A New Method for Solving Interval Neutrosophic Linear Programming Problems

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January 9, 2020

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Abstract: Because of uncertainty in the real-world problems, achieving to the optimal solution is always time consuming and even sometimes impossible.In order to overcome this drawback the neutrosophic sets theory which is a generalization of the fuzzy sets theory is presented that can handle not only incomplete information but also indeterminate and inconsistent information which is common in real-world situations. By considering these conditions in this paper for the first time an interval neutrosophic linear programming model will be presented, where the parameters of the proposed model are represented by triangular interval-valued neutrosophic numbers and call it Interval Neutrosophic Linear Programming (INLP) problems. Furthermore by using a ranking function present a technique to convert every INLP problem into a crisp model and solve it by standard methods.

Keywords: Triangular Interval neutrosophic programming; neutrosophic; neutrosophic linear programming; neutrosophic set; interval- valued neutrosophic number.

1 Introduction

Linear programming is one of the most important usages of operation research methods in our real life [9] [11] [14] [18] that is inclusive of one objective function and one or several constraints which can be in the form of equality and inequality [2] [5] [8] [10]. Most of the problems in the real world are include of inconsistent and astute uncertainty, because of this reason it's not always possible to obtain the optimal solution easily. Uncertainty is an information deficit which is usually one of the inseparable components of real-world problems. The fuzzy sets theory proposed by Zadeh [22] is a practical approach to overcoming uncertainty so as such it assigns membership function to any non-deterministic event. In fuzzy sets, the membership degree of an element in [0,1] expresses the degree of belongingness of that element to a fuzzy set. Sometimes because of uncertainty determining the degree of membership isn't possible. For this reason, in1975 Zadeh

[23] proposed interval fuzzy sets to express the uncertainty in the membership function. An interval value fuzzy set is a fuzzy set where the membership degree is assumed to belong to an interval. In this respect, Attanasov in [3] by adding the degree of non-membership introduced another extension of fuzzy sets namely intuitionistic fuzzy sets. The degree of elements belonging to an intuitionistic fuzzy set are represented by the membership and the non-membership degrees in [0,1], respectively. In 1989 Atanasov and Gragov[4] proposed a generalization of the intuitionistic fuzzy sets called interval-valued intuitionistic fuzzy set where the membership value and the non-membership value of any of its elements are represented by an interval value of [0,1]. For dealing with the indeterminacy Smarandach in [15] [16] introduced the Neutrosophic Sets (NSs) as a new generalization of Intuitionistic Fuzzy Sets. This approach added the indeterminacy membership as an independent factor to the basis of intuitionistic fuzzy sets. A neutrosophic set N in X can be characterized by three membership functions such as truth $T_N(x)$, indeterminacy $I_N(x)$ and falsity $F_N(x)$, membership functions that are completely independent of each other. Recently, NSs have become an interesting research topic and attracted wide attention, since the NS is difficult to be directly used in our real-life applications. In [20] Wang et al proposed a neutrosophic set which is single-valued from a scientific and engineering point of view, as an instance of the neutrosophic set. In this respect, Wang et al. in [19] by considering the degrees of truth, indeterminacy, and falsity membership functions as a subinterval of $[0,1]$ introduced Interval Neutrosophic Sets. By using interval neutrosophic sets in addition to incomplete information we can handle inconsistent and indeterminate information which exist in our real life. This research, as the first time, presents a linear programming problem in a neutrosophic environment with triangular interval-valued neutrosophic numbers and call it Interval Neutrosophic Linear Programming(INLP) problem. As a fact, an INLP problem is a problem with one or more coefficients that are represented by interval neutrosophic numbers. In the same way by introducing a ranking function we will convert INLP problems to crisp problems and will solve it by standard methods. Because by using INLP problems there is no necessity to make a sentient formulation so INLP problems are more useful than crisp linear programming problems. The rest of this paper is marshaled as follows: Section 2 briefly outlines some definitions of single and interval value neutrosophic sets, operational rules between them and will formulate the neutrosophic and interval neutrosophic linear programming models, respectively. In section 3 we present our method for solving INLP problems. In section 4 two examples are presented to illustrate the proposed method. In section 5 we discuss about the efficiency and superiority of our method with respect to other existing methods and the conclusion are discussed in section 6.

2 Preliminaries

This section, presents new concepts and definitions of Interval-Valued

Neutrosophic set N on the real numbers line which is presented in this study and it will be used in throughout of the paper.

Definition1. [12] [16] A Single-Valued Neutrosophic(SVN) set N through X taking the form $N=\{x,T_N(x),\mathrm{F}_N(x),\mathrm{I}_N(x);x\in X\}$, where X be a space of discourse, $T_N(x):X\to[0,1]$, $F_N(x):X\to[0,1]$ and $I_N(x):X\to[0,1]$ with $0<T_N(x)+F_N(x)+I_N(x)<3$ for all $x\in X$. $T_N(x),F_N(x)$ and $I_N(x)$, respectively represent truth membership, falsity membership and indeterminacy membership degree of x to N .

Definition2. A neutrosophic number N is an extended version of the fuzzy set on $\mathbb{R}$ with the following truth, falsity and indeterminacy membership functions:

$$T_N(x)=\begin{cases}\dfrac{x-a^l}{a^m-a^l}, & a^l\le x\le a^m,\\ \dfrac{a^u-x}{a^u-a^m}, & a^m\le x\le a^u,\\ 0, & \text{O.W},\end{cases} \tag{1}$$

$$I_N(x)=\begin{cases}\dfrac{a^m-x}{a^m-a^l}, & \delta a^l+(1-\delta)a^m\le x\le a^m,\\ \dfrac{x-a^m}{a^u-a^m}, & a^m\le x\le(1-\delta)a^m+\delta a^u,\\ \delta, & \text{O.W},\end{cases} \tag{2}$$

$$F_N(x)=\begin{cases}\dfrac{a^m-x}{a^m-a^l}, & a^l\le x\le a^m,\\ \dfrac{x-a^m}{a^u-a^m}, & a^m\le x\le a^u,\\ 1, & \text{O.W},\end{cases} \tag{3}$$

where δ is the maximum degree of indeterminacy, also $a^l\le a^m\le a^u$ and $\delta\in(0,1)$.

The truth, indeterminacy and falsity membership functions of the above triangular neutrosophic number are presented in figure 1.

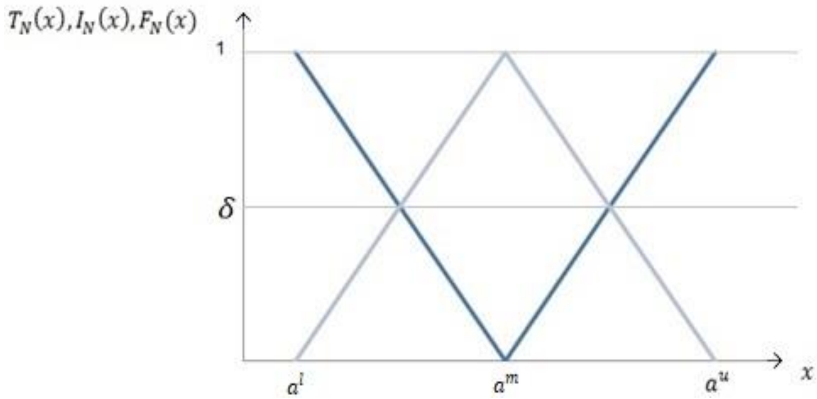

Figure 1
Truth, falsity, and indeterminacy membership functions of triangular neutrosophic number

Definition 3. [17] Let $N=[(a^l,a^m,a^u);\mu_N,\nu_N,\gamma_N]$ and $M=[(\mathrm{b}^l,\mathrm{b}^m,\mathrm{b}^u);\mu_M,\nu_M,\gamma_M]$ be two triangular neutrosophic numbers, the mathematical operations between N and M are as follows:

$$N+M=[(a^l+b^l,a^m+b^m,a^u+b^u);\mu_N\wedge\mu_M,\nu_N\vee\nu_M,\gamma_N\vee\gamma_M], \tag{4}$$

$$N-M=[(a^l-b^l,a^m-b^m,a^u-b^u);\mu_N\wedge\mu_M,\nu_N\vee\nu_M,\gamma_N\vee\gamma_M], \tag{5}$$

$$kN=\begin{cases}[(ka^l,ka^m,ka^u);\mu_N,\nu_N,\gamma_N], & k>0,\\ [(ka^u,ka^m,ka^l);\mu_N,\nu_N,\gamma_N], & k<0,\end{cases} \tag{6}$$

where "$\wedge$" and "$\vee$" are respectively mean that minimum and maximum operators.

Remark1.We show the set of all triangular neutrosophic numbers by $N(\mathbb{R})$.

In the next definition, we present one of the most important concepts which have a key role to preparing the solving methods of mathematical programming where the parameters of the models are considered based on a kind of uncertainty such as fuzziness, intuitionistic, and neutrosophic.

Definition 4. [13] A ranking function R on $N(\mathbb{R})$ is a map from $N(\mathbb{R})$ to the real numbers line, where the natural ordering there exists. Consider two triangular neutrosophic numbers as:

$N=[(a^l,a^m,a^u);\mu_N,\nu_N,\gamma_N]$ and $M=[(\mathrm{b}^l,\mathrm{b}^m,\mathrm{b}^u);\mu_M,\nu_M,\gamma_M]$. Then, the ranking method is defined as follows:

i. If $R(N)>R(M)$ then $N>M$.
ii. If $R(N)<R(M)$ then $N<M$. (7)
iii. If $R(N)=R(M)$ then $N=M$.

Definition 5. [19] let X be a space of discourse, an Interval Neutrosophic Set (INS) N through X taking the form $N=\{x,T_N(x),F_N(x),I_N(x);\ x\in X\}$ where $T_N(x),F_N(x),I_N(x)\subseteq[0,1]$ and $0\le SupT_N(x)+SupF_N(x)+SupI_N(x)\le 3$ for all $x\in X$. $T_N(x)$, $F_N(x)$ and $I_N(x)$ represent truth membership, falsity membership and indeterminacy membership of x to N, respectively.

Definition 6. An interval neutrosophic number N is an extended version of the fuzzy set on $\mathbb{R}$ with the following truth, falsity and indeterminacy membership function:

$$T_N^L(x)=\begin{cases}\dfrac{x-a^l+h_N(a^l-x)}{a^m-a^l}, & a^l\le x\le a^m,\\[2ex] \dfrac{a^u-x+h_N(x-a^u)}{a^u-a^m}, & a^m\le x\le a^u,\\[2ex] 0, & \text{O.W},\end{cases} \tag{8}$$

$$T_N^U(x)=\begin{cases}\dfrac{x-a^l+h_N(a^m-x)}{a^m-a^l}, & a^l\le x\le a^m,\\[2ex] \dfrac{a^u-x+h_N(x-a^m)}{a^u-a^m}, & a^m\le x\le a^u,\\[2ex] h_N, & \text{O.W},\end{cases} \tag{9}$$

where $T_N(x)=[T_N^L(x),T_N^U(x)]$.

$$F_N^L(x)=\begin{cases}\dfrac{a^m-x+h_N(x-a^m)}{a^m-a^l}, & a^l\le x\le a^m,\\[2ex] \dfrac{x-a^m+h_N(a^m-x)}{a^u-a^m}, & a^m\le x\le a^u,\\[2ex] 1-h_N, & \text{O.W},\end{cases}\tag{10}$$

$$F_N^U(x)=\begin{cases}\dfrac{a^m-x+h_N(x-a^l)}{a^m-a^l}, & a^l\le x\le a^m,\\[2ex] \dfrac{x-a^m+h_N(a^u-x)}{a^u-a^m}, & a^m\le x\le a^u,\\[2ex] 1, & \text{O.W},\end{cases}\tag{11}$$

where $F_N(x)=[F_N^L(x),F_N^U(x)]$.

$$I_N^L(x)=\begin{cases}\dfrac{a^m-x+h_N(x-a^m)}{a^m-a^l}, & \delta a^l+(1-\delta)a^m\le x\le a^m,\\[2ex] \dfrac{x-a^m+h_N(a^m-x)}{a^u-a^m}, & a^m\le x\le(1-\delta)a^m+\delta a^u,\\[2ex] \delta, & \text{O.W},\end{cases}\tag{12}$$

$$I_N^U(x)=\begin{cases}\dfrac{a^m-x+h_N(x-a^l)}{a^m-a^l}, & \delta a^l+(1-\delta)a^m\le x\le a^m,\\[2ex] \dfrac{x-a^m+h_N(a^u-x)}{a^u-a^m}, & a^m\le x\le(1-\delta)a^m+\delta a^u,\\[2ex] 1-\delta, & \text{O.W,}\end{cases} \tag{13}$$

where $I_N(x)=[I_N^L(x),I_N^U(x)]$ and $h_N=T_N^L(x)-T_N^U(x)$ such that $\delta\in(0,1)$ and $h_N\le\delta<\dfrac{1}{2}.$

The membership functions of the above symmetric triangular interval neutrosophic number are presented in Figure 2.

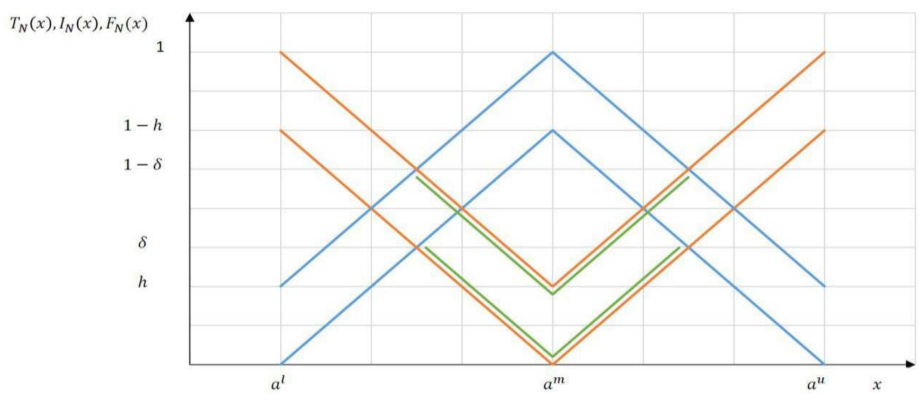

Figure 2

Truth, falsity, and indeterminacy membership functions of triangular interval neutrosophic number

Remark 2. An INS $N=[(a^l,a^m,a^u);[\mu_N^l,\mu_N^u],[\nu_N^l,\nu_N^u],[\gamma_N^l,\gamma_N^u]]$ will be reduced to the NS if $\mu_N^l=\mu_N^u$, $\nu_N^l=\nu_N^u$ and $\gamma_N^l=\gamma_N^u$.

Definition 7. Let $N=[(a^l,a^m,a^u);[\mu_N^l,\mu_N^u],[\nu_N^l,\nu_N^u],[\gamma_N^l,\gamma_N^u]]$ and $M=[(\mathrm{b}^l,\mathrm{b}^m,\mathrm{b}^u);[\mu_M^l,\mu_M^u]\ ,[\nu_M^l,\nu_M^u],[\gamma_M^l,\gamma_M^u]]$ be two triangular interval neutrosophic numbers. The mathematical operation between N and M can be represented as :

$$N+M=[(a^l+b^l,a^m+b^m,a^u+b^u);[\mu_N^l+\mu_M^l-\mu_N^l\mu_M^l,\mu_N^u+\mu_M^u-\mu_N^u\mu_M^u],[\nu_N^l\nu_M^l,\nu_N^u\nu_M^u],[\gamma_N^l\gamma_M^l,\gamma_N^u\gamma_M^u]],$$

$$N-M=[(a^l-b^l,a^m-b^m,a^u-b^u);[\mu_N^l+\mu_M^l-\mu_N^l\mu_M^l,\mu_N^u+\mu_M^u-\mu_N^u\mu_M^u],[\nu_N^l\nu_M^l,\nu_N^u\nu_M^u],[\gamma_N^l\gamma_M^l,\gamma_N^u\gamma_M^u]].$$

3. Proposed interval neutrosophic linear programming method

In this section, by using a new ranking function for interval neutrosophic numbers we suggest a new method for solving INLP problems. The basis of our work will be presented as follows:

Step (1). Let decision makers insert their INLP problem with triangular interval neutrosophic numbers. Because we always want to maximize truth degree, minimize indeterminacy and falsity degree of information, and then inform decision makers to apply this concept when entering triangular neutrosophic numbers of INLP model.

Step (2). Convert interval-valued neutrosophic programming problem to its crisp model by using the following method:

In order to compare any two interval-valued triangular neutrosophic numbers, which are based on the ranking function, let $N=[(a^l,a^m,a^u);[\mu_N^l,\mu_N^u],[\nu_N^l,\nu_N^u],[\gamma_N^l,\gamma_N^u]]$ be a symmetric interval-valued neutrosophic number, where $[\mu_N^l,\mu_N^u],[\nu_N^l,\nu_N^u]$ and $[\gamma_N^l,\gamma_N^u]$ are truth, falsity, and indeterminacy membership degrees of N, respectively. Also a^l,a^m and a^u are lower, median and upper bounds for N, respectively. The ranking function for interval-valued neutrosophic number N will be defined as follow:

$$R(N)=\frac{a^l+a^u+2a^m}{4}+\text{confirmation degree}$$

Mathematically, this function can be written as follows:

$$R(N)=\frac{1}{4}[a^l+a^u+2a^m]+(\bar{\mu}_N-\bar{\nu}_N-\bar{\gamma}_N), \tag{14}$$

Where $\bar{\mu}_N=\frac{\mu_N^l+\mu_N^u}{2}$, $\bar{\nu}_N=\frac{\nu_N^l+\nu_N^u}{2}$ and $\bar{\gamma}_N=\frac{\gamma_N^l+\gamma_N^u}{2}$. where satisfy the situation of definition 4. Moreover, we have :

$$N\geq\tilde{0}\quad\text{if}\quad\frac{a^l+a^u+2a^m}{4}\geq 0.$$

Step (3). By applying the previous ranking function, convert each triangular interval-valued neutrosophic number to its equivalent crisp value. This lead to convert INLP problem to its crisp model.

Step (4). Solve the crisp model using the standard method and obtain the optimal solution of problem.

4 Numerical example

In this section, we present two examples to illustrate the effectiveness of our proposed model of INLP problems. It should be noted that we also can consider the interval neutrosophic programming problems with interval neutrosophic variables, but because in real calculations we always prefer to obtain the crisp value as an optimal solution, so in this research, we always consider the values of x_j as a real numbers. The order of each element for triangular interval neutrosophic numbers in all examples are considered as follows: lower bound, median bound and upper bound.

Example 1.

In this example, we consider the fully INLP problem where all parameters expect x_j are considered as interval neutrosophic numbers.

$$\begin{aligned} &Max\ Z \approx \tilde{7}x_1 + \tilde{6}x_2 + 1\tilde{4}x_3 \\ &s.t. \\ &\quad 1\tilde{5}x_1 + \tilde{1}x_2 \lesssim 1\tilde{0} \\ &\quad \tilde{9}x_1 + \tilde{4}x_2 + \tilde{8}x_3 \lesssim \tilde{2} \\ &\qquad 1\tilde{9}x_2 + 1\tilde{1}x_3 \lesssim \tilde{4} \\ &\qquad x_1, x_2, x_3 \geq 0 \end{aligned}$$

where

$$\tilde{7} = \langle (1,7,13),[0.7,0.9],[0.1,0.4],[0.2,0.4] \rangle$$

$$\tilde{6} = \langle (2,6,10),[0.2,0.6],[0.2,0.5],[0.1,0.8] \rangle$$

$$1\tilde{4} = \langle (7,14,21),[0.5,0.7],[0.4,0.6],[0.3,0.4] \rangle$$

$$1\tilde{5} = \langle (14,15,16),[0.6,0.8],[0.1,0.4],[0.4,0.9] \rangle$$

$$\tilde{1} = \langle (0,1,2),[0.2,0.7],[0.1,0.6],[0.8,0.9] \rangle$$

$1\tilde{0} = \langle (5,10,15),[0.1,0.5],[0.1,0.4],[0.6,0.9] \rangle$

$\tilde{9} = \langle (4,9,14),[0.4,0.5],[0.5,0.8],[0.4,0.8] \rangle$

$\tilde{4} = \langle (1,4,7),[0.1,0.9],[0.4,0.5],[0.3,0.4] \rangle$

$\tilde{8} = \langle (6,8,10),[0.7,0.8],[0.5,0.6],[0.1,0.6] \rangle$

$\tilde{2} = \langle (1,2,3),[0.3,0.6],[0.1,0.9],[0.4,0.6] \rangle$

$1\tilde{9} = \langle (5,19,33),[0.5,0.9],[0.3,0.5],[0.7,0.8] \rangle$

$1\tilde{1} = \langle (8,11,14),[0.6,0.8],[0.4,0.9],[0.6,0.9] \rangle$

$\tilde{4} = \langle (0,4,8),[0.3,0.7],[0.1,0.2],[0.1,0.3] \rangle$

By using the ranking function proposed in Eq.(14), the previous problem will be converted to the crisp model as follows:

$$
\begin{aligned}
& Max\ \text{Z}=6.75x_1 + 5.6x_2 + 13.75x_3 \\
& s.t. \\
& \qquad 14.8x_1 + 0.25x_2 \le 9.3 \\
& \qquad 8.2x_1 + 3.7x_2 + 7.85x_3 \le 1.45 \\
& \qquad 18.55x_2 + 10.3x_3 \le 4.15 \\
& \qquad x_1, x_2, x_3 \ge 0
\end{aligned}
$$

The standard form of the above INLP problem will be constructed as follows, where s_1, s_2 and s_3 are the slack variables

$$
\begin{aligned}
& Max\ \text{Z}=6.75x_1 + 5.6x_2 + 13.75x_3 \\
& s.t. \\
& \qquad 14.8x_1 + 0.25x_2 + s_1 = 9.3 \\
& \qquad 8.2x_1 + 3.7x_2 + 7.85x_3 + s_2 = 1.45 \\
& \qquad 18.55x_2 + 10.3x_3 + s_3 = 4.15 \\
& \qquad x_1, x_2, x_3, s_1, s_2, s_3 \ge 0
\end{aligned}
$$

By using the simplex method we have:

Table 1
Initial Table

Basis	x_1	x_2	x_3	s_1	s_2	s_3	$R.H.S$
s_1	14.8	0.25	0	1	0	0	9.3
s_2	8.2	3.7	7.85	0	1	0	1.45
s_3	0	18.55	10.3	0	0	1	4.15
Z	-6.75	-5.6	-13.75	0	0	0	0

where in the previous table x_3 is a coming variable and s_2 is a leaving variable. The second simplex tableau presented in Table 2 as follows:

Table 2
Optimal Table

Basis	x_1	x_2	x_3	s_1	s_2	s_3	$R.H.S$
s_1	14.8	0.25	0	1	0	0	9.3
x_3	1.04	0.47	1	0	0.13	0	0.18
s_3	-10.76	13.7	0	0	-1.31	1	2.25
Z	7.61	0.88	0	0	1.75	0	2.54

where $Z^* = 2.54$ and Table 2 is the optimal simplex form for our example.

Example 2 (Case Study)

In a computer manufacturing plant, we need to produce four basic units, such as RAMs, graphics cards, hard drives, and CPUs, to produce each computer. All productions have to get through four parts. These four parts include" Design, Fabrication, Probe, and Assembly". The favorable time for each unit manufactured and its profit is presented in table 3. The minimum production amount for supplementing monthly products is presented in table 4. The purpose of the company is producing products in this limit for maximizing the general profits.

Table 3

Products	Design	Fabrication	Probe	Assembly	Unit profit
P_1	0.2	0.5	0.1	0.1	$\widetilde{14}$\$
P_2	0.5	3	2	0.6	$\tilde{7}$\$
P_3	0.4	4	4	0.8	$\tilde{5}$\$
P_4	1	2	0.2	0.2	$\tilde{8}$\$

Table 4

Sector	Capacity	Products	Minimum production level
Design	$\widetilde{1300}$	P_1	$\widetilde{100}$
Fabrication	$\widetilde{3340}$	P_2	$\widetilde{280}$
Probe	$\widetilde{1800}$	P_3	$\widetilde{194}$
Assembly	$\widetilde{2100}$	P_4	$\widetilde{400}$

The values and the degrees of truth, indeterminacy and falsity membership functions for each interval neutrosophic number in the previous tables are represented as follows:

$$\widetilde{14} = \langle (12,14,16), [0.3,0.7], [0.2,0.8], [0.2,0.9] \rangle$$

$$\tilde{7} = \langle (2,7,12), [0.1,0.6], [0.4,0.7], [0.6,0.8] \rangle$$

$$\tilde{5} = \langle (4,5,6), [0.2,0.6], [0.4,0.9], [0.3,0.4] \rangle$$

$$\tilde{8} = \langle (3,8,13), [0.2,0.5], [0.3,0.8], [0.6,0.9] \rangle$$

$\widetilde{1300} = \langle (1000,1300,1600), [0.1,0.6], [0.2,0.7], [0.3,0.8] \rangle$

$\widetilde{3340} = \langle (3215,3340,3465), [0.7,0.9], [0.2,0.7], [0.4,0.9] \rangle$

$\widetilde{1800} = \langle (1390,1800,2210), [0.4,1], [0.2,0.6], [0.1,0.2] \rangle$

$\widetilde{2100} = \langle (1818,2100,2510), [0.3,0.7], [0.1,0.6], [0.4,0.8] \rangle$

$\widetilde{100} = \langle (99,100,101), [0.1,0.7], [0.2,0.6], [0.3,0.4] \rangle$

$\widetilde{280} = \langle (230,280,330), [0.7,0.9], [0.1,0.2], [0.2,0.5] \rangle$

$\widetilde{194} = \langle (184,194,204), [0.1,0.6], [0.3,0.7], [0.1,0.7] \rangle$

$\widetilde{400} = \langle (200,400,600), [0.1,0.4], [0.2,0.6], [0.4,0.8] \rangle$

Let x_1, x_2, x_3 and x_4 represent the number of produced RAMs, graphics cards, hard drives and CPUs, respectively.The above problem can be formulated as follows:

$max\tilde{Z} \approx \widetilde{14}x_1 + \tilde{7}x_2 + \tilde{5}x_3 + \tilde{8}x_4$

s.t.

$0.2x_1 + 0.5x_2 + 0.4x_3 + 1x_4 \leq \widetilde{1300}$

$0.5x_1 + 3x_2 + 4x_3 + 2x_4 \leq \widetilde{3340}$

$0.1x_1 + 2x_2 + 4x_3 + 0.2x_4 \leq \widetilde{1800}$

$0.1x_1 + 0.6x_2 + 0.8x_3 + 0.2x_4 \leq \widetilde{2100}$

$x_1 \geq \widetilde{100}$

$x_2 \geq \widetilde{280}$

$x_3 \geq \widetilde{194}$

$x_4 \geq \widetilde{400}$

$x_1, x_2, x_3, x_4 \geq \tilde{0}$

by using the ranking function proposed in Eq. (14) the previous problem will be converted to a crisp model as follows:

$$max\tilde{Z} \approx 13.45x_1 + 6.1x_2 + 4.4x_3 + 7.05x_4$$

s.t.

$$0.2x_1 + 0.5x_2 + 0.4x_3 + 1x_4 \leq 1299.35$$

$$0.5x_1 + 3x_2 + 4x_3 + 2x_4 \leq 3339.7$$

$$0.1x_1 + 2x_2 + 4x_3 + 0.2x_4 \leq 1800.15$$

$$0.1x_1 + 0.6x_2 + 0.8x_3 + 0.2x_4 \leq 2099.55$$

$$x_1 \geq 99.65$$

$$x_2 \geq 280.3$$

$$x_3 \geq 193.45$$

$$x_4 \geq 399.25$$

$$x_1, x_2, x_3, x_4 \geq 0$$

By using the primal simplex method the optimal solution of the above problem will be obtained as follows:

$$x_1 = 1853$$

$$x_2 = 280.3$$

$$x_3 = 193.45$$

$$x_4 = 399.25$$

$$Z^* = 30298.57.$$

5. Comparison analysis and discussion

In order to illustrate the efficiency and eventuality of our model, a comparative study with other existing methods that were outlined in [1] [6] [7] is conducted.

By comparing our model with Deli and Şubaş in [6] we are noted that:

1. Their model is so complex and too time-consuming to do the calculations, but our model is so simple.
2. Their model is only able to solve the problem with single-valued neutrosophic numbers but by using the proposed model in addition to single-valued neutrosophic numbers we can handle interval-valued neutrosophic numbers.
3. Due to the lack of necessity in determining the truth, falsity, and indeterminacy-membership degrees as crisp values, the proposed model can exhibit reality effectively and is more efficient than their model.

By comparing the present method with Akyar et al. in [1] we also noted that:

1. By considering all aspects of the decision-making process in our calculations such as truthiness, indeterminacy and falsity degrees. The proposed model can exhibit reality efficiently than their model.
2. While in their model only the fuzzy linear programming problems are converted to crisp models but their model is more complex than our model.

Also by comparing our method with Dubei and Mehra in [7] we also founded that:

1. In their model, they only consider truthiness and falsity while in our real-life circumstances the decision-making process has the form “agree, not sure and disagree”, where this drawback can be treated by using of neutrosophic sets.
2. In their model, the degrees of membership and non-membership are considered as a single-valued number, while by considering these degrees as an interval number in our model, we can overcome the uncertainty in determining the membership and non-membership degrees.
3. Their model can only handle incomplete information but by using the proposed method we can handle not only the incomplete information but also the indeterminate and inconsistent information.

6. Conclusion

In this research, by considering a linear programming problem under a neutrosophic environment with triangular interval neutrosophic numbers we have presented a new linear programming model. In this model in view of considering the truthiness, indeterminacy and falsity degrees will be able to cover more real daily life circumstances. It should be noticed there is no necessity the values of these degrees be crisp values. In this respect, we proposed a ranking function that is capable of converting every triangular interval neutrosophic number to its equivalent crisp value. And subsequently, every interval neutrosophic problem will be converted to the crisp model where can be solved by standard methods easily. The proposed model indicates more simplicity applicability and more efficiency in comparison with other existing models.

References

[1] E.Akyar, H. Akyar, & S. A. Düzce, "A new method for ranking triangular fuzzy numbers," International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems, vol. 20(05), pp. 729-740, 2012.
[2] T. Allahviranloo, F. H.Lotfi, M. K. Kiasary, N. A.Kiani, & L. Alizadeh, "Solving fully fuzzy linear programming problem by the ranking function," Applied mathematical sciences, vol.2(1), pp.19-32, 2008.
[3] , K. Atanassov, "Intuitionistic fuzzy sets. Fuzzy Sets and Systems," vol.20, pp. 87–96, 1986.
[4] K.Atanassov, G.Gargov, "Interval-valued intuitionistic fuzzy sets," Fuzzy Sets and Systems, vol. 31(3), pp. 343–349, 1989.
[5] J. J.Buckley, T.Feuring, "Evolutionary algorithm solution to fuzzy problems: fuzzy linear programming," Fuzzy sets and systems, vol.109(1), pp. 35-53, 2000.
[6] I. Deli, Y. Şubaş, "A ranking method of single valued neutrosophic numbers and its applications to multi-attribute decision making problems," International Journal of Machine Learning and Cybernetics, vol 8(4), pp.1309-1322, 2017.
[7] D.Dubey, A.Mehra, "Linear programming with triangular intuitionistic fuzzy number," (Published Conference Proceedings style)," 7th Annu. European Society for Fuzzy Logic and Technology, pp. 563-569, Atlantis Press.
[8] S. M.Hashemi, M.Modarres, E., Nasrabadi, &M. M.Nasrabadi, "Fully fuzzified linear programming, solution and duality," Journal of Intelligent & Fuzzy Systems, vol.17(3), pp.253-261, 2006.
[9] H.Kivija¨rvi, P.Korhonen, & J.Wallenius, "Operations research and its practice in Finland," Interfaces, vol(16), pp.53–59, 1986.

[10] A.Kumar, J. Kaur, & P.Singh, "A new method for solving fully fuzzy linear programming problems," Applied mathematical modelling, vol. 35(2), pp.817-823, 2011.

[11] G.L. Lilien, "MS/OR: a mid-life crisis," Interfaces, vol (17), pp. 35–38, 1987.

[12] M. Mohamed, M. AbdelBasset, A.N. Zaied, & F. Smarandache, "Neutrosophic integer programming problem," Neutrosophic sets and systems,vol 15,pp. 3–7, 2017.

[13] A. Nafei, W. Yuan, H. Nasseri, H. "Group multi-attribute decision making based on interval neutrosophic sets," *Studies in Informatics and Control*, vol. 28(3), pp. 309-316, 2019.

[14] H. M. Z. Selhausen, "Repositioning OR's products in the market," Interfaces, vol.19, pp. 79–87, 1989.

[15] F. Smarandache, " A generalization of the intuitionistic fuzzy set," International journal of Pure and Applied Mathematics, vol. 24, pp. 287-297, 2005.

[16] F. Smarandache, "A geometric interpretation of the neutrosophic set-A generalization of the intuitionistic fuzzy set," arXiv preprint math/0404520.

[17] Y. S¸ubas¸, "Neutrosophic numbers and their application to Multi-attribute decision making problems," (In Turkish) (Masters Thesis, Kilis 7 Aralık University, Graduate School of Natural and Applied Science)

[18] G. A. Tingley, " Can MS/OR sell itself well enough?," Interfaces, vol. 17, pp. 41–52, 1987.

[19] H. Wang, F. Smarandache, Y.Q. Zhang, R. Sunderraman, "Interval neutrosophic sets and logic: Theory and applications in computing," Hexis, Phoenix, AZ, 2005.

[20] H. Wang, F. Smarandache, Y. Q. Zhang, R. Sunderraman, " Single valued neutrosophic sets," Multispace and Multistructure, infinite study.

[21] Y. M. Wang, T. M. Elhag, "A fuzzy group decision making approach for bridge risk assessment," Computers & Industrial Engineering, vol. 53(1), pp.137-148, 2007.

[22] L. A. Zadeh, "Fuzzy sets," Information and control, vol. 8(3), pp. 338-353, 1965.

[23] L. A. Zadeh, "The concept of a linguistic variable and its application to approximate reasoning—I," Information sciences, vol. 8(3), pp. 199-249, 1975.