = U_k, \quad L_k^T = L_k \qquad \text{for truth membership}$$

$$U_k^F = U_k^T, \quad L_k^F = L_k^T + t_k\left(U_k^T - L_k^T\right) \qquad \text{for falsity membership}$$

$$U_k^I = L_k^T + s_k\left(U_k^T - L_k^T\right), \quad L_k^I = L_k^T \qquad \text{for indeterminacy membership}$$

where t_k,s_k are predetermined real numbers in (0,1).

According to the above bounds, the membership functions can be introduced as follows:

$$T_k(Z_k(x)) = \begin{cases} 1 & \text{if } Z_k(x) < L_k^T \\ 1 - \dfrac{Z_k(x) - L_k^T}{U_k^T - L_k^T} & \text{if } L_k^T \le Z_k(x) \le U_k^T \\ 0 & \text{if } Z_k(x) > U_k^T \end{cases} \tag{10}$$

$$I_k(Z_k(x)) = \begin{cases} 1 & \text{if } Z_k(x) < L_k^I \\ 1 - \dfrac{Z_k(x) - L_k^I}{U_k^I - L_k^I} & \text{if } L_k^I \le Z_k(x) \le U_k^I \\ 0 & \text{if } Z_k(x) > U_k^I \end{cases} \tag{11}$$

$$F_k(Z_k(x)) = \begin{cases} 1 & \text{if } Z_k(x) > U_k^F \\ 1 - \dfrac{U_k^F - Z_k(x)}{U_k^F - L_k^F} & \text{if } L_k^F \le Z_k(x) \le U_k^F \\ 0 & \text{if } Z_k(x) < L_k^F \end{cases} \tag{12}$$

Where $U_k^{(.)} \neq L_k^{(.)}$ for all objectives. If $U_k^{(.)} = L_k^{(.)}$ for any membership, then the value of this membership is set to one. Following the Bellman and Zadeh [6], the neutrosophic optimization model of MO-TP (P_2) can be stated as follows:

$$\begin{aligned}
P_2 :\ & Max\ \min_{k=1,2,\ldots,K}\ T_k(Z_k(x)) \\
& Min\ \max_{k=1,2,\ldots,K}\ F_k(Z_k(x)) \\
& Max\ \min_{k=1,2,\ldots,K}\ I_k(Z_k(x)) \\
& S.t. \\
& \sum_{j=1}^{n} x_{ij} = a_i, \qquad i = 1, 2, \ldots, m, \\
& \sum_{i=1}^{m} x_{ij} = b_j, \qquad j = 1, 2, \ldots, n, \\
& x_{ij} \geq 0\ ,\ i = 1, 2, \ldots, m,\ \ j = 1, 2, \ldots, n.
\end{aligned} \tag{13}$$

By using auxiliary parameters, problem P_2 can be transformed to the following form (P_3):

$$\begin{aligned}
P_3 :\ & Max\ \ \alpha \\
& Max\ \ \gamma \\
& Min\ \ \beta \\
& S.t. \\
& T_{Z_k}(x) \geq \alpha\ ,\ \ I_{Z_k}(x) \geq \gamma\ ,\ \ F_{Z_k}(x) \leq \beta\ , \\
& \sum_{j=1}^{n} x_{ij} = a_i, \qquad i = 1, 2, \ldots, m, \\
& \sum_{i=1}^{m} x_{ij} = b_j, \qquad j = 1, 2, \ldots, n, \\
& x_{ij} \geq 0\ ,\ i = 1, 2, \ldots, m,\ \ j = 1, 2, \ldots, n. \\
& \alpha \geq \gamma\ , \alpha \geq \beta,\ \ \alpha + \gamma + \beta \leq 3,\ \ \alpha, \gamma, \beta \in [0, 1],
\end{aligned} \tag{14}$$

The simplified model of MO-TP can be represented as follows:

$$\begin{aligned}
P_4 :\ & Max\ \ \alpha - \beta + \gamma \\
& S.t. \\
& \sum_{j=1}^{n} x_{ij} = a_i, \qquad i = 1, 2, \ldots, m, \\
& \sum_{i=1}^{m} x_{ij} = b_j, \qquad j = 1, 2, \ldots, n, \\
& x_{ij} \geq 0\ ,\ i = 1, 2, \ldots, m,\ \ j = 1, 2, \ldots, n. \\
& Z_k(x) + (U_k^T - L_k^T)\alpha \leq U_k^T\ , \\
& Z_k(x) + (U_k^I - L_k^I)\gamma \leq U_k^I\ , \\
& Z_k(x) - (U_k^F - L_k^F)\beta \leq L_k^F\ , \\
& \alpha \geq \gamma\ , \alpha \geq \beta,\ \ \alpha + \gamma + \beta \leq 3, \\
& \alpha, \gamma, \beta \in [0, 1],\ k = 1, 2, \ldots, K
\end{aligned} \tag{15}$$

The procedures using the neutrosophic optimization approach for solving MO-TP can be summarized as follows:

Step 1. Solve each objective function individually as a single objective transportation problem (SOTP) subject to the constraints. Let $X^1, X^2, \ldots, X^K$ denote the respective ideal solutions for K different objective ($Z_*^k(x),\ k = 1, 2, \ldots, K$) transportation problems. It is supposed that at least two ideal solutions out of the set of all ideal solutions are different and has the different bound values. If all K objectives have the same solutions, $X^1 = X^2 = \ldots = X^K = \{x_{ij}\}_{i,j=1}^{m,n}$ choose one of them as the optimal compromise solution and go to step 6. Otherwise, go to step 2.

Step 2. For the obtained K solutions evaluate all the K objectives functions and then compute the lower and upper bounds for all objectives, $U_k = \max\{F_k(x)\}_{k=1}^{K}$ and $L_k = \min\{F_k(x)\}_{k=1}^{K}$.

Step 3. Define all membership functions using Eqs. (10),(11) and (12).
Step 4. Construct the neutrosophic programming model (P_2) and then formulate its equivalent model (P_4) as in Eq. (15).
Step 5. Solve neutrosophic model using TORA package to obtain the best compromise solution then determine the values of the K objective functions
Step 6. Step 6: Stop.
Step 7. To cover a wide spectrum of compromise solution ask the DM for specified values for t_k and s_k.
Step 8. Use the TOPSIS approach to rank the solutions by different approaches.

3.1. Stage 2: ranking the solutions

In this subsection, the Technique for Order Performance by Similarity to Ideal Solution (TOPSIS) [17] is employed to rank all the compromise solutions with the aim to obtain the best compromise solution among different methods. It is developed based on minimizing the distance from an ideal solution and maximizing the distance from the ant-ideal point solution simultaneously.

(1) Record the compromise solutions Z_{ij} from M methods over K objectives, $i=1, 2, \dots, M$, $j=1, 2, \dots, K$.
(2) Compute the normalized rating r_{ij} for element Z_{ij}.
(3) Ask the DM for the weight w_j that associated with each objective to obtain the weighted matrix $v_{ij}=w_j\, r_{ij}$.
(4) Determine the ideal solution (best performance on each objective) S^+.

$$S^+ = \{v_1^+, v_2^+, .., v_j^+, .., v_K^+\} = \left\{\left(\max \mathrm{v_{ij}} | j \in J_1\right), \left(\min \mathrm{v_{ij}} | j \in J_2\right), i = 1, ..., n\right\} \tag{16}$$

Where J_1 is the benefit values and J_2 is the cost attributes.

(5) Compute the anti-ideal solution (worse performance on each objective) S^-.

$$S^- = \{v_1^-, v_2^-, .., v_j^-, ..v_K^-\} = \left\{\left(\min \mathrm{v_{ij}} | j \in J_1\right), \left(\max \mathrm{v_{ij}} | j \in J_2\right), i = 1, ..., n\right\} \tag{17}$$

(6) Calculate the distances to ideal (D_i^+) and anti-ideal (D_i^-) respectively.

$$D_i^+ = \sqrt{\sum_{j=1}^{K} (v_{ij} - v_j^+)^2} \text{ and } D_i^- = \sqrt{\sum_{j=1}^{K} (v_{ij} - v_j^-)^2} \tag{18}$$

(7) Determine the relative closeness to the ideal solution and the farthest to an anti-ideal solution using the following ratio and observe the compromise solution with the maximum ratio.

$$R = \frac{D_i^-}{D_i^- + D_i^+} \tag{19}$$

Due to due to existence of different methodologies for solving multi-objective transportation problem, ranking and comparing these methods become a new challenge. Thus, it is needed to present a tool to aid in ranking and selecting the best method. Therefore, TOPSIS is introduced to rank the performances among different methods in terms of their optimal solutions, where the alternatives consist of the optimal solutions and the criteria are the objective functions.

4. Illustrative examples

To demonstrate the neutrosophic optimization procedures, two illustrative examples two examples of MO-TP are considered [7].

Example 1. Let us consider a MO-TP with the following characteristics:

Supplies: $a_1=8$, $a_2=19$, $a_3=17$.
Demand: $b_1=11$, $b_2=3$, $b_3=14$, $b_4=16$.
Penalties:

$$C_1 = \begin{bmatrix} 1 & 2 & 7 & 7 \\ 1 & 9 & 3 & 4 \\ 8 & 9 & 4 & 6 \end{bmatrix}, \; C_2 = \begin{bmatrix} 4 & 4 & 3 & 4 \\ 5 & 8 & 9 & 10 \\ 6 & 2 & 5 & 1 \end{bmatrix}$$

Step 1: The MO-TP model

$$Min\ Z_1 = x_{11} + 2x_{12} + 7x_{13} + 7x_{14} + x_{21} + 9x_{22} + 3x_{23} + 4x_{24} + 8x_{31} + 9x_{32} + 4x_{33} + 6x_{34}$$

$$Min\ Z_2 = 4x_{11} + 4x_{12} + 3x_{13} + 3x_{14} + 5x_{21} + 8x_{22} + 9x_{23} + 10x_{24} + 6x_{31} + 2x_{32} + 5x_{33} + x_{34}$$

Subject to:

$$x_{11} + x_{12} + x_{13} + x_{14} = 8$$
$$x_{21} + x_{22} + x_{23} + x_{24} = 19$$
$$x_{31} + x_{32} + x_{33} + x_{34} = 17$$
$$x_{11} + x_{21} + x_{31} = 11$$
$$x_{12} + x_{22} + x_{32} = 3$$
$$x_{13} + x_{23} + x_{33} = 14$$
$$x_{14} + x_{24} + x_{34} = 16$$
$$x_{ij} \geq 0 \text{ and integer } \forall i, j.$$

Step 2: Solve the above model as single objective to find the individual solution as $X_1 = (5,\ 3,\ 0,\ 0,\ 6,\ 0,\ 0,\ 13,\ 0,\ 0,\ 14,\ 3)$ and $X_2 = (0,\ 0,\ 8,\ 0,\ 11,\ 2,\ 6,\ 0,\ 0,\ 1,\ 0,\ 16)$.

Step 3: Compute the objectives using the obtained solutions and then determine the bounds for each objective: $Z_1(X_1) = 143$, $Z_1(X_2) = 208$, $Z_2(X_1) = 167$, $Z_2(X_2) = 265$, i.e., $143 \leq Z_1 \leq 208$ and $167 \leq Z_2 \leq 265$.

Step 4: Formulate the membership functions based on neutrosophic terminology for the two objectives:

For Z_1**:**

$$U_{Z_1}^T = 208, \quad L_{Z_1}^T = 143$$
$$U_{Z_1}^F = U_{Z_1}^T = 208, \quad L_{Z_1}^F = L_{Z_1}^T + t_1 = 143 + t_1$$
$$U_{Z_1}^I = L_{Z_1}^T + s_1 = 143 + s_1, \quad L_{Z_1}^I = L_{Z_1}^T = 143$$

$$T_1(Z_1(x)) = \begin{cases} 1 & if \quad Z_1(x) < 143 \\ 1 - \dfrac{Z_1(x) - 143}{208 - 143} & if \quad 143 \leq Z_1(x) \leq 208 \\ 0 & if \quad Z_1(x) > 208 \end{cases}$$

$$I_1(Z_1(x)) = \begin{cases} 1 & if \quad Z_1(x) < 143 \\ 1 - \dfrac{Z_1(x) - 143}{s_1} & if \quad 143 \leq Z_1(x) \leq 143 + s_1 \\ 0 & if \quad Z_1(x) > 143 + s_1 \end{cases}$$

$$F_1(Z_1(x)) = \begin{cases} 1 & if \quad Z_1(x) > 208 \\ 1 - \dfrac{208 - Z_1(x)}{208 - 143 - t_1} & if \quad 143 + t_1 \leq Z_1(x) \leq 208 \\ 0 & if \quad Z_1(x) < 143 + t_1 \end{cases}$$

For Z_2**:**

$$U_{Z_2}^T = 265, \quad L_{Z_2}^T = 167$$
$$U_{Z_2}^F = U_{Z_2}^T = 265, \quad L_{Z_2}^F = L_{Z_2}^T + t_2 = 167 + t_2$$
$$U_{Z_2}^I = L_{Z_2}^T + s_2 = 167 + s_2, \quad L_{Z_2}^I = L_{Z_2}^T = 167$$

$$T_2(Z_2(x)) = \begin{cases} 1 & if \quad Z_2(x) < 167 \\ 1 - \dfrac{Z_2(x) - 167}{265 - 167} & if \quad 167 \leq Z_2(x) \leq 265 \\ 0 & if \quad Z_2(x) > 265 \end{cases}$$

$$I_2(Z_2(x)) = \begin{cases} 1 & if \quad Z_2(x) < 167 \\ 1 - \dfrac{Z_2(x) - 167}{s_1} & if \quad 167 \le Z_2(x) \le 167 + s_1 \\ 0 & if \quad Z_2(x) > 167 + s_1 \end{cases}$$

$$F_2(Z_2(x)) = \begin{cases} 1 & if \quad Z_2(x) > 265 \\ 1 - \dfrac{265 - Z_2(x)}{265 - 167 - t_1} & if \quad 167 + t_1 \le Z_2(x) \le 265 \\ 0 & if \quad Z_2(x) < 143 + t_1 \end{cases}$$

Step 5: Constitute the neutrosophic model of MO-TP:

$$\begin{aligned}
&Max\ \alpha - \beta + \gamma \\
&Subject\ to: \\
&\quad x_{11} + x_{12} + x_{13} + x_{14} = 8 \\
&\quad x_{21} + x_{22} + x_{23} + x_{24} = 19 \\
&\quad x_{31} + x_{32} + x_{33} + x_{34} = 17 \\
&\quad x_{11} + x_{21} + x_{31} = 11 \\
&\quad x_{12} + x_{22} + x_{32} = 3 \\
&\quad x_{13} + x_{23} + x_{33} = 14 \\
&\quad x_{14} + x_{24} + x_{34} = 16 \\
&\quad x_{11} + 2x_{12} + 7x_{13} + 7x_{14} + x_{21} + 9x_{22} + 3x_{23} + 4x_{24} + 8x_{31} + 9x_{32} + 4x_{33} + 6x_{34} + 65\alpha \le 208 \\
&\quad 4x_{11} + 4x_{12} + 3x_{13} + 3x_{14} + 5x_{21} + 8x_{22} + 9x_{23} + 10x_{24} + 6x_{31} + 2x_{32} + 5x_{33} + x_{34} + 98\alpha \le 265 \\
&\quad x_{11} + 2x_{12} + 7x_{13} + 7x_{14} + x_{21} + 9x_{22} + 3x_{23} + 4x_{24} + 8x_{31} + 9x_{32} + 4x_{33} + 6x_{34} + t_1\gamma - t_1 \le 143 \\
&\quad 4x_{11} + 4x_{12} + 3x_{13} + 3x_{14} + 5x_{21} + 8x_{22} + 9x_{23} + 10x_{24} + 6x_{31} + 2x_{32} + 5x_{33} + x_{34} + t_2\gamma - t_2 \le 167 \\
&\quad x_{11}+2x_{12}+7x_{13}+7x_{14}+x_{21}+9x_{22}+3x_{23}+4x_{24}+8x_{31}+9x_{32}+4x_{33} + 6x_{34} + (\beta - 1)(143 + s_1) - 208\beta \le 0 \\
&\quad 4x_{11}+4x_{12}+3x_{13}+3x_{14}+5x_{21}+8x_{22}+9x_{23} + 10x_{24}+6x_{31}+2x_{32}+5x_{33}+x_{34}+(\beta - 1)(167+s_2)-265\beta \le 0 \\
&\quad \alpha \ge \gamma\ , \alpha \ge \beta, \ \ \alpha + \gamma + \beta \le 3, \ \ \alpha, \gamma, \beta \in [0, 1], \ \ 0 \le t_1, s_1, \le 65, \ \ 0 \le t_2, s_2 \le 98, \\
&\quad x_{ij} \ge 0 \text{ and integer } \forall i, j.
\end{aligned}$$

The above model can be solved using the TORA software for integer values of x_{ij}

$$\begin{aligned}
&x_{11} = 4,\ x_{12} = 3,\ x_{13} = 1,\ x_{14} = 0,\ x_{21} = 7,\ x_{22} = 0,\ x_{23} = 12,\ x_{24} = 0,\ x_{31} = 0, \\
&x_{32} = 0,\ x_{33} = 1,\ x_{34} = 16,\ t_1 = 65,\ t_2 = 100.34,\ s_1 = 18.50,\ s_2 = 50.62,\ \alpha = 0.714,\ \beta = 0,\ \gamma = 0.714. \\
&Z_1 = 160,\ \ Z_2 = 195.
\end{aligned}$$

Example 2. Let us employ another model of MO-TP with the following characteristics.

Supplies: $a_1 = 5$, $a_2 = 4$, $a_3 = 2$, $a_4 = 9$.
Demand: $b_1 = 4$, $b_2 = 4$, $b_3 = 6$, $b_4 = 2$, $b_5 = 4$.
Penalties:

$$C_1 = \begin{bmatrix} 9 & 12 & 9 & 6 & 9 \\ 7 & 3 & 7 & 7 & 5 \\ 6 & 5 & 9 & 11 & 3 \\ 6 & 8 & 11 & 2 & 2 \end{bmatrix} C_2 = \begin{bmatrix} 2 & 9 & 8 & 1 & 4 \\ 1 & 9 & 9 & 5 & 2 \\ 8 & 1 & 8 & 4 & 5 \\ 2 & 8 & 6 & 9 & 8 \end{bmatrix} C_3 = \begin{bmatrix} 2 & 4 & 6 & 3 & 6 \\ 4 & 8 & 4 & 9 & 2 \\ 5 & 3 & 5 & 3 & 6 \\ 6 & 9 & 6 & 3 & 1 \end{bmatrix}$$

This model optimizes three objectives simultaneously.

Step 1: The MO-TP model

$$
\begin{aligned}
&Min\ Z_1 = 9x_{11} + 12x_{12} + 9x_{13} + 6x_{14} + 9x_{15} + 7x_{21} + 3x_{22} + 7x_{23} + \\
&\qquad 7x_{24} + 5x_{25} + 6x_{31} + 5x_{32} + 9x_{33} + 6x_{34} + 3x_{35} + \\
&\qquad 6x_{41} + 8x_{42} + 11x_{43} + 2x_{44} + 2x_{45} \\
&Min\ Z_2 = 2x_{11} + 9x_{12} + 8x_{13} + x_{14} + 4x_{15} + x_{21} + 9x_{22} + 9x_{23} + \\
&\qquad 5x_{24} + 2x_{25} + 8x_{31} + x_{32} + 8x_{33} + 4x_{34} + 5x_{35} + \\
&\qquad 2x_{41} + 8x_{42} + 6x_{43} + 9x_{44} + 8x_{45} \\
&Min\ Z_3 = 2x_{11} + 4x_{12} + 6x_{13} + 3x_{14} + 6x_{15} + 4x_{21} + 8x_{22} + 4x_{23} + \\
&\qquad 9x_{24} + 2x_{25} + 5x_{31} + 3x_{32} + 5x_{33} + 3x_{34} + 6x_{35} + \\
&\qquad 6x_{41} + 9x_{42} + 6x_{43} + 3x_{44} + x_{45} \\
&Subject\ to: \\
&\quad x_{11} + x_{12} + x_{13} + x_{14} + x_{15} = 5 \\
&\quad x_{21} + x_{22} + x_{23} + x_{24} + x_{25} = 4 \\
&\quad x_{31} + x_{32} + x_{33} + x_{34} + x_{35} = 2 \\
&\quad x_{41} + x_{42} + x_{43} + x_{44} + x_{45} = 9 \\
&\quad x_{11} + x_{21} + x_{31} + x_{41} = 4 \\
&\quad x_{12} + x_{22} + x_{32} + x_{42} = 4 \\
&\quad x_{13} + x_{23} + x_{33} + x_{43} = 6 \\
&\quad x_{14} + x_{24} + x_{34} + x_{44} = 2 \\
&\quad x_{15} + x_{25} + x_{35} + x_{45} = 4 \\
&\quad x_{ij} \ge 0 \text{ and integer } \forall i, j.
\end{aligned}
$$

Step 2: Solve the above model as single objective to find the individual solution as $X_1 = (0, 0, 5, 0, 0, 0, 4, 0, 0, 0, 1, 0, 1, 0, 0, 3, 0, 0, 2, 4)$, $X_2 = (3, 0, 0, 2, 0, 0, 0, 0, 0, 4, 0, 2, 0, 0, 0, 1, 2, 6, 0, 0)$ and $X_3 = (3, 2, 0, 0, 0, 0, 0, 4, 0, 0, 0, 2, 0, 0, 0, 1, 0, 2, 2, 4)$.

Step 3: Compute the objectives using the obtained solutions and then determine the bounds for each objective: $Z(X_1) = (102, 148, 100)$, $Z(X_2) = (157, 72, 86)$ and $Z(X_3) = (129, 126, 64)$ i.e., $102 \le Z_1 \le 157$, $72 \le Z_2 \le 148$ and $64 \le Z_3 \le 100$.

Step 4: Formulate the membership functions based on neutrosophic terminology for the three objectives:

For Z_1:

$$
\begin{aligned}
&U_{Z_1}^T = 157, \quad L_{Z_1}^T = 102 \\
&U_{Z_1}^F = U_{Z_1}^T = 157, \quad L_{Z_1}^F = L_{Z_1}^T + t_1 = 102 + t_1 \\
&U_{Z_1}^I = L_{Z_1}^T + s_1 = 102 + s_1, \quad L_{Z_1}^I = L_{Z_1}^T = 102
\end{aligned}
$$

$$
T_1(Z_1(x)) = \begin{cases} 1 & if \quad Z_1(x) < 102 \\ 1 - \dfrac{Z_1(x) - 102}{157 - 102} & if \quad 102 \le Z_1(x) \le 157 \\ 0 & if \quad Z_1(x) > 157 \end{cases}
$$

$$
I_1(Z_1(x)) = \begin{cases} 1 & if \quad Z_1(x) < 102 \\ 1 - \dfrac{Z_1(x) - 102}{s_1} & if \quad 102 \le Z_1(x) \le 102 + s_1 \\ 0 & if \quad Z_1(x) > 102 + s_1 \end{cases}
$$

$$F_1(Z_1(x)) = \begin{cases} 1 & if \quad Z_1(x) > 157 \\ 1 - \dfrac{157 - Z_1(x)}{157 - 102 - t_1} & if \quad 102 + t_1 \le Z_1(x) \le 157 \\ 0 & if \quad Z_1(x) < 102 + t_1 \end{cases}$$

For Z_2:

$$U^T_{Z_2} = 148, \; L^T_{Z_2} = 72$$
$$U^F_{Z_2} = U^T_{Z_2} = 148, \; L^F_{Z_2} = L^T_{Z_2} + t_2 = 72 + t_2$$
$$U^I_{Z_2} = L^T_{Z_2} + s_2 = 72 + s_2, \; L^I_{Z_2} = L^T_{Z_2} = 72$$

$$T_2(Z_2(x)) = \begin{cases} 1 & if \quad Z_2(x) < 72 \\ 1 - \dfrac{Z_2(x) - 72}{148 - 72} & if \quad 72 \le Z_2(x) \le 148 \\ 0 & if \quad Z_2(x) > 148 \end{cases}$$

$$I_2(Z_2(x)) = \begin{cases} 1 & if \quad Z_2(x) < 72 \\ 1 - \dfrac{Z_2(x) - 72}{s_1} & if \quad 72 \le Z_2(x) \le 72 + s_1 \\ 0 & if \quad Z_2(x) > 72 + s_1 \end{cases}$$

$$F_2(Z_2(x)) = \begin{cases} 1 & if \quad Z_2(x) > 148 \\ 1 - \dfrac{148 - Z_2(x)}{148 - 72 - t_1} & if \quad 72 + t_1 \le Z_2(x) \le 148 \\ 0 & if \quad Z_2(x) < 72 + t_1 \end{cases}$$

For Z_3:

$$U^T_{Z_2} = 100, \; L^T_{Z_2} = 64$$
$$U^F_{Z_2} = U^T_{Z_2} = 100, \; L^F_{Z_2} = L^T_{Z_2} + t_2 = 64 + t_2$$
$$U^I_{Z_2} = L^T_{Z_2} + s_2 = 64 + s_2, \; L^I_{Z_2} = L^T_{Z_2} = 64$$

$$T_3(Z_3(x)) = \begin{cases} 1 & if \quad Z_3(x) < 64 \\ 1 - \dfrac{Z_3(x) - 64}{100 - 64} & if \quad 64 \le Z_3(x) \le 100 \\ 0 & if \quad Z_3(x) > 100 \end{cases}$$

$$I_3(Z_3(x)) = \begin{cases} 1 & if \quad Z_3(x) < 64 \\ 1 - \dfrac{Z_3(x) - 64}{s_1} & if \quad 64 \le Z_3(x) \le 64 + s_1 \\ 0 & if \quad Z_3(x) > 64 + s_1 \end{cases}$$

$$F_3(Z_3(x)) = \begin{cases} 1 & if \quad Z_3(x) > 100 \\ 1 - \dfrac{100 - Z_3(x)}{100 - 64 - t_1} & if \quad 64 + t_1 \le Z_3(x) \le 100 \\ 0 & if \quad Z_3(x) < 64 + t_1 \end{cases}$$

Step 5: Constitute the neutrosophic model of MO-TP:

$$
\begin{aligned}
& Max\ \ \alpha-\beta+\gamma \\
& Min\ Z_2 = 2x_{11}+9x_{12}+8x_{13}+x_{14}\ +4x_{15}\ +x_{21}+9x_{22}+9x_{23}+ \\
& \qquad 5x_{24}+\ 2x_{25}+\ 8x_{31}+x_{32}+8x_{33}+4x_{34}+5x_{35}+ \\
& \qquad 2x_{41}+\ 8x_{42}+\ 6x_{43}+9x_{44}+8x_{45} \\
& Min\ Z_3 = 2x_{11}+4x_{12}+6x_{13}+3x_{14}\ +6x_{15}\ +4x_{21}+8x_{22}+4x_{23}+ \\
& \qquad 9x_{24}+\ 2x_{25}+\ 5x_{31}+3x_{32}+5x_{33}+3x_{34}+6x_{35}+ \\
& \qquad 6x_{41}+\ 9x_{42}+\ 6x_{43}+3x_{44}+x_{45} \\
& Subject\ to: \\
& \quad x_{11}+x_{12}+x_{13}+x_{14}+x_{15}=5 \\
& \quad x_{21}+x_{22}+x_{23}+x_{24}+x_{25}=4 \\
& \quad x_{31}+x_{32}+x_{33}+x_{34}+x_{35}=2 \\
& \quad x_{41}+x_{42}+x_{43}+x_{44}+x_{45}=9 \\
& \quad x_{11}+x_{21}+x_{31}+x_{41}=4 \\
& \quad x_{12}+x_{22}+x_{32}+x_{42}=4 \\
& \quad x_{13}+x_{23}+x_{33}+x_{43}=6 \\
& \quad x_{14}+x_{24}+x_{34}+x_{44}=2 \\
& \quad x_{15}+x_{25}+x_{35}+x_{45}=4 \\
& \quad x_{ij}\geq 0 \text{ and integer } \forall i,\ j. \\
& \quad 9x_{11}+12x_{12}+9x_{13}+6x_{14}\ +9x_{15}\ +7x_{21}+3x_{22}+7x_{23}+\ 7x_{24}+\ 5x_{25}+\ 6x_{31}+5x_{32}+9x_{33}+6x_{34}+3x_{35}+ \\
& \quad 6x_{41}+\ 8x_{42}+\ 11x_{43}+2x_{44}+2x_{45}+55\alpha\leq 157 \\
& \quad 2x_{11}+9x_{12}+8x_{13}+x_{14}\ +4x_{15}\ +x_{21}+9x_{22}+9x_{23}+\ 5x_{24}+\ 2x_{25}+\ 8x_{31}+x_{32}+8x_{33}+4x_{34}+5x_{35}+ \\
& \quad 2x_{41}+\ 8x_{42}+\ 6x_{43}+9x_{44}+8x_{45}+76\alpha\leq 148 \\
& \quad 2x_{11}+4x_{12}+6x_{13}+3x_{14}\ +6x_{15}\ +4x_{21}+8x_{22}+4x_{23}+\ 9x_{24}+\ 2x_{25}+\ 5x_{31}+3x_{32}+5x_{33}+3x_{34}+6x_{35}+ \\
& \quad 6x_{41}+\ 9x_{42}+\ 6x_{43}+3x_{44}+x_{45}+36\alpha\leq 100 \\
& \quad 9x_{11}+12x_{12}+9x_{13}+6x_{14}\ +9x_{15}\ +7x_{21}+3x_{22}+7x_{23}+\ 7x_{24}+\ 5x_{25}+\ 6x_{31}+5x_{32}+9x_{33}+6x_{34}+3x_{35}+ \\
& \quad 6x_{41}+\ 8x_{42}+\ 11x_{43}+2x_{44}+2x_{45}+t_1\gamma-t_1\leq 102 \\
& \quad 2x_{11}+9x_{12}+8x_{13}+x_{14}\ +4x_{15}\ +x_{21}+9x_{22}+9x_{23}+\ 5x_{24}+\ 2x_{25}+\ 8x_{31}+x_{32}+8x_{33}+4x_{34}+5x_{35}+ \\
& \quad 2x_{41}+\ 8x_{42}+\ 6x_{43}+9x_{44}+8x_{45}+t_2\gamma-t_2\leq 72 \\
& \quad 2x_{11}+4x_{12}+6x_{13}+3x_{14}\ +6x_{15}\ +4x_{21}+8x_{22}+4x_{23}+\ 9x_{24}+\ 2x_{25}+\ 5x_{31}+3x_{32}+5x_{33}+3x_{34}+6x_{35}+ \\
& \quad 6x_{41}+\ 9x_{42}+\ 6x_{43}+3x_{44}+x_{45}\ +t_3\gamma-t_3\leq 64 \\
& \quad 9x_{11}+12x_{12}+9x_{13}+6x_{14}\ +9x_{15}\ +7x_{21}+3x_{22}+7x_{23}+\ 7x_{24}+\ 5x_{25}+\ 6x_{31}+5x_{32}+9x_{33}+6x_{34}+3x_{35}+ \\
& \quad 6x_{41}+\ 8x_{42}+\ 11x_{43}+2x_{44}+2x_{45}+(\beta-1)(102+s_1)-157\beta\leq 0 \\
& \quad 2x_{11}+9x_{12}+8x_{13}+x_{14}\ +4x_{15}\ +x_{21}+9x_{22}+9x_{23}+\ 5x_{24}+\ 2x_{25}+\ 8x_{31}+x_{32}+8x_{33}+4x_{34}+5x_{35}+ \\
& \quad 2x_{41}+\ 8x_{42}+\ 6x_{43}+9x_{44}+8x_{45}+(\beta-1)(72+s_2)-148\beta\leq 0 \\
& \quad 2x_{11}+4x_{12}+6x_{13}+3x_{14}\ +6x_{15}\ +4x_{21}+8x_{22}+4x_{23}+\ 9x_{24}+\ 2x_{25}+\ 5x_{31}+3x_{32}+5x_{33}+3x_{34}+6x_{35}+ \\
& \quad 6x_{41}+\ 9x_{42}+\ 6x_{43}+3x_{44}+x_{45}+(\beta-1)(64+s_2)-100\beta\leq 0 \\
& \quad \alpha\geq\gamma\ ,\alpha\geq\beta,\ \ \alpha+\gamma+\beta\leq 3,\ \ \alpha,\gamma,\beta\in[0,1],\ \ 0\leq t_1,s_1\leq 55,\ \ 0\leq t_2,s_2\leq 76,\ 0\leq t_2,s_2\leq 36, \\
& \quad x_{ij}\geq 0 \text{ and integer } \forall i,\ j.
\end{aligned}
$$

Table 1
Comparisons between the proposed NCPA with other existing approaches for Example 1.

Methods	Z_1	Z_2	D_i^+	D_i^-	R	Rank
Ideal	143	167	0	2.39E-4	1	1
Anti-ideal	208	265	2.39E-4	0	0	7
Proposed NCPA	160	195	6.59E-05	1.73E-4	0.7241	**2**
Waiel [7]	170	190	7.68E-05	1.66E-4	0.6841	5
Ringuest and Rinks [8]	186	174	1.02E-4	1.78E-4	0.6350	6
FEIGP [10]	176	175	7.93E-05	1.84E-4	0.6994	4
Waiel [9]	168	185	6.79E-05	1.77E-4	0.7226	3
Nomani et al. [11]	176	175	7.93E-05	1.84E-4	0.6994	4
Murthy [12]	160	195	6.59E-05	1.73E-4	0.7241	**2**
Bander et al. [13]	176	175	7.93E-05	1.84E-4	0.6994	4

Table 2
Comparisons between the proposed NCPA with other existing approaches for Example 2.

Methods	Z_1	Z_2	Z_3	D_i^+	D_i^-	R	Rank
Ideal	102	72	64	0	2.90E-4	1	1
Anti-ideal	157	148	100	2.90E-4	0	0	7
Proposed NCPA	132	100	76	1.11E-4	1.82E-4	0.6207	**2**
Ringuest and Rinks [8]	127	104	76	1.15E-4	1.77E-4	0.6064	4
Waiel [7]	122	106	80	1.27E-4	1.64E-4	0.5638	6
Mousa et al. [17]	127	104	76	1.15E-4	1.77E-4	0.6064	4
Zaki et al. [18]	126.79	100	77.52	1.10E-4	1.79E-4	0.6184	3
FEIGP [10]	132	100	76	1.11E-4	1.82E-4	0.6207	**2**
Nomani et al. [11]	127	104	76	1.15E-4	1.77E-4	0.6064	4
Murthy [12]	122	106	80	1.27E-4	1.64E-4	0.5638	6
Zangiabadi et al. [14]	126.79	103.10	77.52	1.17E-4	1.74E-4	0.5979	5

The above model can be solved using the TORA software for integer values of x_{ij}

$$x_{11}=3,\ x_{12}=0,\ x_{13}=0,\ x_{14}=2, x_{15}=0,\ x_{21}=1,\ x_{22}=2,\ x_{23}=1,\ x_{24}=0,\ x_{25}=0,$$
$$x_{31}=0,\ x_{32}=2,\ x_{33}=0,\ x_{34}=0,\ x_{35}=0, x_{41}=0,\ x_{42}=0,\ x_{43}=5,\ x_{44}=0,\ x_{45}=4,$$
$$t_1=33.5,\ t_2=30,\ t_3=12,\ s_1=33.49,\ s_2=30,\ s_3=12,\ \alpha=0.454,\ \beta=0,\ \gamma=0.$$
$$Z_1=132,\ Z_2=100,\ Z_3=76.$$

5. Results and discussion

The proposed neutrosophic compromise programming approach (NCPA) is investigated using two numerical examples. The methodology begins by obtaining the ideal solution (individual minimum), and anti-ideal solution (individual maximum) and then the membership functions for truth, indeterminacy and falsity degrees are constructed. Afterwards, the neutrosophic model for MO-TP is formulated to obtain the best compromise solution. The obtained solution is compared solutions with other existing approaches as in Tables 1 and 2 for Examples 1and 2 respectively. According to Table 1 and Table 2, the results clarify that the NCPA has a better solution than the existing approaches for both examples concerning the ratio (R) that obtained by TOPSIS approach. Although the obtained solution by NCPA is identical to FEIGP [10], the proposed NCPA can save the computations, where FEIGP has been employed interactive procedures that have been skunked the border through this strategy. Due to conflicting among the objectives obtaining any improvement in one objective will lead to sacrificing in the other objectives. So, we have compared the results based on the TOPSIS approach, which may be a best ranking method in a realistic sense. Accordingly, we can conclude that the NCPA proposed is superior to the other existing methods.

6. Conclusion

In many transportation situations, the well-known approaches such as probability theory or fuzzy set are not adequate to deal with these situations in which the indeterminacy inherently is involved. Under this circumstance, this paper presents a novel neutrosophic compromise programming approach (NCPA) to deal with the multi-objective transportation (MO-TP). This approach is characterized by simultaneously maximizing the degrees of truth (satisfaction), falsity (dissatisfaction) and minimizing the degree of indeterminacy (satisfaction to some extent) of a neutrosophic decision. The efficiency of the NCPA is investigated using TOPSIS approach, where the proposed NCPA gets the better outcomes regarding the ranking of values in respect of TOPSIS approach. This result demonstrates the superiority of the results on existing techniques. Although the merits of the neutrosophic set in addressing uncertainty it needs to be specified from a technical point of view, especially when dealing with real applications. Future research can be concentrated on developing generalized framework of neutrosophic set that is suitable for different practical problems such as nonlinear programming, fractional programming, and multilevel programming problems.

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Rizk Masoud Rizk-Allah received his Ph.D. in Engineering Mathematics from Menoufia University, Egypt. He is currently an assistant Professor at Menoufia University, Faculty of Engineering, Department of Basic Engineering Science. His research interests include: nonlinear programming problems, many objectives optimization problems, fuzzy sets, metaheuristic algorithms, and Engineering applications.

Aboul Ella Hassanien is a Professor at Cairo University, Faculty of Computers & Information. He is the founder and Chair of the Scientific Research Group in Egypt (SRGE). He is the Ex-Dean of faculty of computers and Information, Beni-Suef University, Egypt.

Mohamed Elhoseny is as assistant professor at Faculty of Computers and Information, Mansoura University, Egypt. Elhoseny received his Ph.D. from Mansoura University Egypt (in a scientific research channel with Department of Computer Science and Engineering, University of North Texas, USA). Collectively, Elhoseny authored/co-authored over 50 International Journal articles, Conference Proceedings, Book Chapters, and 2 Springer brief books.