An Approach to The Optimization of Multi-Objective Assignment Problems with Neutrosophic Numbers

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Hamiden Abd El- Wahed Khalifa
Cairo University

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RESEARCH PAPER

An Approach to The Optimization of Multi-Objective Assignment Problems with Neutrosophic Numbers

Hamiden Khalifa *

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ABSTRACT
This paper aims to study the multi-objective assignment problem with emphasis on imprecise costs rather than price information. The NMOAS problem is considered by adding single-valued trapezoidal neutrosophic numbers to the elements of cost matrices. After converting the NMOAS problem into the corresponding crisp Multi-Objective Assignment (MOAS) problem based on the score function, an approach to finding the most preferred neutrosophic solution was discussed. The approach was used through a weighting Tchebycheff problem which was applied by defining relative weights and ideal targets. This approach was more flexible than the standard multi-objective assignment problem and it allowed the Decision-Maker (DM) to choose the targets. Finally, a numerical example was given to illustrate the utility, effectiveness, and applicability of the approach.

KEYWORDS: *Multi-objective assignment problem; Neutrosophic numbers; Membership functions; Weighting tchebycheff problem; Optimal compromise neutrosophic solution.*

1. Introduction

Assignment (AS) problem is a well-studied topic in combinatorial optimization and is directly linked to production planning, telecommunication, economy, etc. It deals with the question of how to set n assignees to m tasks in an injective manner for which an optimal assignment can be made in the best possible way. Depending on the objectives, one must optimize different problems ranging from linear AS problem to quadratic and high-dimensional AS problems. Linear AS problem is a particular type of the Linear Programming (LP) problem in which assignees are charged with accomplishing tasks on a one-to-one basis such that the assignment cost (or profit) can be reduced to minimum (or maximum). The best assignee for the task is a perfect description of the AP, where the number of rows and columns is the same (Ehrgot et al., 2016). Bao et al. (2007) developed and solved a multi-objective AS problem. Geetha and Nair (1993) first formulated and solved cost-time AS problem as a multi-criteria decision-making problem.

However, AS problem representing real-world situations involves a set of parameters whose values are assigned by decision-makers. DMs are required to allocate exact values to parameters in conventional approaches. In this case, DM does not precisely know the exact value of parameters; thus, the parameters of the problem are usually defined in an uncertain manner. Bellmann and Zadeh (1970) introduced the concept of fuzzy set theory into the decision-making problem involving uncertainty and imprecision. Zimmermann was the first to solve the LP problem with several objectives through suitable membership functions. Sakawa and Yano (1989) introduced the concept of fuzzy multi-objective linear programming (MOLP) problems. Kiruthiga and Loganathan (2015) reduced the fuzzy MOLP problem to the corresponding ordinary one using the ranking function and, hence, solved it using the fuzzy programming technique. Hamadameen (2018) proposed a technique for solving the fuzzy MOLP problem in which the coefficients of objective functions are triangular fuzzy numbers. Leberling (1981) solved the vector maximum LP problem using a special type of non-linear membership functions. Bit et al. (1992) applied fuzzy programming approach to Multi- Objective Transportation Problem (MOTP). Belacela and Boulasselb (2001) studied multi- criteria AS

* Corresponding author: ***Hamiden Khalifa***
hamiden_2008@yahoo.com

1. *Associate Professor Cairo University*

problem in fuzzy environment. Lin and Wen (2004) designed a labeling algorithm for a fuzzy AS problem. Yang et al. (2005) designed a Tabu search algorithm based on fuzzy simulation to achieve an appropriate best solution to the fuzzy AS problem. De and Yadav (2011) proposed an algorithm for solving MOASP through interactive fuzzy goal programming approach. Kagade and Bajaj (2010) discussed interval numbers including cost coefficients of MOASP. Mukherjee and Basu (2010) solved the fuzzy cost AS problem using the ranking method, introduced by Yager (1981). Pramanik and Biswas (2012) studied multi- objective AS problem with imprecise costs, time, and ineffectiveness. Haddad et al. (2012) discussed two models for the generalized AS problem in the uncertain environment. Emrouznejad et al. (2012) developed an alternative formulation for the fuzzy the AS problem with fuzzy costs or fuzzy profits for each possible assignment based on Data Envelopment Analysis. Kumar and Gupta (2011) developed a solution method for fuzzy AS problems and fuzzy travelling salesman problems with different membership functions with ranking index introduced by Yager (1981). Jayalakshmi and Sujatha (2018) introduced a new method, namely optimal flowing method, to provide the ideal set of all efficient solutions. Medvedeva and Medvedev (2018) applied the properties of primal and dual MOAS problems. Hamou and Mohamed El- Amine (2018) applied a branch-and-bound method to generate a set of all efficient solutions to the MOAS problem.

Neutrosophic set is considered to be a generalization of crisp set, fuzzy set, and intuitionistic fuzzy set to represent the uncertainty, inconsistency, and incomplete knowledge about a real-world problem. Vidhya et al. (2017) studied the neutrosophic MOLP problem. Pramanik and Banerjee (2018) applied a goal programming strategy to MOLP problem with neutrosophic numbers. Rizk- Allah, R. M. (2018) developed a new compromise algorithm for MOTP which was inspired by Zimmermann's fuzzy programming and the neutrosophic set terminology.

This study attempts to study the Multi-Objective Assignment (MOAS) problem in the neutrosophic environment. An approach to finding the most preferred neutrosophic solution is discussed. The approach is used through a weighting Tchebycheff problem which is applied by defining relative weights and ideal targets.

The outlay of the paper is organized as follows: Section 2 present some preliminaries. Section 3 formulates the NMOAS problem. Section 4 introduces an approach to obtain neutrosophic optimal satisfactory solution to the MOAS problem. Section 5 gives a numerical example for illustration. Finally, some concluding remarks are reported in Section 6.

2. Preliminaries

In order to discuss the problem conveniently, basic concepts and results of fuzzy numbers, trapezoidal fuzzy numbers, intuitionistic trapezoidal fuzzy numbers, and neutrosophic set are recalled.

Definition 1 (Fuzzy number). A fuzzy number $\widetilde{B}$ is a convex normalized fuzzy set on the real line $\mathbb{R}$ such that:

1. $\mu_{\widetilde{B}}(x)$ is piecewise continuous,
2. $\exists\, x \in \mathbb{R}$, with $\mu_{\widetilde{B}}(x) = 1$.

Definition 2. (Trapezoidal fuzzy numbers, Kaur and Kumar, 2012). A fuzzy number $\widetilde{B} = (r, s, t, u)$ is a trapezoidal fuzzy number, where $r, s, t, u \in \mathbb{R}$ and its membership function are defined as follows:

$$\mu_{\widetilde{B}}(x) = \begin{cases} \dfrac{x-r}{s-r}, & r \le x \le s, \\ 1, & s \le x \le t, \\ \dfrac{u-x}{u-t}, & t \le x \le u, \\ 0, \text{otherwise}, & \end{cases}$$

Definition 3 (Intuitionistic fuzzy set, Atanason, 1986). A fuzzy set $\widetilde{B}$ is said to be an intuitionistic fuzzy set $\widetilde{B}^{IN}$ of a non-empty set X if $\widetilde{B}^{IN} = \{\langle x, \mu_{\widetilde{B}^{IN}}, \rho_{\widetilde{B}^{IN}}\rangle : x \in X\}$, where $\mu_{\widetilde{B}^{IN}}$ and $\rho_{\widetilde{B}^{IN}}$ are the membership and nonmembership functions such that $\mu_{\widetilde{B}^{IN}}, \rho_{\widetilde{B}^{IN}} : X \to [0,1]$, and $0 \le \mu_{\widetilde{B}^{IN}} + \rho_{\widetilde{B}^{IN}} \le 1$ for all $x \in X$.

Definition 4 (Intuitionistic fuzzy number, Atanason, 1999). An intuitionistic fuzzy set $\widetilde{B}^{IN}$ of $\mathbb{R}$ is called an Intuitionistic fuzzy number if the following conditions hold:

1. There exists $c \in \mathbb{R}$: $\mu_{\widetilde{B}^{IN}}(c) = 1$ and $\rho_{\widetilde{B}^{IN}}(c) = 0$,
2. $\mu_{\widetilde{B}^{IN}} : \mathbb{R} \to [0,1]$ is a continuous function such that
$0 \le \mu_{\widetilde{B}^{IN}} + \rho_{\widetilde{B}^{IN}} \le 1$, for all $x \in X$,
3. The membership and nonmembership functions of $\widetilde{B}^{IN}$ are

$$\mu_{\tilde{B}^{IN}}(x)=\begin{cases}0, & -\infty< x<r\\ h(x), & r\le x\le s\\ 1, & x=s\\ l(x), & s\le x\le t\\ 0, & t\le x<\infty,\end{cases}$$

$$\rho_{\tilde{B}^{IN}}(x)=\begin{cases}0, & -\infty< x<a\\ f(x), & a\le x\le s\\ 1, & x=s\\ g(x), & s\le x\le b\\ 0, & b\le x<\infty,\end{cases}$$

where $f, g, h, l: \mathbb{R}\to[0,1]$, h and g are the strictly increasing functions, l and f are the strictly decreasing functions with the conditions $0\le f(x)+f(x)\le 1$ and $0\le l(x)+g(x)\le 1$.

Definition 5 (Trapezoidal intuitionistic fuzzy number, Jianqiang and Zhong, 2009).
A trapezoidal intuitionistic fuzzy number is denoted by $\tilde{B}^{IN}=(r,s,t,u),(a,s,t,b)$, where $a\le r\le s\le t\le u\le b$ with membership and nonmembership functions are defined as follows:

$$\mu_{\tilde{B}^{INT}}(x)=\begin{cases}\frac{x-r}{s-r}, & r\le x<s\\ 1, & s\le x\le t\\ \frac{u-x}{u-t}, & t\le x\le u\\ 0, & \text{otherwise,}\end{cases}$$

$$\rho_{\tilde{B}^{INT}}(x)=\begin{cases}\frac{s-x}{s-a}, & a\le x<s\\ 0, & s\le x\le t\\ \frac{x-t}{b-t}, & t\le x\le b\\ 1, & \text{otherwise,}\end{cases}$$

Definition 6 (Neutrosophic set, Smarandache, 1998). A neutrosophic set $\overline{B}^{N}$ of non-empty set X is defined as follows:
$\overline{B}^{N}=\{\langle x, I_{\overline{B}^N}(x), J_{\overline{B}^N}(x), V_{\overline{B}^N}(x)\rangle: x\in X, I_{\overline{B}^N}(x), J_{\overline{B}^N}(x), V_{\overline{B}^N}(x)\in\,]0_-,1^+[\}$, where $I_{\overline{B}^N}(x), J_{\overline{B}^N}(x)$, and $V_{\overline{B}^N}(x)$ are truth membership function, an indeterminacy membership function, and a falsity membership function, respectively, and there is no restriction on the sum of $I_{\overline{B}^N}(x), J_{\overline{B}^N}(x)$, and $V_{\overline{B}^N}(x)$; therefore, $0^-\le I_{\overline{B}^N}(x)+J_{\overline{B}^N}(x)+V_{\overline{B}^N}(x)\le 3^+$ and $]0_-,1^+[$ is a nonstandard unit interval.

Definition 7 (Single-valued neutrosophic set, Wang et al., 2010). A single-valued neutrosophic set $\overline{B}^{SVN}$ of a non empty set X is defined as follows:
$\overline{B}^{SVN}=\{\langle x, I_{\overline{B}^N}(x), J_{\overline{B}^N}(x), V_{\overline{B}^N}(x)\rangle: x\in X\}$,
where $I_{\overline{B}^N}(x), J_{\overline{B}^N}(x)$, and $V_{\overline{B}^N}(x)\in[0,1]$ for each $x\in X$ and $0\le I_{\overline{B}^N}(x)+J_{\overline{B}^N}(x)+V_{\overline{B}^N}(x)\le 3$.

Definition 8 (Single-valued neutrosophic number, Thamariselvi and Santhi, 2016). Let $\tau_{\tilde{b}}, \varphi_{\tilde{b}}, \omega_{\tilde{b}}\in[0,1]$ and $r,s,t,u\in\mathbb{R}$ such that $r\le s\le t\le u$. Then, a single-valued trapezoidal neutrosophic number, $\tilde{b}^N=\langle(r,s,t,u):\tau_{\tilde{b}},\varphi_{\tilde{b}},\omega_{\tilde{b}}\rangle$, is a special neutrosophic set on $\mathbb{R}$, whose truth membership, indeterminacy membership, and falsity membership functions are given below:

$$\mu_{\tilde{b}}{}^{N}(x)=\begin{cases}\tau_{\tilde{b}^N}\left(\frac{x-r}{s-r}\right), & r\le x<s\\ \tau_{\tilde{b}}, & s\le x\le t\\ \tau_{\tilde{b}^N}\left(\frac{u-x}{u-t}\right), & t\le x\le u\\ 0, & \text{otherwise,}\end{cases}$$

$$\rho_{\tilde{b}}{}^{N}(x)=\begin{cases}\frac{s-x+\varphi_{\tilde{b}^N}(x-r)}{s-r}, & r\le x<s\\ \varphi_{\tilde{b}^N}, & s\le x\le t\\ \frac{x-t+\varphi_{\tilde{b}^N}(u-x)}{u-t}, & t\le x\le u\\ 1, & \text{otherwise,}\end{cases}$$

$$\sigma_{\tilde{b}}{}^{N}(x)=\begin{cases}\frac{s-x+\omega_{\tilde{b}^N}(x-r)}{s-r}, & r\le x<s\\ \omega_{\tilde{b}^N}, & s\le x\le t\\ \frac{x-t+\omega_{\tilde{b}^N}(u-x)}{u-t}, & t\le x\le u\\ 1, & \text{otherwise.}\end{cases}$$

where $\tau_{\tilde{b}}, \varphi_{\tilde{b}}$, and $\omega_{\tilde{b}}$ denote the maximum truth, minimum indeterminacy, and minimum falsity membership degrees, respectively. A single-valued trapezoidal neutrosophic number $\tilde{b}^N=\langle(r,s,t,u):\tau_{\tilde{b}^N},\varphi_{\tilde{b}^N},\omega_{\tilde{b}^N}\rangle$ may be expressed as an ill-defined quantity of b, which is approximately equal to $[s,t]$.

Definition 9. Let $\tilde{b}^N=\langle(r,s,t,u):\tau_{\tilde{b}^N},\varphi_{\tilde{b}^N},\omega_{\tilde{b}^N}\rangle$ and $\tilde{d}^N=\langle(r',s',t',u'):\tau_{\tilde{d}^N},\varphi_{\tilde{d}^N},\omega_{\tilde{d}^N}\rangle$ be two single-valued trapezoidal neutrosophic numbers and $v\ne 0$. The arithematic operations on $\tilde{b}^N$ and $\tilde{d}^N$are

1. $\tilde{b}^N\oplus\tilde{d}^N=\langle(r+r', s+s', t+t', u+u');\ \tau_{\tilde{b}^N}\wedge\tau_{\tilde{d}^N}, \varphi_{\tilde{b}^N}\vee\varphi_{\tilde{d}^N}, \omega_{\tilde{b}^N}\vee\omega_{\tilde{d}^N}\rangle$,

2. $\tilde{b}^N \ominus \tilde{d}^N = \langle (r-u', s-t', t-s', u'-r);\ \tau_{\tilde{b}^N} \wedge \tau_{\tilde{d}^N}, \varphi_{\tilde{b}^N} \vee \varphi_{\tilde{d}^N}, \omega_{\tilde{b}^N} \vee \omega_{\tilde{d}^N} \rangle,$

3. $$\tilde{b}^N \otimes \tilde{d}^N = \begin{cases} \langle (rr', ss', tt', uu');\ \tau_{\tilde{b}^N} \wedge \tau_{\tilde{d}^N}, \varphi_{\tilde{b}^N} \vee \varphi_{\tilde{d}^N}, \omega_{\tilde{b}^N} \vee \omega_{\tilde{d}^N} \rangle, u,\ u' > 0 \\ \langle (ru', st', st', ru');\ \tau_{\tilde{b}^N} \wedge \tau_{\tilde{d}^N}, \varphi_{\tilde{b}^N} \vee \varphi_{\tilde{d}^N}, \omega_{\tilde{b}^N} \vee \omega_{\tilde{d}^N} \rangle, u < 0,\ u' > 0 \\ \langle (uu', ss', tt', rr');\ \tau_{\tilde{b}^N} \wedge \tau_{\tilde{d}^N}, \varphi_{\tilde{b}^N} \vee \varphi_{\tilde{d}^N}, \omega_{\tilde{b}^N} \vee \omega_{\tilde{d}^N} \rangle, u < 0,\ u' < 0, \end{cases}$$

4. $$\tilde{b}^N \oslash \tilde{d}^N = \begin{cases} \langle (r/u', s/t', t/s', u/r');\ \tau_{\tilde{b}^N} \wedge \tau_{\tilde{d}^N}, \varphi_{\tilde{b}^N} \vee \varphi_{\tilde{d}^N}, \omega_{\tilde{b}^N} \vee \omega_{\tilde{d}^N} \rangle, u,\ u' > 0 \\ \langle (u/u', t/t', s/s', r/r');\ \tau_{\tilde{b}^N} \wedge \tau_{\tilde{d}^N}, \varphi_{\tilde{b}^N} \vee \varphi_{\tilde{d}^N}, \omega_{\tilde{b}^N} \vee \omega_{\tilde{d}^N} \rangle, u < 0,\ u' > 0 \\ \langle (u/r', t/s', s/t', r/u');\ \tau_{\tilde{b}^N} \wedge \tau_{\tilde{d}^N}, \varphi_{\tilde{b}^N} \vee \varphi_{\tilde{d}^N}, \omega_{\tilde{b}^N} \vee \omega_{\tilde{d}^N} \rangle, u < 0,\ u' < 0, \end{cases}$$

5. $$k\tilde{d}^N = f(x) = \begin{cases} \langle (kr, ks, kt, k);\ \tau_{\tau_{\tilde{d}^N}}, \varphi_{\tau_{\tilde{d}^N}}, \omega_{\tau_{\tilde{d}^N}} \rangle, k > 0, \\ \langle (ku, kt, ks, k\,r);\ \tau_{\tau_{\tilde{d}^N}}, \varphi_{\tau_{\tilde{d}^N}}, \omega_{\tau_{\tilde{d}^N}} \rangle, k < 0, \end{cases}$$

6. $\tilde{d}^{N^{-1}} = \langle (1/u', 1/t', 1/s', 1/r');\ \tau_{\tau_{\tilde{d}^N}}, \varphi_{\tau_{\tilde{d}^N}}, \omega_{\tau_{\tilde{d}^N}} \rangle, \tilde{d}^N \neq 0.$

Definition 10 (Score and Accuracy functions of single valued trapezoidal neutrosophic number). A two single-valued trapezoidal neutrosophic numbers, $\tilde{b}$, and $\tilde{d}$, can be compared based on the score and accuracy functions as follows:

1. Accuracy function $AC(\tilde{b}^N) = \left(\frac{1}{16}\right)[r+s+t+u] * [\mu_{\tilde{b}^N} + (1 - \rho_{\tilde{b}^N}(x) + (1 + \sigma_{\tilde{b}^N}(x)],$
2. Score function $SC(\tilde{b}^N) = \left(\frac{1}{16}\right)[r+s+t+u] * [\mu_{\tilde{b}^N} + (1 - \rho_{\tilde{b}^N}(x) + (1 - \sigma_{\tilde{b}^N}(x)].$

Definition 11. The order relations between $\tilde{b}^N$ and $\tilde{d}^N$ based on $SC(\tilde{b}^N)$ and $AC(\tilde{b}^N)$ are defined as follows:

1. If $SC(\tilde{b}^N) < SC(\tilde{d}^N)$, then $\tilde{b}^N < \tilde{d}^N$
2. If $SC(\tilde{b}^N) = SC(\tilde{d}^N)$, then $\tilde{b}^N = \tilde{d}^N$,
3. If $AC(\tilde{b}^N) < AC(\tilde{d}^N)$, then $\tilde{b}^N < \tilde{d}^N$,
4. If $AC(\tilde{b}^N) > AC(\tilde{d}^N)$, then $\tilde{b}^N < \tilde{d}^N$,
5. If $AC(\tilde{b}^N) = AC(\tilde{d}^N)$, then$\tilde{b}^N = \tilde{d}^N$.

3. Problem Definition and Solution Concepts

3.1. Assumptions, index, and notation

3.1.1. Assumption

Assume that there are n jobs that must be performed by n persons, where the costs depend on specific assignments. Each job must be assigned to one and only one person and each person must perform one and only one job.

3.1.2. Index

i: Persons
j: Jobs
k: Number of objective functions

3.1.3. Notation

c_{ij} : Cost of the ith person assigned to the jth job
x_{ij}: Number of the jth jobs assigned to the ith person

Consider the following single-valued trapezoidal neutrosophic (NMOAS) problem below:

(NMOAS) min $\tilde{Z}_k{}^N = \sum_{i=1}^n \sum_{j=1}^n (\tilde{c}^N)_{ij}^k\ x_{ij}, k = 1, 2, \dots, K$

Subject to

$\sum_{i=1}^n x_{ij} = 1, j = 1, 2, \dots, n$ (only one person would be assigned the jth job)

$\sum_{j=1}^n x_{ij} = 1, i = 1, 2, \dots, n$(only one job selected by the ith person)

$x_{ij} = 0$ or 1 .

where $(\tilde{c}^N)_{ij}^k$ ($i = j = 1, 2, 3, \dots, n; 1, 2, 3, \dots, K$) are single-valued neutrosophic numbers.

Definition 12. A point *x* that satisfies the constraints in the NMOAS problem is said to be a neutrosophic feasible point.

Definition 13. A neutrosophic feasible point x° is called single-valued trapezoidal neutrosophic efficient solution to Problem (1) if and only if there does not exist another x such that

$\tilde{Z}(x,\tilde{c}^N) \le \tilde{Z}(x^\circ,\tilde{c}^N)$, and$\tilde{Z}(x,\tilde{c}^N) \ne \tilde{Z}(x^\circ,\tilde{c}^N)$.

According to the score function in Definition 10, the NMOAS problem is converted into the following crisp MOAS problem as follows:

(MOAS) min $Z = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}^{k}\, x_{ij}, k = 1,2,\dots,K$

Subject to

$\sum_{i=1}^{n} x_{ij} = 1, j = 1,2,\dots,n$ (only one person would be assigned the jth job)

$\sum_{j=1}^{n} x_{ij} = 1, i = 1,2,\dots,n$(only one job selected by the ith person)

$x_{ij} = 0$ or 1.

Definition 14 (Compromise solution, Leberling, 1981). A feasible vector $X^* \in S$ is called a compromise solution to the MOSA problem if and only if $X^* \in M$ and $Z(X^*) \le \wedge_{X\in S} Z(X)$, where $\wedge$ stands for " minimum" and M is the set of efficient solutions.

The MOAS problem will be solved by the weighting Tchebycheff method as follows:

$\min_x \max_{1\le k\le K}\{\gamma_k(Z_k - Z_k^*)\}$,

Or equivalently

$\min_x\{\beta: \gamma_k(Z_k - Z_k^*) \le \beta, k = 1,2,\dots,K\}$.

where $\gamma_k \ge 0, k = 1,2,3,\dots,K$, and $Z_k^*, k = 1,2,\dots,K$ are the ideal targets.

4. Solution Procedure

This solution procedure is based on the premise that the best-compromise neutrosophic solution has the minimum combined deviation from the ideal point, Z^*, where

$Z_k^* = \min_x Z_i(x),\ \ k = 1,2,3,\dots,K$.

The steps of the solution procedure are given below:

Step1: Formulate the NMOAS problem,

Step2: Convert the NMOAS problem into the corresponding crisp MOAS problem using the score function,

Step3: Calculate the individual minimum and maximum values of each objective function of the MOAS problem under the given constraints,

Step4: Compute the weight through the relation

$$\gamma_k = \frac{\overline{z}_k - \underline{z}_k}{\sum_{k=1}^{K}(\overline{z}_k - \underline{z}_k)}. \quad (1)$$

where $\overline{z}_k$ is the individual maximum and $\underline{z}_k$ is the individual minimum.

Step5: Formulate the following problem

$\min_x \beta$

Subject to

$$\gamma_k(Z_k - Z_k^*) \le \beta, k = 1,2,\dots,K, \quad (2)$$

$\sum_{i=1}^{n} x_{ij} = 1, j = 1,2,\dots,n$

(only one person would be assigned the jth job)

$$\sum_{j=1}^{n} x_{ij} = 1, i = 1,2,\dots,n$$

(only one job selected by the ith person)

$x_{ij} = 0$ or 1.

Step 6: Solve the above problem using Lingo Package to obtain the best compromise solution x_{ij}° and the corresponding optimum value β°.

5. Numerical Example

Consider the following cost matrices

$$\tilde{c}^{N^1} = \begin{bmatrix} \widetilde{10}^N & \tilde{8}^N & \widetilde{15}^N \\ \widetilde{13}^N & \widetilde{12}^N & \widetilde{13}^N \\ \tilde{8}^N & \widetilde{10}^N & \tilde{9}^N \end{bmatrix}, \text{ and} \tilde{c}^{N^2} = \begin{bmatrix} \widetilde{13}^N & \widetilde{15}^N & \tilde{8}^N \\ \widetilde{10}^N & \widetilde{20}^N & \widetilde{12}^N \\ \widetilde{18}^N & \widetilde{10}^N & \widetilde{12}^N \end{bmatrix},$$

Then, the mathematical model of NMOAS problem can be formulated as follows:

$$\min_x \tilde{z}_1^N = \left(\widetilde{10}^N x_{11} + \tilde{8}^N x_{12} + \widetilde{15}^N x_{13} + \widetilde{13}^N x_{21} + \widetilde{12}^N x_{22} + \widetilde{13}^N x_{23} + \tilde{8}^N x_{31} + \widetilde{10}^N x_{32} + \tilde{9}^N x_{33}\right)$$

$$\min_x \tilde{z}_2^N = \left(\widetilde{13}^N x_{11} + \widetilde{15}^N x_{12} + \tilde{8}^N x_{13} + \widetilde{10}^N x_{21} + \widetilde{20}^N x_{22} + \widetilde{12}^N x_{23} + \widetilde{15}^N x_{31} + \widetilde{10}^N x_{32} + \widetilde{12}^N x_{33}\right)$$

Subject to (3)

$$\sum_{i=1}^{3} x_{ij} = 1, \quad j = 1,2,3; \sum_{j=1}^{3} x_{ij} = 1, \quad i = 1,2,3,$$

$x_{ij} = 0$ or 1

where
$\tilde{8} = \langle(13,18,20,24);0.6,0.4,0.5\rangle, \tilde{9} = \langle(14,16,21,23);0.7,0.5,0.3\rangle,$
$\widetilde{10} = \langle(14,17,21,28);0.8,0.2,0.6\rangle, \widetilde{12} = \langle(6,10,13,15);0.7,0.3,0.4\rangle,$
$\widetilde{13} = \langle(15,18,23,30);0.9,0.2,0.3\rangle, \widetilde{15} = \langle(20,25,30,35);0.8,0.4,0.2\rangle,$
$\widetilde{20} = \langle(28,32,35,40);0.9,0.3,0.2\rangle.$
By using the score function of the single-valued trapezoidal neutrosophic number, the above problem becomes as follows:

$$\min_x z_1 = \begin{pmatrix} 10\,x_{11} + 8x_{12} + 15x_{13} + 13x_{21} + 12x_{22} + 13x_{23} + 8x_{31} \\ + 10x_{32} + 9x_{33} \end{pmatrix}$$
$$\min_x z_2 = \begin{pmatrix} 13x_{11} + 15x_{12} + 8x_{13} + 10x_{21} + 20x_{22} + 12x_{23} + 15x_{31} \\ + 10x_{32} + 12x_{33} \end{pmatrix}$$
Subject to (4)
$$\sum_{i=1}^{3} x_{ij} = 1, \quad j = 1,2,3; \sum_{j=1}^{3} x_{ij} = 1, \quad i = 1,2,3,$$
$$x_{ij} = 0 \text{ or } 1$$

The solution of each objective function of Problem (2) is given under the given constraints as follows:

$$z_1^{min} = 29, \ z_1^{max} = 38, \ z_2^{max} = 42, \ z_2^{min} = 28. \quad (5)$$
Use Relation (1) to calculate the weights

$$\gamma_1 = \frac{38-29}{(38-29)+(42-28)} = \frac{9}{23}, \quad \text{and} \quad \gamma_2 = \frac{42-28}{(38-29)+(42-28)} = \frac{14}{23} \quad (6)$$
Substituting from (5) and (6) into (1), we obtain:
$$\min_x \beta$$
Subject to
$$\frac{9}{23}\begin{pmatrix} 10\,x_{11} + 8x_{12} + 15x_{13} + 13x_{21} + 12x_{22} + 13x_{23} + 8x_{31} \\ + 10x_{32} + 9x_{33} - 29 \end{pmatrix} \le \beta, \quad (7)$$
$$\frac{14}{23}\begin{pmatrix} 13x_{11} + 15x_{12} + 8x_{13} + 10x_{21} + 20x_{22} + 12x_{23} + 15x_{31} \\ + 10x_{32} + 12x_{33} - 28 \end{pmatrix} \le \beta,$$
$x_{11} + x_{12} + x_{13} = 1,$
$x_{21} + x_{22} + x_{23} = 1,$
$x_{31} + x_{32} + x_{33} = 1,$
$x_{11} + x_{21} + x_{31} = 1,$
$x_{12} + x_{22} + x_{32} = 1,$
$x_{13} + x_{23} + x_{33} = 1,$
$x_{ij} = 0$ or 1 .

Tab. 1. The optimal compromise solution to Problem (7)

Variables	Objective
$x_{12}^* = 1$	$\beta^* = 0.28$
$x_{21}^* = 1$	$z_1^* = 30$
$x_{33}^* = 1$	$z_2^* = 37$

Tab. 2. The optimal compromise neutrosophic solution to Problem (3)

Variables	Objective
$x_{12}^* = 1$	$z_1^* = \langle(42,52,64,77);0.6,0.5,0.5\rangle$
$x_{21}^* = 1$	$z_2^* = \langle(40,52,64,78);0.7,0.4,0.6\rangle$
$x_{33}^* = 1$	

6. Concluding Remarks

In this paper, interval-valued trapezoidal Neutrosophic Multi-Objective Assignment (NMOAS) problem was studied. A new approach was proposed to solve the crisp (MOAS) problem. The approach was used by a weighting

Tchebycheff method which was applied by defining relative weights and ideal targets. The advantage of this approach is more flexible than the standard multi-objective assignment problem, where it allows the decision-maker (DM) to choose the desired targets.

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